# A Study on the Boundary Conditions of 90° Paper Pop-up Structures

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*Abstract* — **The design of a pop-up book or card has hitherto been labour intensive with tasks of trials and errors. The constructions of collapsible pop-up structures can be demanding and inefficient without adequate knowledge of their geometric properties.**

**This paper examines the properties of creases in 90° pop-up structures. A 90° pop-up structure is one that erects fully when two adjacent base pages, on which it sits, are opened to a right angle. In particular, we define a boundary region for creating 90° pop-ups. Similarly, paper folds are able to achieve pop-up effects and can be integrated with 90° pop-up constructions. The development of these pop-up structures can be represented graphically.**

**Through this study, a fundamental foundation for pop-up topology and geometry is built. This foundation would be vital for understanding the applications of pop-up making techniques. The mathematical relationships devised would be useful for developing computer-enhanced pop-up design.**

***Index Terms*—Computer aided design, geometry, pop-up structures, paper folding.**

## I. Introduction

MOVABLE pop-ups not only need creative minds but also require engineering skills. Required skills include the method to accomplish flat foldability of paper mechanisms when pages are closed. Specialists who design and create such mechanisms are known as paper engineers. However, the design in pop-up crafting has so far been primary manual and based on traditional crafting methods. Due to the extensive manual process, the method of trials and errors was particularly evident in the design stage. Trials and errors are common in the sizing of the sketches, folding and aligning the paper pieces. A misfit of the pop-ups on the pages would result in repeated work. As such, pop-up making can be labour intensive, time consuming and burdened with repetitive tasks, especially for the less experienced pop-up enthusiasts.

Books had been published to teach novices pop-up constructions by following given step-by-step instructions with templates and illustrations. However, they were only able to make structures from specific designs and are unable to understand the techniques behind the constructions. Realizing the limitation of previous craft books and the need to educate enthusiasts in pop-up techniques, which encompass comprehension in geometry, paper engineers have published several books [1], [2] in recent years on pop-up techniques with explanations of simple geometric rules. But the design of pop-up still involves heavy manual work and is inefficient.

With the advent of computational science, many computer aided design tools have evolved from traditional craftworks to enhance the needs in digital living. Examples include architectural drafting and tailoring. Likewise for paper crafts, CAD tools for paper models and origami as such have been developed in recent years. In pop-up designs, paper engineers use a few graphics softwares like FreeHand in place of traditional methods of drawing, tracing and colouring. But for the engineering work in pop-up design, a reduction of manual effort is still desirable.

A CAD design approach could minimize unnecessary manual work to save time and cost. Nevertheless, before a new approach can be proposed, a thorough study on pop-up techniques and their geometries is necessary. Knowledge in pop-up techniques and their geometries would be vital for the development of a CAD system. The rest of this paper is outlined as follows. The next section surveys related fields. Section III introduces the types and techniques of pop-ups. Section IV further discusses 90° pop-up structures. Section

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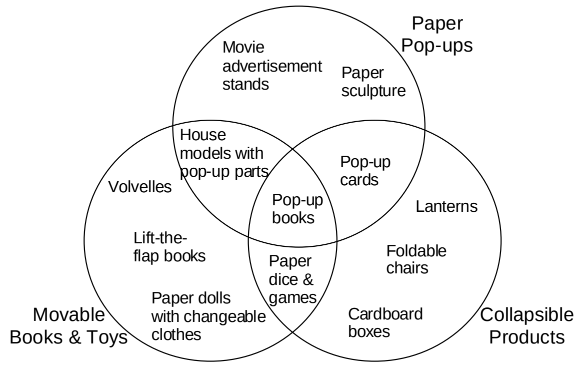



Fig. 1. A pop-up book and its association with collapsible products, movable book and toys and other paper crafts.

V examines boundary conditions of these structures. Section VI looks into structures with added pop-up effects by paper folding and Section VII explains the inversion of the structures. We conclude in Section VIII.

## II. RELATED FIELDS

Pop-up structures, when broadly examined, provide an ample study ground in several relevant areas, as shown in fig.1. One of these areas is collapsible design, one main attribute of pop-up structures. Like pop-ups, crease lines are vital embodiments in collapsible paper product and packaging. Collapsible design has also been applied to a wide range of products, which include foldable wheelchairs, retractable poles and inflatable chairs. Hence pop-up constructions are not confined to the school of handicrafts but it is a field that entails collapsible design and development.

Pop-ups are also often classified as part of movable books and toys. Other than pop-ups, movable books contain paper mechanisms like sliders, rotating wheels, cams and pull-tabs. Hence, many of the books are not solely made up of pop-ups but a combination of these engineered designs.

Pop-up techniques have been used in many other types of paper crafts. They can be constructed as part of a foldable paper sculpture or exist as glued-on pieces on paper dollhouses. Though not common, the use of pop-up is also noted in paper cutting and origami. Pop-up techniques have been integrated into other types of paper crafts. See fig. 2.

In addition, the development of paper crafts in recent years is notable in computational science, which can be further divided into three areas, namely computer aided design, the use of crafts in science applications and findings in paper crafts' geometries. This development signifies an increased level of integration between science and crafts.

### *A. CAD tools for Paper Crafts*

There have been a growing number of software tools to aid the design of craftwork. Blauvelt, Wrensch and Eisenberg [3] have envisioned such development in computer-aided design as a strategy for blending crafts and computers. For example, Nishioka and Eisenberg have developed HyperGami, a software for creating paper polyhedral models [4]. Pepakura Designer [5], a tool developed in Japan, and Touch 3D [6], a commercial program, also generate unfolded paper models.

In the field of origami, Robert Lang has developed TreeMaker [7], which can generate crease patterns for origami. Another program, Mathematica [8], is able to simulate paper-folding steps. For pop-up designs, Jun Mitani created the 3D Card Maker [9], which generates crease and cut lines for one-piece pop-up structures. The software is capable of creating and animating the double slit.

### *B. Paper Crafts and Applications*

Paper crafts have alternatively been used in mathematic education and physics applications. Particularly in origami, such developments have rapidly grown over the past decade. For example, mathematician Humiaki Huzita [10] has developed a set of origami axioms that describe geometric constructions usually done with compasses and rulers.

Origami crease patterns have been explored for foldable designs e.g. that of telescopes and safety airbags in vehicles [11]. The ability to store in compactness and the method for expansion are vital aspects in the design of these applications. In the craft of pop-up, it has been used as an educational aid to provide visual effects in many children books. Diego Uribe has attempted to stimulate interests in fractals through pop-up card constructions [12]. Paper crafts have also been used as engineering models, e.g. the study of the WISE craft [13].

### *C. Geometries of Paper Crafts*

Many of the computer-aided tools are developed from intrinsic geometric properties of the crafts. In single flat vertex folds, Maekawa [14], [15], [16] determined a basic relationship between mountain and valley creases for flat foldability. (Refer to Section IV for definitions.) Kawasaki [14], [15], [16] asserted that the sum of the alternate angles about the vertex fold is 180°. Similarly, Justin [17] yield

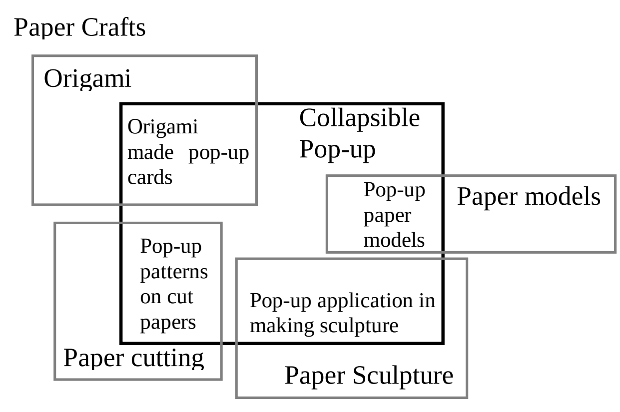



Fig. 2. Uses of collapsible pop-ups in other paper crafts.

valuable results in the topology of flat folds.

Based on previous ideas of vertex folds, Gibson and Huband [18] further discovered properties of fold-cut angles. Hull [16] formulated a method to count possible number of combinations of mountain and valley crease assignments for flat vertex fold. In paper cutting, Demaine [19], [20] devised an approach to obtain a desired shape by one straight cut after folding specific crease patterns.

Previous geometric studies into collapsible pop-ups have also led to interesting results. Glassner [21], [22] has modeled the movement of pop-ups by examining three intersecting spheres. Lee, Tor and Soo [23] animate pop-up structures by investigating the angles between planar pop-up pieces. However, much of the study on pop-ups' properties is in its infancy stage. More remains to be unraveled.

## III. POP-UP TYPES AND TECHNIQUES

Pop-up structures come in numerous variations. They can be categorized as collapsible and non-collapsible. Pop-ups in cards and books are collapsible but the pop-up advertisement stands and many paper sculptures are not.

Collapsible pop-ups can be further classified by the angle of opening two base pages, on which the pop-up structure sits, specifically at 90°, 180° and 360°. These are the angles at which the pop-ups are fully erected. Some like Masahiro Chatani [24] have termed pop-ups with 0°. In other words, there is no folding but layers of cut papers overlapping on top of one another with some protruding portions.

A simpler alternative is to organize pop-up according to number of sheets used: the one-piece type and the multi-piece type. The one-piece type is found in pop-ups whose constructions are entirely made on a single piece of paper. Multi-pieces refer to two or more pieces used to make a pop-up structure. See fig 3.

Paul Jackson [2] defined a pop-up as a self-erecting, three-dimensional structure, formed by the action of opening a crease. These refer to the paper pop-ups found in cards and books. In this study, we follow this definition. But it is also noted that some collapsible pop-up are not self-erected and require manual assistance to raise them. Fig. 4 illustrates the classification of pop-up structures.

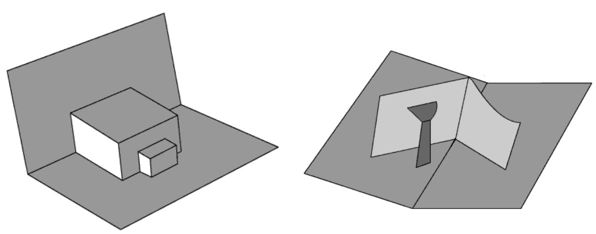
Techniques with slitting are mainly for 90° one-piece type. These include the single slit and double slits. Variations can be achieved by altering the length and shape of the slit lines. Under the multi-piece type, pop-ups include Floating Layers, the Horizontal V and Boxes. Tabbing and gluing are usually needed to construct such pop-ups. In particular, this paper examines 90° one-piece pop-up structures, of which some examples are shown in fig. 5.

## IV. 90° POP-UP STRUCTURES

Before we examine further into 90° pop-up structures, let us define some commonly used terms in this study.

### *A. Mountain and Valley Creases*

A crease is a fold line between two movable planar pieces. It can be a mountain crease or valley crease depending on the side of the planar pieces facing up. If the angle θ between the two up-facing sides is less than 180°, the crease between them is a valley crease. On the other hand, if the angle is more than 180°, the crease is a mountain crease. See fig. 6. Though the crease exists at



(a) (b)

Fig. 3. (a) A 90° one-piece pop-up structure. (b) A 180° multi-piece pop-up structure.

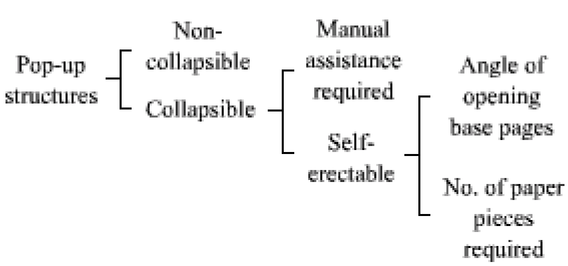
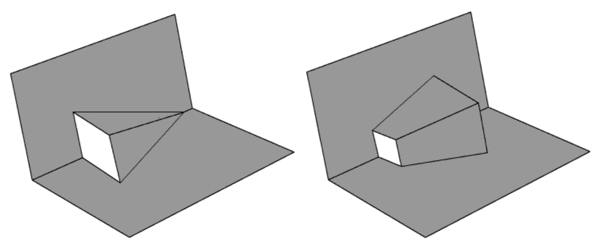
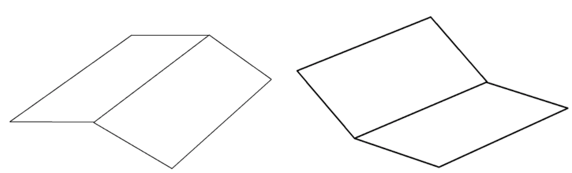



Fig. 4. Classification of pop-up structures



(a) (b)

Fig. 5. (a) A pop-up made from a single slit. (b) A pop-up constructed from double slits.



(a) (b)

Fig. 6. (a) A mountain crease. (b) A valley crease

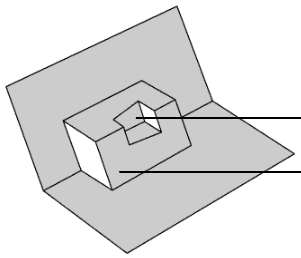



Fig. 7. The first pop-up level is a valley crease initiation and the second level is a mountain crease initiation.

180°, it does not have a mountain or valley crease assignment. The crease between the two base pages is known as the gutter crease [2]. For this paper, the gutter crease is taken to be a valley crease for all cases in the following cases, unless otherwise stated.

### B. Pop-up Levels

A pop-up level is a layer of pop-up pieces built over the gutter crease or existing creases on other pop-up layers. A pop-up structure can be made up of different number of pop-up levels. For example, the structure in fig. 3a is made up of two-level. More than one level can be built upon the same crease.

### C. Faces

A face refers to a planar piece or section of a pop-up structure. Curved faces are not considered in the investigation of 90° pop-up structures in this paper.

### D. Crease Initiations

The process of developing a pop-up level over a crease is termed as a crease initiation. There are two types of crease initiations, for a pop-up level can be developed over a mountain crease or a valley crease. A level developed over a mountain crease is known as the mountain crease initiation. That over a valley crease is known as the valley crease initiation. Fig. 7 further explains crease initiations.

### E. Basic Mathematical Relationships

Creases can be related to the pop-up levels and faces (or planar pieces) in mathematical forms. The mathematical relationships are devised from integer sequences [25] observed in 90° pop-up structures.

1) *Crease-Level Relation:*

If $N$ is the number of creases on a pop-up structure and $n$ is the number of pop-up levels, then

$$N = 3n + 1,\ n \in Z^{+}. \qquad (1)$$

2) *Crease-Face Relation:*

If $F$ is the number of planar faces on the pop-up structure and $n$ is the number of pop-up levels, then

$$F = 2n + 2,\ n \in Z^{+}. \qquad (2)$$

For example, (1) is obtained by examining the number of creases added for each additional pop-up levels. For 90° pop-up structures, three creases are added on a new pop-up level. Hence, an integer sequence $N = \{1, 4, 7, 10, \ldots\}$ is formed. $N = 1$ refers to the gutter crease before any pop-up is constructed ($n = 0$). Combining (1) and (2) yields another relationship between the number of faces, $F$, and the number of creases, $N$.

$$F = \frac{2}{3}(N + 2),\ F \in Z^{+},\ N \geq 1 \qquad (3)$$

(3) is used to derive the boundary constraints in the next section.

## V. Boundary conditions For 90° pop-up structures

With different combination of folds and number of pop-up levels, a 90° pop-up structure can take up many interesting forms. However, there are instances when the constructions do not enable the structures to collapse flat between the base pages. From this problem, a question arises: What are the mathematical constraints that determine the flat foldabilty of a pop-up structure?

In early 1980s, Maekawa has shown that to achieve flat foldability in a vertex fold, the difference between the numbers of mountain and valley creases is always 2. This has since been known as the Maekawa's Theorem [11], [15]. At the same time, Justin [15, 17] has also contributed similar findings in flat vertex folds. Likewise, the creases on a pop-up structure affect its ability to collapse flat. Note that there can be other causes to the problem of pop-up structure's inability to collapse flat, like the material used for pop-up constructions and interferences with other paper pieces on the pages. But in this paper, we examine, in particular, the properties of the creases on a pop-up structure that determine its flat foldability.

### A. Successive Crease Initiations

As we take the gutter crease as a valley crease, the first pop-up level would always be developed over a valley crease. But for subsequent levels, they can be constructed over a combination of mountain and valley creases. Thus, we look specifically into two boundary cases where the successive pop-up levels are developed solely from a type of crease initiation, i.e. solely mountain crease initiations and solely valley crease initiations.

Let us denote $M$ and $V$ as the number of mountain and valley creases respectively. Given that the type of crease initiations for subsequent level is the same and the increment in the number of creases is constant (as described earlier on the crease-level relation), $F$, $M$ and $V$ would form linear relationships in both cases. After derivation, for successive mountain crease initiations, the relationship is

$$F = \frac{2}{5}(3V + M). \qquad (4)$$

For successive valley crease initiations, the relationship becomes

$$F = 2V - 2M\ . \tag{5}$$

(4) and (5) describe the two boundary cases.

*B. Crease Constraints*

Now refer to (3). It gives the general relationship between the number of faces and the number of creases. Since the total number of creases, *N*, is the sum of mountain and valley creases, $N = M + V$. Then

$$F = \frac{2}{3}(M + V + 2). \tag{6}$$

When (4) is substituted into (6), a new expression on mountain and valley creases would be formed.

$$M = 2V - 5,\ V \geq 3 \tag{7}$$

(7) represents the relationship between mountain and valley creases for the boundary case of successive mountain crease initiaitions. It forms the first mathematical constraint for creases on 90° pop-up structures. Similarly, the second mathematical constraint is obtained by looking into the boundary case of successive valley initiations. Combining (5) and (6) results in

$$V = 2M + 1,\ M \geq 0. \tag{8}$$

The third and last mathematical constraint for creases is observed from the first pop-up level. Four planar faces form the first level of any 90° pop-up structures. Hence, the minimum number of faces to form a 90° pop-up structure is 4. From (6), an inequality can be formed as follows.

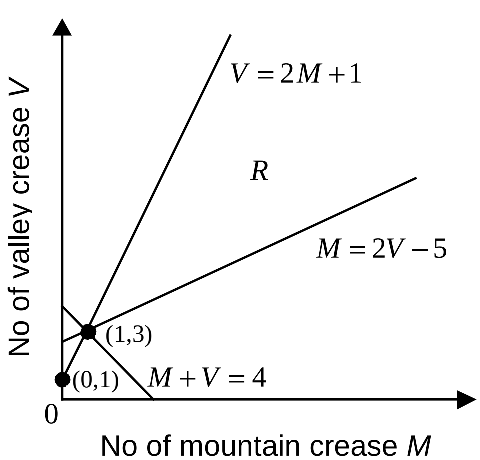



Fig. 8. Boundary region for 90° pop-up structures. Coordinates (0,1) represents the gutter crease and (1,3) represents the first pop-up level.

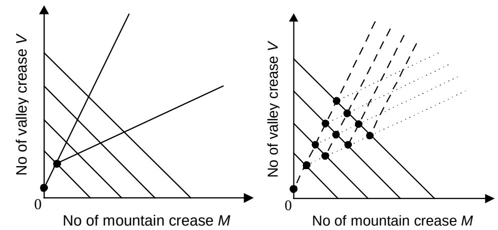



Fig. 9. (a) Parallel level lines $M + V = N$. (b) Dotted lines are lines of mountain crease initiations and dashed lines are those of valley crease initiations. Dots on line intersections are possible crease combinations to create 90° pop-up structures.

$$\frac{2}{3}(M + V + 2) \geq 4$$

$$M + V \geq 4 \tag{9}$$

Therefore, to form a 90° pop-up structure, there must be at least four creases. $M + V = 4$ is the third mathematical constraint on creases.

*C. Graph of Mountain and Valley Creases*

The three crease constraints forms a semi-infinite region on a mountain-valley crease graph. See fig 8. In short, we term it as a MV graph. The region is a boundary area where the construction of collapsible 90° pop-up structures is feasible. Hence, it also represents a region where flat foldability of 90° pop-up structures is attainable. We are, thus, able to predict the number combination of mountain and valley creases required for a 90° pop-up construction and verify if a structure with a fixed number of mountain and valley creases is flat foldable. However, only specific integer coordinates in the boundary region represent the number of mountain and valley creases feasible for flat foldability. To locate these coordinates, we need to examine two other types of lines on the MV graph. They are the level lines and crease initiation lines.

*1) Level Lines*

Level lines represent the pop-up levels. The lines are represented by $M + V = N$ and *N* is determined from (1). The third crease constraint $M + V = 4$ is the level line for the first level. Possible integer coordinates are located on the level lines, as shown in fig. 9a. To determine the exact positions of the coordinates on the level lines, we need to examine the crease initiation lines.

*2) Crease Initiation Lines*

These lines represent how a pop-up level is developed. The representations of the crease initiation lines are shown in fig 9b. As there are two types of crease initiations, the lines for mountain crease initiations and valley crease initiations have different properties. They differ in their slopes (or gradients). We term the slopes of the lines as MV slopes. For a mountain crease initiation, there is an increment of two mountain creases and one valley crease.

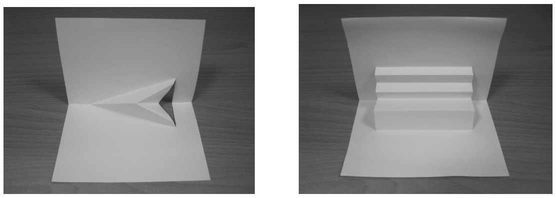



Fig. 10. (a) A single slit pop-up structure with a vertex fold. (b) A double slit pop-up structure with pleats.

Hence the MV slope is $\frac{1}{2}$. For a valley crease initiation, there is an increment of one mountain crease and two valley creases. The MV slope is thus 2. The crease initiation lines cut across the level lines and the points of intersection are the positions of the required coordinates, and so the combination of mountain and valley creases, to achieve flat foldability of 90° pop-up structures.

## VI. Added Pop-up effects by paper folding

So far, we have examined creases on basic 90° pop-up structures. After a basic pop-up structure has been developed, it is possible to further fold the planar pieces to create additional pop-up effects. However, there is a specific technique to fold the planar pieces, for other folding methods would hinder the structure from erecting properly. The feasible method is by changing the mountain-valley assignment of an existing crease on the pop-up structure and creating two new creases at the same time. In origami idiom, this folding method is known as reverse folding. Reverse folding on pop-up structures can produce flat vertex folds or pleats. A flat vertex fold has crease lines that meet at a vertex whereas those of pleats do not. Fig. 10 illustrates some examples of these pop-up structures.

Paper folds added to pop-up structures are not governed by mathematical relationships of 90° pop-up structures discussed in previous sections. But since the resulting pop-up structure is flat foldable, can its crease properties be satisfied within the boundary region? This means that in order to be within the boundary region, the lines representing the flat vertex fold or pleats on the pop-up structures on the MV graph must have MV slopes between $\frac{1}{2}$ and 2.

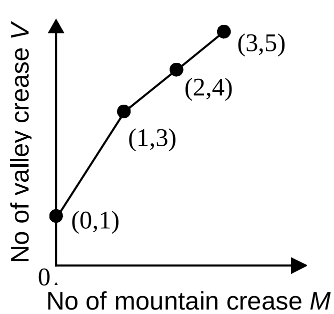



Fig. 11. A graph representation of a 90° pop-up structure. Coordinate pair (1,3) represent a pop-up level and (3,5) represents the full structure after pleats are added onto the pop-up level.

TABLE I
MV Slopes for Added Folds to 90° Pop-up Structures

| Type of folds | Fold level | Developed from | MV slope |
|---|---|---|---|
| Flat vertex fold | First fold | Mountain crease | 1/2 |
| | | Valley crease | 2 |
| | Successive folds | Any crease | 1 |
| Pleats | All folds | Any crease | 1 |

By observation, for the first fold, a flat vertex fold would result in an increment of two mountain creases and one valley crease if it were developed from a mountain crease on the pop-up structure. The MV slope for this case is thus $\frac{1}{2}$. If the first fold is developed from a valley crease, then the MV slope is 2. For successive fold levels, the MV slope is 1 regardless of the type of creases on the pop-up structure, mountain or valley, which the flat vertex fold is developed from. (Crease lines of successive levels meet at the same vertex.)

Similarly, adding a reverse-folded pleat to a pop-up structure would result in a MV slope of 1, regardless of the type of creases, which it is developed from. Hence, the corresponding graph of 90° pop-up structures with added flat vertex folds and pleats would not diverge out of the boundary region. Table I summarizes the type of folds and their MV slopes. Fig. 11 illustrates a MV graph corresponding to the pop-up structure in Fig. 10b. However, note that other folding methods can also result in a MV slope between $\frac{1}{2}$ and 2 but do not enable a pop-up structure to erect properly or collapse flat.

## VII. Inverted 90° pop-up structures

The gutter crease has been taken to be a valley crease for the above explanations. To represent a mountain gutter crease or inverted 90° pop-up structures, including those with added folding, they can be represented on a MV graph as a reflection along the line $V = M$. The inversion can also be represented by a matrix transformation $T$ as follows.

$$T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and

$$\begin{pmatrix} M \\ V \end{pmatrix}_{Inverted} = T \begin{pmatrix} M \\ V \end{pmatrix} \qquad (10)$$

## VIII. Conclusion

One significant finding in this paper is the identification of a boundary region where 90° pop-up constructions are feasible. A 90° pop-up structure that is flat foldable satisfies the boundary conditions. But the reverse is not true. That is, a graph constructed within the boundary region does not necessary produce a flat-foldable 90° pop-up structure. For future research, further investigation would look into the flat foldability of other types of pop-up structures as well as their geometric properties.

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