View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by DSpace@MIT

Optimization of Passive Constrained Layer Damping Treatments for Vibration Control of Cylindrical Shells

H. Zheng G. S. H. Pau and G. R. Liu

Abstract—**This paper presents the layout optimization of passive constrained layer damping (PCLD) treatment for vibration control of cylindrical shells under a broadband force excitation. The equations governing the vibration responses are derived using the energy approach and assumed-mode method. These equations provided relationship between the integrated displacement response over the whole structural volume, i.e. the structural volume displacement (SVD), of a cylindrical shell to structural parameters of base structure and multiple PCLD patches, Genetic algorithms (GAs) based penalty function method is employed to find the optimal layout of rectangular PCLD patches with minimize the maximum displacement response of PCLD-treated cylindrical shells. Optimization solutions of PCLD patches' locations and shape are obtained under the constraint of total amount of PCLD in terms of percentage added weight to the base structure. Examination of the optimal layouts reveals that the patches tend to increase their coverage in the axial direction and distribute over the whole surface of the cylindrical shell for optimal control of the structural volume displacement.**

***Index Terms*—Vibration analysis, nonlinear optimization, cylindrical shells, constrained layer damping.**

I. Introduction

Constrained layer damping (CLD) treatment has been regarded as an effective way to suppress vibrations of and sound radiation from various structures. Literature survey shows that the pioneer work in the study of CLD treatment could be traced back to 1950's when DiTaranto [1] and Mead and Markus [2] developed the theoretical models respectively for the axial and bending vibrations of sandwich beams with viscoelastic core. After that, many researchers have reported different formulations and techniques in this field, e.g., Douglas and Yang [3], and Rao [4], Yan and Dowell [5], and Rao and He [6]. The problem of computing damped natural frequencies and loss factors is explicitly solved [7,8] for both beams and plates when simply support end conditions are assumed. Analytical-numerical procedures are proposed to solve the problem when different boundary conditions are assumed [9]. The finite element procedure has also been adopted on the basis of design consideration [10~12].

Most of these early works dealt with full coverage CLD treatments that are evidently not practical in application. In partially covered viscoelastically damped sandwich beams or plates with constrained type of treatment, only a portion of the base structure is covered with CLD. Nokes and Nelson [13] were among the earliest investigators to provide the solution to the problem of a partially covered sandwich beam. And a more thorough analytical study was carried out by Lall *et al*. Who solved, by using Rayleigh-Ritz approach, the eigenvalue problem for a beam [14] and for a plate [15] with a single damping patch. Kung & Singh [16] presented a refined method for analyzing the modal damping of beams with multiple constrained-layer viscoelastic patches.

In addition to all above-described works on PCLD treatments for vibration suppression of beams and plates, the study of vibration and damping in shells with added damping treatment has also been of interest to many researchers. Pan [17] studied the axisymmetrical vibration of a finite length cylindrical shell with a viscoelastic core. Jones & Salerno [18] investigated the effect of damping on the forced axisymmetrical vibration of cylindrical shells with a viscoelastic core. Alam & Asnani [19] carried out the vibration and damping analysis of a general multilayered cylindrical shell consisting of an arbitrary number of elastic and viscoelastic layers with simply supported end conditions. Ramesh & Ganesan [20] used a finite element method to solve for a cylinder-absorber system with thin axial strips which bonded to the cylinder with a thin viscoelastic layer. Liu *et al* [21] presented a finite element method for vibration control simulation of laminated composite structures with integrated piezoelectronics. Hu and Huang [22] developed a generic theory for the CLD treated shell with full coverage. Recently, Chen and Huang investigated the damping effects of CLD treatment of strip type along longitudinal direction [23] and along circumference [24], respectively, on the forced response of a cylindrical shell. A thin shell theory in conjunction with the Donnell-Mushtari-Vlasov assumptions is employed to

Manuscript received October 31, 2002.

G. R. Liu is with the Department of Mechanical Engineering of National University of Singapore, 10 Kent Ridge Crescent Singapore 119260. He is SMA Fellow (e-mail: mpeliugr@ nus.edu.sg).

H. Zheng is with the Institute of High Performance Computing, #01-01 The Capricorn, Science Park II, Singapore 117528. (e-mail: zhengh@ihpc.a-star.edu.sg).

G. S. H. Pau is the SMA graduate (HPCES). He is currently with Motorola Electronics Pte Ltd (Singapore). (e-mail: shpau@motorola.com).

yield their mathematical model. Employing the assumed-mode method, the discretized equations of motion in terms of shell transverse modal coordinates are derived. Their parametric studies showed that thicker or stiffer CL warrants better damping, and thicker VEM does not always give better damping than thinner ones when CL exceeds a certain thickness.

These theoretical works and parametric studies on PCLD treatments for vibration and noise suppression really assist design decision. However, studies based on the optimization are very few, particularly, in the optimum design of partial PCLD treatment of cylindrical shell, there has been no existing literature to the author's best knowledge. Marcelin *et al* [25] considered both partial covering and optimization with design variables being the dimensions and the locations of all the viscoelastic layers. Special beam finite elements were used to represent the behavior of the sandwich parts of the beam. Both theory and experiment [26] show that for stiff viscoelastic layers the loss factor is greater for partial coverage than for full coverage. Chen & Huang [27] presented a study on optimal placement of PCLD treatment for vibration suppression of plates. In their optimization, the structural damping plays the main performance index and the frequencies' shift and CLD thickness play as penalty functions. Topographical and complex optimal solution techniques were employed in searching for the optimal value of CLD treatment.

The study presented in this paper attempts to arrive at an optimum design of partial PCLD treatment of cylindrical shell by finding an optimal layout of multiple rectangular PCLD patches of fixed thickness and material properties to minimize the forced vibration response under a broadband transverse excitation. The equations relating the integrated displacement response over the whole structural volume, i.e., the structural volume displacement (SVD), of a cylindrical shell to structural parameters of base structure and multiple PCLD patches are derived using energy based approach and assumed-mode method. Genetic algorithms (GAs) based penalty function method [28,29] is employed to find the optimal layout of rectangular PCLD patches with aim to minimize the SVD of PCLD-treated cylindrical shell. Optimization solutions of PCLD patches' locations and shapes are obtained under the constraint of total amount of PCLD in terms of the percentage of the added weight to the base structure. Effects of the total number of added weight due to PCLD treatment are also studied, towards maximum vibration attenuation using minimum amount of PCLD patches.

II. ANALYTICAL MODEL OF CYLINDER WITH MULTIPLE PCLD PATCHES

A. Kinematic Relation

A cylinder treated with multiple PCLD patches is modeled as a composite cylindrical shell consisting of 3 layers, namely the base, constraining and viscoelastic layers, each referred to by using the subscripts/superscripts *s*, *c* and *v* respectively. A general configuration for a simply-supported cylinder treated with a PCLD patch *p* is shown in Fig. 1, with labeled design parameters and design variables. The layers have different thickness denoted by h_i, where $i = s, c$ or v. Similarly, Y_i is the elastic modulus of the *i*th layer. Then, the mathematical model for the cylinder is derived based on classical procedure [22,23]. Under the Donnell-Mushtari-Vlasov assumptions [30], the stress-strain relationship in cylindrical shell and in constraining layer are described by

$$\begin{aligned}
\sigma^i_{xx} &= \frac{E_i}{1-\upsilon_i^2}\left(\varepsilon^i_{xx}+\upsilon_i\varepsilon^i_{\theta\theta}\right)\\
\sigma^i_{\theta\theta} &= \frac{E_i}{1-\upsilon_i^2}\left(\varepsilon^i_{\theta\theta}+\upsilon_i\varepsilon^i_{xx}\right) \quad i=s,c \\
\sigma^i_{x\theta} &= G_i\varepsilon^i_{x\theta}
\end{aligned} \tag{1}$$

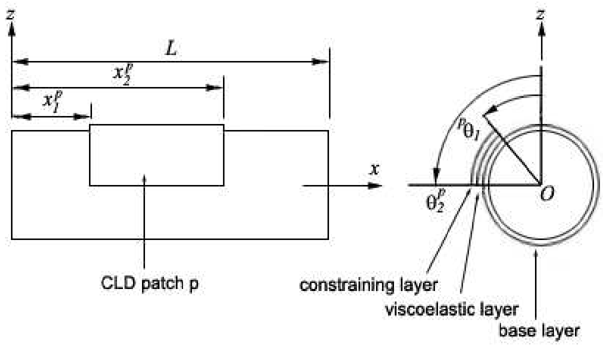

Fig. 1. A simply supported cylinder with one partial PCLD patch

As the cylinder is approximated by a thin shell, the displacements in x and θ directions are assumed to vary linearly through the shell thickness, and the displacement in the transverse direction is independent of z. Thus, the strain-displacement relations is given by

$$\begin{aligned}
\varepsilon^i_{xx} &= \frac{\partial u^i_x}{\partial x} - z\frac{\partial^2 u^i_x}{\partial x^2}\\
\varepsilon^i_{\theta\theta} &= \frac{1}{r_i}\frac{\partial u^i_\theta}{\partial\theta}+\frac{u^i_z}{r_i}-\frac{z}{r_i^2}\frac{\partial^2 u^i_z}{\partial\theta^2} \qquad i=s,c\\
\varepsilon^i_{x\theta} &= \frac{\partial u^i_\theta}{\partial x}+\frac{1}{r_i}\frac{\partial u^i_x}{\partial\theta}-2\frac{z}{r_i^2}\frac{\partial^2 u^i_z}{\partial x\partial\theta}
\end{aligned} \tag{2}$$

For the viscoelastic layer, the stress relation is given by [23]

$$\begin{aligned}
\sigma^v_{xz} &= G_v\varepsilon^v_{xz}\\
\sigma^v_{\theta z} &= G_v\varepsilon^v_{\theta z}
\end{aligned} \tag{3}$$

and its strain-displacement relation is given by

$$\begin{aligned}
\varepsilon^v_{xz} &= \beta^v_x + \frac{\partial u^v_z}{\partial x}\\
\varepsilon^v_{\theta z} &= \beta^v_\theta - \frac{u^v_\theta}{r_v}+\frac{1}{r_v}\frac{\partial u^v_z}{\partial\theta}
\end{aligned} \tag{4}$$

The deformation pattern of three layers in axial direction is shown in Fig. 2. Taking into consideration the Love simplifications, the assumption of no-slip condition between layers and $u_z^i = u_z$, where $i = s, c, v$, yields

$$\varepsilon_{xz}^{v} = \frac{1}{h}\left(u_x^c - u_x^s\right) + \frac{1}{2h_v}\left(h_c + h_z + 2h_v\right)\frac{\partial u_z}{\partial x}$$

$$\varepsilon_{\theta z}^{v} = \left(\frac{1}{h_v} - \frac{1}{2r_v}\right)u_\theta^c - \left(\frac{1}{h_v} + \frac{1}{2r_v}\right)u_\theta^s + \left(\frac{h_c}{2h_v r_c} + \frac{h_s}{2h_v r_s} - \frac{h_c}{4r_v r_c} + \frac{h_s}{4h_v r_s} + \frac{1}{r_v}\right)\frac{\partial u_z}{\partial \theta} \tag{5}$$

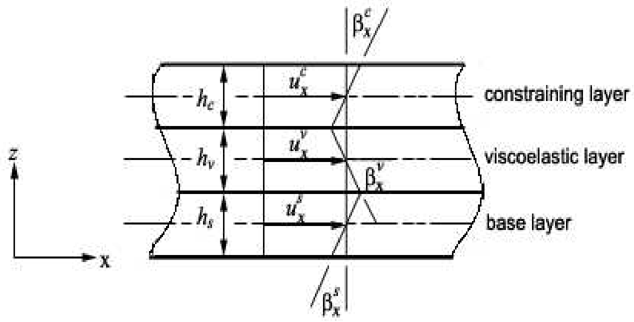

Fig. 2. The deformation pattern of the layers in x direction for cylindrical shell with PCLD treatment

B. Energy Expressions

The kinetic energies of the layers with neglected in-plane inertia are

$$T_s = \frac{1}{2} r_s \rho_s h_s \int_0^L \int_0^{2\pi} \dot{u}_z^2 dx$$

$$T_v = \frac{1}{2} r_v \rho_v h_v \sum_{p=1}^{n_p} \int_{x_1^p}^{x_2^p} \int_{\theta_1^p}^{\theta_2^p} \dot{u}_z^2 dx \tag{6}$$

$$T_c = \frac{1}{2} r_c \rho_c h_c \sum_{p=1}^{n_p} \int_{x_1^p}^{x_2^p} \int_{\theta_1^p}^{\theta_2^p} \dot{u}_z^2 dx$$

where $p = 1, 2, \ldots, n_p$ and n_p is the number of PCLD patches. The strain energies are

$$U_s = \frac{r_s}{2} \int_{-h_s/2}^{h_s/2} \int_0^L \int_0^{2\pi} \Big[\frac{E_s}{1-\upsilon_s^2}\left({\varepsilon_{xx}^s}^2 + {\varepsilon_{\theta\theta}^s}^2\right) + \frac{2E_s \upsilon_s}{1-\upsilon_s^2}\left(\varepsilon_{xx}^s \varepsilon_{\theta\theta}^s\right) + G_s {\varepsilon_{x\theta}^s}^2 \Big] d\theta dx dz$$

$$U_v = \frac{r_v}{2} \sum_{p=1}^{n_p} \int_{-h_v/2}^{h_v/2} \int_{x_1^p}^{x_2^p} \int_{\theta_1^p}^{\theta_2^p} \left(G_v {\varepsilon_{xz}^v}^2 + G_v {\varepsilon_{\theta z}^v}^2\right) d\theta dx dz$$

$$U_c = \frac{r_c}{2} \sum_{p=1}^{n_p} \int_{-h_c/2}^{h_c/2} \int_{x_1^p}^{x_2^p} \int_{\theta_1^p}^{\theta_2^p} \Big[\frac{E_c}{1-\upsilon_c^2}\left({\varepsilon_{xx}^c}^2 + {\varepsilon_{\theta\theta}^c}^2\right) + \frac{2E_c \upsilon_c}{1-\upsilon_c^2}\left(\varepsilon_{xx}^c \varepsilon_{\theta\theta}^c\right) + G_c {\varepsilon_{x\theta}^c}^2 \Big] d\theta dx dz \tag{7}$$

Assuming an external transverse load of $f(x, t)$ applied on the cylinder surface, the work done by this force can be expressed as

$$P = \int_0^L \int_0^{2\pi} r_s f(x, \theta, t) w(x, \theta, t) d\theta dx \tag{8}$$

C. Equation of Motion

The dynamic response of the PCLD treated cylinder excited by the external transverse load can be calculated by substituting the kinetic and strain energies into Lagrange's equation [30]

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i \tag{9}$$

where q_i represents the ith generalized coordinate and Q_i is the ith generalized force, T and U are respectively the kinetic and strain energy of the whole system expressed by

$$T = T_s + T_v + T_c \qquad U = U_s + U_v + U_c \tag{10}$$

For a cylinder, the displacements can be approximated by

$$w(x,\theta,t) = \sum_m \sum_n W_{mn}(x,\theta)\zeta_{mn}(t) = W^T \zeta$$

$$u_x^s(x,\theta,t) = \sum_m \sum_n U_{mn}^s(x,\theta)\eta_{imn}^s(t) = U^{s^T} \eta^s$$

$$u_\theta^s(x,\theta,t) = \sum_m \sum_n V_{mn}^s(x,\theta)\xi_{mn}^s(t) = V^{s^T} \xi^s \tag{11}$$

$$u_x^c(x,\theta,t) = \sum_m \sum_m U_{mn}^c(x,\theta)\eta_{mn}^c(t) = U^{c^T} \eta^c$$

$$u_\theta^c(x,\theta,t) = \sum_m \sum_n V_{mn}^c(x,\theta)\xi_{mn}^c(t) = V^{c^T} \xi^c$$

Here W, U^i and V^i $(i=s,\ c)$ are the assumed displacement shapes and ζ, η^i and ξ^i are the generalized coordinates of the plate response in cylinder radial, axial and circumferential directions respectively.

D. Solutions of Displacement Response

In the case of simply supported ends, the mode shape functions are as follows

$$W_{mn}(x,\theta) = \sin\left(\frac{m\pi x}{L}\right)\cos n(\theta - \phi)$$

$$U_{mn}^i(x,\theta) = \cos\left(\frac{m\pi x}{L}\right)\cos n(\theta - \phi) \quad i = s, c \tag{12}$$

$$V_{mn}^i(x,\theta) = \sin\left(\frac{m\pi x}{L}\right)\sin n(\theta - \phi)$$

Using above shape functions and substituting equations (6~8) into Lagrange's equation (9) yield the equation of motion of the cylinder in the form

$$[M]\{\ddot{q}\} + [K]\{q\} = \{F\} \tag{13}$$

where $[M] = \left[M^s + M^v + M^c\right]$, the mass matrix, and $[K] = \left[K^s + K^v + K^c\right]$, the stiffness matrix of the PCLD-treated cylinder. Vector $\{q\} = \left[q_\zeta^T, q_\eta^{s^T}, q_\xi^{s^T}, q_\eta^{c^T}, q_\xi^{c^T}\right]^T$ is a column vector containing the modal coefficients and $\{F\}$ is the vector of generalized force, of which the first $m \times n$ elements can be written as

$$\{F\}_{m\times n} = \left[-\int_0^L \int_0^{2\pi} r_s\, f(x,\theta,t) W_{ij}(x,\theta)\, d\theta\, dx \right]^T \tag{14}$$
$$i = 1,2,\ldots,m; \qquad j = 1,2,\ldots,n$$

All other are zeros since only a transverse load applied on the cylinder surface. Further assuming this transverse load is a unit time-harmonic point force at the location x^*, θ^*, the (i,j)th modal force is

$$F_{ij} = -r_s W_{ij}(x^*, \theta^*) e^{j\omega t} = F_{0ij} e^{j\omega t} \tag{15}$$

where ω is the circlar frequency of the transverse force. Under the excitation of this time-harmonic force, the system equation can be written as

$$\left[-\omega^2 M + K\right]\{q_0\} = \{F_0\} \tag{16}$$

Solving this system equation yields the solution of generalized displacement at circular frequency, ω. Multiplying them by the assumed modes for the structure, the physical displacement response at any location, (θ, x), of the cylinder can be calculated.

III. PCLD LAYOUT OPTIMIZATION

A. Objective Function

A general nonlinear optimization problem can be defined as follows [31].

Find an n-vector of design variables to minimize an objective function

$$f(\vec{x}) = f(x_1, x_2, \Lambda, x_n) \tag{16}$$

subjected to the equality and inequality constraints

$$h_j(\vec{x}) = 0 \qquad j = 1 \, to \, p \tag{17}$$

$$g_i(\vec{x}) \le 0 \qquad i = 1 \, to \, q \tag{18}$$

and simple bounds on the design variables

$$x_{il} \le x_i \le x_{iu} \qquad i = 1 \, to \, n \tag{19}$$

Since our objective is to reduce the vibration response of the cylinder, the out-of-plane displacement complex amplitude is the quality of interest. Obviously, this quality is location-dependent. A structural volume displacement (SVD) is therefore defined as the objective function to be minimized as

$$D = \int_0^L \int_0^{2\pi} |w(\theta, x)| r_s\, d\theta dx \tag{20}$$

where $|w(\theta, x)|$ is the module of out-of-plane displacement complex amplitude at cylinder surface location (θ, x). This SVD is a function of the layout of the PCLD patches and the total amount of PCLD material used. Obviously, minimizing the SVD would lead to significant reductions of the vibrational energy of whole cylinder.

Furthermore, as the SVD depends upon the frequency, an integral criterion over an appropriate frequency is required for the case of broadband excitation. A solution that meets technical interest is

$$f = \frac{1}{\omega_{\max} - \omega_{\min}} \int_{\omega_{\min}}^{\omega_{\max}} D(\omega) d\omega \tag{21}$$

Here ω_{min} and ω_{max} and respectively the minimum and maximum frequencies of interest. In the optimization that follows, these two parameters are set 0Hz and 3kHz

B. Variables and Constraints

Assuming that only rectangular patches are used, the layout of one PCLD patch can be completely defined by four design variables, namely the axial location, angular location, axial length and angular length, as shown in Fig. 3. For the convenience of fabrication, it is also assumed that sizes of all patches are same. Meanwhile, all the PCLD patches are kept same thickness and Young's modulus for constraining layer and viscoelastic layer, respectively. In this circumstance, the number of variables of optimization for the problem, i.e., the design parameters to be optimized, are $4\times n_p$ provided that n_p patches are used for the treatment.

In real-life vibration control design, the added weight to the base structure owing to CLD treatment is always restricted to a small amount of percentage of the structure. Thus, in the layout optimization, the total amount of PCLD material is fixed in each computation run. The problem is then constrained to ensure physical feasibility in which the patches are bounded within the surface of the cylinder and do not overlap each other.

C. Optimization Strategy

Several optimization algorithms/methods are available to solve the problem defined by expressions (16)~(19). Most algorithms are designed so far to find a local optimum. One example is the sequential quadratic programming, SQP, algorithm which has shown to be robust and efficient for most optimization problems [31]. However, many optimization problems have several local optima and it is often of interest to find the best optimum in the whole feasible design domain, i.e., the global optimum. Since no mathematical conditions for global optimality exist, a global optimum is usually more difficult and time-consuming to find than a local optimum. Some methods have, nevertheless, been developed to find an approximation of the global optimum without scanning the whole feasible design domain. The genetic algorithm, or shortly GA, is such a method that is developed to search for the approximation of global optimum. The GA has been used previously by a lot of researchers to solve various nonlinear optimization problems [28,29]. Here the GA based penalty function method is also employed for the PCLD layout optimization.

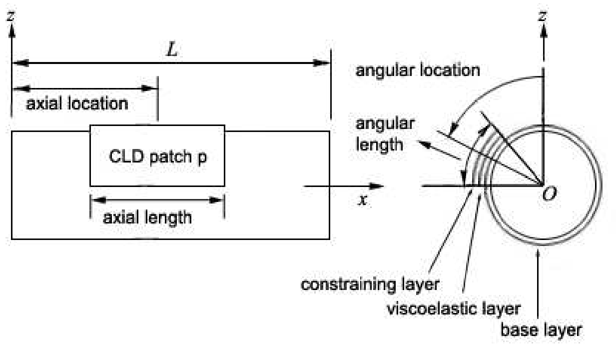

Fig. 3. The four variables to be optimized: axial location, angular location, axial length and angular length

With above definition of the bonds on the design variables, the optimization problem leads to a large design domain. Thus, direct application of genetic algorithms may not yield good results. In order to obtain a solution closest to global optimum, the design space have to be further constrained. Therefore, three different approaches are used to restrict the number of design variables. In the first approach, the surface of the cylinder is divided into three segments along axial direction such that the patches are evenly distributed in each segment. The approach 2 is similar to approach 1 except that the cylinder is divided into two segments. In approach 3, there is no cylinder dividing, but the axial length and angular length of each PCLD patch are fixed. So the axial location and the angular location constitute two design variables of each PCLD patch.

IV. RESULTS AND DISCUSSION

The geometric and material properties of the cylinder to be controlled are shown in Table 1. The length and radius of the cylinder are respectively 0.35 m and 0.1 m. The loss factor of viscoelastic material, η_v, and also its shear modulus, G_v, are assumed invariant with frequency. For comparison purpose, a small structural damping is introduced in the form of a complex Elastic Modulus for the base shell and the constraining layers:

$$\tilde{E} = E(1 + i\eta)$$

where η is the structural loss factor. A η value of 0.0001 is used. With this setup, genetic algorithms is applied to the optimization problem for each of the approaches outlined. For each set of parameters, defined by the total amount of PCLD material, the number of patches and the approach used, 5 runs are executed to arrive at the layout with the lowest SVD of the cylinder.

Table 1 Material properties used in analysis of cylindrical shell

PROPERTIES	SHELL	CL (PZT-5H)	VE MATERIAL
Elastic Modulus, $\tilde{E}$ (GPa)	70(1 + 0.0001i)	49(1 + 0.0001i)	–
Density, ρ (kg/m^3)	2.71×10^3	7.50×10^3	1.00×10^3
Thickness, h (m)	0.002	0.0002	0.0002
Shear Modulus, G (MPa)	–	–	0.896(1 + 0.5i)

A unit harmonic transverse force is applied at the middle cylinder, i.e, $\theta^*=0$ and $x^*=L/2$, and the excitation frequency is from 0 to 3.5kHz. Before performing the optimization, the analytical model and associated solution procedure are validated by comparing the natural frequencies of bare cylinder with the theoretical predictions given in [27]. For cylinder with PCLD treatment, the frequency response at the force location is compared to results obtained a multiple physics finite element code. The cylinder with single PCLD patch treatment is considered. Good agreements between the values are observed for both cases.

A. *Comparison of Three Approaches*

For each of above-described optimization approaches, the optimal layouts is obtained for a fixed number of patches and a fixed total amount of PCLD material equivalent to an added weight of 2.4%. It is shown that approach 1 is the best approach since the optimal layouts obtained consistently give the largest reduction in SVD. With approach 3, although it gives an optimal layout with largest reduction in SVD using the least number of patches, regularity in the reduction achieved cannot be easily achieved. As for approach 2, the reduction achieved is consistently lower than approach 3 and approach 1. This result thus indicates that a well-behaved problem must be suitably constrained to reduce the possible design space. But the constraints imposed must not preclude the actual optimal solution. The layouts obtained using approach 1 are shown in Fig. 4 for the cases of 3, 6, 9 and 12 patches for PCLD treatment.

Nevertheless, three approaches show similar trends in the results in which the structural volume displacement decreases with diminishing reduction when the number of patches increases. Beyond a certain number of patches, further reduction of SVD becomes negligible. The fact that the trend is not dependent on the approach used implies that this is an intrinsic property of the system.

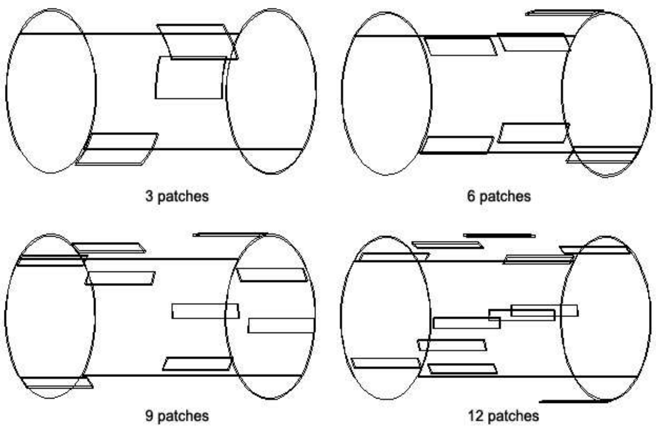

Fig. 4. PCLD patches layout obtained using 3, 6, 9 and 12 patches using approach 1

B. *Attributes of Optimal Layout*

Optimal layouts with 12 patches obtained using three approaches are examined to determine the attributes of an optimal layout. As shown in Fig. 5, in the layout obtained using approach 1 which gives the largest reduction, the patches tend to increase coverage in the axial direction rather than the angular direction. On the other hand, in approach 2 in which the patches cannot maximize its coverage in axial direction due to the constraints imposed, lower reduction in SVD is observed. In addition, approach 3, in which the patches' positions are not constrained, also arrived at similar layout obtained using approach 1. This shows that maximization of coverage in the axial direction is an attribute of an optimal layout since it is independent of the approach used.

Furthermore, in all three layouts shown in Fig. 5, the patches tend to distribute over the whole surface of the cylinder. This is thus another attribute of an optimal layout.

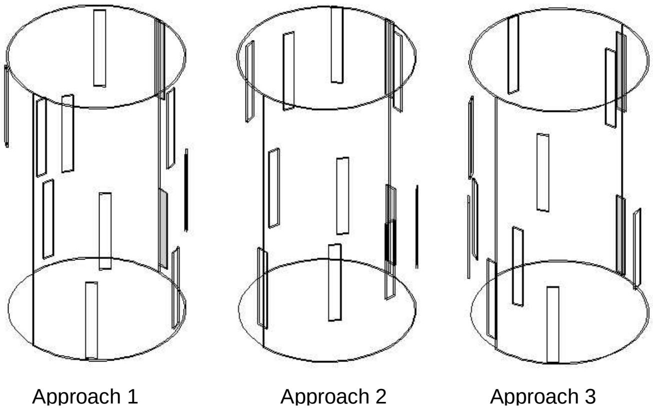

Fig. 5. Comparison of optimal layout for configuration using 12 patches

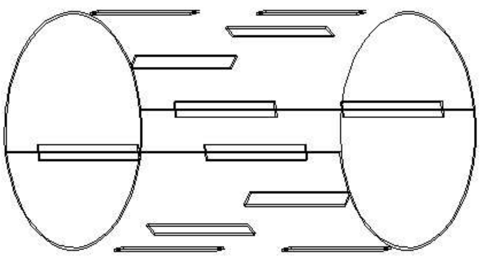

Fig. 6. PCLD patches layout approximated by the spatially distributed cosine shaped layout

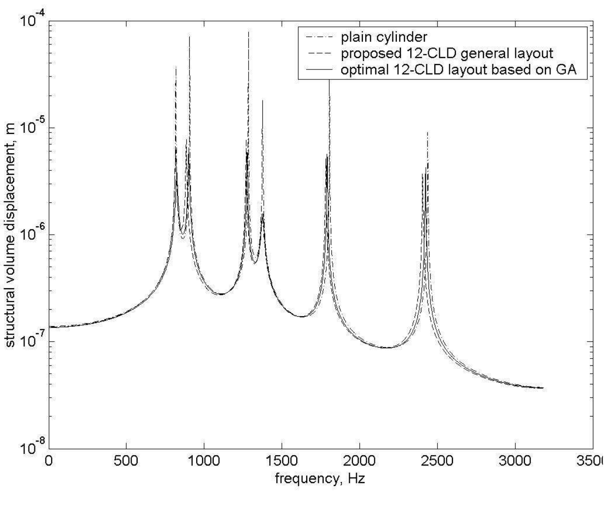

Fig. 7. Frequency response of the layouts obtained using (i) approximation of the spatially distributed cosine shaped layout and (ii) approach 1

Based on the above observations, a general layout shown in Fig. 6 is proposed for a configuration with 12 patches. It resembles the spatially distributed cosine shaped sensor layout developed in [32]. The reduction of SVD obtained based on this general layout is 20.0 dB. This reduction is 2.4 dB more than reduction resulting from the best layout obtained based on approach 1. Frequency responses of the two layouts (Fig. 7) demonstrated similar response.

C. Effects of the Patches' Shape

To determine the influence of the patches' shape on the SVD reduction achieved and since only rectangular patches are used, a dimensionless parameter, ratio of axial length to angular length, is defined to characterize the shape. The results, as shown in Fig. 8, indicate that the reduction of the SVD decreases when the ratio is decreased. This is consistent with the previous observation where we found that in an optimal layout, the patches tend to increase coverage in the axial direction rather than the angular direction. Thus, patches with large ratio of axial length to the angular length should be used in the layout design.

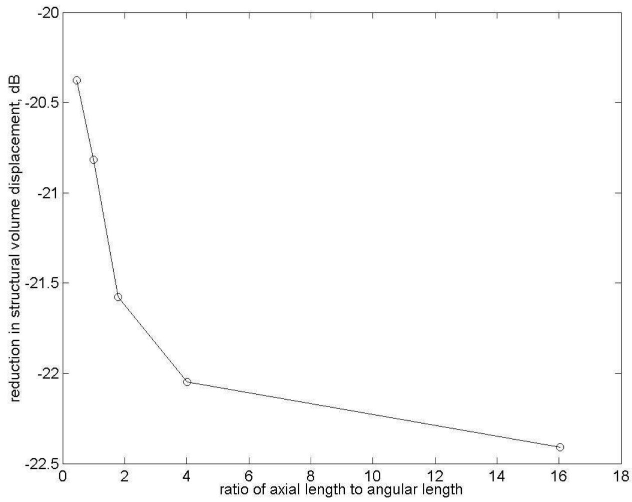

Fig. 8. Variation of SVD with the ratio of axial length to angular length of the patches

D. Effects of Total Amount of PCLD Used

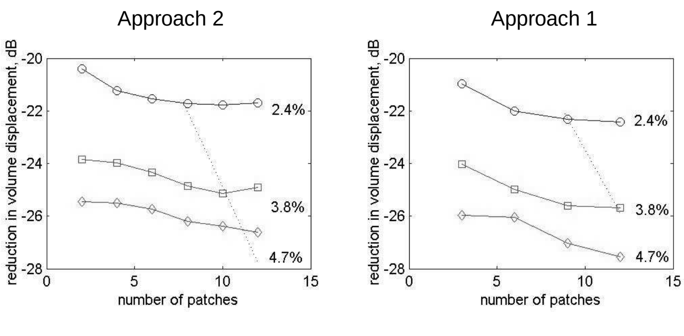

Fig. 9. Effects of increasing the total amount of PCLD patches used

The result is shown in Fig. 9 where the percentage shown is the percentage of added weight to the base cylinder. It can be seen that the degree of vibration attenuation achieved increases as the total amount of PCLD materials used. The optimal numbers of patches are indicated by the dashed line in the figure. The results imply that greater structural volume displacement reduction can be achieved when the PCLD patches are spread out over the whole surface of the cylinder. It also indicates that for all the cases, there is an optimal patch size of which any further reduction will lead to greater difficulty in obtaining a better optimal solution.

V. CONCLUSION

The optimization of the layout of PCLD patches for structural volume displacement (SVD) reduction of a simply-supported cylinder excited by a broadband transverse fore is presented in this paper. An analytical model is developed using the energy method to relate the SVD of cylinder and geometric and physical parameters of both the base cylindrical shell and all PCLD patches for the treatment. GA-based penalty function method is employed to optimize PCLD patches' locations and shapes with the aim of minimizing the structural displacement response under given constraint of total amount of PCLD used. The optimal analyses indicate that for a fixed number of patches and amount of PCLD material, there are two attributes of an optimal layout. First, the patches tend to increase their coverage in the axial direction; and second, the patches tend to distribute over the whole surface of the cylinder. Other optimal analysis findings include that rectangular patches with large ratio of axial length to angular length produce better damping effects, and the degree of vibration attenuation of the cylinder with PCLD treatment significantly increases with the increase of the total amount of PCLD materials.

REFERENCES

[1] R. A. DiTaranto, 1965, Theory of vibratory bending for elastic and viscoelastic layered finite-length beams. *ASME Journal of Applied Mechanics* 881-6

[2] D. J. Mead & S. Markus, 1969, The forced vibration of a three-layer, damped sandwich beam with arbitrary boundary conditions. *AIAA Journal* 10(2), 163-175

[3] A. Bhimaradi, 1994, Sandwich beam theory and the analysis of constrained layer damping. *Journal of Sound and Vib*ration. 179(4), 591-602

[4] D. K. Rao, 1978, Frequency and loss factors of sandwich beams under various boundary conditions. *International Journal of Mechanical Engineering Science* 20, 271-278

[5] M.J. Yan & E.H. Dowell, 1972, Governing equations of vibrating constrained-layer damping sandwich plates and beams. *ASME Journal of Applied Mechanics*, 1041-1046

[6] M.D. Rao & S. He, 1993, Dynamic analysis and design of laminated composite beams with multiple damping layers. *AIAA Journal* 31(4), 736-745

[7] K. S. Rao & B.C. Nakra, 1973, Theory of vibratory bending of unsymmetrical sandwich plates. *Archives of Mechanics* 25, 213-225

[8] T. Sakiyama, H. Matsuda & C. Morita 1996 Free vibration analysis of sandwich beam with elastic or viscoelastic core by applying the discrete Green function. *Journal of Sound and Vibration* 191, 189-206

[9] K. M. Ahmed, 1972, Dynamic analysis of sandwich beams. *Journal of Sound and Vibration* 21, 263-276

[10] M.L. Soni & F.K. Bogner, 1982, Finite element vibration analysis of damped structures. *AIAA Journal*, 20, 700-707

[11] C. D. Johnson and D. A. Kienholz 1982, Finite element prediction of damping in structures with constrained viscoelastic layers. *AIAA Journal* 20, 1284-1290

[12] X.Q. Peng, K.Y. Lam and G.R. Liu, 1998, Active vibration control of composite beams with piezoelectrices: A finite element model with third-order theory. *Journal of Sound and Vibration* 209, 635-650

[13] D.S. Nokes & F.C. Nelson 1968, Constrained layer damping with partial coverage. *Shock and Vibration Bulletin* 38, 5-10

[14] A.K. Lall, N.T. Asnani & B.C. Nakra 1988, Damping analysis of partially covered sandwich beams. *Journal of Sound and Vibration* 123(2), 247-259

[15] A.K. Lall, N.T. Asnani & B.C. Nakra 1988, Vibration and damping analysis of rectangular plate with partially covered constrained viscoelastic layer. *ASME Journal of Vibration, Acoustics, Stress, and Reliability in Design* 109, 241-247

[16] S.W. Kung & R. Singh 1998, Vibration analysis of beams with multiple constrained layer damping pactches. *Journal of Sound and Vibration* 212(5), 781-805

[17] H.H. Pan 1969, Axisymmetrical vibrations of a circular sandwich shell with a viscoelastic core layer. *Journal of Sound and Vibration* 9(2), 338-348

[18] I.W. Jones & V.L. Salerno 1990, Analysis of free damped vibration of laminated composite plates and shells. *International Journal of Solids and Structures* 25, 129-149

[19] N. Alam & N.T. Asnani 1984, Vibration damping analysis of a multilayered cylindrical shell, Part I: Theoretical Analysis. *AIAA Journal* 22(6), 803-810

[20] T.C. Ramesh & N. Ganesan 1995, Vibration and damping analysis of cylindrical shells with constrained damping treatment – a comparison of threetheories. *Journal of Vibration and Acoustics* 117, 213-219

[21] G.R. Liu, X.Q. Peng, K.Y. Lam and J. Tani, 1999, Vibration control simulation of laminated composite plates with piezoelectrices. *Journal of Sound and Vibration* 220, 827-846

[22] Y.C. Hu & S.C. Huang 2000, The frequency response and damping effect of three-layer thin shell with viscoelastic core. *Computers and Structures* 76, 577-591

[23] L.H. Chen & S.C. Huang 1999, Vibrations of a cylindrical shell with partially constrained layer damping (CLD) treatment. *International Journal of Mechanical Science* 41, 1485-1498

[24] L.H. Chen & S.C. Huang 2001, Vibration attenuation of a cylindrical shell with partially constrained layer damping strips treatment. *Computers and Structures* 79, 1355-1362

[25] J. L. Marcelin, P. Trompette & A. Smati 1992, Optimal constrained layer damping with partial coverage. *Finite elements in Analysis and Design.* 12, 273-280

[26] J. L. Marcelin, S. Shakhesi & F. Pourroy 1995, Optimal constrained layer damping of beams: experimental numerical studies. *Shock and Vibration* 2(6), 445-450

[27] Y.C. Chen & S.C. Huang 2002, An optimal placement of CLD treatment dor vibration suppression of plates. *International Journal of Mechanical Science* 44, 1801-1821

[28] G.R. Liu, X. Han, and K.Y. Lam, 2002 A combined genetic algorithm and nonlinear least squares method for material characterization using elastic waves. *Computer methods in Appl. Mech. and Eng*., 191, 1909-1921

[29] Z.L. Yang, G.R. Liu, and K.Y. Lam, 2002, An inverse procedure for crack detection using integral strain measured by optical fibers, *Smart Materials and Structures*. 11, 72-78

[30] W. Soedel 1993, *Vibrations of Plates and Shells*, 2nd ed. New York: Marcel Dekker, 1993

[31] S.S Rao, 1996, *Engineering Optimization – Theory and Practice.* 3rd ed. New York: Wiley

[32] H.S. Zhou, J.P. Zhong and M. Natori, 1993, Sensor mechanics of distributed shell convolving sensors applied to flexible rings. *ASME Journal of Vibration and Acoustics*, 115, 40-46.