Quasi-2D Resistivity Model from Inversion of Vertical Electrical Sounding (VES) Data using Guided Random Search Algorithm

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Abstract. Vertical electrical sounding (VES) data are usually interpreted in terms of a 1D resistivity model using linearized inversion. The local approach of a non-linear inverse problem has fundamental limitations, i.e. the necessity of a starting model close to the solution and possible convergence to a local rather than a global minimum solution. We studied the application of a global search approach for non-linear inversion using the guided random search method to model VES data. A quasi-2D resistivity model can be created by stitching 1D models obtained from VES data along a profile. Both vertical and lateral resistivity variations are minimized to incorporate a 2D smoothness constraint. The proposed method was applied to invert synthetic VES data as well as field data from a sedimentary environment. Both synthetic and field data inversions resulted in models that correlated well with the known synthetic model and with the geology of the study area, respectively.

Keywords: *2D model; DC resistivity; global search; Monte Carlo method; non-linear inversion.*

1 Introduction

In geophysics, the so-called direct current (DC) geo-electrical method can be used to infer the resistivity structure of the subsurface. A pair of electrodes is employed to inject an electrical current into the ground after which the potential difference at another pair of electrodes is measured. The electrode array is generally set up along a certain linear configuration for meaningful results and simplicity in interpretation of the data. In the resistivity sounding technique, measured parameters are apparent resistivities as a function of electrode spacings that represent resistivity variations with depth. Resistivity is an important physical property of the subsurface that can be related to the existence of ground-water bearing formations, variations in agricultural soil moisture, underground contamination plumes having conductive or resistive anomalies, or in general to the lithology of rock formations [1].

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Improvements in the geo-electrical method include both instrumentation for data acquisition and software development for modeling the data. The use of a multi-electrode data acquisition system with digital multi-channel recording has become the standard for efficient and cost-effective 2D resistivity imaging [2,3]. However, 2D resistivity imaging is mostly applied in investigations with relatively shallow depth targets, such as in geotechnical and environmental studies [4-6], archaeology [7] and agriculture [8]. Although it is also possible to image deeper sections of the subsurface with a multi-electrode system, this is less efficient due to the long multi-core wires and the large amount of electrodes required. Logistics are also more demanding and in most cases more powerful and specially designed instruments are necessary [9,10].

To probe deeper parts of the subsurface, the vertical electrical sounding (VES) technique is considered more practical, especially in areas with difficult access (e.g. rough topography, dense vegetation, etc.) [11,12]. With the Schlumberger electrode configuration, only the distance of the outer (i.e. current) electrodes has to be increased to achieve a greater investigation depth. VES data are commonly interpreted using a 1D model in which resistivity varies only with depth [13,14]. A quasi-2D resistivity model can be constructed by correlating the layered earth model at a VES site along a profile [15,16]. The full 2D resistivity modeling may also be applied to VES data [17,18]. Riss, *et al.* [19] have proposed to interpolate VES data and used 2D modeling software to interpret the data. In all cases, the non-linear inverse problems are solved by using linearization or a local search approach.

The inverse problem of VES data is highly non-linear, even for a simple 1D model. The common practice is to linearize the misfit function at the vicinity of a starting model and then update the model iteratively until a convergence to an optimum solution is reached [13,14]. In this study, we developed our algorithm [20] to simultaneously invert the VES data sets distributed along a profile. To obtain a quasi-2D resistivity model, the algorithm minimizes the misfit as well as vertical and lateral resistivity variations of the model. The inclusion of lateral smoothness constraint modifies the original Markov Chain Monte Carlo (MCMC) algorithm. Therefore, we call our method a guided random search algorithm.

2 Inversion Algorithm

A geo-electrical 1D model is defined by a number of layers with different thicknesses and resistivities. We use a large number of "thin" layers with thickness increasing logarithmically with depth to represent the decreasing resolution with depth. The thicknesses are fixed during inversion, such that the model parameters to be estimated are the layers' resistivities $\mathbf{m} = [m_j]$; $j = 1, 2,$

..., *NL*, where *NL* is the number of layers. Resistivity variation with depth is represented by changes of resistivity from layer to layer. The observed VES data are apparent resistivities **d** = [d_i]; i = 1, 2, ..., *ND*, where *ND* is the number of data associated with the number of electrode spacings (AB/2). The misfit $E(\mathbf{m})$ is expressed by

$$E(\mathbf{m}) = \sum_{i=1}^{ND} (d_i - g_i(\mathbf{m}))^2 \tag{1}$$

where $\mathbf{g} = [g_i(\mathbf{m})]$ is the 1D model response calculated by using the well-known Gosh-Koefoed filters for the Schlumberger array [14,21]. Basically, resolving an inverse problem means finding models associated with the minimum of the misfit or the objective function.

A set of possible values [ρ_k]; k = 1, 2, ..., *NP* is proposed for the layers' resistivities, where *NP* is the number of *a priori* resistivity values. The relative probability of resistivity values for a layer m_j can be written as

$$P(\rho_k) = \exp(-E(\mathbf{m} \,|\, m_j = \rho_k))\,; \;\; k = 1, 2, ..., NP \tag{2}$$

where $E(\mathbf{m} \,|\, m_j = \rho_k)$ is the misfit related to a model **m** in which $m_j = \rho_k$; k = 1, 2, ..., *NP*, while other layers (m_l; $l \neq j$) are fixed at their current resistivity values. Starting with an arbitrary resistivity value for all layers, iterative refinement of the model proceeds by choosing the resistivity for each layer from the *a priori* resistivity values with their relative probabilities in Eq. (2) used as weight. A resistivity value for a particular layer m_j has a higher probability to be chosen if it is associated with a lower misfit, and vice versa. With a large number of iterations, the resulting models will converge and vary slightly around an optimum model. The solution of the inverse problem is obtained by averaging the models after convergence.

Due to equivalence problems where there are different models with almost identical misfits, the data cannot adequately constrain the model in the inversion process [20]. Therefore, we include smoothness constraints, such that the spatial resistivity variations are minimized both vertically and laterally. The vertical smoothness constraint is justified to obtain a meaningful model from the VES data at an individual station, while the lateral smoothness constraint is used to facilitate the correlation of the resistivity models from station to station along a traverse line. The result is a quasi-2D resistivity model where the resistivity variations are smooth [22], similar to laterally constrained inversion (LCI) as proposed by Auken, *et al*. [15,23] using a linearized inversion approach.

The modified relative probability for the *a priori* resistivities can be expressed by

$$P(\rho_k) = \exp(-E(\mathbf{m} \mid m_j = \rho_k) - \alpha V - \beta L); \; k = 1, 2, ..., NP \tag{3}$$

where αV and βL are terms associated with the vertical and lateral smoothness constraints respectively. In order to have a higher relative probability, the resistivity values for the j-th layer that minimizes the misfit $E(\mathbf{m} \mid m_j = \rho_k)$ should also minimize the resistivity differences between the j-th layer and the layers above and below it, i.e.

$$V = (\log m_{j-1}^i - \log m_j^i)^2 + (\log m_{j+1}^i - \log m_j^i)^2 \tag{4}$$

where m_j^i is the resistivity of the j-th layer at the i-th sounding point. In addition, the resistivity values for the j-th layer should also minimize the resistivity differences between the same j-th layers at three adjacent sounding sites, i.e. the i-th site and its surrounding area such that,

$$L = (\log m_j^{i-1} - \log m_j^i)^2 + (\log m_j^{i+1} - \log m_j^i)^2 \tag{5}$$

The relative weights of the smoothness constraint (α and β) are determined by trial and error. Terms in Eq. (3) can be normalized to become more or less equivalent in magnitude such that α and β can be chosen in the interval between 1 to 20. In a quasi-2D model, the choice of the vertical smoothness factor α is less critical than the lateral smoothness factor β.

3 Synthetic Data

A relatively simple synthetic 2D model representing a sedimentary environment without topographic variation is presented in Figure 1(a). The model response was calculated using the 2D forward modeling scheme [18] and 5% noise with Gaussian distribution was added. The pseudo-section in Figure 1(b) presents the apparent resistivity data at 12 VES sites with 200 m spacing for AB/2, which varies from 50 to 2500 m. Qualitatively, the pseudo-section shows only a gradual variation of resistivities from shallow (more than 100 Ohm.m) to deeper parts of the section (less than 25 Ohm.m). The faulted layers appear only as a lateral thickness variation of the superficial layers.

We divided *a priori* resistivity values between 1 and 1000 Ohm.m into 20 discrete values homogeneously separated in the logarithmic scale. The 1D model under each VES station was limited in the interval between 0 to 1000 m depth and was divided into 20 layers with homogeneous intervals in the logarithmic scale. Inversion of the synthetic data from each VES site was performed with up to 50 iterations, where the models from the 10 last iterations were averaged. All VES data were inverted sequentially with up to 100 iterations from which the final model was obtained by averaging only the 25 last iterations. A pseudo-section of the inverse model response with an overall misfit of 11.8% is shown in Figure 2(a). The quasi-2D resistivity model (Figure 2(b)) was produced by contouring the resistivity values of the 1D models obtained from inversion of each VES synthetic data set.

(a)

(b)

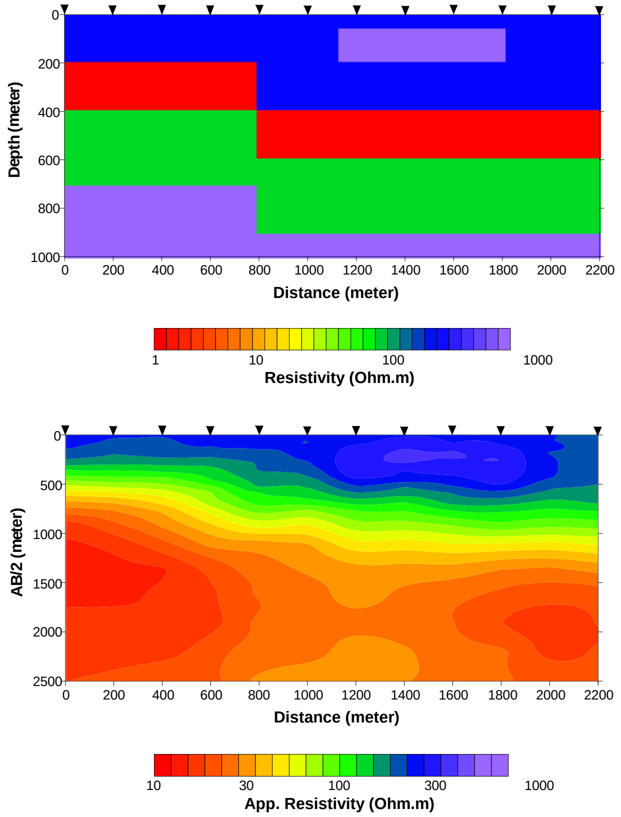


Figure 1 (a) 2D synthetic model used to generate the synthetic VES data at 12 sounding sites with 200 m spacing (shown as inverted triangles). (b) Apparent resistivity pseudo-section associated with the synthetic model, 5% Gaussian noise added to simulate real data.

In the inverse model (Figure 2(b)), the resistive (300 Ohm.m or more) superficial layer and the shallower part of the conductive layer (5 to 30 Ohm.m) are relatively well resolved with their actual thicknesses and depths. The resistive near-surface inclusion embedded in the first layer is fairly well resolved. The lateral discontinuity of the conductive layer confirms the normal fault. The deeper part of the conductive layer is less well resolved due to a combination of (i) the screening effect of the conductive layer and (ii) the limit of data coverage in terms of maximum electrode spacing (AB/2). However, larger electrode spacings would not ensure the resolution of the deeper layers, since there are also averaging effects (both vertically and laterally) of 2D resistivity variations contained in data with a large AB/2.

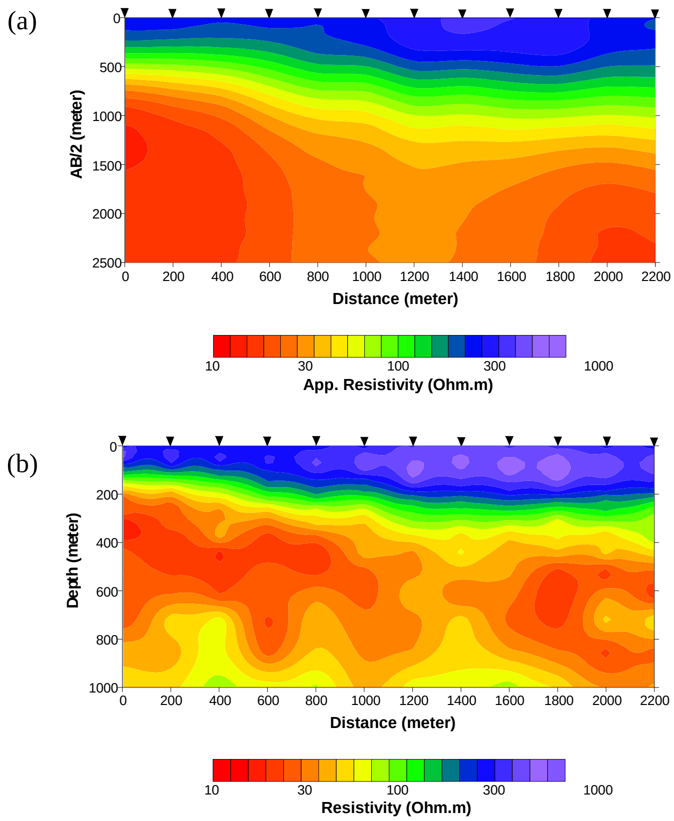


Figure 2 (a) Calculated theoretical response of the inverse model with an overall misfit of 11.8% relative to the synthetic data. (b) Quasi-2D resistivity model obtained from inversion of the synthetic data.

4 Field Data

The field data consisted of 16 VES sites along a 6.8 km profile with station spacing between 400 to 600 m in a sedimentary environment dominated by limestone (reef) formations in East Java, Indonesia (Figure 3). The measured data up to AB/2 = 1500 m were laterally interpolated to obtain more regular VES data at every 200 m along the profile. The field data show a relatively smooth resistivity variation from resistive at the surface to conductive at depth. The superficial resistive layers become thinner toward the South-Eastern part of the profile (Figure 4(a)). Similar inversion parameters as for the inversion of the synthetic data were used for the field data, i.e. maximum depth and thickness of layers, *a priori* resistivity interval and its discretization, number of iterations and models for averaging. A calculated pseudo-section of the inverse model is presented in Figure 4(b) with 13.3% RMS error, while the averaged inverse model is shown in Figure 4(c).

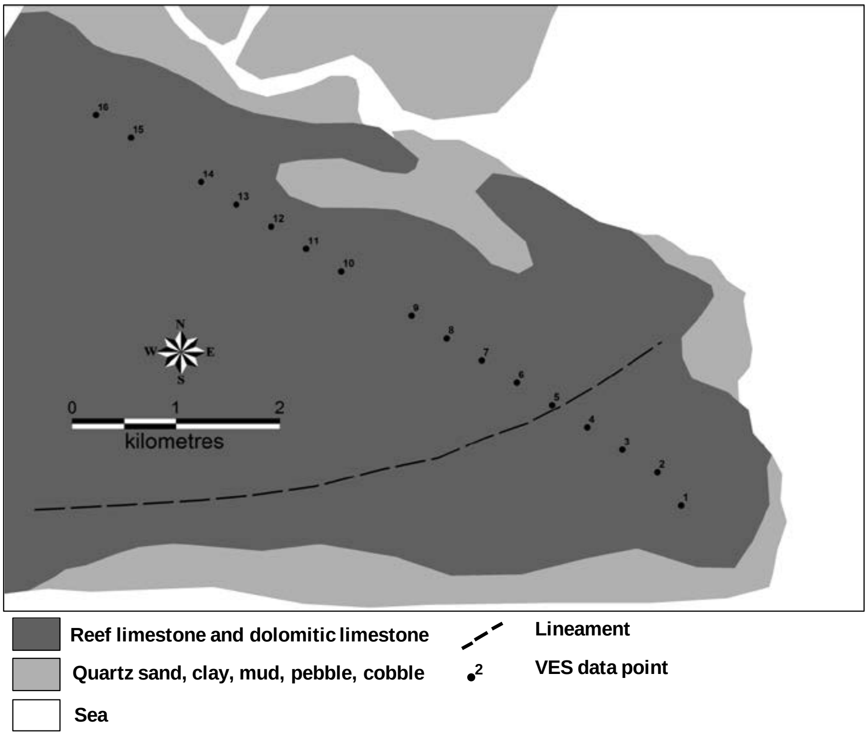
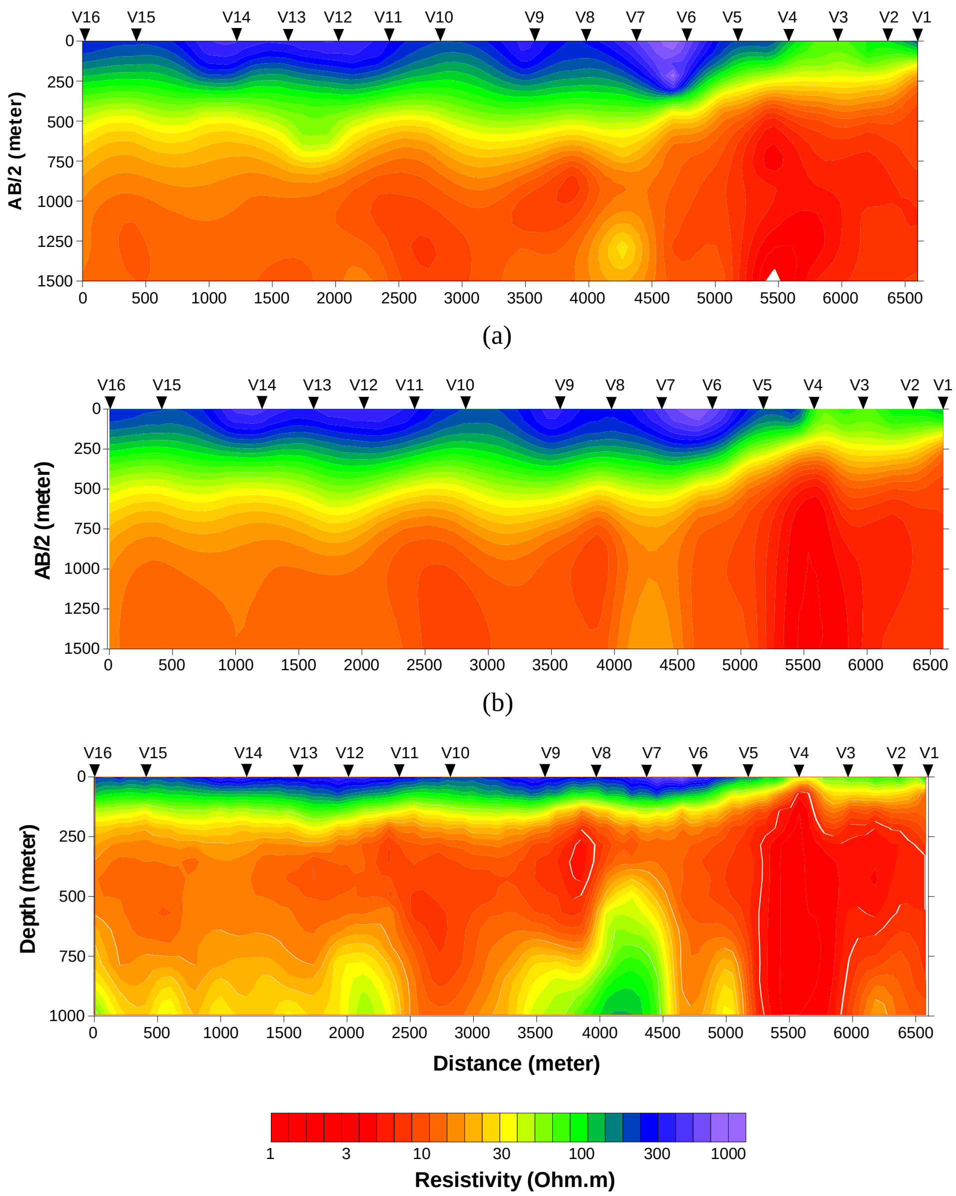


Figure 3 Simplified geological map of the survey area with distribution of VES stations along a traverse line.

The interpreted section of the resistivity model is presented in Figure 5. The superficial resistive layers (more than 100 Ohm.m) corresponding to the limestone formation become thinner toward the right of the profile (South-East). The shallow resistivity inhomogeneities represent fractures mainly due to water intrusion observed in the field. The reef formation is underlain by conductive layers (less than 10 Ohm.m) that can be associated with clayey formations with significant lateral heterogeneities. The lateral discontinuity of the resistivity distribution can be interpreted as geological structures (faults). The fault reaching the surface between VES stations 4 and 5 is well correlated with the surface geology (see Figure 3). The interpreted subsurface faults mostly affect the moderate resistivity layer underneath (30-50 Ohm.m). The East-West tensional regime responsible for the fault system is likely related to a subduction zone South of the area, with a North-South main tectonic stress.



(c)

Figure 4 (a) Pseudo-section of the field data. (b) Calculated theoretical response of the inverse model with 13.3% misfit relative to the observed data. (c) Quasi-2D model from inversion representing the resistivity section. Note that all contours use the color scale shown at the bottom of the figure.

Based on the local geology and the marine seismic data towards the South-Eastern part of the study area (not shown), the low resistivity clayey layers are part (north-western flank) of anticline structures. These conductive layers may serve as a "cap" for a relatively shallow hydrocarbon bearing (i.e. carbonate) formation as indicated by similar formation in the surrounding area [24].

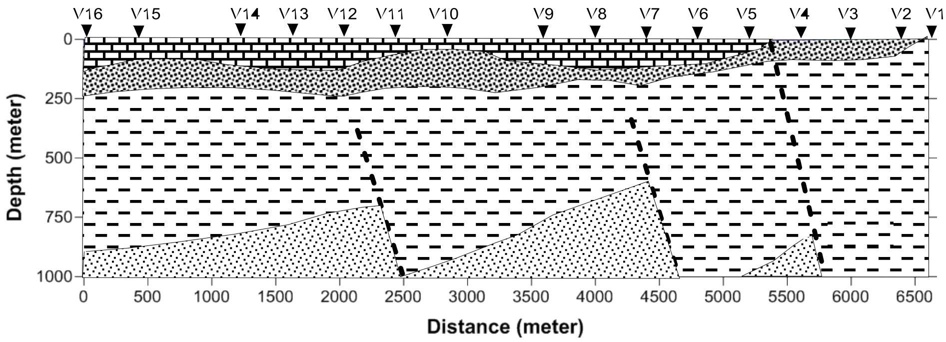


Figure 5 Interpreted section of the resistivity model in Figure 4©, with reef formations (first and second layers), clayey formation (dashed), carbonate formation (small dotted).

5 Conclusion

The guided random search algorithm is a computer intensive technique that necessitates many forward modeling calculation to estimate misfit and search for optimum models [25]. The use of 1D forward modeling in our approach is intended to approximate the relative probability of *a priori* resistivity values for a model parameter. The relative probability serves as a sampling probability for the model space exploration. The simultaneous inversion of VES data sets along a profile with vertical and lateral smoothing constraints does not allow to recast our algorithm in the "strict" MCMC context [26]. We focused more on applying the method to obtain a quasi-2D resistivity section from 1D models along a profile. However, the inversion of synthetic data resulted in optimum models that recovered the synthetic model relatively well with a good misfit value. The results may be considered an empirical validation of our approach.

The application of a geo-electrical method was intended to overcome the lack of subsurface information from other geological and geophyscal data in this area. The limestone cover with high seismic velocity at the surface leads to difficulties in applying a seismic method. Although further implications of our results for exploration purposes are beyond the scope of this paper, the quasi-2D resistivity model from inversion of field data is in good agreement with the known local geology of the survey area.

The use of full 2D geo-electrical resistivity forward modeling in the inversion algorithm was also investigated to obtain a more reliable resistivity image of the subsurface. Such an approach allows to incorporate the variation of the topography prevailing in geothermal and volcanic areas. Preliminary results were encouraging, although additional (geometry) constraints were necessary to further constrain the inversion. The algorithm is able to invert VES data from non-sedimentary environments such as geothermal prospects with a rather non-layered character of the subsurface [22,27].

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