# Application of the EM Algorithm for the Multitarget/Multisensor Tracking Problem

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**Abstract**

An important problem in surveillance and reconnaissance systems is the tracking of multiple moving targets in cluttered noise environments using outputs from a number of sensors possessing wide variations in individual characteristics and accuracies. A number of approaches have been proposed for this multitarget/multisensor tracking problem ranging from reasonably simple, though ad-hoc, schemes to fairly complex, but theoretically optimum, approaches. In this paper we describe a new iterative procedure for multitarget/multisensor tracking based upon use of the expectation-maximization (EM) algorithm. More specifically, we pose the multitarget/multisensor tracking problem as an incomplete data problem with the observable sensor outputs representing the incomplete data while the target-associated sensor outputs constitute the complete data. This formulation then allows a straightforward application of the EM algorithm which provides an iterative solution to the simultaneous maximum-likelihood (ML) and/or maximum *a posteriori* (MAP) estimate of the target states, under the assumption of appropriate motion models, based upon the outputs of disparate sensors. The advantage of this EM-based approach is that it provides a computationally efficient means for approaching the performance offered by theoretically optimum, but computationally infeasible, simultaneous ML estimator. We provide selected results illustrating the performance/complexity characteristics of this EM-based approach compared to competing schemes.

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# I Introduction

An important problem in surveillance and reconnaissance systems is the simultaneous state estimation of a multiple number of moving targets under motion in cluttered noise environments. The data used to perform the target state estimation originates from a multiple number of sensors, where the sensors may have widely different characteristics and/or accuracies. Based upon the output of these multiple sensors it's desired to estimate and track the states of a possibly unknown number of targets. This is the multitarget/multisensor tracking problem, and arises in application areas such as air defense, battlefield surveillance, air-traffic control, etc.

A number of approaches have been proposed for the multitarget/multisensor tracking problem, ranging from reasonable ad-hoc schemes to fairly sophisticated theoretically optimum schemes. The former are easy to implement and reasonably robust but provide relatively poor overall performance, while the latter require highly complex implementations but are capable of good performance under *idealized* modeling assumptions although generally sensitive to modeling mismatch problems.

Approaches that are *optimal* in some sense perform state estimation given data from a set of observed measurements indexed through time and compute an estimate, or a set of estimates, using the entire set of data. These batch processing approaches are often difficult to implement because the memory and/or time required to process the large amount of data makes them infeasible. Typically, the problem can be reformulated into a Baysian recursive approach which allows data to be processed in an iterative manner that yields the optimal result. Since processing begins before all of the data has been collected, processing time may be saved by using the recursive approach, though memory requirements are not reduced by the recursion. Alternatively, sub-optimal schemes can be derived from the optimal approaches that are more easily implementable but do not perform as well as their optimal counterparts. The resulting state estimates reported by the different approaches generally take one of two forms. The first approach is to perform the state estimation by evaluating the expected value of a targets state given the measurement data (minimum variance estimate), while the second is to report the state that is the maximum-likelihood (ML) estimate of the state given the measurement data. If prior state probabilities are known then the maximum *a posteriori* (MAP) estimate can alternatively be reported.

Examples of optimal schemes are described in [1, 2, 3]. These schemes are Baysian recursive

approaches which keep a history of measurement-to-track hypotheses formed from previous estimates. This history is used to compute state updates given new measurements. These optimal schemes are generally not implementable in a high clutter/dense target environment since an exponentially growing memory is required to keep all of the hypotheses created since initialization. Though the number of hypotheses must be carefully managed, these schemes provide a coherent framework for initializing new tracks by simply creating a new track hypothesis from a new measurement. The reported state output is a list of hypotheses which can be ranked by probability estimates. Sub-optimal versions that are implementable are generally described with particular attention being paid to hypotheses management schemes that restrict the number of possible hypotheses from becoming too large.

Another class of state estimation approaches are those that compute the ML (or MAP) estimate for a number of targets given measurements indexed through time [4, 5, 6]. These approaches are optimal in the sense that they maximize the posterior state distribution conditioned upon the available measurements. Dynamic programming is often used in [4] to compute the best paths through consecutive frames of measurement data. In this approach the state-space is quantized into a discrete space and dynamic programming is used to find the best path that leads to the largest *score* for each element in the state-space. The *score* for a target at a particular location in the state-space is derived from the probability distribution of the targets motion from a previous position in state-space and the measurement distribution for a target at this location. At completion of the algorithm, the *scores* for each element in the state-space of the last frame can be ranked to determine the number of targets present and the path through the frames that led to these scores is reported. The disadvantage of this approach is that false paths that are in close proximity to the path of maximum *score* are also given large *scores* making it difficult to accurately identify the true target path.

A similar application of dynamic programming is given in [5], where measurements in the form of state-space data are arranged in a trellis. A *score* for a directed path between two nodes in the trellis is the path metric and is based upon the probability distribution of the predicted target motion between the two nodes. Each path is considered a directed path where the direction is determined by the temporal relationship between the two nodes. This problem is similar in nature that treated in [4]. Instead of the entire state-space being quantized into many discrete parts, the possible state-space

locations are described as a trellis. The difference lies in the application of the dynamic programming, since here it is applied to find the best $K$ non-intersecting paths through the trellis of state-space measurements. This will eliminate the effect of propagating false paths that have nodes in common to the target paths which are contained in the set of paths that have the largest *score*, but is somewhat restrictive in that a 1-1 mapping from measurement space to target state-space is assumed.

Another approach that maximizes the posterior state distribution conditioned upon the available measurements is described in [6]. In this approach the problem is formulated using the following steps: First, a set of feasible tracks is constructed based upon a hypothesis test of the log-likelihood function from the innovations sequence of measurements. Each feasible track is identified by the set of measurements that belong to it and has a corresponding cost assigned. The problem is then reformulated as the set-partitioning problem of finding the subset of feasible tracks (hypothesis) that partitions the measurements into the set that has the smallest cost. This is solved as a $0-1$ integer programming problem, yielding the hypothesis with the smallest cost. One difficulty in this approach is that the set of feasible tracks has to be generated before the $0-1$ integer programming can be used. In highly non-linear situations this will prove to be the most difficult part of the problem. Similarly, it is not obvious how to determine a threshold for the hypothesis test, though this can be chosen so a fixed number of feasible tracks are generated.

The above approaches share one common aspect; they all compute the state estimate update for a given track by assigning each measurement to a specific target or to a separate class representing false alarms. This type of measurement-to-track association is considered a *hard* decision approach. A number of suboptimal approaches that do not make a hard decision in the measurement-to-track assignment problem are described in [7, 8, 9]. Given a specified number of existing targets, these recursive approaches keep a single state estimate for each track at any time. As new measurements are received the state estimate is formed by computing the probabilities that the measurements originated from existing targets and then approximating the minimum variance estimate using these association probabilities. These approaches can be computationally efficient; however, in the situation that joint measurement probabilities are computed, the processing time increases exponentially with the number of measurements to be processed. Also note that these are "$N$-scan" algorithms. Measurements are processed only from the past $N$ measurement sets to determine the current state estimate and the

state estimate from the previous $(N+1)$'st measurement set is assumed to be the correct prior initial state.

Estimating the state of a number of unknown targets under uncertain measurement origin is a *non-classical* filtering problem, the *classical* filtering problem arising when the measurements origins are known. The *non-classical* filtering problem can also be considered an *incomplete* data problem [10, 11]. To develop the idea of *complete* data let $\mathbf{Y}$ be the observed or *incomplete* data and $\mathbf{Z}$ represent some unobserved data which, if available, simplifies the estimation problem. Then the *complete* data can be represented by $\mathbf{X}$, where $\mathbf{X} = (\mathbf{Y}, \mathbf{Z})$. In the above state estimation problem, the observed data, $\mathbf{Y}$, are the measurement returns from sensors over the observation time while the unobserved data, $\mathbf{Z}$, are the associations between the measurements and the set of possible classes from which the measurements can originate. Looking at the *non-classical* filtering problem as an *incomplete* data problem, we can draw upon solution techniques for parameter estimation from this domain.

Recently, there has been much interest in the literature regarding the maximum-likelihood (ML) estimation of parameters from *incomplete* data by use of the Expectation-Maximization (EM) algorithm. Formalized by Dempster, Laird, and Rubin [12], the EM algorithm is an iterative procedure that estimates both the parameters and the missing or unobservable data during an iteration. The approach first computes an approximation to the expectation of the log-likelihood functional of the complete data conditioned on the current parameter estimate. This is called the expectation step (E-step) and here the current *incomplete* data estimate is calculated. Next, a new parameter estimate is computed by finding the value of the parameter that maximizes the functional found in the E-step. This is called the maximization step (M-step). The EM algorithm has been found to have the advantages of reliable global convergence properties in most instances, although it can exhibit seeming slow convergence in some applications [10]. Furthermore, it is shown in [13, 14] that the EM algorithm is a particular example of a more general approach that minimizes the Kullbach-Leibler informational divergence between two distributions [13, 14].

The EM approach can also be used to compute penalized likelihood estimates [15, 16, 17], leading to MAP estimates, by placing a prior distribution on the parameter to be estimated. Applications of the EM algorithm include its use in: [18] to compute maximum-likelihood (ML) estimates of unknown

means of Poisson distributions in emission tomography; in [10] and [19] to compute ML parameter estimates from finite mixture distributions; and in [11] to compute ML parameter estimates of image parameters in emission tomography and gamma-ray astronomy.

Another application in which the EM algorithm has been used is the area of image segmentation [20, 21, 15, 22]. In the *unsupervised* segmentation problem, training data is not available and feature values which characterize different classes of regions must be estimated from the data directly. This problem is cast as a missing data problem where the image intensities are observable, the state assignments are *missing*, and the model parameters need to be estimated. The EM algorithm performs the parameter estimation and segmentation for images that have a number of different intensity and state process stochastic models. The more complex stochastic models pose a problem in that the analytical expression for the conditional expectation required in the E-Step can only be found for rather simple models. However, approximate techniques are shown to be effective [22] in overcoming these difficulties. Recently, iteration of the expectation step is shown to result in a *self-consistent* mean-field approximation [23] of the state process [24, 25, 26, 27, 28] and offers significant performance advantages in complex stochastic models.

Formulation of the multitarget tracking problem as an incomplete data problem and use of the EM algorithm is not new. For example, Avitzour [29] has considered a block formulation where the complete data history is taken as the incomplete data and all possible measurement-to-target associations are taken as the missing unobservable data. This results in an extremely large underlying state-space and places an unreasonable computational burden on the E-step of the EM algorithm. By contrast, the incomplete data problem is formulated here in a recursive manner. More specifically, target state estimates are updated for each set of new measurements based upon estimates obtained from past measurements. As a result, the missing or unobservable data in our recursive formulation is the set of all measurement-to-target associations for a single scan. This results in a much more manageable computational burden in the E-step. Furthermore, this computational burden is further reduced through our use of a Markov random field (MRF) model for the underlying association process. This allows us to avoid explicit enumeration of all possible joint measurement-to-target associations as required in the approach described in [29]. In particular, in [29] the computational complexity of calculating the joint association process in the E-step is shown to grow exponentially

as the number of targets increases, and is computationally more complex than the M-step. For the recursive approach employing a MRF modeling assumption as described here, we show by contrast that the E-step is relatively simple and considerably less complex than the M-step.

The remainder of this paper is organized as follows. Section II formulates the problem and introduces the state and observation process models. Also in this section a recursive form of state estimation is discussed. Section III introduces the EM approach of recursive state estimation and discusses calculation of both steps. Section IV reports the results of the EM approach on two examples. The first example considers measurements which are linear with respect to the estimated state. The second example is a non-linear example taken from the literature and the EM approach is compared to other approaches reported in that example. Finally, we conclude in section V.

## II Problem Formulation

In this section a thorough theoretical formulation of the target state process and the observation process will be described. In the state process formulation we assume a maximum number of targets is present during the entire observation period and to handle a varying number of observable targets during this period the idea of *active* targets is presented. This is formalized in the next section. In the formulation of the observation process a multiple number of sensors is assumed to present measurement data at discrete instants in time, though the presented data may itself have been processed from raw measurements collected throughout a time interval. This is made more precise in the section on the observation process.

### State Process Formulation

We suppose that there are at most $N_{t,max}$ targets present at any time. These potential targets will be indexed appropriately, although the particular index assigned to any target is not important. The system state, $\mathbf{S}_k$, at time $k$ is then given by

$$\mathbf{S}_k = (\mathbf{\Phi}_k, \boldsymbol{\zeta}_{t,k}); \qquad k = 0, 1, 2, \ldots, \tag{1}$$

where $\boldsymbol{\zeta}_{t,k} = (\zeta_{t,k}^{(1)}, \zeta_{t,k}^{(2)}, \ldots, \zeta_{t,k}^{(N_{t,max})})$ is a *binary* vector of dimension $N_{t,max}$ representing the target activity status and $\mathbf{\Phi}_k^T = (\mathbf{\Phi}_k^{(1)}, \mathbf{\Phi}_k^{(2)}, \ldots, \mathbf{\Phi}_k^{(N_{t,max})})$ represents the quantitative target state information. More precisely, if $\zeta_{t,k}^{(i)} = 1$, then the $i$'th target is active at time $k$ while $\zeta_{t,k}^{(i)} = 0$ indicates that

the target is inactive. The total number of targets present at time $k$ is then

$$N_{t,k} = \sum_{i=1}^{N_{t,max}} \zeta_{t,k}^{(i)}; \qquad k = 0, 1, 2, \ldots \tag{2}$$

The corresponding element $\mathbf{\Phi}_k^{(i)}$ then carries *useful* quantitative state information concerning the $i$'th target if active ($\zeta_{t,k}^{(i)} = 1$) while this data is *irrelevant* if the corresponding target is inactive ($\zeta_{t,k}^{(i)} = 0$). Alternatively, if ($\zeta_{t,k}^{(i)} = 0$) the corresponding element $\mathbf{\Phi}_k^{(i)}$ represents a default entry.

We assume that the target state $\mathbf{\Phi}_k$ can be broken down into a *geometrical* state $\mathbf{G}_k$ and a *feature* or *attribute* state $\mathbf{F}_k$. We will write $\mathbf{\Phi}_k = (\mathbf{G}_k, \mathbf{F}_k)$. Each of these components is likewise of dimension $N_{t,max}$. Alternatively, the $i$'th component of $\mathbf{\Phi}_k^T$ is given by $\mathbf{\Phi}_k^{(i)} = (\mathbf{G}_k^{(i)}, \mathbf{F}_k^{(i)})$ having meaning only when target $i$ is active.

It will be useful to allow the target state activity process $\{\boldsymbol{\zeta}_{t,k}\}$ to possess a causal Markov structure. More specifically, the state space for the process $\{\boldsymbol{\zeta}_{t,k}\}$ consists of

$$M = \sum_{l=0}^{N_{t,max}} \binom{N_{t,max}}{l} = 2^{N_{t,max}} \tag{3}$$

values corresponding to the number of possible activity patterns of $N_{t,max}$ targets. Then, let $\mathbf{Q}_{t,k}^T = (Q_{t,k}(0), Q_{t,k}(1), \ldots, Q_{t,k}(M))$ represent the probability mass function (p.m.f.) of $\boldsymbol{\zeta}_{t,k}$. That is

$$Pr\{\boldsymbol{\zeta}_{t,k} = m\} = Q_{t,k}(m); \qquad m = 0, 1, \ldots, M-1; \quad k = 0, 1, \ldots \tag{4}$$

We assume that $\mathbf{Q}_{t,0}$ is given and that $\mathbf{Q}_{t,k}$ is generated recursively according to

$$\mathbf{Q}_{t,k+1} = \mathbf{A}\mathbf{Q}_{t,k}; \qquad k = 0, 1, \ldots, \tag{5}$$

where $\mathbf{A}$ is the $M \times M$ state transition matrix with $(m, n)$ element

$$a_{m,n} = Pr\{\boldsymbol{\zeta}_{t,k+1} = m | \boldsymbol{\zeta}_{t,k} = n\}; \qquad m, n = 0, 1, \ldots, M-1. \tag{6}$$

Clearly, the elements of $\mathbf{A}$ satisfy

$$\sum_{m=0}^{M-1} a_{m,n} = 1; \qquad n = 0, 1, \ldots, M-1. \tag{7}$$

This model includes the situation where a *fixed* subset of $N_t$ targets is always active, with $0 \leq N_t \leq N_{t,max}$, and yet is simple enough to include the situation where targets can be *born* or *die* over the course of time.

Let $\mathbf{S}^k = (\mathbf{S}_0, \mathbf{S}_1, \ldots, \mathbf{S}_k)$ represent the state history up to the $k$'th sequential observation time with similar notation for the components $\mathbf{\Phi}^k$ and $\boldsymbol{\zeta}_{t,k}$. We will assume a Markov structure for $\mathbf{\Phi}^k$ such that

$$p(\mathbf{\Phi}^k|\boldsymbol{\zeta}_t^k) = \prod_{j=0}^k p(\mathbf{\Phi}_j|\mathbf{\Phi}_{j-1}, \boldsymbol{\zeta}_{t,j-1}, \boldsymbol{\zeta}_{t,j}); \qquad k = 0, 1, \ldots \tag{8}$$

This requires knowledge of the probability density function (p.d.f.) $p(\mathbf{\Phi}_j|\mathbf{\Phi}_{j-1}, \boldsymbol{\zeta}_{t,j-1}, \boldsymbol{\zeta}_{t,j})$ of the state transition $\mathbf{\Phi}_{j-1} \rightarrow \mathbf{\Phi}_j$ given the target activity status $\boldsymbol{\zeta}_{t,j-1}$ and $\boldsymbol{\zeta}_{t,j}$ at times $j-1$ and $j$, respectively. Clearly, this also imposes a Markov structure on $\{\mathbf{S}_k\}$ in the sense that

$$p(\mathbf{S}^k) = \prod_{j=0}^k p(\mathbf{S}_j|\mathbf{S}_{j-1}); \qquad k = 0, 1, \ldots, \tag{9}$$

where

$$p(\mathbf{S}_j|\mathbf{S}_{j-1}) = p(\mathbf{\Phi}_j|\mathbf{\Phi}_{j-1}, \boldsymbol{\zeta}_{t,j-1}, \boldsymbol{\zeta}_{t,j}) Q(\boldsymbol{\zeta}_{t,j}|\boldsymbol{\zeta}_{t,j-1}), \tag{10}$$

with

$$Q(\boldsymbol{\zeta}_{t,j}|\boldsymbol{\zeta}_{t,j-1}) = a_{m,n}; \qquad m, n = 0, 1, \ldots, M-1, \tag{11}$$

the one-step transition probability of the activity process $\{\boldsymbol{\zeta}_{t,k}\}$ in going from state $n$ to state $m$.

**Observation Process Formulation**

Next, consider the observation process $\{\mathbf{Y}_k\}$ with

$$\mathbf{Y}_k = (\mathbf{y}_k, N_{m,k}, \mathbf{s}_k, t_k); \qquad k = 0, 1, \ldots \tag{12}$$

Here, $N_{m,k}$, represents the *number* of measurements available at the $k$'th time instant with $\mathbf{y}_k$ representing the actual *measurement data* which is supplied by *sensor* $\mathbf{s}_k$ at the precise time $t_k$. The measurement data $\mathbf{y}_k^T = (\mathbf{y}_{k,1}, \mathbf{y}_{k,2}, \ldots, \mathbf{y}_{k,N_{m,k}})$ consists of $N_{m,k}$ components which itself could vary with time. Similarly, $\mathbf{s}_k^T = (s_{k,1}, s_{k,2}, \ldots, s_{k,N_{m,k}})$ consists of $N_{m,k}$ components where the element $s_{k,i}$ indicates which sensor supplied the measurement $\mathbf{y}_{k,i}$. The ordering or indexing of the measurement data bears no relationship to the indexing of targets as is implicit in the definition of the activity status vector $\boldsymbol{\zeta}_{t,k}$. Indeed, this assignment of measurements to targets is the most difficult aspect of the multitarget tracking problem. The situation is compounded by the fact that spurious measurements may exist which are in no way related to an active target. This is a *false alarm* event. Likewise, a target may be active but not generate a corresponding measurement. This is a *missed*

*target* event. Similarly, more than one active target can lead to the same measurement which we will call an *unresolved target* event. Finally, a single active target can lead to multiple measurements, a situation which will be called a *multiple measurement* event. An appropriate formulation of the multiple target tracking problem must be capable of dealing with all such eventualities.

At this point, it's useful to add a *fictitious* measurement $\mathbf{y}_{k,0}$ to the $N_{m,k}$ measurements $\mathbf{y}_k$ so that $\mathbf{y}_k^T = (\mathbf{y}_{k,0}, \mathbf{y}_{k,1}, \ldots, \mathbf{y}_{k,N_{m,k}})$ with $\mathbf{y}_{k,0}$ representing missed observations, i.e., the missed target event. Likewise, $\boldsymbol{\zeta}_{t,k}$ is augmented to include a *fictitious* target so that $\boldsymbol{\zeta}_{t,k}^T = (\zeta_{t,k}^{(0)}, \zeta_{t,k}^{(1)}, \ldots, \zeta_{t,k}^{(N_{t,max})})$ with the component $\zeta_{t,k}^{(0)}$ used to represent the false-alarm event in a manner to be made precise in what follows. In particular, we introduce a *measurement-to-target* association process $\mathbf{Z}_k$ to represent the true, but unknown origin of the measurements. Here $\mathbf{Z}_k$ is defined as

$$\mathbf{Z}_k = (\mathbf{z}_k, N_{m,k}, N_{t,max}, \mathbf{s}_k, t_k); \qquad k = 0, 1, \ldots \tag{13}$$

where $N_{m,k}$, $N_{t,max}$, $\mathbf{s}_k$, and $t_k$ have been previously defined. We define the associations, $\mathbf{z}_k$, specifically as an $(N_{t,max}+1) \times (N_{m,k}+1)$ matrix with $(i,j)$ element $z_k(i,j)$ for $i = 0, 1, \ldots, N_{t,max}$, $j = 0, 1, \ldots, N_{m,k}$ such that

$$z_k(i,j) = \begin{cases} 1; & \text{if measurement } \mathbf{y}_{k,j} \text{ originated from target } i \\ 0; & \text{otherwise .} \end{cases} \tag{14}$$

It will be useful in the various tracking algorithms to be able to partition the matrix $\mathbf{z}_k$ along its rows or columns. Partitioning $\mathbf{z}_k$ along its columns, we let $\mathbf{z}_k = (\mathbf{z}_{k,0}, \mathbf{z}_{k,1}, \ldots, \mathbf{z}_{k,N_{m,k}})$. Each $\mathbf{z}_{k,i}$ corresponds to measurement $\mathbf{y}_{k,i}$ and describes the targets that contribute to the $i$'th measurement. Partitioning $\mathbf{z}_k$ along its rows yields $\mathbf{z}_k = (\mathbf{z}_{k,0}^T, \mathbf{z}_{k,1}^T, \ldots, \mathbf{z}_{k,N_{t,max}}^T)^T$. Each $\mathbf{z}_{k,j}$ now corresponds to the $j$'th target and describes the measurements that originate from this target. It should be noted that the various partitions of $\mathbf{z}_k$ are equivalent with respect to the true associations in the matrix $\mathbf{z}_k$, and the concept is introduced here to allow various tracking algorithms to use the association matrix in different forms while adhering to this same basic framework.

To avoid any ambiguities for matrix $\mathbf{z}_k$, we assume that $z_k(0,0) = 0$ for $k = 0, 1, \ldots$. If $z_k(0,j) = 1$ for some $1 \leq j \leq N_{m,k}$ then the $j$'th measurement represents a false alarm. Likewise, if $z_k(i,0) = 1$ for some $1 \leq i \leq N_{t,max}$ then the $i$'th target was active at the $k$'th sequential observation time but resulted in a missed detection. Similarly, if for some $1 \leq i \leq N_{t,max}$ we have $z_k(i,j)$ for more than one value of $j$ in the range $1 \leq j \leq N_{m,k}$ then this represent a multiple measurement event. Finally,

if for some $1 \leq j \leq N_{m,k}$ we have $z_k(i,j) = 1$ for more than one value of $i$ in the range $1 \leq i \leq N_{t,max}$ then this represents an unresolved target event.

As an example, suppose that $N_{t,max} = 6$ and $N_{m,k} = 5$. Then a typical $\mathbf{z}_k$ matrix might look like the following

$$\mathbf{z}_k = \left[\begin{array}{c|ccccc} 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]. \tag{15}$$

Here we find there was one false alarm, corresponding to measurement 2, while likewise there was a single missed detection on target 2. Measurement 1 is uniquely associated with target 3 while targets 1 and 4 have contributed to measurement 3 and target 5 has contributed to both measurements 4 and 5. Finally, target 6 is inactive.

It should be noted that the target activity status vector $\boldsymbol{\zeta}_{t,k}$ is easily evaluated from the matrix $\mathbf{z}_k$. In particular,

$$\zeta_{t,k}^{(i)} = \begin{cases} 1; & \text{if } \sum_{j=0}^{N_{m,k}} z_k(i,j) \geq 1 \\ 0; & \text{otherwise,} \end{cases} \tag{16}$$

for $i = 0, 1, \ldots, N_{t,max}$. If $\zeta_{t,k}^{(0)} = 1$ this simply implies that at least one false alarm has occurred. Indeed, the false alarm probability is given by

$$\alpha_k = E\left\{\zeta_{t,k}^{(0)}\right\}; \qquad k = 0, 1, \ldots \tag{17}$$

Similarly, define

$$\xi_{m,k}^{(0)} = \begin{cases} 1; & \text{if } \sum_{i=0}^{N_{t,max}} z_k(i,0) \geq 1 \\ 0; & \text{otherwise.} \end{cases} \tag{18}$$

Then the detection probability is simply

$$\beta_k = 1 - E\left\{\xi_{m,k}^{(0)}\right\}; \qquad k = 0, 1, \ldots \tag{19}$$

**Recursive State Estimation**

Having formulated the multitarget, multisensor modeling assumptions, it is now of interest to estimate the state of the targets at a specific time $k$ given cumulative measurements up to and including time $k$. It is desired to develop recursive target state estimates, for which it is necessary to develop a recursive update rule for $p(\mathbf{S}_k|\mathbf{Y}^k)$, the *a posteriori* probability of the state given cumulative measurements

up to and including time $k$. This update rule is developed in the following section, and then used in the EM algorithm to evaluate the MAP estimate for each target state at time $k$ taking the previous state estimates at time $k-1$ as the prior state information. In order to develop the recursive update rule for the *a posteriori* state probability, $p(\mathbf{S}_k|\mathbf{Y}^k)$, first observe that

$$p(\mathbf{S}_k|\mathbf{Y}^k) = \frac{1}{p(\mathbf{Y}^k)} \int_{\bar{\mathbf{\Omega}}^{k-1}(\mathbf{S}_k)} p(\mathbf{S}^k, \mathbf{Y}^k) d\mathbf{S}^{k-1}, \tag{20}$$

where $\bar{\mathbf{\Omega}}^{k-1}(\mathbf{S}_k) \equiv \{\mathbf{S}^{k-1} \in \mathbf{\Omega}^{k-1} : (\mathbf{S}^{k-1}, \mathbf{S}_k) \in \mathbf{\Omega}^k\}$, with $\mathbf{\Omega}^k$ the state-space for $\mathbf{S}^k$. The state space $\bar{\mathbf{\Omega}}^{k-1}(\mathbf{S}_k)$ then consists of all sequences $\mathbf{S}^{k-1} \in \mathbf{\Omega}^{k-1}$ *compatible* with $\mathbf{S}_k$ in the sense that the concatenation $(\mathbf{S}^{k-1}, \mathbf{S}_k) \in \mathbf{\Omega}^k$ for $\mathbf{S}_k \in \mathbf{\Omega}_k$. This last expression can be rewritten as:

$$p(\mathbf{S}_k|\mathbf{Y}^k) = \frac{1}{p(\mathbf{Y}^k)} \int_{\bar{\mathbf{\Omega}}^{k-1}(\mathbf{S}_k)} p(\mathbf{Y}^k|\mathbf{S}^k) p(\mathbf{S}^k) d\mathbf{S}^{k-1}. \tag{21}$$

Furthermore, we assume that[1]

$$\begin{aligned} p(\mathbf{Y}^k|\mathbf{S}^k) &= \prod_{j=0}^{k} p(\mathbf{Y}_j|\mathbf{S}_j) \\ &= p(\mathbf{Y}_k|\mathbf{S}_k) p(\mathbf{Y}^{k-1}|\mathbf{S}^{k-1}) \end{aligned} \tag{22}$$

describes the probability of the measurements $\mathbf{Y}^k$ given the state $\mathbf{S}^k$. Likewise, from (9)

$$p(\mathbf{S}^k) = p(\mathbf{S}_k|\mathbf{S}_{k-1}) p(\mathbf{S}^{k-1}), \tag{23}$$

while finally

$$p(\mathbf{Y}^k) = p(\mathbf{Y}_k|\mathbf{Y}^{k-1}) p(\mathbf{Y}^{k-1}). \tag{24}$$

Substitution of (22) through (24) into (21) yields

$$p(\mathbf{S}_k|\mathbf{Y}^k) = \frac{p(\mathbf{Y}_k|\mathbf{S}_k)}{p(\mathbf{Y}_k|\mathbf{Y}^{k-1})} \int_{\mathbf{\Omega}_{k-1}(\mathbf{S}_k)} p(\mathbf{S}_k|\mathbf{S}_{k-1}) p(\mathbf{S}_{k-1}|\mathbf{Y}^{k-1}) d\mathbf{S}_{k-1}, \tag{25}$$

where $\mathbf{\Omega}_{k-1}(\mathbf{S}_k) = \{\mathbf{S}_{k-1} \in \mathbf{\Omega}_{k-1} : \mathbf{S}^{k-1} \in \bar{\mathbf{\Omega}}^{k-1}(\mathbf{S}_k)\}$. Observe that

$$\begin{aligned} &\int_{\mathbf{\Omega}_{k-1}(\mathbf{S}_k)} p(\mathbf{S}_k|\mathbf{S}_{k-1}) p(\mathbf{S}_{k-1}|\mathbf{Y}^{k-1}) d\mathbf{S}_{k-1} \\ &\quad = \int_{\mathbf{\Omega}_{k-1}(\mathbf{S}_k)} p(\mathbf{S}_k|\mathbf{S}_{k-1}, \mathbf{Y}^{k-1}) p(\mathbf{S}_{k-1}|\mathbf{Y}^{k-1}) d\mathbf{S}_{k-1} \\ &\quad = p(\mathbf{S}_k|\mathbf{Y}^{k-1}), \end{aligned} \tag{26}$$

[1]This, of course, places some constraints on the observation process.

which is considered the state prediction probability at time $k$ given the past measurement data. Likewise

$$\begin{aligned} p(\mathbf{Y}_k|\mathbf{Y}^{k-1}) &= \int_{\mathbf{\Omega}_k} p(\mathbf{Y}_k, \mathbf{S}_k|\mathbf{Y}^{k-1}) d\mathbf{S}_k \\ &= \int_{\mathbf{\Omega}_k} p(\mathbf{Y}_k|\mathbf{S}_k, \mathbf{Y}^{k-1}) p(\mathbf{S}_k|\mathbf{Y}^{k-1}) d\mathbf{S}_k \\ &= \int_{\mathbf{\Omega}_k} p(\mathbf{Y}_k|\mathbf{S}_k) p(\mathbf{S}_k|\mathbf{Y}^{k-1}) d\mathbf{S}_k. \end{aligned} \tag{27}$$

It follows that the desired recursive updating rule is

$$p(\mathbf{S}_k|\mathbf{Y}^k) = \frac{p(\mathbf{Y}_k|\mathbf{S}_k)}{p(\mathbf{Y}_k|\mathbf{Y}^{k-1})} p(\mathbf{S}_k|\mathbf{Y}^{k-1}), \tag{28}$$

with $p(\mathbf{S}_k|\mathbf{Y}^{k-1})$ and $p(\mathbf{Y}_k|\mathbf{Y}^{k-1})$ given by (26) and (27), respectively. We assume that $p(\mathbf{S}_0|\mathbf{Y}^0) = p_0(\mathbf{S}_0)$, the *a priori* distribution of the initial state $\mathbf{S}_0$, which is assumed known. All that is required is specification of $p(\mathbf{S}_k|\mathbf{S}_{k-1})$ and $p(\mathbf{Y}_k|\mathbf{S}_k)$ and the ability to perform the integrations in (26) and (27).

Evaluation of $p(\mathbf{S}_k|\mathbf{S}_{k-1})$ follows directly from (10). Now consider $p(\mathbf{Y}_k|\mathbf{S}_k)$. Note that

$$\begin{aligned} p(\mathbf{Y}_k|\mathbf{S}_k) &= \int p(\mathbf{Y}_k, \mathbf{Z}_k|\mathbf{S}_k) d\mathbf{Z}_k \\ &= \int p(\mathbf{Y}_k|\mathbf{Z}_k, \mathbf{S}_k) p(\mathbf{Z}_k|\mathbf{S}_k) d\mathbf{Z}_k. \end{aligned} \tag{29}$$

Now, given $\mathbf{Z}_k$, we find that

$$p(\mathbf{Y}_k|\mathbf{Z}_k, \mathbf{S}_k) = p(\mathbf{Y}_k|\mathbf{Z}_k, \mathbf{\Phi}_k) \tag{30}$$

which is generally not difficult to evaluate. The quantity $p(\mathbf{Z}_k|\mathbf{S}_k)$ allows for a state-dependent data assignment matrix, which can be used to impose scenario-dependent constraints on this matrix.

The evaluations of (26), (27), and (29) is often difficult to implement and the specific assumptions chosen lead to the different solution techniques discussed previously. For example, the multiple hypothesis approach evaluates (29) given different combinations of the matrix $\mathbf{z}_k$, keeping those realizations of $\mathbf{z}_k$ that are highly probable. Introducing the association process $\mathbf{Z}_k$ into the solution in equation (29) is general in that the minimum variance or MAP estimate can be generated from (28). Alternatively, only the MAP estimate may be desired, in which case the association process can be viewed as *missing* data in an *incomplete* data problem. This approach is described in the next section.

## III EM Approach to Recursive State Estimation

Considering the measurements up to and including time $k$ to be the entire set of observed data, we let the association process $\{\mathbf{Z}_k\}$ be the corresponding *missing* data. Then the entire set of *complete* data is $\mathbf{X}^k = (\mathbf{Y}^k, \mathbf{Z}^k)$ where $\mathbf{Z}^k = (\mathbf{Z}_0, \mathbf{Z}_1, \ldots, \mathbf{Z}_k)$. The *complete* data for the update at time $k$ is $\mathbf{X}_k = (\mathbf{Y}_k, \mathbf{Z}_k)$ where $\mathbf{Z}_k$ is the association matrix discussed previously. We are now interested in computing the MAP estimate of the state $\mathbf{S}_k$ given this *complete* data. Using an identical approach to that used in deriving (28) we obtain

$$p(\mathbf{S}_k|\mathbf{X}^k) = \frac{p(\mathbf{X}_k|\mathbf{S}_k)}{p(\mathbf{X}_k|\mathbf{X}^{k-1})} p(\mathbf{S}_k|\mathbf{X}^{k-1}). \tag{31}$$

Using Bayes rule and expanding $\mathbf{X}^k = (\mathbf{Y}^k, \mathbf{Z}^k)$ gives

$$\begin{aligned} p(\mathbf{S}_k|\mathbf{X}^k) &= \frac{p(\mathbf{Y}_k|\mathbf{Z}_k, \mathbf{S}_k) p(\mathbf{Z}_k|\mathbf{S}_k) p(\mathbf{S}_k|\mathbf{Z}^{k-1}, \mathbf{Y}^{k-1})}{p(\mathbf{X}_k|\mathbf{X}^{k-1})} \\ &= \frac{p(\mathbf{Y}_k|\mathbf{Z}_k, \mathbf{\Phi}_k) p(\mathbf{Z}_k|\mathbf{S}_k) p(\mathbf{S}_k|\mathbf{Z}^{k-1}, \mathbf{Y}^{k-1})}{p(\mathbf{X}_k|\mathbf{X}^{k-1})}. \end{aligned} \tag{32}$$

The MAP estimate is now found using (32) by computing

$$\hat{\mathbf{S}}_k = \arg \max_{\mathbf{S}_k} \log p(\mathbf{S}_k|\mathbf{X}^k). \tag{33}$$

Note that since $\mathbf{S}_k = (\mathbf{\Phi}_k, \zeta_{t,k})$ the MAP estimate includes the problem of validating the number of active target classes in the target activity status $\zeta_{t,k}$. This is important in situations where the number of targets is changing due to targets entering/leaving the field of view, or the birth/death of targets.

In this paper the use of the EM algorithm for the MAP estimation of the target states given a known number of targets will be described. This is the situation encountered in maintaining a fixed number of target tracks where $\zeta_{t,k}$ is known, and the measurements will be assigned only to the active target classes. Also, the unresolved target events will not be considered here and will be addressed at a later time.

With $\zeta_{t,k}$ given equation (32) can now be written as

$$p(\mathbf{\Phi}_k|\mathbf{X}^k) = \frac{p(\mathbf{Y}_k|\mathbf{Z}_k, \mathbf{\Phi}_k) p(\mathbf{Z}_k|\mathbf{\Phi}_k) p(\mathbf{\Phi}_k|\mathbf{Z}^{k-1}, \mathbf{Y}^{k-1})}{p(\mathbf{X}_k|\mathbf{X}^{k-1})}, \tag{34}$$

and the MAP estimate of the target states $\mathbf{\Phi}_k$ at time $k$ is

$$\begin{aligned}\hat{\mathbf{\Phi}}_k &= \arg\max_{\mathbf{\Phi}_k} \log p(\mathbf{\Phi}_k|\mathbf{X}^k) \\ &= \arg\max_{\mathbf{\Phi}_k} \left[\log p(\mathbf{X}_k|\mathbf{\Phi}_k) + \log p(\mathbf{\Phi}_k|\mathbf{X}^{k-1})\right]. \end{aligned} \tag{35}$$

A penalized EM approach [15, 16, 17] can now be used with equation (35) to compute $\hat{\mathbf{\Phi}}_k$. The penalized EM approach is described in two steps

$$\text{E-Step}: \quad Q(\mathbf{\Phi}_k \mid \hat{\mathbf{\Phi}}_k^{(p)}) = E[\log p(\mathbf{X}_k|\mathbf{\Phi}_k) \mid \mathbf{Y}_k, \hat{\mathbf{\Phi}}_k^{(p)}] \tag{36}$$

$$\text{M-Step}: \quad \hat{\mathbf{\Phi}}_k^{(p+1)} = \arg\max_{\mathbf{\Phi}_k}[Q(\mathbf{\Phi}_k \mid \hat{\mathbf{\Phi}}_k^{(p)}) + \log p(\mathbf{\Phi}_k|\mathbf{X}^{k-1})], \tag{37}$$

where $p$ corresponds to the $p$'th iteration of the algorithm. As in [22], implicit in the E-step is the computation of the conditional expectation of $\mathbf{Z}_k$ given $\mathbf{Y}_k$ and $\hat{\mathbf{\Phi}}_k^{(p)}$. This is equivalent to computing the probabilities of each of the individual measurements belonging to each of the target classes. These probabilities are then used in the M-step as soft-decisions to update the $p$'th iterative state estimate of each target.

The term $\log p(\mathbf{\Phi}_k|\mathbf{X}^{k-1})$ is analogous to (26) in that it is the probability of the state at time $k$ given the past *complete* data $\mathbf{X}^{k-1}$. To form a recursive algorithm, the MAP estimate of $\mathbf{\Phi}_{k-1}$ at time $k-1$ is found and used to compute $\hat{\mathbf{\Phi}}_k$ at time $k$. Alternatively, a non-recursive or *block* approach at time $k$ could be used to compute the MAP estimate of $\hat{\mathbf{\Phi}}^k = (\hat{\mathbf{\Phi}}_1, \hat{\mathbf{\Phi}}_2, \ldots, \hat{\mathbf{\Phi}}_k)$ taking into account all the associations $\mathbf{Z}^k$. The recursive approach is discussed here, with the block approach to be addressed at a later time.

### Calculation of E-Step

In this section the calculation of the expectation of the log-likelihood functional given in (36) is described. To calculate this functional a column partition of the association matrix of $\mathbf{Z}_k$ will be used and corresponds to $\mathbf{z}_k = (\mathbf{z}_{k,0}, \mathbf{z}_{k,1}, \ldots, \mathbf{z}_{k,N_{m,k}})$. Thus, each $\mathbf{z}_{k,i}$ corresponds to the $i$'th measurement $\mathbf{y}_{k,i} \in \mathbf{y}_k$.

The individual measurements $\mathbf{y}_{k,i}$ are assumed to be conditionally independent given the associations $\mathbf{z}_k$. In this case the log-likelihood functional of $\mathbf{X}_k$ can be written as

$$\log p(\mathbf{X}_k|\mathbf{\Phi}_k) = \log p(\mathbf{Y}_k, \mathbf{Z}_k|\mathbf{\Phi}_k)$$

$$
\begin{aligned}
&= \log p(\mathbf{Y}_k|\mathbf{Z}_k, \mathbf{\Phi}_k) + \log p(\mathbf{Z}_k|\mathbf{\Phi}_k) \\
&= \log p(\mathbf{y}_k|\mathbf{z}_k, \mathbf{\Phi}_k) + \log p(\mathbf{z}_k|\mathbf{\Phi}_k) \\
&= \sum_{i=1}^{N_{m,k}} \log p(\mathbf{y}_{k,i}|\mathbf{z}_{k,i}, \mathbf{\Phi}_k) + \log p(\mathbf{z}_k|\mathbf{\Phi}_k). \qquad (38)
\end{aligned}
$$

In order to compute the functional $Q(\mathbf{\Phi}_k \mid \hat{\mathbf{\Phi}}_k^{(p)})$ it is desired to separate $\mathbf{z}_k$ and $\mathbf{\Phi}_k$. More specifically, under the assumption that a measurement $\mathbf{y}_{k,i}$ originates from one target, each $\mathbf{z}_{k,i}$ is a unit vector with one target class equal to unity and all other target classes equal to zero. Now we can write

$$\log p(\mathbf{y}_{k,i}|\mathbf{z}_{k,i}, \mathbf{\Phi}_k) = \mathbf{z}_{k,i}^T U(\mathbf{y}_{k,i}|\mathbf{\Phi}_k), \qquad (39)$$

where[2]

$$U(\mathbf{y}_{k,i}|\mathbf{\Phi}_k) = (\log p(\mathbf{y}_{k,i}|\mathbf{e}_0, \mathbf{\Phi}_k), \log p(\mathbf{y}_{k,i}|\mathbf{e}_1, \mathbf{\Phi}_k), \ldots, \log p(\mathbf{y}_{k,i}|\mathbf{e}_{N_t}, \mathbf{\Phi}_k))^T. \qquad (40)$$

Next, the functional $\log p(\mathbf{z}_k|\mathbf{\Phi}_k)$ is considered. First, recognize that individual associations are not independent under certain assumptions. For example, if it is assumed that a sensor will detect target $t$ only once at time $k$, then the assignment of target $t$ to measurement $\mathbf{y}_{k,i}$, designated by $\mathbf{z}_{k,i} = \mathbf{e}_t$, will influence the associations $\mathbf{z}_{k,j}$ for $j \neq i$. This implies that it is necessary to impose constraints upon the assignment of individual associations in some manner. This is investigated in [30], where an *energy* functional is used to impose constraints upon the association matrix in order to estimate measurement-to-target association values. This *energy* functional is minimized by a Hopfield analog network [31] in order to generate the association values which are then used in a PDA tracker [7].

Note that the constraints imposed in [30] are not directly available in a form of probability distributions that can be incorporated into a Baysian framework. What is desired is a method for mapping constraints on the association process into distributions that can be used for $\log p(\mathbf{z}_k|\mathbf{\Phi}_k)$ in equation (38). In fact, we show that the association matrix partitioned as $\mathbf{z}_k = (\mathbf{z}_{k,0}, \mathbf{z}_{k,1}, \ldots, \mathbf{z}_{k,N_{m,k}})$ is a Markov random field (MRF) with a corresponding Gibb's distribution [32].

In order to develop the MRF formulation of the association matrix $\mathbf{z}_k$, note that in a tracking system for time $k$ each measurement $\mathbf{y}_{k,i}$ is considered as either a possible candidate for updating the state of each target $t \in \{1, 2, \ldots, N_{t,max}\}$, or belonging to the false alarm event $t = 0$. We will

[2]Here, $p(\mathbf{y}_{k,i}|\mathbf{e}_t, \mathbf{\Phi}_k)$ represents the conditional pdf $p(\mathbf{y}_{k,i}|\mathbf{z}_{k,i} = \mathbf{e}_t, \mathbf{\Phi}_k)$, $\mathbf{e}_t$ being the vector with a one in the $t$'th component, $t \in \{0, 1, \ldots, N_{t,max}\}$, and zeros elsewhere.

consider the elements of the matrix $\mathbf{z}_k$ as the vertices of an undirected graph where the edges of the node $\mathbf{z}_{k,i}$ form connections to other nodes that will affect the association of measurement $i$ with target $t$. These edge connections are described as follows:

i) Node $z_k(t,i)$ is connected to all nodes $z_k(t,j)$ for $j \neq i$. This connects node $z_k(t,i)$ all other nodes that can be associated with the same target $t$.

ii) Node $z_k(t,i)$ is connected to all nodes $z_k(s,i)$ where $s \in \{0,1,\ldots,N_{t,max}\}$. This connects node $z_k(t,i)$ to all nodes that can be associated with measurement $i$.

Using the above edge connections, each node in the graph representing $\mathbf{z}_k$ connects nodes that belong in the same row or column of matrix $\mathbf{z}_k$. Also, the second condition allows a node to be connected to itself. The node $z_k(t,i)$ will be considered to have a value in the range $[0,1]$ which will be the probability that measurement $i$ originated from target $t$.

Typically not all measurements are considered as possible updates for a specific target $t$ during the $k$'th time interval. For each target $t \neq 0$ the subset from the entire set of measurements at time $k$ that pass a *gating* criteria will be considered as possible candidates for updating the state of target $t$. The edges of the graph are thus modified to only allow connections between two nodes as follow:

i) For $t \neq 0$, two nodes $z_k(t,i)$ and $z_k(t,j)$, where $j \neq i$, are connected if measurements $\mathbf{y}_{k,i}$ and $\mathbf{y}_{k,j}$ both pass the gating criteria for target $t$ at time $k$.

ii) For $t = 0$, node $z_k(0,i)$ is connected to $z_k(0,j)$ for all $j \neq i$.

iii) For $t, s \neq 0$, two nodes $z_k(t,i)$ and $z_k(s,i)$ are connected if measurement $\mathbf{y}_{k,i}$ passes the gating criteria for both targets $t$ and $s$ at time $k$.

iv) For $t \neq 0$ and $s = 0$, two nodes $z_k(t,i)$ and $z_k(s,i)$ are connected if measurement $\mathbf{y}_{k,i}$ passes the gating criteria for target $t$ at time $k$.

A specific node $z_k(t,i)$ may be isolated from all other nodes, and in the case this node is not connected to itself, the association between measurement $i$ and target $t$ is considered not to be possible and the value of node $z_k(t,i)$ equals zero. Having defined this graph structure, we consider the association process $\mathbf{z}_k$ partitioned as $(\mathbf{z}_{k,0}, \mathbf{z}_{k,1}, \ldots, \mathbf{z}_{k,N_{m,k}})$. For each $i \in \{0,1,\ldots,N_{m,k}\}$ the nodes $z_k(t,i)$ with

$t \in \{0, 1, \ldots, N_{t,max}\}$ will be grouped together to form a larger node representing the association vector $\mathbf{z}_{k,i}$. For each $\mathbf{z}_{k,i}$ the connections from $\mathbf{z}_{k,i}$ to the other $\mathbf{z}_{k,j}$ for $j \neq i$ define a *local* neighborhood structure for the $i$'th association vector. As an example, consider the matrix $\mathbf{z}_k$ from equation (15), (where the unresolved target event is allowed). The graph structure for this true association matrix is shown in Figure 1. Note that the unmarked entries in the matrix indicate that these specific associations are not possible.

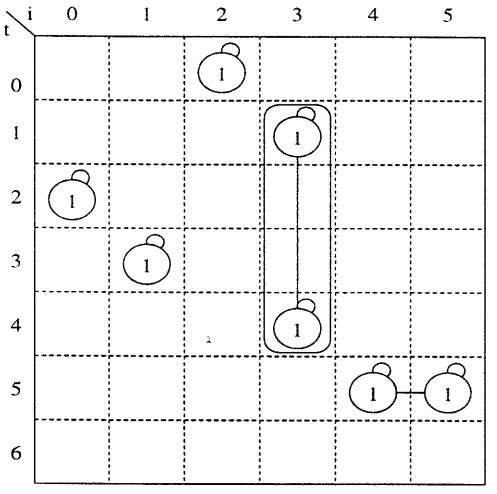


Figure 1: Example Association Graph

The local neighborhood of $\mathbf{z}_{k,i}$ is designated as $\eta_{\mathbf{z}_{k,i}}$ and consists of those $\mathbf{z}_{k,j}$ for those $j \neq i$ that have edge connections to $\mathbf{z}_{k,i}$. Letting $P$ be the probability measure assigned to the set of all possible configurations of $\mathbf{z}_k$, the *local* characteristics of $P$ on this set are conditional probabilities of the form $p(\mathbf{z}_{k,i}|\mathbf{z}_{k,j}, j \neq i, \mathbf{\Phi}_k)$. The probability measure defines $\mathbf{z}_k$ as a Markov random field since the *local* characteristics of site $\mathbf{z}_{k,i}$ depends only upon the knowledge of the outcomes at the neighboring sites $\mathbf{z}_{k,j}$, $j \in \eta_{\mathbf{z}_{k,i}}$. The pdf. $p(\mathbf{z}_{k,i}|\mathbf{z}_{k,j}, j \neq i, \mathbf{\Phi}_k)$ is then written as

$$p(\mathbf{z}_{k,i}|\mathbf{z}_{k,j}, j \neq i, \mathbf{\Phi}_k) = p(\mathbf{z}_{k,i}|\mathbf{z}_{k,j}, j \in \eta_{\mathbf{z}_{k,i}}, \mathbf{\Phi}_k). \tag{41}$$

Noting the MRF - Gibbs distribution equivalence [32, 33], $p(\mathbf{z}_k|\mathbf{\Phi}_k)$ is given by

$$p(\mathbf{z}_k|\mathbf{\Phi}_k) = \frac{1}{\mathcal{Z}} \exp(-V(\mathbf{z}_k)). \tag{42}$$

Thus, with this MRF formulation of $\mathbf{z}_k$ the $Q$-function becomes

$$Q(\mathbf{\Phi}_k \mid \hat{\mathbf{\Phi}}_k^{(p)}) = \sum_{i=1}^{N_{m,k}} E\left[\mathbf{z}_{k,i}^T \mid \mathbf{y}_k, \hat{\mathbf{\Phi}}_k^{(p)}\right] U(\mathbf{y}_{k,i} \mid \mathbf{\Phi}_k) - E\left[V(\mathbf{z}_k \mid \mathbf{\Phi}_k) + \log \mathcal{Z} \mid \mathbf{y}_k, \hat{\mathbf{\Phi}}_k^{(p)}\right]. \tag{43}$$

The main problem is now to compute the $Q$-function efficiently. Writing $p(\mathbf{z}_k|\mathbf{y}_k, \mathbf{\Phi}_k)$ as

$$\begin{aligned} p(\mathbf{z}_k|\mathbf{y}_k, \mathbf{\Phi}_k) &= \frac{p(\mathbf{y}_k|\mathbf{z}_k, \mathbf{\Phi}_k)p(\mathbf{z}_k|\mathbf{\Phi}_k)}{p(\mathbf{y}_k|\mathbf{\Phi}_k)} \\ &= \prod_{i=1}^{N_{m,k}} p(\mathbf{y}_{k,i}|\mathbf{z}_{k,i}, \mathbf{\Phi}_k)\mathcal{Z}^{-1} \exp(-V(\mathbf{z}_k))p(\mathbf{y}_k|\mathbf{\Phi}_k)^{-1} \\ &= \exp\left[\sum_{i=1}^{N_{m,k}} \log p(\mathbf{y}_{k,i}|\mathbf{z}_{k,i}, \mathbf{\Phi}_k) - V(\mathbf{z}_k)\right] [\mathcal{Z}p(\mathbf{y}_k|\mathbf{\Phi}_k)]^{-1}, \end{aligned} \tag{44}$$

we notice that this is also a Gibbs distribution where the log-likelihood functional in the exponent of (44) is considered as part of the first-order, or *singleton*, clique function.

Taking the approach used in [22], the approximate techniques of Besag [34] are used instead of the Monte-Carlo method of [32]. In particular, the *pseudo-likelihood* approximation of $p(\mathbf{z}_k|\mathbf{\Phi}_k)$ is used in the form

$$p(\mathbf{z}_k|\mathbf{\Phi}_k) = \prod_{i=1}^{N_{m,k}} p(\mathbf{z}_{k,i}|\mathbf{z}_{k,j}, j \in \eta_{\mathbf{z}_{k,i}}, \mathbf{\Phi}_k), \tag{45}$$

using the local neighborhood structure of $\mathbf{z}_{k,i}$ discussed previously. It follows that under this approximation

$$\begin{aligned} p(\mathbf{z}_{k,i}|\mathbf{\Phi}_k) &= \sum_{\substack{\mathbf{z}_{k,j} \\ j \neq i}} p(\mathbf{z}_k|\mathbf{\Phi}_k) \\ &\approx \sum_{\substack{\mathbf{z}_{k,j} \\ j \neq i}} \prod_{j=1}^{N_{m,k}} p(\mathbf{z}_{k,j}|\mathbf{z}_{k,l}, l \in \eta_{\mathbf{z}_{k,j}}, \mathbf{\Phi}_k). \end{aligned} \tag{46}$$

Next, suppose that $\hat{\mathbf{z}}_{k,l}$ is some previously obtained estimate of $\mathbf{z}_{k,l}$. This yields the approximate result

$$\begin{aligned} p(\mathbf{z}_{k,i}|\mathbf{\Phi}_k) &\approx \sum_{\substack{\mathbf{z}_{k,j} \\ j \neq i}} \prod_{j=1}^{N_{m,k}} p(\mathbf{z}_{k,j}|\hat{\mathbf{z}}_{k,l}, l \in \eta_{\mathbf{z}_{k,j}}, \mathbf{\Phi}_k) \\ &= p(\mathbf{z}_{k,i}|\hat{\mathbf{z}}_{k,l}, l \in \eta_{\mathbf{z}_{k,i}}, \mathbf{\Phi}_k). \end{aligned} \tag{47}$$

Assume now that for the $p$'th iteration of the EM algorithm the estimate of $\mathbf{z}_{k,l}$ during the $(p-1)$'st EM iteration is used as $\hat{\mathbf{z}}_{k,l}$. This gives

$$p(\mathbf{z}_{k,i}|\mathbf{\Phi}_k) = p(\mathbf{z}_{k,i}|\hat{\mathbf{z}}_{k,l}^{(p-1)}, l \in \eta_{\mathbf{z}_{k,i}}, \mathbf{\Phi}_k). \tag{48}$$

To proceed from here we consider $\mathbf{z}_k$ to be a second-order MRF with a Gibbs energy term $V(\mathbf{z}_k \mid \mathbf{\Phi}_k)$. A second-order MRF has only *singleton* and pair-wise, or *doubleton*, cliques which can be used to represent the desired constraints on the association process. In this case, $V(\mathbf{z}_k \mid \mathbf{\Phi}_k)$ is written as

$$V(\mathbf{z}_k \mid \mathbf{\Phi}_k) = \sum_{i=1}^{N_{m,k}} \mathbf{z}_{k,i}^T V_1(\mathbf{\Phi}_k) + \sum_{i,j \in \mathcal{C}} \mathbf{z}_{k,i}^T V_2(\mathbf{\Phi}_k) \mathbf{z}_{k,j}, \tag{49}$$

where $V_1(\mathbf{\Phi}_k)$ is a vector with entries $\log p(\mathbf{z}_{k.i} = \mathbf{e}_t | \mathbf{\Phi}_k)$ for $t = 0, 1, \ldots, N_{t,max}$ and $V_1(\mathbf{\Phi}_k)$ is a $(N_{t,max}+1) \times (N_{t,max}+1)$ matrix with the $(t,s)$ element $\log p(\mathbf{z}_{k.i} = \mathbf{e}_t, \mathbf{z}_{k.j} = \mathbf{e}_s | \mathbf{\Phi}_k)$. Using this notation, the $Q$-function becomes

$$\begin{aligned} Q(\mathbf{\Phi}_k \mid \hat{\mathbf{\Phi}}_k^{(p)}) &= \sum_{i=1}^{N_{m,k}} E\left[\mathbf{z}_{k,i}^T \mid \mathbf{y}_k, \hat{\mathbf{\Phi}}_k^{(p)}\right] \left[U(\mathbf{y}_{k,i} | \mathbf{\Phi}_k) + V_1(\mathbf{\Phi}_k)\right] \\ &\quad - \sum_{i,j \in \mathcal{C}} E\left[\mathbf{z}_{k,i}^T V_2(\mathbf{\Phi}_k) \mathbf{z}_{k,j} \mid \mathbf{y}_k, \hat{\mathbf{\Phi}}_k^{(p)}\right] \\ &\quad - E\left[\log \mathcal{Z} \mid \mathbf{y}_k, \hat{\mathbf{\Phi}}_k^{(p)}\right]. \end{aligned} \tag{50}$$

In order to evaluate $Q(\mathbf{\Phi}_k \mid \hat{\mathbf{\Phi}}_k^{(p)})$ for the MRF model the expected values $E\left[\mathbf{z}_{k,i}^T \mid \mathbf{y}_k, \hat{\mathbf{\Phi}}_k^{(p)}\right]$ and $E\left[\mathbf{z}_{k,i}^T V_2(\mathbf{\Phi}_k) \mathbf{z}_{k,j} \mid \mathbf{y}_k, \hat{\mathbf{\Phi}}_k^{(p)}\right]$ must be computed. Taking the approach described in [27], the approximation

$$E\left[\mathbf{z}_{k,i}^T V_2(\mathbf{\Phi}_k) \mathbf{z}_{k,j} \mid \mathbf{y}_k, \hat{\mathbf{\Phi}}_k^{(p)}\right] = \hat{\mathbf{z}}_{k,i}^{(p)\,T} V_2(\mathbf{\Phi}_k) \hat{\mathbf{z}}_{k,j}^{(p)} \tag{51}$$

is used where $\hat{\mathbf{z}}_{k,i}^{(p)} = E\left[\mathbf{z}_{k,i} \mid \mathbf{y}_k, \hat{\mathbf{\Phi}}_k^{(p)}\right]$. Thus, it is necessary only to evaluate the vector $\hat{\mathbf{z}}_{k,i}^{(p)}$, whose $t$'th element is $E\left[\mathbf{z}_{k,i} = \mathbf{e}_t \mid \mathbf{y}_k, \hat{\mathbf{\Phi}}_k^{(p)}\right]$ assuming that $\mathbf{z}_{k,i} \in (\mathbf{e}_0, \mathbf{e}_1, \ldots, \mathbf{e}_{N_{t,max}})$. Denoting this element as $\mathbf{z}_{k,i}(t)$ and taking the expected value gives

$$\begin{aligned} \hat{\mathbf{z}}_{k,i}^{(p)}(t) &= E\left[\mathbf{z}_{k,i} = \mathbf{e}_t \mid \mathbf{y}_k, \hat{\mathbf{\Phi}}_k^{(p)}\right] \\ &= p(\mathbf{z}_{k,i} = \mathbf{e}_t \mid \mathbf{y}_k, \hat{\mathbf{\Phi}}_k^{(p)}) \\ &= \frac{\pi_{k,i}^{(p)}(t) p(\mathbf{y}_{k,i} \mid \mathbf{z}_{k,i} = \mathbf{e}_t, \hat{\mathbf{\Phi}}_k^{(p)})}{\sum_{s=0}^{N_{t,max}} \pi_{k,i}^{(p)}(s) p(\mathbf{y}_{k,i} \mid \mathbf{z}_{k,i} = \mathbf{e}_s, \hat{\mathbf{\Phi}}_k^{(p)})}, \end{aligned} \tag{52}$$

where $\pi_{k,i}^{(p)}(t) = p(\mathbf{z}_{k,i} = \mathbf{e}_t \mid \hat{\mathbf{\Phi}}_k^{(p)})$ is the assumed stationary probability that $\mathbf{z}_{k,i} = \mathbf{e}_t$ at iteration $p$. Using the MRF formulation of $\mathbf{z}_k$, $\pi_{k,i}^{(p)}(t)$ is evaluated as

$$\begin{aligned} \pi_{k,i}^{(p)}(t) &= p(\mathbf{z}_{k,i} = \mathbf{e}_t \mid \hat{\mathbf{\Phi}}_k^{(p)}) \\ &= \frac{\exp(-V(\mathbf{z}_{k,i} = \mathbf{e}_t \mid \hat{\mathbf{\Phi}}_k^{(p)}))}{\sum_{s=0}^{N_{t,max}} \exp(-V(\mathbf{z}_{k,i} = \mathbf{e}_s \mid \hat{\mathbf{\Phi}}_k^{(p)}))}. \end{aligned} \tag{53}$$

However, using the *local* neighborhood approximation previously defined gives the estimate

$$\begin{aligned}\pi_{k,i}^{(p)}(t) &\approx \hat{\pi}_{k,i}^{(p)}(t) \\ &= p(\mathbf{z}_{k,i} = \mathbf{e}_t \mid \hat{\mathbf{z}}_{k,j}^{(p)}, j \in \eta_{\mathbf{z}_{k,i}}, \hat{\mathbf{\Phi}}_k^{(p)}) \\ &= \frac{\exp(-V(\mathbf{z}_{k,i} = \mathbf{e}_t \mid \hat{\mathbf{z}}_{k,j}^{(p)}, j \in \eta_{\mathbf{z}_{k,i}}, \hat{\mathbf{\Phi}}_k^{(p)}))}{\sum_{s=0}^{N_{t,max}} \exp(-V(\mathbf{z}_{k,i} = \mathbf{e}_s \mid \hat{\mathbf{z}}_{k,j}^{(p)}, j \in \eta_{\mathbf{z}_{k,i}}, \hat{\mathbf{\Phi}}_k^{(p)}))}. \end{aligned} \tag{54}$$

The approximation in (54) is similar to Besag's iterated conditional mode (ICM) technique; however, *soft* rather than *hard* assignment values of $\mathbf{z}_{k,j}$ are used. Furthermore, these *soft* assignment values are shown to result in a self-consistent mean-field approximation [23, 24, 27]. Note that the particular form of $V$ depends upon the constraints imposed upon the association process and that the constraints described in [30] can be written in the form of equation (49). Also note that the computation of $\hat{\mathbf{z}}_{k,i}^{(p)}$ is performed once for each possible measurement-to-target association and the joint association are not enumerated by this approach.

### Calculation of M-Step

The maximization of the $Q$-function is straightforward in that $\mathbf{\Phi}_k$ is solved for after setting the derivatives of (50) equal to zero. In the case there are constraints among the components of $\mathbf{\Phi}_k$ this results in the more difficult *constrained* optimization problem and will not be discussed here. A special case is when the parameters for $\mathbf{z}_k$ and $\mathbf{y}_k$ are separable, i.e., $\mathbf{\Phi}_k = (\mathbf{\Phi}_{k,y}, \mathbf{\Phi}_{k,z})$, which is assumed here. Dropping the subscript $k$ here for notational convenience, the new parameters $\mathbf{\Phi}^{(p+1)} = (\mathbf{\Phi}_y^{(p+1)}, \mathbf{\Phi}_z^{(p+1)})$ can be found by solving

$$\sum_{i=1}^{N_{m,k}} \left[\nabla_{\mathbf{\Phi}_y} U^T(\mathbf{y}_i \mid \mathbf{\Phi})\right] \hat{\mathbf{z}}_i^{(p)} = \mathbf{0}, \tag{55}$$

and

$$\sum_{i=1}^{N_{m,k}} \sum_{j \in \eta_{z_i}} \left[\nabla_{\mathbf{\Phi}_z} (\hat{\mathbf{z}}_i^{(p)\,T} V_2(\mathbf{\Phi}) \hat{\mathbf{z}}_j^{(p)})\right] + \sum_{i=1}^{N_{m,k}} \left[\nabla_{\mathbf{\Phi}_z} V_1^T(\mathbf{\Phi})\right] \hat{\mathbf{z}}_i^{(p)} = \mathbf{0}. \tag{56}$$

The quantity $\nabla_{\mathbf{\Phi}_y} U^T(\mathbf{\Phi})$ is a block diagonal matrix of dimension $M_y \times N_{t,max}$. More specifically, suppose $\mathbf{\Phi}_y = (\mathbf{\Phi}_{y,1}, \mathbf{\Phi}_{y,2}, \ldots, \mathbf{\Phi}_{y,N_{t,max}})$ where $\mathbf{\Phi}_{y,t}$ is a vector of dimension $M_{y,t}$ representing the parameters associated with target $t$, $t = 1, 2, \ldots, N_{t,max}$. Then with

$$U_t(\mathbf{y}_i|\mathbf{\Phi}) = \log p(\mathbf{y}_i|\mathbf{z}_i = \mathbf{e}_t, \mathbf{\Phi}) \tag{57}$$

representing the $t$'th component of $U_t(\mathbf{y}_i|\mathbf{\Phi})$ defined according to (40), we have

$$\nabla_{\mathbf{\Phi}_y} U^T(\mathbf{y}_i|\mathbf{\Phi}) = \begin{bmatrix} \nabla_{\mathbf{\Phi}_{y,1}} U_1(\mathbf{y}_i|\mathbf{\Phi}) & 0 & \cdots & 0 \\ 0 & \nabla_{\mathbf{\Phi}_{y,2}} U_2(\mathbf{y}_i|\mathbf{\Phi}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \nabla_{\mathbf{\Phi}_{y,N_{t,max}}} U_{N_{t,max}}(\mathbf{y}_i|\mathbf{\Phi}) \end{bmatrix}, \tag{58}$$

which has

$$M_y = \sum_{t=1}^{N_{t,max}} M_{y,t} \tag{59}$$

rows and $N_{t,max}$ columns. Each of the blocks along the main diagonal in (58) is the gradient of the individual components of $U_t(\mathbf{y}_i \mid \mathbf{\Phi})$ taken with respect to the corresponding parameter vector $\mathbf{\Phi}_{y,t}$.

Similarly, let $\mathbf{\Phi}_z = (\mathbf{\Phi}_{z,1}, \mathbf{\Phi}_{z,2}, \ldots, \mathbf{\Phi}_{z,N_{t,max}})$ with $\mathbf{\Phi}_{z,t}$ a vector of dimension $M_{z,t}$ describing the parameters associated with target $t$ for $t = 1, 2, \ldots, N_{t,max}$. Then with

$$V_{1,t}(\mathbf{\Phi}) = \log p(\mathbf{z}_i = \mathbf{e}_t \mid \mathbf{\Phi}) \tag{60}$$

representing the $t$'th component of $V_1(\mathbf{\Phi})$ defined by (42), we have

$$\nabla_{\mathbf{\Phi}_z} V_1^T(\mathbf{\Phi}) = \begin{bmatrix} \nabla_{\mathbf{\Phi}_{z,1}} V_{1,1}(\mathbf{\Phi}) & 0 & \cdots & 0 \\ 0 & \nabla_{\mathbf{\Phi}_{z,2}} V_{1,2}(\mathbf{\Phi}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \nabla_{\mathbf{\Phi}_{z,N_{t,max}}} V_{1,N_{t,max}}(\mathbf{\Phi}) \end{bmatrix}, \tag{61}$$

which consists of

$$M_z = \sum_{t=1}^{N_{t,max}} M_{z,t} \tag{62}$$

rows and $N_{t,max}$ columns. Each column represents the gradient of individual components of $V(\mathbf{\Phi})$ with respect to the corresponding parameter vectors $\mathbf{\Phi}_{z,t}$ for $t = 1, 2, \ldots, N_{t,max}$. The *doubleton* term for $\mathbf{z}_i$ and $\mathbf{z}_j$ requires a slightly more complicated approach. The term $V_2(\mathbf{\Phi})$ is given by

$$V_2(\mathbf{\Phi}) = \begin{bmatrix} V_2(\mathbf{e}_0, \mathbf{e}_0|\mathbf{\Phi}) & V_2(\mathbf{e}_0, \mathbf{e}_1|\mathbf{\Phi}) & \cdots & V_2(\mathbf{e}_0, \mathbf{e}_{N_{t,max}}|\mathbf{\Phi}) \\ V_2(\mathbf{e}_1, \mathbf{e}_0|\mathbf{\Phi}) & V_2(\mathbf{e}_1, \mathbf{e}_1|\mathbf{\Phi}) & \cdots & V_2(\mathbf{e}_1, \mathbf{e}_{N_{t,max}}|\mathbf{\Phi}) \\ \vdots & \vdots & \ddots & \vdots \\ V_2(\mathbf{e}_{N_{t,max}}, \mathbf{e}_0|\mathbf{\Phi}) & V_2(\mathbf{e}_{N_{t,max}}, \mathbf{e}_1|\mathbf{\Phi}) & \cdots & V_2(\mathbf{e}_{N_{t,max}}, \mathbf{e}_{N_{t,max}}|\mathbf{\Phi}) \end{bmatrix}, \tag{63}$$

where

$$V_2(\mathbf{e}_t, \mathbf{e}_s|\mathbf{\Phi}) = p(\mathbf{z}_i = \mathbf{e}_t, \mathbf{z}_j = \mathbf{e}_s|\mathbf{\Phi}). \tag{64}$$

Thus,

$$\hat{\mathbf{z}}_i^{(p)\,T} V_2(\mathbf{\Phi}) \hat{\mathbf{z}}_j^{(p)} = tr\{\mathbf{K}_{ij} V_2(\mathbf{\Phi})\}, \tag{65}$$

where $tr\{\cdot\}$ represents the trace of a matrix and

$$\mathbf{K}_{ij} = \hat{\mathbf{z}}_i^{(p)} \hat{\mathbf{z}}_j^{(p)\,T}. \tag{66}$$

The other difficulty in maximizing the $Q$-function involves the partition function term $\log \mathcal{Z}$. As an approximation, the maximization is performed using only the energy function terms of the MRF ignoring $\log \mathcal{Z}$. Typically, a more convenient approximation is to assume that $\mathbf{\Phi}_z$ is known by selecting reasonable parameters.

## IV Results

In this section the EM tracking algorithm is applied to the scenario discussed in [8]. In this problem, two measurement types are received and used to track two targets that cross in both state and measurement space. Receive measurements consist of both [ bearing, frequency ] reports from three different sensors and [ doppler difference, time delay difference ] measurements between sensor pairs. For this problem, the tracking of the two crossing targets is examined and the EM tracking results are described and compared to the results from two other tracking filters presented in [8].

More specifically, the problem consists of the tracking of two crossing targets using measurements from a number of passive sensors. The targets are assumed to follow straight-line trajectories with no process noise, and the true target trajectories are shown in Figure 2 for a six-hour long scenario. The target initial positions (longitude, latitude) are at $(1.0°, 3.95°)$ and $(1.0°, 4.05°)$ moving at a speed of 6 knots and traveling on courses of 100° and 80°, respectively. The initial target positions are marked by triangles, final target positions are marked by diamonds, and the sensor positions are again marked by squares.

For this example each target is assumed to radiate energy at one narrow-band frequency and the corresponding frequency is coupled with the targets speed. In [8] the target state space relationships are summarized by the following equations

$$\begin{array}{lcll}
\dot{L} = \frac{V \cos C}{60} & + & \nu_L & \text{(latitude)} \\
\dot{M} = \frac{V \sin C}{60 \cos L} & + & \nu_M & \text{(longitude)} \\
\dot{C} = 0 & + & \nu_C & \text{(course)} \\
\dot{V} = (FK - V)/c_t & + & \nu_V & \text{(speed)} \\
\dot{K} = 0 & + & \nu_K & \text{(speed-frequency coupling)} \\
\dot{F} = 0 & + & \nu_F & \text{(propulsion frequency),}
\end{array} \tag{67}$$

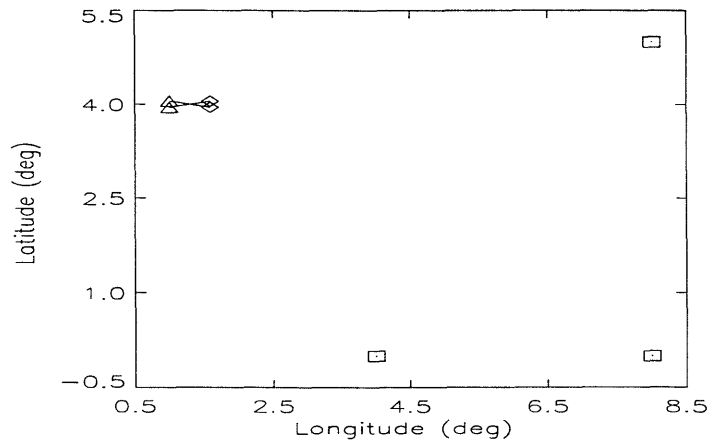


Figure 2: True Trajectories

where $c_t$ is a time constant. The above state equations correspond to the state vector $\mathbf{\Phi}_{t,k}$ for target $t$ at time $k$ given by

$$\mathbf{\Phi}_{t,k} = [L_k, M_k, C_k, V_k, K_k, F_k]^T . \tag{68}$$

The state equation for the above relationships is written as

$$\dot{\mathbf{\Phi}}_{t,k} = F(\mathbf{\Phi}_{t,k}) + \boldsymbol{\nu}_k. \tag{69}$$

The three sensors shown in Figure 2 report bearing-frequency measurements that have the form

$$\mathbf{y}_{n,k} = \begin{bmatrix} b_{n,k} \\ f_{n,k} \end{bmatrix} = \begin{bmatrix} \beta_n(L_k, M_k) \\ F_k \left[1 + V_k \cos \alpha_n(L_k, M_k, C_k)/c_s\right] \end{bmatrix} + \mathbf{w}_{n,k}, \tag{70}$$

where $\beta_n(L_k, M_k)$ and $\alpha_n(L_k, M_k, C_k)$ are, respectively, a targets bearing and aspect angle with respect to sensor $n$, and $c_s$ is the speed of sound. The measurement noise is assumed to be zero-mean, white, and Gaussian with covariance

$$E\{\mathbf{w}_{n,k}\mathbf{w}_{n,j}^T\} = \begin{bmatrix} \sigma^2_{b_{n,k}} & 0 \\ 0 & \sigma^2_{f_{n,k}} \end{bmatrix} \delta(k,j). \tag{71}$$

Along with the individual single-sensor bearing-frequency measurement from each of the three sensors, measurements can be constructed from pairs of sensors. These measurements are formed by cross-correlation of the data between two sensors, and the reported measurements are time delay difference and Doppler frequency difference. These measurements take the form

$$\mathbf{y}_{n,m,k} = \begin{bmatrix} \Delta\tau_{n,m,k} \\ \Delta f_{n,m,k} \end{bmatrix} = \begin{bmatrix} \left[r_n(L_k, M_k) - r_m(L_k, M_k)\right]/c_s \\ F_k V_k \left[\cos \alpha_n(L_k, M_k, C_k) - \cos \alpha_m(L_k, M_k, C_k)\right]/c_s \end{bmatrix} + \mathbf{w}_{n,m,k}, \tag{72}$$

where $r_n(L_k, M_k)$ is the targets range from sensor $n$. The measurement noise is assumed to be zero-mean, white, and Gaussian with covariance

$$E\{\mathbf{w}_{n,m,k}\mathbf{w}_{n,m,j}^T\} = \begin{bmatrix} \sigma^2_{\Delta\tau_{n,m,k}} & 0 \\ 0 & \sigma^2_{\Delta f_{n,m,k}} \end{bmatrix} \delta(k,j). \tag{73}$$

These measurements are also assumed to be independent from the individual bearing/frequency measurements reported by each sensor. Synthetic measurements were created for the six-hour scenario and are reported at five-minute intervals throughout this period. Measurement noise values are

$$\begin{array}{lcl} \sigma_{b_{n,k}} & = & 5^\circ \\ \sigma_{f_{n,k}} & = & 80 \text{ mHz} \\ \sigma_{\Delta\tau_{n,m,k}} & = & 3.6 \text{ s} \\ \sigma_{\Delta f_{n,m,k}} & = & 4 \text{ mHz}. \end{array} \tag{74}$$

The probability of detection for the true measurements is given as $P_d = 0.7$ for all times and all sensors. The number of clutter induced false alarms is assumed to have a Poisson distribution and the placement of the corresponding measurements (once the number is known) is uniformly distributed about the measurement region. The clutter density parameters are

$$\begin{array}{lcll} \lambda_{b,f} & = & 0.25/\text{deg} \cdot \text{Hz} & \text{in bearing/frequency space} \\ \lambda_{\Delta\tau,\Delta f} & = & 0.25/\text{sec} \cdot \text{Hz} & \text{in } \Delta\tau/\Delta f \text{ space.} \end{array} \tag{75}$$

As in [8], the initial target position uncertainty for each target is assumed to be uncorrelated with standard deviations of 2 nautical miles in $x$ and $y$ position, 10° in bearing, and 1.5 knots in speed. These standard deviations indicate that the target tracks have been established previously, and are being tracked accurately before their trajectories cross.

A constant velocity target motion model used to predict the target state one step ahead from time $k-1$ to time $k$ is given by

$$\mathbf{\Phi}_{t,k} = \mathbf{\Phi}_{t,k-1} + \dot{\mathbf{\Phi}}_{t,k-1}\Delta_{k-1,k} + \boldsymbol{\nu}_k, \tag{76}$$

where the target state vectors $\mathbf{\Phi}_{t,k}$ and $\mathbf{\Phi}_{t,k-1}$ are given by (68) and the velocity components $\dot{\mathbf{\Phi}}_{t,k-1}$ are given by (67). The innovations noise component $\boldsymbol{\nu}_k$ values used are

$$\begin{array}{lcl} \nu_C & = & 0.2^\circ \\ \nu_V & = & 0.2 \text{ knots} \\ \nu_F & = & 0.01 \text{ mHz}. \end{array} \tag{77}$$

Tracking results from various algorithms are shown in Figures 3 through 6. In each figure the true target position (longitude, latitude) are shown by the straight lines, with diamonds marking the

hour time intervals. The estimated target positions are shown also with solid circles marking the corresponding hour time intervals. The $2-\sigma$ confidence ellipse is shown for each target estimate for two hour intervals. The tracking algorithms used are the extended Kalman filter with true associations, the nearest-neighbor filter, the Probabilistic Data Association (PDA) filter, and the EM tracking algorithm. The probability that measurements fall within the validation gate are set to be 0.9999, and the target state initial conditions are the same for each filter type.

Figure 3 shows the resulting target track estimates using a first-order extended Kalman filter assuming that the true target-to-measurement associations are known. Since true associations are known, this result provides the most accurate performance achievable with this set of data.

Figure 4 shows the result of tracking the two targets using a nearest-neighbor filter described in [35]. In this approach the validated measurement $\mathbf{y}_{k,i}$ that is *closest* to the predicted measurement, $\hat{\mathbf{y}}_{k/k-1,i}$, for a target is used to update the state estimate for that target. The distance measure that is used to find the *closest* target is the weighted norm of the innovation, which is

$$l_{t,i}^2(\mathbf{y}_{k,i}) = \left[\mathbf{y}_{k,i} - \hat{\mathbf{y}}_{k/k-1,i}\right]^T \mathbf{S}_{t,i}^{-1} \left[\mathbf{y}_{k,i} - \hat{\mathbf{y}}_{k/k-1,i}\right], \tag{78}$$

where $\mathbf{S}_{t,i}$ is the covariance matrix of the innovation component. The nearest-neighbor approach is one that makes use of *hard* decisions, and the results of Figure 4 show that incorrect decisions near the crossing target trajectory quickly tend to degrade the track accuracy, eventually leading to the two separate tracks merging and the resulting error large enough such that the true target positions are far from the estimated tracks. The uncertainty ellipse in this approach is sometimes smaller than that of the extended Kalman filter estimate since a targets true measurement is not necessarily the measurement with the *closest* distance to the predicted target track. By accepting such false alarm measurements as correct the uncertainty ellipse decreases more rapidly than when the true measurements are used.

Figure 5 shows the result obtained with the Probabilistic Data Association (PDA) filter described in [7, 35] using a Poisson clutter model with the clutter density parameters described previously. In performing the estimate of the target tracks for time $k$ using measurements from the three sensors and three sensor pairs, the PDA approach is used for measurements from each sensor (or sensor pair) individually. This is necessary since the association weights for a target must sum to one, and this only holds if the measurements come from the same sensor. This allows for the possibility that

different estimation results will occur depending upon the *order* that the sensors are processed. For an individual target being tracked with the PDA approach, the measurements about the predicted measurement are treated as one true measurement plus clutter measurements. Since the interfering target produces measurements in the same gate validation area, the PDA approach handles this model mismatch by computing a weighted average of the two true target tracks. The confidence ellipse is larger than the extended Kalman filter estimate, reflecting the uncertainty in the target estimation: however, the approach cannot recover the true tracks since the state estimates are too far away from the true target states.

Figure 6 shows the results obtained from using the EM tracking approach. In this approach the measurements from all sensors and sensor pairs are used simultaneously to estimate the target states at time $k$. This occurs since for each measurement the associations are probability vectors across the possible target and false alarm events, while the interaction between individual measurements are accounted for by the energy function $V(\mathbf{z}_k|\Phi_k)$ and the constraints it imposes. In the E-step; the values of $U(\mathbf{y}_{k,i}|\Phi|k)$ and $V(\mathbf{z}_k|\Phi_k)$ are calculated in the same manner as in the previous example. The constraints use to calculate $V(\mathbf{z}_k|\Phi_k)$ are that no two measurements from the same sensor can come from the same target, and that the number of false alarms from any particular sensor is equal to the expected number of false alarm measurements predicted for that sensor given the gate probability, detection probability, and the target validation gate volumes. As shown in Figure 6, the resulting target state estimates are very close to that of the extended Kalman filter estimate. The resulting confidence ellipses are larger, however, resulting from the uncertainty in the association probabilities. The result is highly effective in dealing with this particular crossing target scenario. In [8], the Joint Probabilistic Data Association (JPDA) algorithm is discussed, which is similar to the PDA approach except that JPDA replaces the target-to-measurement probability estimates for independent targets with estimates calculated using the joint measurement probabilities for all targets. Using the JPDA approach, the probabilities of the joint measurement events must be calculated and leads to exponential growth when enumerating these events. The results of the EM approach are comparable to that of the JPDA algorithm; however, we note that the explicit enumeration of the joint probabilities is not required and offers computational advantage as the number of ambiguous measurements increases.

## V Conclusion

A novel approach for recursively estimating the states of a multiple number of targets using data from multiple sensors based upon the EM procedure has been presented which offers an attractive alternative to existing multi-target tracking procedures. This estimation problem is treated as an *incomplete* data problem, where the associations between targets and measurements are considered to be the unobserved data, and the association process in modeled as a second-order Markov Random Field which reduces the complexity of the calculations in the E-step. With known associations, this EM approach reduces to the iterated extended Kalman filter, while with unknown associations the algorithm provides an approximation to the MAP solution of the incomplete data problem for each new set of measurement data. Detailed experimental results have been provided to demonstrate the effectiveness of this approach, and the results compare favorable to known algorithms reported in the literature. The algorithm is robust in that it provides a theoretical basis for incorporating constraint information into the problem, and while the examples here have had a $1-1$ target to measurement correspondence, more complex constraints can easily be realized. This new EM-based scheme is used here to compute the state updates as part of a Bayesian recursive estimation approach, and can be used either as a stand-alone procedure with initial target estimates provided for the algorithm, or as part of a more comprehensive tracking scheme.

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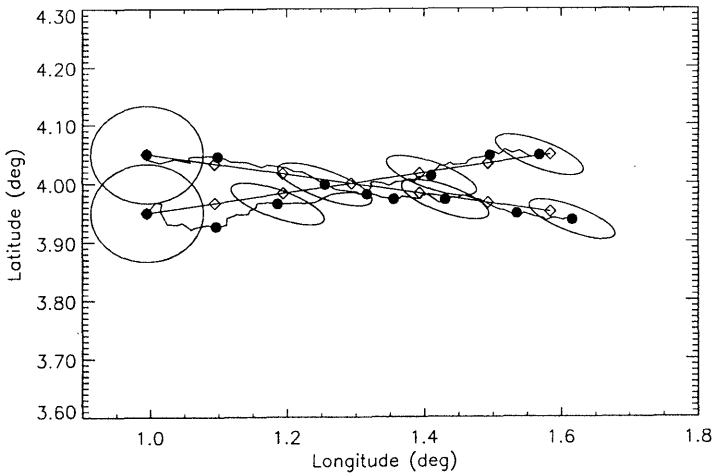


**Figure 3: Extended Kalman Filter Estimate w/ Known Assoc.**

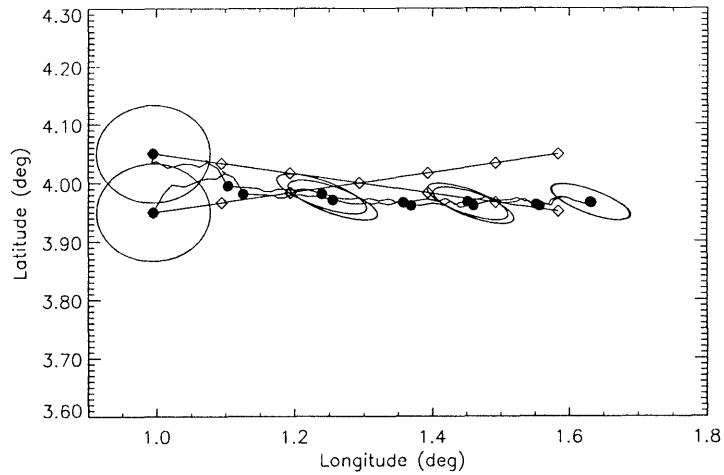


**Figure 4: Tracking Estimate w/ Nearest Neighbor Assoc.**



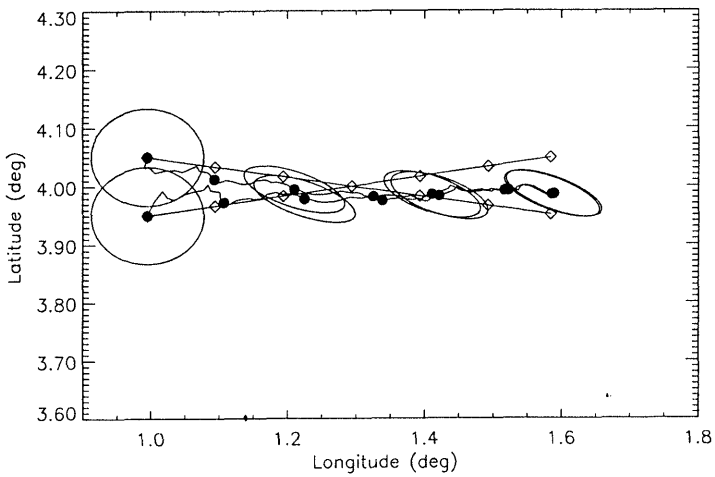


**Figure 5: Tracking Estimate w/ PDA Associations**

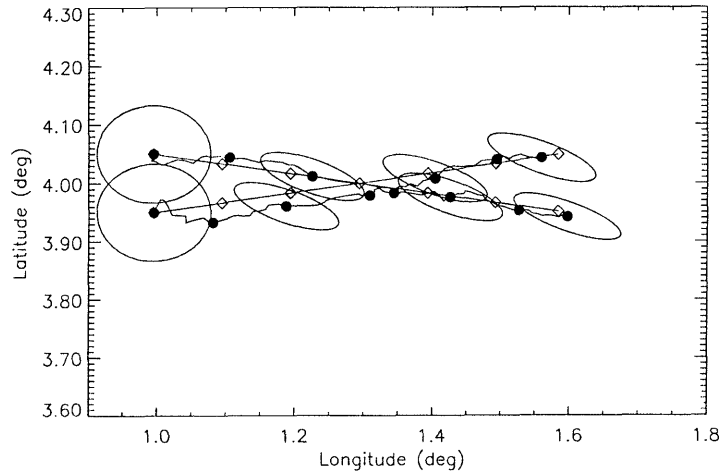


**Figure 6: Tracking Estimate w/ EM Associations**