Methods for Calculating the Optical Band Structure of Photonic Composites

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I. Introduction

Lately, there has been an increasing interest in studying the propagation of electromagnetic waves in periodic dielectric structures (photonic crystals). Like the electron propagation in semiconductors, these structures are represented by band diagrams in which gaps can be found where the electromagnetic propagation is forbidden. Much effort is dedicated to find structures that can prohibit the propagation of light in all directions. This effect could lead to light localization.

II. Plane-Wave Method and Finite Element Method applied to 1-D and 2-D photonic crystals

By using the plane wave method or the finite element method, band diagrams for photonic crystals can be obtained. The propagation of the electromagnetic waves is studied in periodic dielectric structures by numerically solving Maxwell's equations. The photonic structure is characterized by the real dielectric constant $\varepsilon(\vec{r})$. The magnetic permeability $\mu(\vec{r})$ is considered constant and equal to one throughout the structure. By combining Maxwell equations the fundamental equation for the electric field $\vec{E}$ is obtained:

$$\nabla \times \nabla \times \vec{E} = \frac{\omega^2}{c^2}\varepsilon(\vec{r})\vec{E} \tag{1}$$

where, ω is the angular frequency and c is the speed of light. In the case of the plane-wave method, the dielectric constant and the electric field are expanded in a sum of plane waves and substituted into the fundamental equation. The following generalized eigenvalue problem is obtained:

$$\begin{aligned}&\left(\vec{k}+\vec{G}\right)[(\vec{k}+\vec{G})\times\vec{E}(\vec{G})]\\&+\tfrac{\omega^2}{c^2}\textstyle\sum_{\vec{G}'}\varepsilon(\vec{G}-\vec{G}')\vec{E}(\vec{G}')=0\end{aligned} \tag{2}$$

where $\vec{k}$ is the wave vector and $\vec{G}$ are the reciprocal lattice vectors. If real dielectric constants are considered the two resultant matrices are hermitian. The eigenvalue matrix equation is solved to obtain the propagating frequencies for the corresponding values of the wave vector k. For particular structures, gaps can be found where the electromagnetic propagation is forbidden.

In the case of the finite element method, the fundamental equation is solved by using the standard techniques provided by the finite element approximation. The addition of periodic boundary conditions to the standard formulation is necessary. Like the plane-wave method, the eigenvalue matrix equation is solved to obtain the propagating frequencies. The results produced by this formulation are in complete agreement with those from the plane-wave method.

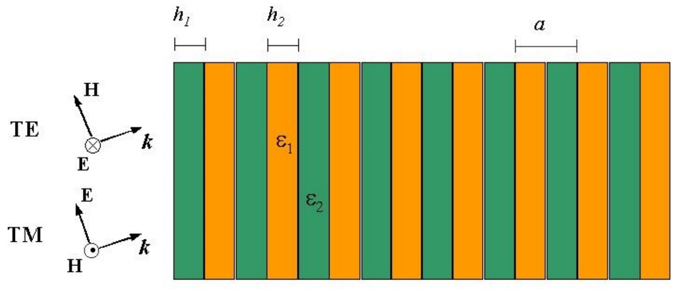

Figure 1 One dimensional photonic crystal formed by alternating layers of different dielectric constants and thickness.

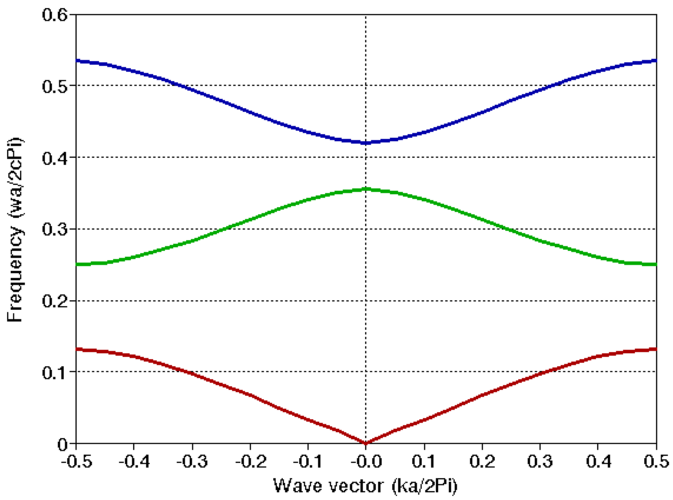

Figure 2 Band diagram for perpendicular incidence. The photonic crystal is characterized by h1=0.8mm h2=1.65mm a=2.45mm e1=21.16 e2=2.56. The first three bands are shown.

III. Translation matrix method applied to 1-D isotropic or anisotropic photonic crystals

This formulation is used to study the propagation of electromagnetic waves in finite periodic structures. In this case the 1-D photonic crystal is composed of a finite number of layers. The basis of the method is to solve the wave equation in each layer and to match the fields on the interfaces

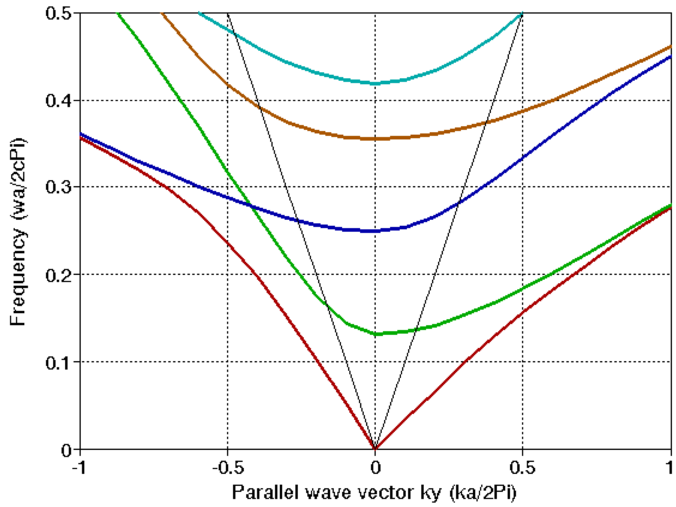

Figure 3 Projected band diagram for a one dimensional photonic crystal composed of alternating layers with thickness h1=0.8mm, h2=1.65mm and dielectric constants e1=21.16, e2=2.56. Allowed states for external light incident on the photonic crystal are represented for the interior area of the light cone. The structure presents an omnidirectional gap.

according to Maxwell's matching conditions. By using this formulation, reflectivity can be obtained as a function of the wave length and the direction of the incoming wave on the surface of the photonic crystal. The formulation is extended to treat anisotropic photonic crystals. The code can be applied to model liquid crystals which display different dielectric constants along and perpendicular to the director.

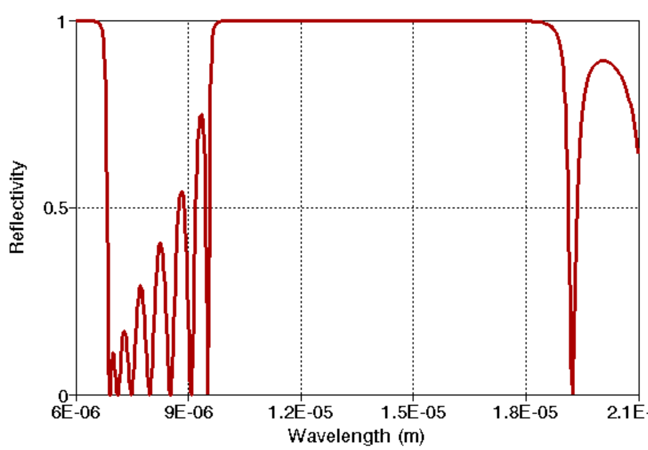

Figure 4 Normal incidence. TE polarization.

Figures 4 to 9 show reflectivity as a function of the wavelength of the incident wave. Different directions and polarizations are shown. The multilayer system consist of 14 layers of alternating media 1 and 2 with h1=0.8mm, h2=1.65mm and e1=21.16, e2=2.56.

It can be seen that a range of wavelenghts exist for which

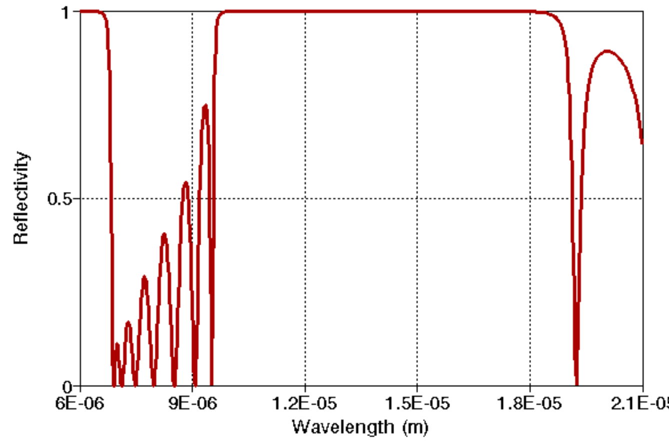

Figure 5 Normal incidence. TM polarization.

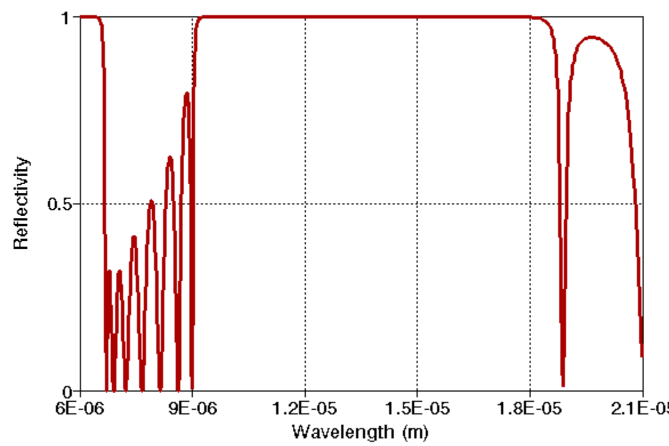

Figure 6 45^o incidence. TE polarization.

the reflectivity is equal to one. In this case the finite photonic crystal behaves as a perfect mirror as demonstrated by Winn et al. Light with frequencies within the range is not allowed to enter the structure. An omnidirectional gap is obtained when a given frequency is forbidden for all directions and polarizations.

IV. Plane-Wave Method for 3-D periodic structures

3-D structures are studied by using the plane wave method in the full vectorial formulation. 3-D structures are being actively studied due to the possibility of obtaining structures with complete band gaps. Some 3-D structures with complete band gaps have already been reported. An important area of interest in 3-D photonic crystals is the study of self assembled structures. Since fabrication on the length scales of interest is extremely difficult self assembly affords a simple route to acheive 3-D photonic crystals. The work in this are has involved the study of such self assembled structures which result in complete band gaps. The results obtained are being submitted for publication.

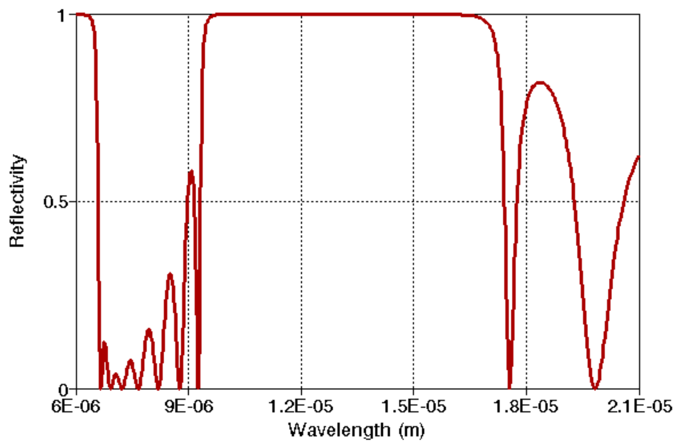

Figure 7 45^o incidence. TM polarization.

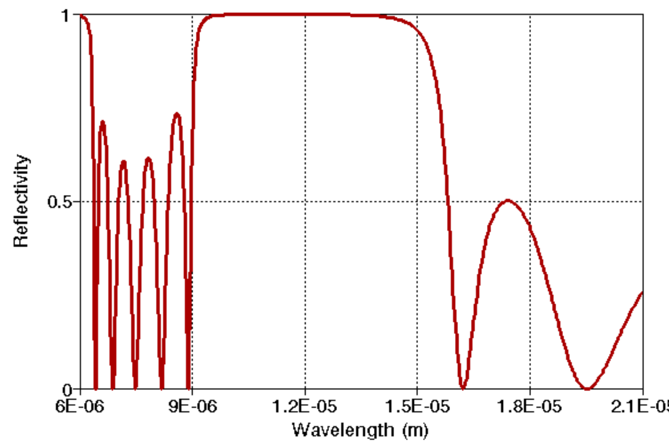

Figure 9 80^o incidence. TM polarization.

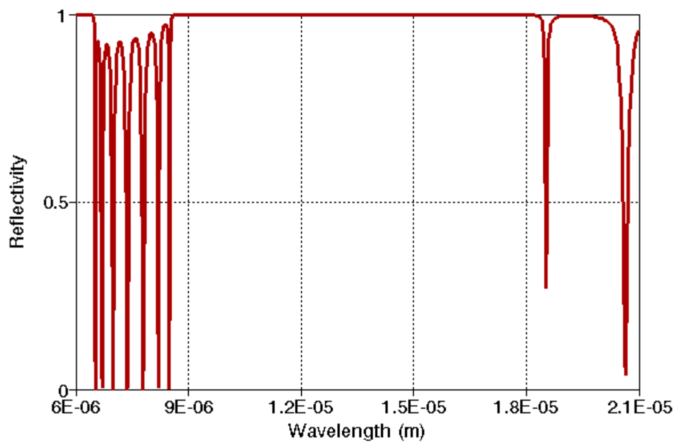

Figure 8 80^o incidence. TE polarization.

V. Finite Element Method for studying finite 2-D photonic crystals

The finite element method is used to obtain reflectivity for 2-D finite systems. The standard finite element technique is applied. In this case absorbing boundary conditions are applied. The 2-D photonic crystals that are considered are composed of an array of dielectric rods on a variety of lattice arrangements. For certain values of the dielectric constant of the rods, the structure does not allow the propagation of electromagnetic waves inside. As a consequence, for a particular geometry an incoming electromagnetic wave is reflected or transmitted by the photonic crystal depending on the value of the dielectric constant of the rods. This formulation can handle complex dielectric constants, dielectric constants depending on frequency and metallic components. Figure 10 shows a photonic crystal that completely reflects the incoming polarized wave. It can be observed that the electromagnetic wave is not allowed to enter the photonic crystal. For this case, the structure behaves as an efficient mirror reflecting the electromagnetic radiation. Figure 11 shows a photonic crytal from which a row of dielectric rods was removed. The electromagnetic wave is still not allowed to travel within the array of dielectric rods. However the wave propagates along the channel created by the missing rods. Such a structure will be used to study waveguiding structures. This method will be also used to study the effect of metallic components inside of the photonic crystals.

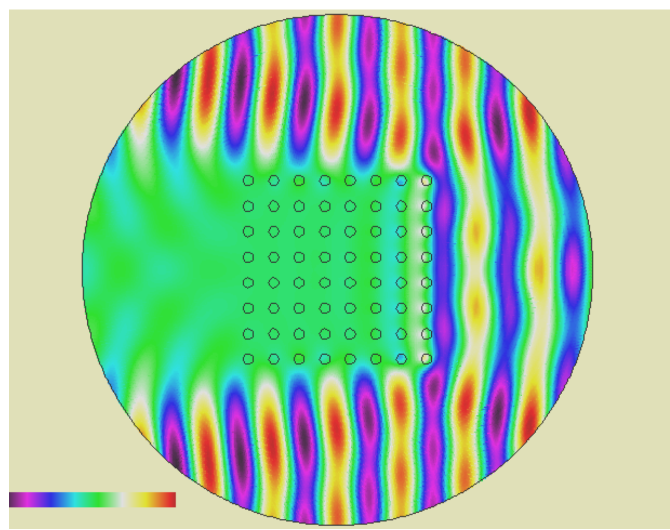

Figure 10

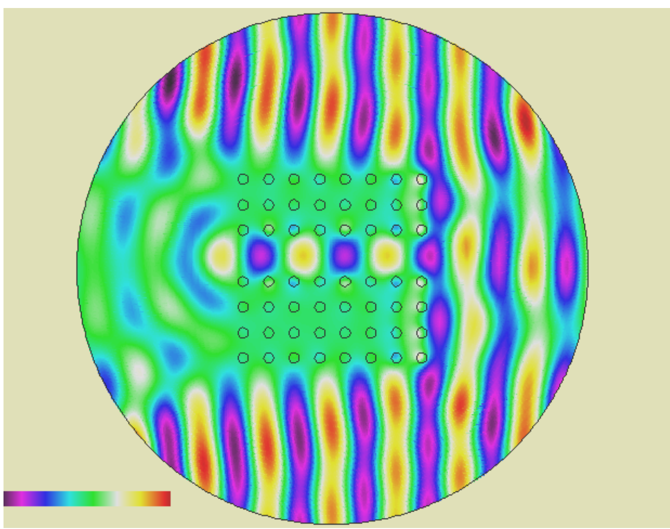

Figure 11