LMS Electrical Filters to Reduce Intersymbol Interference in Direct Detection Optical Systems

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Abstract

Intersymbol interference in direct detection optical systems can limit the channel spacing in frequency division multiplexing systems, and limit data rates and transmission distances in long distance transmission systems. We present minimum mean-square-error electrical filters to compensate for optical intersymbol interference. The performance of these filters is compared with a matched filter and the performance of an electrical filter with a rectangular response, for M-ary amplitude modulation.

1 Introduction

We consider direct detection optical systems where the receiver front-end is composed of an optical filter, a photodetector and a low-pass electrical filter. In frequency division multiplexing (FDM) systems, an optical filter is placed before the photodetector to suppress adjacent channel interference. When the channel spacing gets small, the filter bandwidth needs to be narrow. This introduces intersymbol interference (ISI) for the desired signal. If this distortion is not equalized or compensated for, it can be a limiting factor in spacing the channels.

In long distance transmission, the fiber itself plays the role of an optical filter as it disperses the pulse. The amount of dispersion is proportional to transmission distance and to the square of the baud rate. This impairment affects the system in scaling to higher data rates and longer distances.

The system model is described in Section 2. In this paper we are concerned with reducing the effect of intersymbol interference by using an electrical filter located after the photodiode. The non-linearity of the photodiode makes the problem non-classical [5, 4]. Similar approaches have been presented in [6, 1]. The originality of this work is to design optimal filters from basic principles. Examples of a Fabry-Perot interferometer for optical filtering in a FDM system and a rectangular pulse passing through a dispersive fiber in long-distance transmission are presented in Section 3. The performance of these filters is discussed in Section 4.

2 Model

The receiver front-end is composed of an optical filter, a photodetector and a low-pass electrical filter as shown in Figure 1. The received signal after the optical filter is

$$x(t) \quad = \quad \sqrt{2}\,\mathrm{Re}\,\{\sum_k a_k p(t-kT)e^{j2\pi f_c t}\}, \qquad (1)$$

where $p(t)$ is the complex envelope of the received pulse taking into account the transmit pulse and the channel response, a_k (taken as real here) is the kth transmit symbol, and f_c is the carrier frequency. We assume pulse amplitude modulation without chirping, e.g., by using an external modulator. The sampler after the electrical filter operates at baud rate, $1/T$. The square

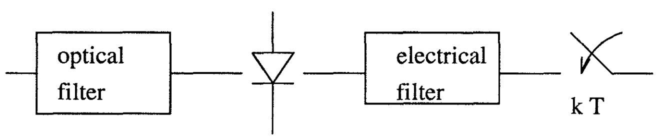

Figure 1: The receiver front-end includes an optical filter, a photodiode and a low-pass electrical filter followed by a sampler.

of the signal envelope appears at the output of the photodiode, together with noise. The noise is modelled

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as additive white Gaussian noise with spectral density $\frac{N_o}{2}$ at the input to the electrical filter. This models the thermal noise from the electronics as the dominant noise. The signal then passes through a low-pass electrical filter and is sampled at time kT where k is an integer.

3 Filter construction

We start with the signal at the detector output, which we write as $r(t) = |x(t)|^2 + n(t)$, where $n(t)$ is the additive white Gaussian noise. In taking the magnitude square, we drop the double frequency terms since they do not appear at the photodetector output. We can write the post-detection received signal as

$$r(t) = \sum_i \sum_j a_i a_j p(t - iT) p^*(t - jT) + n(t), \quad (2)$$

where $p^*(t)$ is the complex conjugate of $p(t)$. Since the left hand side of the above equation is real, we can take the real part of the right hand side. We can rewrite the above equation as

$$r(t) = \sum_i \sum_j a_i a_j p_{i,j}(t) + n(t), \quad (3)$$

where $p_{i,j}(t) = \mathrm{Re}\{p(t - iT)p^*(t - jT)\}$.

Without loss in generality, we can design the electrical filter for time $t = 0$, i.e., we consider $p_{0,0}(t)$ as the desired signal and the other waveforms $p_{i,j}(t)$'s as intersymbol interference waveforms.

$$r(t) = \underbrace{a_0 a_0 p_{0,0}(t)}_{\text{signal}} + \underbrace{\sum_{i\neq 0 \text{ or } j\neq 0} a_i a_j p_{i,j}(t)}_{\text{ISI}} + \underbrace{n(t)}_{\text{noise}} \quad (4)$$

One sees that the signal and ISI terms lie in the space spanned by the $p_{i,j}(t)$'s. As the noise is white, a sufficient statistics is obtained by projecting $r(t)$ on that space, by using correlators or matched filters. In effect, we transform the one dimensional non-linear system into a multi-dimensional linear system. For a fixed value of k, the $p_{i,i+k}(t)$'s are time-translations of each other, i.e., $p_{i,j}(t) = p_{0,j-i}(t - iT)$. It follows that the number of matched filters needed is no more than the dimension of the space spanned by the $p_{0,k}(t)$'s. We define the *ISI space*, I, as the space spanned by $p_{i,j}(t)$'s without $p_{0,0}(t)$, the desired signal. We define the *signal space* to be the space spanned by I and $p_{0,0}$(t). In this work we limit ourselves to least-mean-square linear structures. Zero-forcing, decision-feedback and maximum likelihood structures will be reported elsewhere.

Let $r(t)$ pass through the electrical filter. The sampled output at time $t = 0$ is

$$Y_0 = \sum_i \sum_j a_i a_j \int p_{i,j}(\sigma)\, h(\sigma) d\sigma + N, \quad (5)$$

where $h(t)$ is the *time-reversed* impulse response of the electrical filter and N is the noise at the sampled output. We want the electrical filter to minimize the mean-square-error at the sampled output. A filter response $h(t)$ can be decomposed into a component in the signal space and a component orthogonal to the signal space. The orthogonal component allows only noise through, so we set this component to zero, leaving the signal space as the relevant space to consider in designing the filter. We let the number of transmitted symbols be finite, but large so that it approximates well the infinite case as far as the reception of symbol a_0 is concerned. The corresponding signal space is a finite dimensional space, say, spanned by m $p_{i,j}(t)$'s including $p_{0,0}(t)$. Let n be the dimension of this space. We call this space S_n and reindex the $p_{i,j}(t)$ into $p'_k(t)$. Let $\psi_k(t)$'s be a set of linearly independent waveforms that span S_n. Then the filter response can be written as

$$h(t) = \sum_{k=1}^{n} h_k \psi_k(t). \quad (6)$$

At time $t = 0$, the output of the filter is

$$Y_0 = \sum_k b_k \langle p'_k(t), h(t)\rangle + N, \quad (7)$$

where the $a_i a_j$'s are reindexed as b_k's and $\langle x(t), y(t)\rangle \equiv \int x(t)y(t)dt$. Let the error be the difference between the filter output Y_0 and the desired output a_0^2. The mean-square-error is $M = \mathrm{E}[(Y_0 - a_0^2)^2]$. Using the above values for Y_0 and $h(t)$,

$$\begin{aligned} M &= \mathrm{E}[(\sum_{i=1}^{n}\sum_{j=1}^{m} h_i \langle \psi_i(t), p'_j(t)\rangle b_j - a_0^2)^2] \\ &\quad + \frac{N_0}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} h_i \langle \psi_i(t), \psi_j(t)\rangle h_j, \end{aligned} \quad (8)$$

where $\mathrm{E}[N^2] = \frac{N_o}{2}\langle h(t), h(t)\rangle$. Using matrix notation,

$$\begin{aligned} M &= \mathrm{E}[(\mathbf{h}^T\mathbf{W}\mathbf{b} - a_0^2)(\mathbf{b}^T\mathbf{W}^T\mathbf{h} - a_0^2)] \\ &\quad + \frac{N_o}{2}\mathbf{h}^T\mathbf{A}\mathbf{h}, \end{aligned} \quad (9)$$

where $\mathbf{W}_{i,j} = \langle \psi_i(t), p'_j(t)\rangle$, $\mathbf{A}_{i,j} = \langle \psi_i(t), \psi_j(t)\rangle$, $\mathbf{h}_i = h_i$ and $\mathbf{b}_i = b_i$. The bold face letters are matrices or column vectors. We rearrange the above quantities

$$M = \mathbf{h}^T(\overline{\mathbf{W}\mathbf{b}\mathbf{b}^T\mathbf{W}^T} + \frac{N_o}{2}\mathbf{A})\mathbf{h} \\ -2\mathbf{h}^T\overline{\mathbf{W}\mathbf{b}a_0^2} + \overline{a_0^4}, \quad (10)$$

where $\overline{x}$ denotes the expectation of x. Let $\mathbf{B} = 2(\overline{\mathbf{W}\mathbf{b}\mathbf{b}^T\mathbf{W}^T} + \frac{N_o}{2}\mathbf{A})$.

In Appendix A, we show that $\mathbf{B}$ is a positive definite matrix even when $\frac{N_o}{2} = 0$. The minimum of the mean-square-error occurs at

$$\mathbf{h}_o = \mathbf{B}^{-1}\mathbf{c}, \quad (11)$$

where $\mathbf{c} = 2\overline{\mathbf{W}\mathbf{b}a_0^2}$. Substitute $\mathbf{h}_o$ into M, the mean-square-error at the minimum is

$$M_o = \overline{a_0^4} - \frac{1}{2}\mathbf{c}^T\mathbf{B}^{-1}\mathbf{c}. \quad (12)$$

If a zero-forcing filter exists, it satisfies $\mathbf{h}^T\mathbf{W}\mathbf{b} = a_0^2$ for all $\mathbf{b}$ and it clearly minimizes M when $N_o = 0$. Thus $\mathbf{h}_o$ is the zero-forcing filter (if it exists) when $N_o = 0$.

3.1 Fabry-Perot interferometer

From [2, 3], the complex envelope of the impulse response of the Fabry-Perot interferometer is well approximated by $\frac{1}{\tau T}e^{-\frac{t}{\tau T}}$ for $t \geq 0$, and $\tau T = \frac{1}{\pi F}$ where F is the 3 dB bandwidth of the filter. Letting the transmit pulse equal to 1 in the interval [0,T] and 0 otherwise, the received pulse envelope is

$$p(t) = \begin{cases} 1 - e^{-\frac{t}{\tau T}} & 0 \leq t < T \\ (e^{\frac{1}{\tau}} - 1)e^{-\frac{t}{\tau T}} & t \geq T \end{cases} \quad (13)$$

The $p_{i,j}(t)$'s can be expressed as linear combinations of time translations of only $p_{0,0}(t)$ and $p_{0,-1}(t)$, due to their exponential tails. In Figure 2, we plot the impulse and frequency responses of the electrical filter when SNR = 20 dB and $\tau = 1$, as well as the eye diagram. We let SNR $= \frac{\langle p_{0,0},h\rangle^2}{\frac{N_o}{2}\langle h,h\rangle}$. Note the relatively low D.C. response of the filter.

3.2 Dispersive channel

Let the optical filter model a dispersive channel with frequency response $e^{-j\alpha f^2}$, where

$$\alpha = \frac{\pi R^2 \lambda^2 DL}{c}. \quad (14)$$

R is the bit rate and f is normalized to the bit rate. D is the dispersion of the fiber in units of (ps/km$\cdot$ nm),

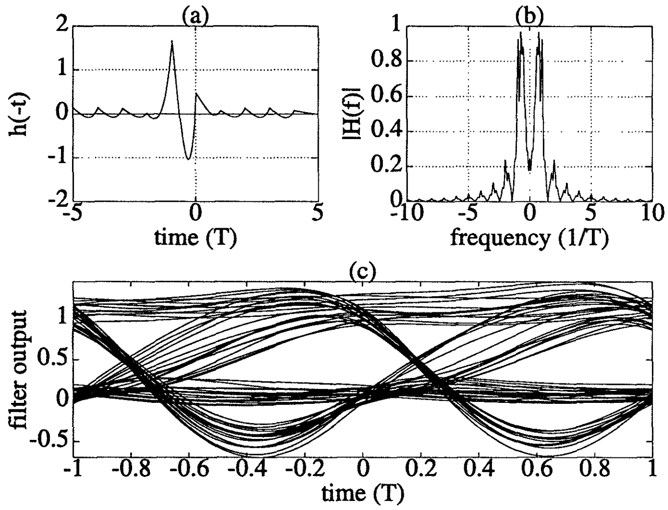

Figure 2: The optical filter is a Fabry-Perot interferometer. The plots show the impulse response (a) and the magnitude of the frequency response (b) of the optimal electrical filter when $\tau = 1$. The eye diagram is shown in (c).

λ is the carrier wavelength, c is the speed of light, and L is the length of the fiber. The transmit pulse is a rectangular pulse equal to 1 for t in $[0, T]$ and zero otherwise. In Figure 3, we plot the impulse and frequency responses of the electrical filter when SNR = 20 dB and $\alpha = 1$, as well as the eye diagram.

4 Performance evaluation

In the following we consider four different performance curves for situations in Sections 3.1 and 3.2. Extension to multilevel signaling is considered for four and eight level signaling. The bit rate is the same. The baud rate is decreased by $\log_2 M$ as compared to the two level case, where M is the number of signaling levels. The transmit symbols $s_0, \ldots, s_{M-1}$ are $s_i = \sqrt{\frac{i}{M-1}}$ for i between 0 and $M-1$. The s_i's have equal probability of being transmitted. With these choices, the average energy per bit and power level stay constant with M.

We define the eye opening to be the minimum vertical opening in an eye diagram at the sampling time. When the filter is normalized to have unit energy, we refer to the eye opening as *normalized eye opening.* When there is only a single pulse transmission, the optimal filter is a filter matched to the electrical pulse $p_{0,0}(t)$. The normalized eye opening for this case appears in the matched filter curve (curve M).

With the mean-square-error formulation, the eye

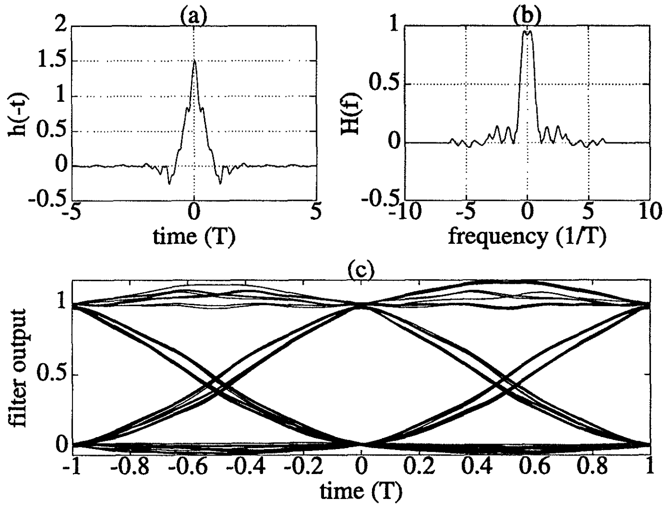

Figure 3: Dispersive channel and rectangular transmit pulse: plots (a) and (b) show the impulse and frequency responses of the optimal electrical filter when SNR = 20 dB and $\alpha = 1$, along with the eye diagram in plot (c).

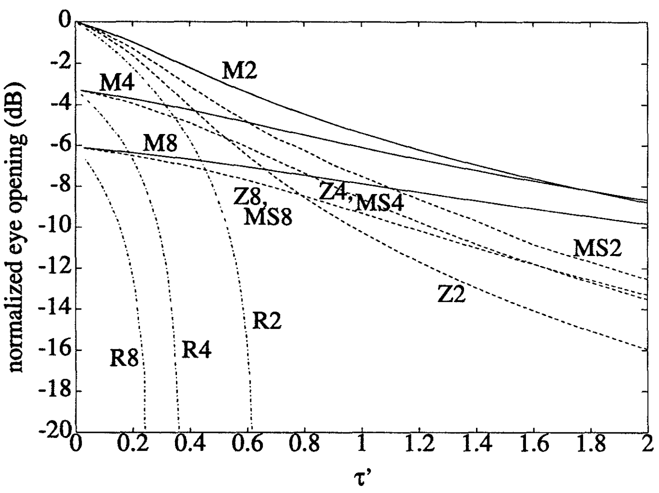

Figure 4: The optical filter is a Fabry-Perot interferometer. The plots have the following labels: (M) matched filter curve, (Z) LMS filter with no noise and (MS) LMS filter with maximum eye opening, (R) rectangular filter and (2,4,8) signaling levels.

opening depends on the amount of noise present. When the noise is set to zero (curve Z), the filter is also a zero-forcing filter, if it exists. As the noise spectral density increases, the normalized eye opening is observed to increase and then to decrease. There is a noise level that corresponds to the maximum normalized eye opening (curve MS). In the case of M-ary signaling with $M = 4$ and $M = 8$, this noise level is very small, and the MS curves almost coincide with the Z curves. We also consider rectangular filters where the impulse response is equal to $\frac{1}{\sqrt{T}}$ in the interval $[0, T]$ and zero otherwise for the Fabry-Perot case, and shifted by $-\frac{T}{2}$ for the dispersion case (curve R).

In Figure 4, we plot curves for the case of Fabry-Perot interferometer (Section 3.1). For the vertical axis, the units are in dB using the definition $10 \log_{10}$(normalized eye opening). The zero dB point is the eye opening for the binary case without ISI. Let $\tau' = \frac{\tau T}{T'}$ where T' is the inverse of the bit rate. From Section 3.1, the quantity τT is inversely proportional to the filter bandwidth and has units of (1/Hz). Thus the bandwidth efficiency (in $\frac{\mathrm{b/s}}{\mathrm{Hz}}$) increases with τ'. The vertical offset when $\tau' = 0$ is due to a $\frac{\sqrt{\log_2 M}}{M-1}$ factor to keep the bit rate and average energy per bit constant for different M. As τ' increases the filter bandwidth becomes smaller and there is more intersymbol interference. The eye openings close for the rectangular filter for large values of τ'. There is no advantage to multilevel signaling for the rectangular filter, same conclusion as in [2]. On the other hand, with equalization such as using the zero-forcing filter, the performance is much better than the rectangular filter. In terms of multilevel signaling, there is advantage to change to four level signaling from two level signaling in the case of the zero-forcing filter for large values of τ'.

In Figure 5, we plot curves for the dispersive channel case (Section 3.2). The vertical axis shows the normalized eye opening in dB. The parameter α for the horizontal axis is a dispersion parameter (14). Although the optical channel is lossless, the matched filter curve decreases with α, because dispersion reduces the electrical energy of the pulse. We see that multilevel signaling is advantageous at high bit rates when a zero-forcing filter is used.

A Appendix

We show that the two matrices in $\mathbf{B} = 2(\mathbf{W}\overline{\mathbf{b}\mathbf{b}^T}\mathbf{W}^T + \frac{N_o}{2}\mathbf{A})$ are positive definite. Since $\psi_i(t)$'s are linearly independent waveforms, $\mathbf{x}^T\mathbf{A}\mathbf{x} = \int(\sum_i x_i\psi_i(t)dt)^2 > 0$ for any non-zero $\mathbf{x}$, and $\mathbf{A}$ is positive definite. Next we show that $\mathbf{x}^T\mathbf{W}\overline{\mathbf{b}\mathbf{b}^T}\mathbf{W}^T\mathbf{x} > 0$ for $\mathbf{x} \neq 0$. For a given $\mathbf{b}$ and $\mathbf{x}$,

$$\mathbf{x}^T\mathbf{W}\mathbf{b} = \int (\sum_i x_i\psi_i(t))(\sum_j b_j p'_j(t))dt \quad (15)$$

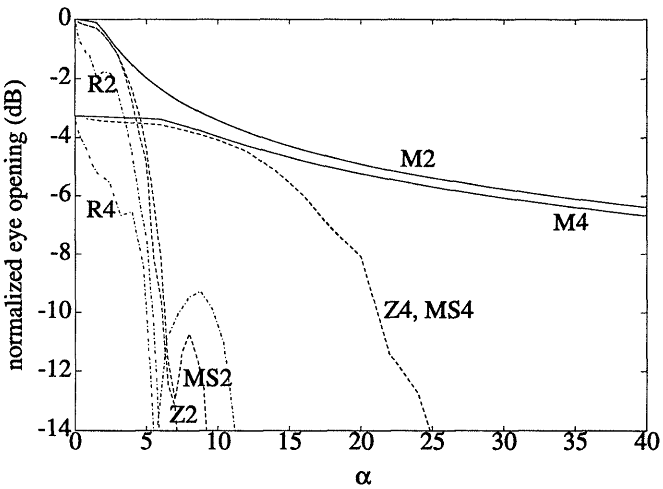

Figure 5: Dispersive channel with rectangular transmit pulses. The plots have the following labels: (M) matched filter curve, (Z) LMS filter with no noise and (MS) LMS filter with maximum eye opening, (R) rectangular filter and (2,4) signaling levels.

where $\sum_i x_i\psi_i(t)$ is in S_n. We will show that S_n is spanned by the set $\{\sum_j b_j p'_j(t)\}$, and thus the right hand side of (15) is non-zero for at least one $\mathbf{b}$. For one symbol transmission, the $p_{i,i}(t)$'s are clearly spanned by $\sum_j b_j p'_j(t)$'s. We need to check that the $p_{i,j}(t)$'s are spanned as well for $i \neq j$. For two symbol transmissions, the output is $a_i a_i p_{i,i}(t) + 2a_i a_j p_{i,j}(t) + a_j a_j p_{j,j}(t)$ and the new component $p_{i,j}(t)$ is spanned. Hence $\mathbf{x}^T\mathbf{W}\mathbf{b}$ is non-zero for some $\mathbf{b}$ when $\mathbf{x} \neq 0$, and $\mathbf{x}\mathbf{W}\mathbf{b}\mathbf{b}^T\mathbf{W}^T\mathbf{x} > 0$ for $\mathbf{x} \neq 0$ as desired.

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