Optimal Capacity Adjustments for Supply Chain Control

Benny Budiman
MIT

Abstract— **Decisions on capacity are often treated separately from those of production and inventory. In most situations, capacity issues are longer-term, so capacity-related decisions are considered strategic and thus not part of supply planning. This research focuses on optimal supply planning with emphasis on variable capacity to meet uncertain demand. It also defines three levels of capacity change: operating hours, labor availability and production hardware availability. The work presented here deals with the fundamental decisions to determine capacity, production, and inventory to meet customer demand while optimizing revenue and costs over a planning horizon (typically the life of the product). With the Lagrangian technique for constrained optimization, it can be shown that the optimal supply capacity has upper and lower bounds. The optimal feedback policy prescribes increasing the supply capacity when at the beginning of the planning interval it is below the lower bound. Similarly, the supply capacity should be decreased to the upper bound when it is above the upper bound. This paper will present arguments for characterizing forecast evolution and information sharing in the supply chain to obtain a predictor-corrector approach to supply chain control.**

Index Terms— **capacity planning, supply chain, inventory control, optimal control, extended enterprise.**

I. Introduction

THE primary goal of this research is to set supply capacity optimally by adjusting capacity variables consisting of plant and equipment, operating duration, and workforce level within each planning interval. However, this paper only presents the optimal gross capacity adjustment.

Demand lead time is the time end consumers expect to wait before fulfilling their demand for products. End consumers will want to fill their needs instantaneously for some products, but are willing to wait for others. By contrast, supply lead time depends on the operating production capacity, production lot size, production cycle time, plant layout, raw material inventory, finished goods inventory and transportation from one-node to another in the supply chain. This research focuses on optimal adjustment of operating production capacity to meet uncertain product demand so as to meet an expected demand lead time at the least cost.

Supplying products to the end consumers requires a network of independently run companies. The supply network may have a number of companies (the supply base) producing parts that go into the product that a manufacturer produces and that the distribution channels take to the end consumers for consumption or use (Fig. 1). Each supply node in the supply network has its supply lead time; therefore, the likelihood that the integrated supply lead time will exceed the demand lead time is high.

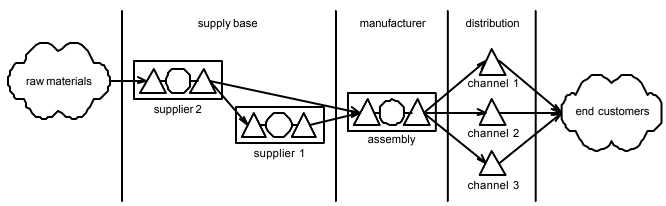

Fig. 1. A simplified structure of a supply network consists of the supply base, the manufacturer, and the distribution channels.

When the demand lead time is shorter than its supply counterpart is, it is customary to meet demand from on-hand inventory. Customer demand is inherently uncertain; therefore, a high level of on-hand inventory or safety stock may be required to hedge against demand uncertainty. The costs of having insufficient supply to meet demand include lost revenues and profits as well as other "hidden" costs such as expediting, loss of reputation that may result in loss of future sales, etc. The following cases[1] demonstrate the importance of simultaneous optimal planning of capacity, production, and inventory:

1. Cisco: Shortages of memory and optical components paralyzed one path of production and hurt earnings. However, Cisco had to write down US$2.25 billion in inventory when it was too late to turn off its supply chain from piling up inventory!
2. The Sony Corporation: Sony shipped only half of the planned PlayStation 2 shipment during the U.S. launch due to a shortage of graphic chips.

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Benny Budiman is with the Mechanical Engineering Department, Massachusetts Institute of Technology, 77 Massachusetts Ave Room 35-023, Cambridge, MA 02139, USA. E-mail: bbudiman@mit.edu.

[1] Unless indicated otherwise, the cases are described in detail by Lakenan *et al.* [20].

3. Apple Computer: Apple filled only half of its orders in late 1999 due to shortage of G4 chips.
4. Koninklijke Phillips Electronics NV: Shortage of flash memory chips almost disrupted production of 18 million telephone units in 2000.
5. Palm: Analysts predicted Palm's revenues could have been 10 to 40 percent higher had Palm not experienced a shortage of liquid crystal displays.
6. Boeing Commercial Airplane: When Boeing tried to increase its production rate to maintain market share, part shortages caused a US$1.6 billion pretax charge in the third quarter of 1997 to cover penalty payments to airlines for late deliveries, overtime, and other unexpected production-related expenses accumulated since the beginning of that year as reported by Biddle [1] and Cole [2] in the Wall Street Journal.

Although the ultimate goal is to perform simultaneous capacity, production, and inventory planning for the supply chain, the work described here focuses on the optimal capacity adjustment problem at a single node within the supply chain. In practice, capacity adjustments occur at three levels: plant and equipment, operating time, and workforce level. The analysis in this paper includes optimal gross capacity adjustment, formulation of the optimal level of operating hours, labor availability, and production hardware availability to achieve that capacity.

II. Research Review: Simultaneous Capacity, Production and Inventory Planning

The literature containing work on simultaneous planning of production, capacity and inventory has the following focal points:

- Single-enterprise aggregate and capacity planning
- Single-time period coordinated planning
- Supplier capacity reservation game
- Capacity subcontracting problem

In single-enterprise aggregate and capacity planning, Bradley and Arntzen [3] optimize the return on operating assets in simultaneous planning of capacity, production schedule, and inventory. Bean *et al.* [4] use a deterministically equivalent demand process to solve for an optimal capacity growth plan that meets the growing demand for capacity over an infinite horizon. Burnetas and Gilbert [5] present a newsvendor-like optimal procurement policy to procure capacity in advance of the selling season, before the price of capacity becomes very expensive. Bard *et al.* [6] optimize capacity expansion at semiconductor manufacturing facilities by using a nonlinear integer program to determine the number of tools at a workstation. Khmelnitsky and Kogan [7] present an optimal control approach to a continuous-time aggregate production planning problem on one hierarchical level with production, overtime, and capacity expansion rates as decision variables on one hierarchical level. Angelus and Porteus [8] report an optimal capacity band that prescribes positive adjustment to capacity if the supply level is below the lower bound and negative adjustment if above the upper bound for a triangular- demand curve during the product life. Bradley and Glynn [9] develop an approximate solution to the GI/M/1 model for managing capacity and inventory jointly by using Brownian motion process. Rajagopalan and Swaminathan [10] present an analysis that solves for the optimal time and amount of capacity adjustment, the optimal production quantities, and the optimal lot sizes for a firm producing several products, using a Lagrangian relaxation procedure to calculate the lower bound and two heuristics based on both the Lagrangian and the dynamic programming approach. Holt *et al.* [11] present a technique to optimally set the aggregate production rate and the size of the work force of a paint factory using quadratic cost functions. Holt *et al.* [12] present the mathematical derivation of the optimal linear decision rule based on quadratic cost functions for regular payroll; hiring and layoff; overtime; and inventory, backorder, and machine setup costs.

In single-period coordinated planning, Zimmer [13] presents an optimal coordination mechanism in a decentralized supply chain facing uncertain availability of capacity to achieve the performance of a centralized supply chain. Van Mieghem and Rudi [14] present a news vendor formulation of the stochastic capacity investment and inventory procurement problem.

In the supplier capacity reservation game, Erhun *et al.* [15] describe the role of capacity spot markets as compared to that of advance capacity reservation in reducing double marginalization in coordinated-decentralized supply chain management. Van Mieghem [16] employs a game theoretic model of contracts for analyzing the effect of state-dependent contracts in eliminating decentralization costs and in coordinating capacity investment decisions.

In the capacity subcontracting problem, Buzacott and Chaouch [17] argue that optimal capacity expansion, to meet growing demand with periods of stochastically disrupted growth, can be achieved by either building up or outsourcing. Tan and Gershwin [18]) perform profit maximization using subcontractors with a different cost structure when customer order is backlog-dependent. Atamturk and Hochbaum [19] analyze the tradeoffs between acquiring capacity, subcontracting, determining production lot size, and holding inventory to meet non-stationary demand.

III. Capacity-Production-Inventory Planning

Gross capacity-production-and-inventory planning involves setting the appropriate level of supply capacity throughout the planning horizon[2]. While the primary goal of the research is to set supply capacity optimally by adjusting capacity variables consisting of plant and equipment, operating duration, and

[2] A planning horizon typically spans the life of the product that ranges from a few quarters to a few years. It is also commonly divided into several planning intervals.

workforce level within each planning interval, this paper only presents the optimal gross capacity adjustment[3].

The analysis starts with a derivation of optimal capacity adjustment for a two-period planning problem. In a two-period planning problem, the cost function consists of the sum of the costs of operating capacity, production (including material), holding inventory, and suffering from insufficient supply. The loss function takes into account revenues generated in both periods as well as the salvage value of remaining inventory at the end of the second (last) interval by subtracting the revenue terms from the cost function above. The optimal capacity adjustments must yield the minimum loss function expressed mathematically as

$$\begin{aligned} E[z^*] = \min_{u_1,\Delta v_1, u_2, \Delta v_2} \sum_{j=1}^{2} & E[-r_j \min(i_j + u_j, x_j + b_j) \\ & + h_j \cdot (z_j + u_j - x_j)^+ + p_j \cdot (x_j - z_j - u_j)^+ \\ & + c_{uj}\, u_j + k_{vj} \cdot (v_j + \Delta v_j) + j_{\Delta j}(\Delta v_j)] \\ & - s_2 \cdot (z_2 + u_2 - x_2)^+ \end{aligned} \tag{1}$$

i_j :On-hand inventory at beginning of period j $(i_j \geq 0)$

b_j :Backlog at beginning of j $(b_j \geq 0)$

z_j :Inventory position at beginning of j

$z_j = i_j - b_j$

v_j :Capacity at beginning of j

u_j :Production level in j $(u_j \geq 0)$

Δv_j : Capacity adjustment in j

x_j :Demand in j $(x_j \geq 0)$

h_j :Unit inventory holding cost j $(h_j \geq 0)$

p_j :Unit penalty cost for lost sales in j $(p_j \geq 0)$

r_j :Unit revenue (price) in j $(r_j \geq 0)$

$j_{\Delta vj}(\Delta v_j)$:Cost of capacity adjustment in j

$$j_{\Delta vj}(\Delta v_j) = \begin{cases} k^{+}_{\Delta vj} \Delta v_j, & \Delta v \geq 0 \\ -k^{-}_{\Delta vj} \Delta v_j, & \Delta v < 0 \end{cases}$$

k_{vj} :Unit cost of operating capacity in j $(k_{vj} \geq 0)$

c_{uj} :Unit cost of production in j $(c_{uj} \geq 0)$

$k^{+/-}_{\Delta vj}$:Unit cost of adjusting capacity in j $(k^{+/-}_{\Delta vj} \geq 0)$

Note that demand x_j is a random variable with a probability density function $f_{xj}(x_j)$. It is assumed to be independently distributed, i.e., demand in a planning interval is not correlated or independent from that in the following planning interval.

The system dynamics describe the inventory dynamics resulting from filling demand and replenishing as well as the capacity dynamics caused by adjustments. The inventory and capacity dynamics is

$$\begin{aligned} z_{j+1} &= z_j + u_j - x_j \\ b_{j+1} &= x_j - z_j - u_j \\ v_{j+1} &= v_j + \Delta v_j \end{aligned} \tag{2}$$

In (2), the inventory position can be negative, but the backlog must always be positive. A positive backlog signifies waiting customer orders when supply is insufficient. In other words, the model presented in (2) allows unfilled demand in an interval to be backlogged for fulfillment in the following interval. The expression for the above constraint on backlog is, therefore,

$$b_{j+1} \geq 0 \tag{3}$$

The production, always positive, never exceeds the available capacity. The capacity adjustment is also assumed available for use in the same period as that in which it is ordered. The above assumptions and constraints are expressed as

$$\begin{aligned} u_j &\leq v_j + \Delta v_j \\ u_j &\geq 0 \end{aligned} \tag{4}$$

Slack variables $e_{b(j+1)}$, e_{vj}, and e_{uj} can transform the above inequality constraints (3) and (4) to

$$\begin{aligned} 0 &= -b_{j+1} + e^2_{b(j+1)} \\ 0 &= u_j - v_j - \Delta v_j + e^2_{vj} \\ 0 &= -u_j + e^2_{uj} \end{aligned} \tag{5}$$

Applying Bellman's optimality principle to rewrite the minimum loss function (1) yields

$$\begin{aligned} E[z^*] = \min_{u_1,\Delta v_1} \Big\{ & -r_1 \left[b_1 + E_1(x_1) \right] + (r_1 + p_1 + h_1) \\ & \cdot \int_0^{z_1 + u_1} (z_1 + u_1 - x_1) \cdot f_{x1}(x_1)\, dx_1 - (r_1 + p_1) \\ & \cdot \left[z_1 + u_1 - E_1(x_1) \right] + c_{u1}\, u_1 + k_{v1} \cdot (v_1 + \Delta v_1) \\ & + j_{\Delta 1}(\Delta v_1) + E_2\left[z_2^*(z_2, v_2, b_2) \right] \Big\} \end{aligned} \tag{6}$$

Note $z_2^*(i_2, v_2)$ is the minimum cost-to-go function defined as:

$$\begin{aligned} E[z_2^*(z_2, v_2, b_2)] = \min_{u_2,\Delta v_2} \Big\{ & -r_2 \left[b_2 + E(x_2) \right] + (r_2 + p_2 - s_2 \\ & + h_2) \cdot \int_0^{\infty} (z_2 + u_2 - x_2)^+ \cdot f_{x2}(x_2)\, dx_2 \\ & - (r_2 + p_2) \cdot \left[z_2 + u_2 - E_2(x_2) \right] + c_{u2}\, u_2 \\ & + k_{v2} \cdot (v_2 + \Delta v_2) + j_{\Delta 2}(\Delta v_2) \Big\} \end{aligned} \tag{7}$$

Subject to

$$\begin{aligned} 0 &= u_2 - v_2 - \Delta v_2 + e^2_{v2} \\ 0 &= -u_2 + e^2_{u2} \end{aligned} \tag{8}$$

Applying the Lagrangian technique to solve for the optimal controls yields

$$\begin{aligned} u_2^* &= v_2 + \Delta v_2^* \\ \Delta v_2^* &= F^{-1}_{x2}\left(g_2^*\right) - i_2 - v_2 \\ g_2^* &= \frac{r_2 + p_2 - c_{u2} - k_{v2} - \left(d j_{\Delta 2} / d \Delta v_2\right)_{\Delta v_2^*}}{r_2 + p_2 - s_2 + h_2} \end{aligned} \tag{9}$$

Note that g_2^* is the cumulative probability that demand will be

[3] Capacity is the maximum number of units that can be produced during each planning interval or period.

less than the optimal supply level

$$S_2^* = i_2 + v_2 + \Delta v_2^* \tag{10}$$

The optimal controls above determine an optimal supply level in the second period that is dual valued, i.e., one corresponding to a positive adjustment (addition) of capacity $(S_2^*)^+$ and the other to a negative adjustment (reduction) of capacity $(S_2^*)^-$ as shown on Fig. 2. The region determines the action to take at the beginning of Period 2, i.e., if the supply at the beginning of Period 2 is less than the lower bound $(S_2^*)^+$, then it is optimal to increase the supply capacity up to the level of the optimal supply lower bound. Similarly, if the supply at the beginning of Period 2 is higher than the upper bound $(S_2^*)^-$, it is optimal to reduce the supply capacity to the level of the optimal supply upper bound.

$$\Delta v_2^* = \begin{cases} (S_2^*)^- - i_2 - v_2, & (S_2^*)^- \le i_2 + v_2 \\ 0, & (S_2^*)^+ \le i_2 + v_2 < (S_2^*)^- \\ (S_2^*)^+ - i_2 - v_2, & i_2 + v_2 < (S_2^*)^+ \end{cases} \tag{11}$$

For a positive capacity adjustment in the last period, the following conditions must hold:

$$k_{\Delta 2}^+ \le r_2 + p_2 - c_{u2} - k_{v2} \tag{12a}$$

$$s_2 \le h_2 + c_{u2} + k_{v2} \tag{12b}$$

Similarly, for a negative capacity adjustment, the conditions are

$$k_{\Delta 2}^- \le -s_2 + h_2 + c_{u2} + k_{v2} \tag{13a}$$

$$r_2 \ge c_{u2} + k_{v2} - p_2 \tag{13b}$$

Note that conditions (12b) and (13b) are satisfied automatically because they are also operating conditions, which require that salvage be less than procurement and holding costs and price be larger than costs, respectively.

Substituting the optimal controls (9) into (6) and solving for the optimal controls in Period 1 yield

$$u_1^* = v_1 + \Delta v_1^*$$

$$\Delta v_1^* = F_{x1}^{-1}(g_1^*) - i_1 - v_1$$

$$g_1^* = \frac{(r_1 - r_2) + p_1 - (c_{u1} - c_{u2}) - (k_{v1} - k_{v2}) + j'_{\Delta 2}(\Delta v_2^*)}{r_1 + p_1 + h_1} - \frac{j'_{\Delta 1}(\Delta v_1^*) - j'_{\Delta 2}(\Delta v_2^*)}{r_1 + p_1 + h_1} \tag{14}$$

Where g_1^* is the cumulative probability that demand will be less than the optimal supply level

$$S_1^* = i_1 + v_1 + \Delta v_1^* \tag{15}$$

The optimal controls in Period 1 above show similar properties to those of the optimal controls in Period 2, i.e., there exist upper and lower bounds of optimal supply capacity $(S_1^*)^+$ and $(S_1^*)^-$, respectively, such that

$$\Delta v_1^* = \begin{cases} (S_1^*)^- - i_1 - v_1, & (S_1^*)^- \le i_1 + v_1 \\ 0, & (S_1^*)^+ \le i_1 + v_1 < (S_1^*)^- \\ (S_1^*)^+ - i_1 - v_1, & i_1 + v_1 < (S_1^*)^+ \end{cases} \tag{16}$$

Where

$$(S_1^*)^- = i_1 + u_1^* = i_1 + v_1 - \Delta v_1^* = F_{x1}^{-1}(g_1^*)$$

$$(g_1^*)^- = \frac{(r_1 - r_2) + p_1 - (c_{u1} - c_{u2}) - (k_{v1} - k_{v2}) + j'_{\Delta 2}(\Delta v_2^*)}{r_1 + p_1 + h_1} + \frac{k_{\Delta 1}^- + j'_{\Delta 2}(\Delta v_2^*)}{r_1 + p_1 + h_1} \tag{17a}$$

and

$$(S_1^*)^+ = i_1 + u_1^* = i_1 + v_1 + \Delta v_1^* = F_{x1}^{-1}\left[(g_1^*)^+\right]$$

$$(g_1^*)^+ = \frac{(r_1 - r_2) + p_1 - (c_{u1} - c_{u2}) - (k_{v1} - k_{v2}) + j'_{\Delta 2}(\Delta v_2^*)}{r_1 + p_1 + h_1} - \frac{k_{\Delta 1}^+ - j'_{\Delta 2}(\Delta v_2^*)}{r_1 + p_1 + h_1} \tag{17b}$$

If the operating and production costs are constants, it can be shown that

$$j'_{\Delta 2}(\Delta v_2^*) \le \frac{1}{2}\left[j'_{\Delta 1}(\Delta v_1^*) + h_1\right] \tag{18}$$

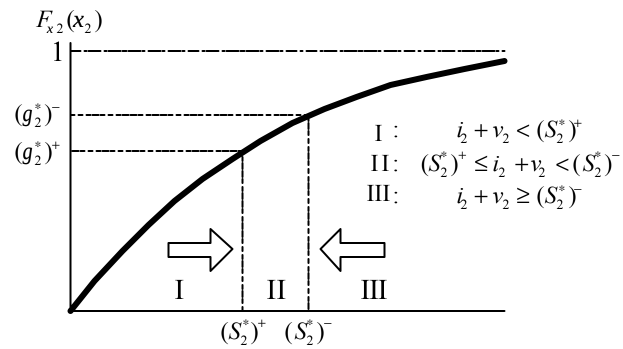

Fig. 2. Regions of optimal capacity adjustment in the second (last) period.

The recursive form of the Bellman's principles of optimality and that of the optimal controls in Period 1 suggest extension of the solution for the multi-period problem. The optimal controls for a multi-period problem are outlined below:

1. Optimal controls for the last period are:

$$u_T^* = v_T + \Delta v_T^*$$

$$\Delta v_T^* = F_{xT}^{-1}\left(g_T^*\right) - i_T - v_T \tag{19a}$$

$$g_T^* = \frac{r_T + p_T - c_{uT} - k_{vT} - \left(d j_{\Delta T}/d\Delta v_T\right)_{\Delta v_T^*}}{r_T + p_T - s_T + h_T}$$

The feedback control law for this period is

$$\Delta v_T^* = \begin{cases} (S_T^*)^- - i_T - v_T, & (S_T^*)^- \le i_T + v_T \\ 0, & (S_T^*)^+ \le i_T + v_T < (S_T^*)^- \\ (S_T^*)^+ - i_T - v_T, & i_T + v_T < (S_T^*)^+ \end{cases} \tag{19b}$$

2. Optimal controls for the earlier periods $j = 1, \ldots, T-1$ are:

$$
\begin{aligned}
u_j^* &= v_j + \Delta v_j^* \\
\Delta v_1^* &= F_{xj}^{-1}(g_j^*) - i_j - v_j \\
g_j^* &= \frac{(r_j - r_{j+1}) + p_j - (c_{uj} - c_{u(j+1)}) - (k_{vj} - k_{v(j+1)})}{r_j + p_j + h_j} \\
&\quad - \frac{j'_{\Delta j}(\Delta v_j^*) - 2 j'_{\Delta(j+1)}(\Delta v_{(j+1)}^*)}{r_j + p_j + h_j}
\end{aligned}
\tag{19c}
$$

The feedback control law is

$$
\Delta v_j^* = \begin{cases} (S_j^*)^- - i_j - v_j, & (S_j^*)^- \le i_j + v_j \\ 0, & (S_j^*)^+ \le i_j + v_j < (S_j^*)^- \\ (S_j^*)^+ - i_j - v_j, & i_j + v_j < (S_j^*)^+ \end{cases} \tag{19d}
$$

If, on the other hand, the cost of capacity adjustment is [4]

$$
\tilde{j}_{\Delta j}(\Delta v_2) = \begin{cases} K_j^+ + k_{\Delta j}^+ \Delta v_j, & \Delta v_j > 0 \\ 0, & \Delta v_j = 0 \qquad \forall j = 1,2 \\ K_j^- - k_{\Delta j}^- \Delta v_j, & \Delta v_j < 0 \end{cases} \tag{20}
$$

The objective function (1) is now

$$
E[\tilde{z}^*] = E[z^*] + \sum_{j=1}^{2} \begin{cases} K_j^+ & \Delta v_j^* > 0 \\ 0\ , & \Delta v_j^* = 0 \\ K_j^- & \Delta v_j^* < 0 \end{cases} \tag{21}
$$

It can be shown that the optimal supply levels S_j^* $\forall j$ (10 and (15) clearly minimize (21) as well if capacity adjustments have been decided (see also Bramel and Simchi-Levi [21], pp. 180–181). With the capacity adjustment cost (20), it can be shown that the optimal capacity band defined in (19) will have an outer band (Fig. 3) such that the lower value of the outer-band limits determines the critical supply value $(\bar{S}_j^*)^+$ below which it will be optimal to increase the supply level to $(S_j^*)^+$. Similarly, the upper value of the outer-band limits determines the critical supply value $(\bar{S}_j^*)^-$ above which it will be optimal to reduce the supply level to $(S_j^*)^-$. The above critical supply values determining the outer band satisfy

$$
z[(\bar{S}_j^*)^{+/-}] = K_j^{+/-} + z[(S_j^*)^{+/-}] \tag{22}
$$

The above formulation (19) depends on knowledge of demand uncertainties, characterized by the cumulative probability distribution of demand in each period. In practice, the uncertainty is greater the further away a period is from the beginning of the planning horizon. Therefore, it is critical to have information sharing in the supply chain that helps characterize demand and throughout on the planning horizon. Consequently, this research will investigate the effect of forecast evolution in characterization of demand uncertainty (Graves *et al.* [22]). For details on information sharing in supply chain, see Katz *et al.* [23], Barlas and Aksogan [24], and Street [25], who present work in sharing demand data; Dobson and Pinker [26], who study the sharing of state-dependent lead time information as compared with that of lead time information based on general probability distribution; and Swaminathan *et al.* [27] , who present work on sharing the capacity information. However, the research effort on forecast information sharing and forecast evolution is beyond the scope of this paper (see Budiman [28] for details).

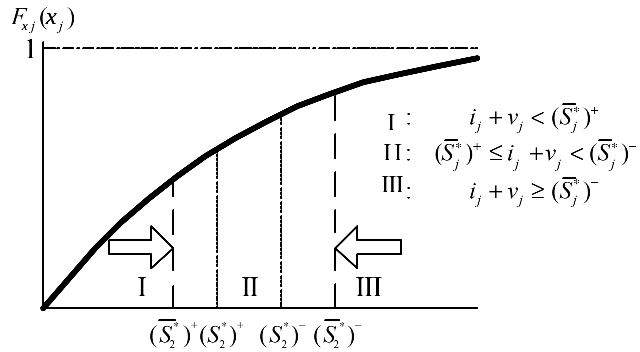

Fig. 3. Upper and lower values of the outer-band limits resulting from taking into account the setup cost in the cost of adjusting capacity.

IV. Capacity Adjustment Model

The previously derived optimal capacity adjustment prescribes only the aggregate adjustment to capacity with an implicit assumption of immediate availability of the adjustment. Implementation of the above prescription requires a similar prescription for capacity adjusting knobs such as plant and equipment, operating time, and workforce. The optimal adjustments to capacity adjusting knobs should take into account the delay between the decision time and the time the adjustments are operational.

In a production interval, the quantity of goods produced depends primarily on the available time for production and on the production cycle time. In turn, the available time for production depends on the availability of production hardware, the availability of labor, and the operating time. The production cycle time depends on the setup time, batch size, and unit production time. The mathematical expression for the production quantity in a production interval is [5]

$$
v_j = \frac{\ell'_j \min[t'_{Rj}, t_H, (w'_j / w_l \ell'_j) q_W (1 + a'_{W\max})]}{T_S / q_j + t} \tag{23}
$$

[4] Cost functions of the form $j(\Delta) = K\, d(\Delta) + c\, \Delta$ are commonly used to model setup cost that results in lower unit cost for higher number of units deployed or produced. In this situation, it can be used to model the setup cost of adding production lines or hardware and the cost of adding people in the form of setup and training, respectively.

[5] Prime indicates the value of the respective variable at the beginning of the period after including the effective adjustments; unprimed variables represent the value prior to making any adjustments.

ℓ_j :Production lines at beginning of period j

ℓ'_j :Effective available production lines at start of j

t_{Rj} :Operating time in j

t'_{Rj} :Effective operating time in j

t_H :Available physical time in j

w_j :Workforce level at beginning of j

w'_j :Effective workforce level at start of j

q_W :Work time per period for the workforce

w_ℓ :Required workforce level per production line

a'_{Wj} :Overtime quotient in period j

$a_{W\max}$:Maximum allowable overtime quotient

T_S :Setup time

q_j :Batch size in period j (assumed constant)

t :Unit processing time for the product

It can be shown that if

$$\begin{aligned} \ell'_j &= \ell_j + d\,\ell^{(+)}_{j-L} - d\,\ell^{(-)}_j \\ w'_j &= w_j + d\,w^{(+)}_{j-L} - d\,w^{(-)}_j \\ t'_{Rj} &= t_{Rj} + d\,t_{Rj} \end{aligned} \tag{24}$$

L_ℓ :Delay in adjusting production lines

L_W :Delay in adjusting workforce level

$d\,\ell^{(+)}_{j-L_\ell}$:Additional lines online at start of j ordered at $j - L_\ell$

$d\,\ell^{(-)}_j$:Lines decommisioned at beginning of j

$d\,w^{(+)}_{j-L_W}$:Additional labor online at start of j ordered at $j - L_W$

$d\,w^{(-)}_j$:Labor laid-off at beginning of j

$d\,t_{Rj}$:PChange in operating time at start of j

Then the labor-hardware-time relationship with capacity (23) can be rewritten as

$$\begin{aligned} v_j &= [w_j + d\,w^{(+)}_{j-L_W} - d\,w^{(-)}_j]\cdot\frac{q_W}{w_\ell}\cdot\frac{(1+a'_{Wj})}{T_S/q_j + t} \\ a'_{Wj} &= \frac{w_\ell}{h_W}\cdot\frac{[\ell_j + d\,\ell^{(+)}_{j-L_\ell} - d\,\ell^{(-)}_j]}{[w_j + d\,w^{(+)}_{j-L_W} - d\,w^{(-)}_j]}\cdot(t_{Rj} + d\,t_{Rj}) - 1 \end{aligned} \tag{25}$$

q_W :Normal work time per period for the workforce

Subject to

$$\begin{aligned} t_{Rj} + d\,t_{Rj} &\le t_H \\ a'_{Wj} &\le a_{W\max} \end{aligned} \tag{26}$$

The aforementioned capacity model is generic because it takes into account the delay in adjusting capacity. This paper, however, presents discussion on how the above capacity model can be included in an optimal capacity planning exercise presented in Section III with the assumption that adding capacity is instantaneous, i.e.,

$$\begin{aligned} \ell'_j &= \ell_j + d\,\ell_j \\ w'_j &= w_j + d\,w_j \\ t'_{Rj} &= t_{Rj} + d\,t_{Rj} \end{aligned} \tag{27}$$

Substituting (27) into (25) to calculate the effect of adjusting the capacity levers on the production capacity yields

$$\begin{aligned} \Delta v_j &= d\,w_j\cdot(1 + a'_j) + w_j\,\Delta a'_j \\ \Delta a'_j &= \frac{w_\ell\,\ell_j\,t_{Rj}}{q_W\cdot(w_j + dw_j)}\left(\frac{dt_{Rj}}{t_{Rj}} + \frac{d\ell_j}{\ell_j} - \frac{dw_j}{w_j} + \frac{dt_{Rj}\,d\ell_j}{t_{Rj}\,\ell_j}\right) \end{aligned} \tag{28}$$

To account for the delay requires only rewriting of the change in the capacity variables in (27) with the appropriately delayed change in the capacity variables in (24); or (28) can be rewritten as

$$\begin{aligned} \Delta v_j &= [d\,w^{(+)}_{j-L} - d\,w^{(-)}_j]\cdot(1 + a'_j) + w_j\,\Delta a'_j \\ \Delta a'_j &= \frac{w_\ell\,\ell_j\,t_{Rj}}{q_W\cdot[w_j + d\,w^{(+)}_{j-L} - d\,w^{(-)}_j]}\left(\frac{dt_{Rj}}{t_{Rj}} + \frac{d\,\ell^{(+)}_{j-L} - d\,\ell^{(-)}_j}{\ell_j}\right. \\ &\quad \left. - \frac{d\,w^{(+)}_{j-L} - d\,w^{(-)}_j}{w_j} + \frac{d\,t_{Rj}\,[d\,\ell^{(+)}_{j-L} - d\,\ell^{(-)}_j]}{t_{Rj}\,\ell_j}\right) \end{aligned} \tag{29}$$

V. Summary and Conclusion

The optimal capacity adjustment is of the form of an outer-bounded capacity band. The optimal capacity formulation is in feedback form in that the capacity adjustment in each period depends on the states: inventory and capacity at the beginning of the period. It is also in feed-forward form because it takes into account the demand uncertainties in the following periods. The work presented here distinguishes three major variables affecting the production capacity: plant and equipment, workforce, and operating time. The formulation shows the importance of understanding demand uncertainties throughout the planning horizon, thus underlining the importance of information sharing throughout and forecast evolution in the supply chain.

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