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Stable Teleoperation with Scaled Feedback

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Abstract

This paper introduces a controller design methodology for scaling force and motion for bilateral teleoperation. In order to handle the stability analysis for force-reflective teleoperation, a new concept, *structured passivity*, is developed based on μ analysis. By designing the teleoperator system as a structured passive system, the force and the motion feedback scaling ratios can be determined independently while maintaining robust stability of teleoperation. Moreover, by closing separate force loops around the master and the slave manipulators, the dynamic masking effect in bilateral teleoperation is reduced. Thus, the fidelity of force transmission can be improved. By combining the structured passive design and the local force loops, a teleoperator system with high fidelity, scaled force reflection is achieved.

1 Introduction

Current implementation of telerobotic technology for remote handling typically attempt to create a sense of *telepresence* for the human operator. This is done by offering force feedback to the master manipulator from the slave manipulator, such that the operator can "feel" what is happening at the remote site. However, in terms of improving maneuverability, there are two very different philosophies. One philosophy [Handlykken & Turner, '80] suggests that a teleoperator can do everything a person could normally do to perform a task as long as the operator has a full sense of being at the remote site. Thus, the teleoperator should be transparent to the operator with zero inertia and infinite stiffness. This idea basically sees the teleoperator as an extension of the human arm and hand. The other philosophy [Raju, '88] suggests that to improve performance, a teleoperator should have adjustable manipulation characteristics just as one would use a racquet for tennis and a bat for baseball. This idea basically sees the teleoperator as a tool instead of the human arm extension.

In addition to the fact that it is impossible to design a system with zero inertia and infinite stiffness, a transparent design will also confine the capability of a teleoperator system to that of the human operator. For instance, nobody will be able to operate a force-reflective crane. Consequently, a dexterous teleoperator system with adjustable manipulation characteristics would be preferable in order to handle a wider range of tasks in unstructured and unpredictable environments. At least, the concept of adjustable design also include the possibility of mimicking the transparent design. Moreover, through impedance adjustment, the feedback force level and the motion between the master and the slave can be scaled. Hence, a master manipulator can be easily interfaced to slave manipulators of different sizes, shapes and capacities. For example, a dexterous hand controller can be used to control a heavy capacity crane by attenuating feedback force and amplifying command motion. The same hand controller can also be used to control a micro-robot by amplifying feedback force and scaling down the command motion.

This paper presents a teleoperator controller design methodology which allows feedback force and motion to be scaled independently with different scaling factor while maintaining robust stability. In the next section, we will first review recent theoretical developments of teleoperator stability analysis based on the *structured passivity* theory. By choosing a suitable controller, the teleoperator system is structured passive and robustly stable. In § 3, the effect of controller gains on system behavior is discussed. By varying the gains of a structured passive controller in a certain way, the feedback force level and the motion are scaled either simultaneously or independently. However, the dynamics of the master and the slave could reduce the fidelity of force transmission when the scaling ratio for force feedback is too large. In § 4, local force loops around the master and the slave are introduced to reduce the dynamic masking effect. Thus, a high fidelity teleoperator system with scaled feedback is achieved by combining the structured passive design with local force loops.

2 Robust Stability of Structured Passivity Systems

During force-reflective teleoperation, the dynamics of the master and the slave manipulators are coupled with a human operator and a environment, or task. Researchers have attempted to model this dynamic interaction problem in order to develop controllers for stable teleoperation.

2.1 Passive networks

Raju ['86, '88] first introduced a two-port network model of bilateral teleoperation. In a two-port network model, a teleoperator can be visualized as a device with two ports at which power is exchanged: the *master port*, where the operator interacts with the master arm, and the *slave port*, where the end-effector manipulates the environment. This model is depicted in Fig. 1.

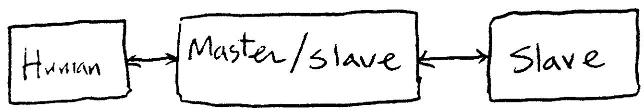

Figure 1: A teleoperator system as a two-port.

The system model is then divided up into three dynamic subsystems which interact through the two ports:

1. the human operator, who manipulates the master device of the teleoperator;
2. a teleoperator which transmits power (forces and motion) between the human operator and the remote task;
3. the task object in the remote environment that is being manipulated by the slave device of the teleoperator.

Stable teleoperation requires not only stability of the master/slave manipulator system, but stability of the "coupled" system, where the human and the task interact dynamically with the master/slave manipulator.

Raju used an impedance matrix for the teleoperator two-port. (Hannaford ['88] introduced a similar analysis using hybrid impedance/admittance two-port networks.) If we presume that the environment and the operator impose motion (velocities and positions) onto the teleoperator's slave and master, respectively, then the teleoperator imposes forces back onto the environment and human according to it's impedance dynamics. Thus, the environment and operator act as admittances, or inverse impedances. This can be represented by

$$\begin{Bmatrix} f_h \\ -f_t \end{Bmatrix} = Z(s) \begin{Bmatrix} v_m \\ v_s \end{Bmatrix} \tag{1}$$

where:

f_h, f_t	are external forces exerted by the human operator and the task;
v_m, v_s	are velocities of the master and the slave manipulators;
Z	is the impedance matrix of the teleoperator system.

Raju showed how to design a stable controller when the loads generated by the human operator and by the task are only known to have arbitrarily passive impedances[5]. He showed that by connecting the teleoperator hardware to a reciprocal proportional-derivative (PD) controller, the teleoperator system will be passive and stable. Raju also solved for the impedances at the master and the slave ports when the system is controlled by the PD controller. Consequently, while maintaining the system stability and passivity, the impedance that the human operator is feeling and the impedance to which the task is subjected can be adjusted by changing the PD gains of the controller.

2.2 Structured Passivity

Recently, Chin ['91a, '91b] explored extra margins of stability for force- reflective teleoperation. He used a different way to look at a teleoperator system: take the human and the task as a combined external load coupled with the underlying master/slave manipulator. This view is depicted in Fig. 2.

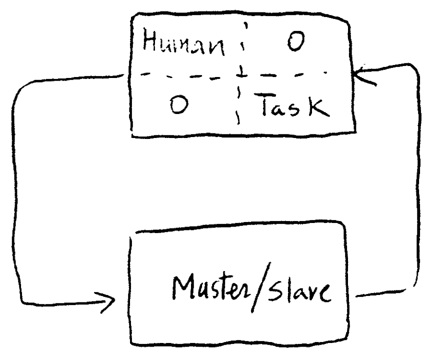

Figure 2: A teleoperator system as a feedback system.

Note that the human operator only directly interacts with the master port, and the task only directly interacts with the slave port. If we examine the combined admittance matrix for the human and task combination, the entire load on the teleoperator, we get the following equation for the load admittance:

$$\begin{Bmatrix} v_m \\ v_s \end{Bmatrix} = \begin{bmatrix} z_h^{-1} & 0 \\ 0 & z_t^{-1} \end{bmatrix} \begin{Bmatrix} f_h \\ -f_t \end{Bmatrix} \tag{2}$$

[5]The human and the task can be active systems. However, we assume that the active forces generated by the human and the task are motion-independent (state-independent). Thus, their impedances (or admittances) are equivalent to that of passive systems.

where z_h and z_t are impedances of the human and the task respectively. This creates a block diagonal structure in the matrix formulation of the dynamics. By exploiting this increased level of system knowledge, Chin was able to increase the envelope of stable performance. Using the structured singular value, μ, a measure developed in the field of robustness control by Doyle ['82], Chin ['91a] and Colgate ['91] showed that when coupled with arbitrarily passive human and task impedances, a loaded teleoperator system is stable if and only if the structured singular value of the scattering matrix of the teleoperator system is less than 1:

$$\text{The coupled system is stable} \iff \mu[(Z-I)(Z+I)^{-1}] < 1$$

This condition can be used to determine the stability of a given controller design. However, it is very difficult to design impedance- based controllers such as PD controllers using this condition. Chin ['91b] later introduced a new concept, the *structured passivity*, by transforming this scattering matrix-based condition into an impedance-based condition for n-port's robust stability. By definition, a system is structured passive when the structured singular value of the scattering matrix of the system is less than or equal to 1. Moreover, he showed that for a two-port (e.g., a teleoperator system), a structured passive system is a system whose impedance matrix[6] can be scaled into a passive impedance matrix by a diagonal matrix which has positive real entries. That is, for a two-port,

$$DZD^{-1} \text{ is passive for some } D \in \mathcal{D} \iff Z \text{ is structured passive,}$$

where

$$\mathcal{D} = \{diag(d_1, d_2, \cdots, d_m) | d_i \text{ are positive real numbers.}\} \qquad (3)$$

A detailed proof of the above theorem can be found in the Appendix.

For example, the impedance matrix of a one degree-of-freedom (DOF) teleoperator system can be represented by the following matrix:

$$Z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$$

This system is structured passive if and only if we can find a positive real number d such that the following matrix

$$\begin{bmatrix} z_{11} & dz_{12} \\ \frac{1}{d}z_{21} & z_{22} \end{bmatrix}$$

is passive. It is obvious that a passive system is always structured passive (when $d = 1$). On the other hand, a certain class of active system can also be structured passive (when $d \neq 1$). Consequently, a loaded structured passive teleoperator system can be active and still be robustly stable.

Since the structured passivity can be derived easily from the impedance of a system, controllers can be designed directly based on the impedance of the system. In the next section, this extra margin of stability is shown to provide individual scaling of the feedback force and/or position. In comparison, a passive system can only scale feedback force and position simultaneously, such that the power transmission from operator to environment is constant when damping is negligible.

[6] The impedance means open loop impedance of a network. The stated condition is also true by replacing the impedance matrix with the admittance or the hybrid impedance/admittance matrix of the system.

3 Feedback Scaling

In this section, we will show that by varying the magnitude of the controller gains, the feedback force and motion in a structured passive system can be scaled independently with different scaling factors.

Let Z_P and Z_C denote the impedances of the plant (master/slave hardware) and the controller respectively:

$$Z_P = \begin{bmatrix} m_m s + b_m & 0 \\ 0 & m_s s + b_s \end{bmatrix} ; \quad Z_C = \begin{bmatrix} \frac{k_{mm}}{s} & -\frac{k_{ms}}{s} \\ -\frac{k_{sm}}{s} & \frac{k_{ss}}{s} \end{bmatrix} \tag{4}$$

where:

m_m, b_m	are mass and damping of the master arm;
m_s, b_s	are mass and damping of the slave arm;
k_{mm}, k_{ms}, k_{sm}, k_{ss},	are proportional gains of the controller. They are positive real numbers.

Thus, the impedance matrix of the master/slave manipulator system is

$$Z = \begin{bmatrix} m_m s + b_m + \frac{k_{mm}}{s} & -\frac{k_{ms}}{s} \\ -\frac{k_{sm}}{s} & m_s s + b_s + \frac{k_{ss}}{s} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \tag{5}$$

In order to present the essence of the effect of controller on feedback scaling, the following analysis assumes that the system is at steady-state condition, and that the inertia and the damping terms are negligible. As a result, the stiffness terms will dominate the system behavior. Then, the equations of motion of the teleoperator system become

$$\begin{Bmatrix} f_h \\ -f_t \end{Bmatrix} = \begin{bmatrix} k_{mm} & -k_{ms} \\ -k_{sm} & k_{ss} \end{bmatrix} \begin{Bmatrix} x_m \\ x_s \end{Bmatrix} \tag{6}$$

The effect of the gain (stiffness) matrix is presented in the following:

Singularity In order to allow free motion (multiple solutions of x_m and x_s) for the master and the slave, the above stiffness matrix must be singular. Otherwise, there will be only one set of possible positions for the master and the slave for a given set of external forces. For example, if the stiffness matrix is not singular, then the master and the slave will both move back to an initial position (0,0) when the external forces is removed. Hence, the following must be true:

$$k_{mm} k_{ss} = k_{ms} k_{sm}$$

Passivity The teleoperator system will be passive if the stiffness matrix is symmetric. That is,

$$k_{ms} = k_{sm}$$

Based on the singularity and the symmetry requirements, the only possible set of proportional gains for a passive teleoperator system satisfy the following condition:

$$k_{ms} = k_{sm} = \sqrt{k_{mm} k_{ss}}$$

Thus, the scaling factors for feedback force and motion can be determined:

$$\frac{x_s}{x_m} = \frac{f_h}{f_t} = \sqrt{\frac{k_{mm}}{k_{ss}}} \implies x_m f_h = x_s f_t$$

This result shows that when the teleoperator system is passive, the energy transmission is not scaled. That is, the teleoperator system works like a level: it will scale up (down) the feedback force and scale down (up) the position by the same ratio, $\sqrt{\frac{k_{mm}}{k_{ss}}}$. It is impossible to scale the force and the position at two different ratios when the teleoperator system is passive. On the other hand, in a structured passive impementation, it is possible to scale feedback force and motion at different ratios.

Structured passivity When a teleoperator system is structured passive, its stiffness matrix does not have to be symmetric. However, as mentioned in the previous section, its impedance matrix (in here, stiffness matrix) has to be able to be scaled into a symmetric matrix in the following way:

$$\begin{bmatrix} k_{mm} & -\frac{1}{d}k_{ms} \\ -dk_{sm} & k_{ss} \end{bmatrix} \text{ is symmetric,} \tag{7}$$

where d is some positive real constant number. Since ratio of rows in a gain matrix will determine the scaling factor for the feedback force, and the ratio of columns will determine the scaling factor of the positions of the master and the slave, a gain matrix in the following form will scale the feedback force and position independently:

$$k\begin{bmatrix} 1 & -\alpha_x \\ -\alpha_f & \alpha_x\alpha_f \end{bmatrix} \tag{8}$$

where $k_{mm} = k$, $k_{ms} = \alpha_x k$, $k_{sm} = \alpha_f k$, and $k_{ss} = \alpha_x\alpha_f k$. By choosing $d = \sqrt{\frac{\alpha_x}{\alpha_f}}$ in Eqn. (7), this matrix can be scaled into a symmetric matrix. Hence, the teleoperator system is structured passive. The effects of this gain matrix on feedback force and position are shown in the following equations:

$$x_m = \alpha_x x_s \tag{9}$$

$$\alpha_f f_h = f_t \tag{10}$$

Hence, the motion of the slave is scaled by a factor of α_x and the reflected force at the master sided is scaled by a factor of α_f.

In this section we have shown that a structured passive teleoperator system enjoys the benefit of the added stability margin over a passive design. That is, while maintaining the system stability, a structured passive system allows independently scaling of force and motion. A passive system only allows reciprocal scaling of force and motion. This result can be explained by the definition of a passive system which always preserves or disspiates power. On the other hand, a structured passive system allows power transmission to be scaled. Although extra energy is added to the system, a structured passive teleoperator system is as stable, and as easy to design, as a passive system.

4 High Fidelity Force Scaling

In a human operated system such as a bilateral telemanipulator system, the subjective "feel" of the system is very important. In the following section, we will first showed that when the force scaling is large, the *dynamic masking* effect become prominent and noticeable. In § 4.2, we will show that by closing local force loops around the master and the slave, the dynamic masking effect can be effectively reduced. Finally, a teleoperator system with high fidelity, scaled force reflection is achieved in § 4.3 by combining the structured passive gain matrix and the local force loops.

4.1 Dynamic Masking Effect

Although a human operator only interacts directly with the master manipulator in bilateral teleoperation, she actually "feels" the entire dynamics generated by the master/slave manipulator as well as the remote task. Since feedback scaling will also scale the dynamics of the manipulator hardware, the masking effect from the manipulator dynamics becomes noticeable when the force scaling factor, α_f, is large. In this section, we will demonstrate the dynamic masking effect when force feedback between the master and the slave is scaled.

In § 3, the force balancing equations for the force reflective teleoperation in Eqn. (4) are:

$$\begin{aligned} f_h &= (m_m \ddot{x}_m + b_m \dot{x}_m) + (k_{mm} x_m - k_{ms} x_s) \\ &= f_{mp} + f_{mc} \\ -f_t &= (m_s \ddot{x}_s + b_s \dot{x}_s) + (k_{ss} x_s - k_{sm} x_m) \\ &= f_{sp} + f_{sc} \end{aligned}$$

where f_{mp} and f_{sp} are forces generated by the master and the slave hardware, and f_{mc} and f_{sc} are forces generated by the controller. By choosing $k = k_{mm}$ and $\alpha_x = 1$ in the structured passive gain matrix in Eqn. (8), we have:

$$k_{mm} \begin{bmatrix} 1 & -1 \\ -\alpha_f & \alpha_f \end{bmatrix} \tag{11}$$

By substituting this gain matrix into the force balancing equations, the force reflection, but not the motion, in teleoperation is scaled. The force balancing equations become:

$$\begin{array}{c} f_h = f_{mp} + f_{mc} \\ -f_t = f_{sp} - \alpha_f f_{mc} \end{array} \quad \Longrightarrow \quad \begin{array}{c} f_h = f_{mp} + \frac{1}{\alpha_f}(f_{sp} + f_t) \\ f_t = \alpha_f (f_h - f_{mp}) - f_{sp} \end{array} \tag{12}$$

The dynamic masking effect is most perceptible when we try to magnify the feedback to the master. This is the case when a regular master is used to drive a micro-robot. By choosing very small α_f, the force felt by the operator will be dominated by the reflected forces:

$$\alpha_f \to 0 \implies f_h \sim \frac{1}{\alpha_f}(f_{sp} + f_t)$$

Due to the magnified slave dynamics, the operator will feel "sluggish" even when the slave is not in contact with the environment ($f_t = 0$). On the other hand, when α_f is very large, the reflected forces may be completed masked by the master dynamics:

$$\alpha_f \to \infty \implies f_h \sim f_{mp}$$

It possible to attenuate the dynamic masking effect by the combination of proper local impedance shaping and local force feedback loop. In the next section, we will discuss how to increase the fidelity of force transmission through local force feedback loops.

4.2 Local Force Loop For Increased Sensitivity

Anderson and Spong [88] suggested to close a local force loop around the slave, such that the sensitivity to the environment is obtained. By assuming no scaling in force reflection ($f_c = f_{mc} = -f_{sc}$), the force balancing equations become:

$$\begin{aligned} f_h &= f_{mp} + f_c \\ -f_t &= f_{sp} - f_c + k_t \hat{f}_t \end{aligned}$$

where $\hat{f}_t$ is a measured contact force at the slave, and k_t is the gain of the local force loop at the slave. Assuming that measured force is close to the real environmental force ($f_t \sim \hat{f}_t$), the system passivity and stability is maintained. Under this assumption, the operator will feel:

$$f_h \sim f_{mp} + f_{sp} + (1 + k_t) f_t$$

Thus, for the operator, the dynamic masking is in effect attenuated since the environmental force is amplified. However, this method will not attenuate the dynamic masking seen from the task, where:

$$f_t \sim \frac{f_h - f_{mp} - f_{sp}}{1 + k_t}$$

A similar local force loop can be closed around the master to increase the overall fidelity of force transmission. With both master and slave local force loop, the force balancing equations become:

$$\begin{aligned} f_h &= f_{mp} + f_c - k_h \hat{f}_h \\ -f_t &= f_{sp} - f_c + k_t \hat{f}_t \end{aligned}$$

By choosing equivalent gains for the master and the slave force loop ($k_h = k_t = k_f$), the force equations become:

$$\begin{aligned} f_h &\sim f_t + \frac{f_{mp} + f_{sp}}{1 + k_f} \\ f_t &\sim f_h - \frac{f_{mp} + f_{sp}}{1 + k_f} \end{aligned}$$

Hence, when k_f is large enough, the dynamics of both the master and the slave hardware are attenuated, and the force transmission between the human and the task is more faithful. However, in this case, the force transmission is not amplified. In the next section, we will show that by combining the local force loop and the structured passive gain matrix, high fidelity force scaling can be achieved with reduced dynamic masking.

4.3 Force Scaling with Reduced Dynamic Masking

In § 3, we have shown that feedback force and motion in bilateral teleoperation can be scaled independently by applying a structured passive gain matrix. However, we have also point out that when force scaling factor is large, the dynamic masking effect becomes more noticeable. Since local force loop can reduce the dynamic masking effect, it is apparent that large force scaling should be accomplished by combining the local force feedback and the structured passive gain matrix. With the local force feedback discussed in the previous section, the force balancing equations in Eqn. (12) becomes

$$\begin{aligned} f_h &= \frac{1}{\alpha_f} f_t + \frac{1}{1+k_f}(f_{mp} + \frac{1}{\alpha_f} f_{sp}) \\ f_t &= \alpha_f f_h - \frac{1}{1+k_f}(\alpha_f f_{mp} + f_{sp}) \end{aligned}$$

By choosing a large k_f (preferable $k_f > \alpha_f$), the reflected forces become

$$f_h \sim \frac{1}{\alpha_f} f_t \; ; \quad f_t \sim \alpha_f f_h$$

Hence, the feedback forces are scaled with the desired ratio, α_f, while the dynamics of the master and the slave are attenuated. Likewise, the local force loop can be applied to reduce the masking effect when motion is scaled ($\alpha_x \neq 1$).

5 Discussion and Conclusion

This paper has introduced a controller design methodology for scaling force and motion for bilateral teleoperation. By designing the teleoperator system as a structured passive system, the force and the motion scaling ratios can be determined independently. This freedom in choosing scaling factors is not available in a passive design. Moreover, the added flexibility will not compromise the robustness stability of the system. In fact, the structured passive design is as stable as a passive design.

When the force scaling ratio in the structured passive teleoperator system is large, the dynamic masking will become prominent and noticeable. In this paper, we have also shown how to reduced the dynamic masking effect in bilateral teleoperation by closing force loops around the master and the slave. Thus, the fidelity of force transmission can be improved. By combining the structured passive design and the local force loops, a teleoperator system with high fidelity, scaled force reflection is achieved.

Appendix: Characteristics of a structured passive system

In this section, the definition of *structured passivity* is introduced. Two theorems are presented to characterize the properties of a structured passive system using its impedance/admittance matrix.

Definition 1 *'The Structured Passive System'*

Let Z be the impedance/admittance matrix of a system, and S be the scattering matrix of the system, where

$$S = (Z - I)(Z + I)^{-1}$$

A structured passive system is a system that satisfies

$$1 \geq \mu[S]$$

where $\mu[S]$ denotes the structured singular value of S. Moreover, the system is strictly structured passive if $1 > \mu[S]$.

That is, the structured singular value, μ, of the scattering matrix of any structured passive system is always less than or equal to 1. In comparison, note that the maximum singular value, $\bar{\sigma}$, of the scattering matrix of any passive system is always less than or equal to 1 ($1 \geq \bar{\sigma}[S]$.) Moreover, a passive system is always structured passive because the following is always true:

$$1 \geq \bar{\sigma}[S] \geq \mu[S]$$

The following two theorems established the properties of the impedance/admittance matrix of a structured passive system based on the properties of μ described in Doyle ['82]. Let $D \in \mathcal{D}$ be a constant diagonal matrix with positive real entries as defined in Eqn. (3):

Theorem 1 *For n-port: DZD^{-1} is passive for some $D \in \mathcal{D} \implies Z$ is structured passive.*

Theorem 2 *For 1, 2, and 3-port: When the uncertainties have 3 blocks or less, DZD^{-1} is passive for some $D \in \mathcal{D} \iff Z$ is structured passive.*

To prove the above theorems, two computational properties of μ are used (see Doyle, '82). The first property applies to uncertainties with unlimited number of blocks (an n-port in a network formulation):

$$\mu[S] \leq \inf_{D \in \mathcal{D}} \bar{\sigma}[DSD^{-1}] \tag{13}$$

On the other hand, when the uncertainty only has three or fewer blocks (a 1, 2, and 3-port in a network formulation), the above 'less than or equal to' condition becomes an 'equal to' condition:

$$\mu[S] = \inf_{D \in \mathcal{D}} \bar{\sigma}[DSD^{-1}] \tag{14}$$

Expanding the right-hand side of the above equation, we have

$$\begin{aligned} DSD^{-1} &= D(Z-I)D^{-1}D(Z+I)^{-1}D^{-1} \\ &= (DZD^{-1}-I)(DZD^{-1}+I)^{-1} \end{aligned} \tag{15}$$

The proofs are obtained by substituting Eqn. (15) into Eqn. (13) and Eqn. (14):

1. Proof of Theorem 1 (for n-port):

   According to Eqn. (13),

   $$\mu[S] \leq \inf_{D \in \mathcal{D}} \bar{\sigma}[DSD^{-1}] = \inf_{D \in \mathcal{D}} \bar{\sigma}[(DZD^{-1}-I)(DZD^{-1}+I)^{-1}]$$

   Also, the following is true for any passive system Z_p,

   $$1 \geq \bar{\sigma}[S_p] = \bar{\sigma}[(Z_p - I)(Z_p + I)^{-1}]$$

If $DZD^{-1} = Z_p$ is passive, then

$$\begin{aligned}\mu[S] &\leq \inf_{D \in \mathcal{D}} \bar{\sigma}[(DZD^{-1} - I)(DZD^{-1} + I)^{-1}] \\ &\leq 1\end{aligned}$$

Hence, the system Z is structured passive if DZD^{-1} is passive.

2. Proof of Theorem 2 (for 1, 2, and 3-port):

According to Eqn. (14), when the uncertainties have 3 blocks or less,

$$\begin{aligned}\mu[S] &= \inf_{D \in \mathcal{D}} \bar{\sigma}[DSD^{-1}] \\ &= \inf_{D \in \mathcal{D}} \bar{\sigma}[(DZD^{-1} - I)(DZD^{-1} + I)^{-1}]\end{aligned}$$

- The sufficient condition: If $DZD^{-1} = Z_p$ is passive, then

$$\begin{aligned}\mu[S] &= \inf_{D \in \mathcal{D}} \bar{\sigma}[(DZD^{-1} - I)(DZD^{-1} + I)^{-1}] \\ &\leq 1\end{aligned}$$

  Hence, the system Z is structured passive.
- The necessary condition: If $Z_a = DZD^{-1}$ is not passive for any $D \in \mathcal{D}$, then

$$\begin{aligned}\mu[S] &= \inf_{D \in \mathcal{D}} \bar{\sigma}[(DZD^{-1} - I)(DZD^{-1} + I)^{-1}] = \bar{\sigma}[(Z_a - I)(Z_a + I)^{-1}] \\ &> 1\end{aligned}$$

  Hence, the system Z is not structured passive if DZD^{-1} is not passive for any $D \in \mathcal{D}$.

To conclude, when the uncertainty has 3 blocks or less, the system Z is structured passive if and only if DZD^{-1} is passive.

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