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# EXPERIMENTAL DESIGN AND EVALUATION OF BOUNDED RATIONALITY USING DIMENSIONAL ANALYSIS*

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## ABSTRACT

Dimensional analysis is a method used in the design and analysis of experiments in the physical and engineering sciences. When a functional relation between variables is hypothesized, dimensional analysis can be used to check the completeness of the relation and to reduce the number of experimental variables. The approach is extended to include dimensions pertaining to cognitive processes so that it can be used in the design of multi-person experiments. The proposed extension is demonstrated by applying it to a single decisionmaker experiment already completed; new results from that experiment are described. It is then applied to the design of a multi-person experiment.

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# EXPERIMENTAL DESIGN AND EVALUATION OF BOUNDED RATIONALITY USING DIMENSIONAL ANALYSIS[1]

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Abstract. Dimensional analysis is a method used in the design and analysis of experiments in the physical and engineering sciences. When a functional relation between variables is hypothesized, dimensional analysis can be used to check the completeness of the relation and to reduce the number of experimental variables. The approach is extended to include dimensions pertaining to cognitive processes so that it can be used in the design of multi-person experiments. The proposed extension is demonstrated by applying it to a single decisionmaker experiment already completed; new results from that experiment are described. It is then applied to the design of a multi-person experiment.

Keywords. Man-machine systems; dimensional analysis; experimental design; cognitive workload.

## INTRODUCTION

In the last few years, a mathematical theory for the analysis and design of information processing and decisionmaking organizations has been developed based on the model of interacting human decisionmakers (DMs) with bounded rationality (Levis, 1984; Boettcher, 1982). While this model was motivated by empirical evidence from a variety of experiments and by the concept of bounded rationality (March,1978), there were no direct experimental data to support it. An experimental program was undertaken to test the theory and obtain values for the model parameters (Louvet et al.,1988).

One of the major difficulties in developing a model-driven experimental program is the large number of parameters that have to be specified and varied. The resulting problem has two aspects: (a) The parametrization of the experimental conditions leads to a very large number of trials, a situation that is not really feasible when human subjects are to be used, and (b) Not all experimental variables can be set at the values required by the experimental design because of the lack of direct control of the cognitive variables.

Consequently, some orderly procedure is needed that will allow the reduction of the number of experimental variables and, more importantly, that will lead to variables that are easier to manipulate. Such an approach, called dimensional analysis, has been in use in the physical and engineering sciences (Hunsacker, 1947; Gerhart, 1985). The purpose of this paper is to extend the approach to problems that have cognitive aspects so that it can be used for the design and analysis of experiments. The class of problems we are interested in are those that relate organizational structure directly to performance, as measured by accuracy and timeliness and, more indirectly, to cognitive workload.

A special class of organizations will be considered - a team of well-trained decisionmakers executing repetitively a set of well-defined cognitive tasks under severe time pressure. The cognitive limitations of decisionmakers imposes a constraint on the organizational performance. Performance, in this case, is assumed to depend mainly on the time available to perform a task and on the cognitive workload associated with the task. When the time available to perform a task is very short (time pressure is very high), decisionmakers are likely to make mistakes so that performance will degrade.

Dimensional analysis will be introduced briefly in the next section. The approach is then extended to include cognitive variables and a completed experiment will be used as an example to demonstrate the approach. Then, the application of dimensional analysis to the design of experiments for the analysis and evaluation of distributed decisionmaking organizations will be described.

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## DIMENSIONAL ANALYSIS AND EXPERIMENTAL DESIGN

Dimensional analysis is a method for reducing the number and complexity of experimental variables which affect a given physical phenomenon. A detailed introduction to dimensional analysis can be found in Hunsacker (1947); Gerhart (1985).

*Dimensions and Units.* A dimension is the measure which expresses a physical variable qualitatively. A unit is a particular way to express a physical quantity, that is, to relate a value to a dimension. *Fundamental dimensions* are the primary dimensions which characterizes all variables in a physical system. For example, length, mass, and time are fundamental dimensions in mechanical systems. A dimension such as length per time is a secondary or derived dimension. If the dimension of a physical variable cannot be expressed by the dimensions of others in the same equation, then this variable is dimensionally independent.

The foundation of dimensional analysis is the Principle of Dimensional Homogeneity, which states that if an equation truly describes a physical phenomenon, it must be dimensionally homogeneous, i.e., each of its additive terms should have the same dimension. The basic theorem of dimensional analysis is the $\pi$ theorem, also called Buckingham's theorem:

> **$\pi$ theorem:** If a physical process is described by a dimensionally homogeneous relation involving n dimensional variables, such as
>
> $$x_1 = f(x_2, x_3, ..., x_n) \quad (1)$$
>
> then there exists an equivalent relation involving (n-k) dimensionless variables, such as
>
> $$\pi_1 = F(\pi_2, \pi_3, ..., \pi_{n-k}) \quad (2)$$

where k is usually equal to, but never greater than, the number of fundamental dimensions needed to describe all x's.

Each of the $\pi$'s in Eq. (2) is formed by combining (k+1) x's to form dimensionless variables. Comparing Eqs. (1) and (2), it is clear that the number of independent variables is reduced by k, where k is the maximum number of dimensionally independent variables in the relation. The proof of the $\pi$ theorem can be found in Gerhart (1985).

The $\pi$ theorem provides a more efficient way to organize and manage the variables in a specific problem and guarantees a reduction of the number of independent variables in a relation. Dimensionless variables, also called dimensionless groups, are formed by grouping primary variables with each one of the secondary variables.

To apply dimensional analysis to decisionmaking organizations, the fundamental dimensions of the variables that describe their behavior must be determined. A system of three dimensions is shown in Table 1 that is considered adequate for modeling cognitive workload and bounded rationality. An experiment conducted at MIT (Louvet et al., 1988) is used to demonstrate the application of dimensional analysis to the experimental investigation of bounded rationality. The purpose of this single-person experiment was to investigate the bounded rationality constraint. The experimental task was to select the smallest ratio from a sequence of comparisons of ratios consisting of two two-digit integers. Two ratios were presented to a subject at each time. The subject needed to decide the smaller one and compare it with the next incoming ratio until all ratios were compared and the smallest one was found. The controlled variable (or manipulated variable) was the amount of time allowed to perform the task. The measured variable was the accuracy of the response, i.e., whether the correct ratio was selected.

TABLE 1 Dimensions for Cognitive Problems

| Dimension | Symbol | Units |
|---|---|---|
| Time | T | second |
| Information (uncertainty) | I | bit |
| Task | S | symbol |

The controlled variables were the number of comparisons in a sequence, denoted by N, and the allotted time to do the task, denoted by $T_w$. For each value of N, where N could take the value of three or six, $T_w$ took twelve values with constant increment in the following way:

$T_w = \{ 2.25, 3, 3.75, ..., 10.5 \}$ second for N = 3;
$T_w = \{ 4.50, 6, 7.50, ..., 20.1 \}$ second for N = 6.

The performance was considered to be accurate or correct if the sequence of comparisons was completed *and* if the smallest ratio selected was correct. The details of the experiment can be found in Louvet (1988).

The hypothesis is that there exists a maximum processing rate for human decision makers. When the allotted time is decreased, there will be a time beyond which the time spent doing the task will have to be reduced, if the execution of the task is to be completed. This will result in an increase in the information processing rate F, if the workload is kept constant. However, the bounded rationality constraint limits the increase of F to a maximum value $F_{max}$. When the allotted time for a particular task becomes so small that the processing rate reaches $F_{max}$, further decrease of the allotted time will cause performance to degrade. It was hypothesized that the bounded rationality constraint $F_{max}$ is constant for each individual DM, but varies from individual to individual. The bounded rationality constraint can be expressed as

$$F_{max} = G / T_w^* \qquad (3)$$

where $T_w^*$ is the minimum allotted time before performance degrades significantly. G and $T_w^*$ vary for different tasks, but $F_{max}$ is constant for a decision maker, no matter what kind of tasks he does. Therefore, significant degradation of performance indicates that the allotted time approaches $T_w^*$. Observation of this degradation during the experiment allows the determination of the time threshold and, therefore, the maximum processing rate, provided the workload associated with a specific task can be estimated or calculated.

The retroactive application of dimensional analysis to this experiment will be shown step by step.

Step 1 Write a dimensional expression.

In the experiment, accuracy, $N_c$, of information processing and decisionmaking is defined as the number of correct decisions, that is, the number of correct results in a sequence of comparisons. Therefore, $N_c$ has the dimension of symbol and depends on the following variables:

N: number of comparisons in each trial;
$T_w$: allotted time to do N comparisons;
H: uncertainty of input, that is, the uncertainty of the ratios to be compared in a trial;

The dimensional expression is

$$N_c = f(T_w, N, H) \qquad (4)$$

First, dimensional analysis checks whether this functional relation could describe the relation between $N_c$ and other variables. The dimensions of the variables in Eq. (4) are the following:

$[N_c] = S \qquad [T_w] = T$
$[N] = S \qquad [H] = I$

Since the dimension of $N_c$ is S, the right hand side of Eq. (4) has to be of the same dimension regardless of what the function f is. However, all three fundamental dimensions are represented by the three independent variables. There is no way to combine these variables to obtain a term of dimension S only. Therefore, according to the Principle of Dimensional Homogeneity, this functional relation is *not* a correct expression of the relation under the investigation.

There are two approaches to obtain the correct relation. The first is to delete $T_w$ and H. This is not acceptable because the allotted time is a critical factor in this experiment. The other approach is to add some variables or dimensional constants to satisfy the requirement for dimensional homogeneity. Dimensional constants are physical constant such as gravity, the universal gas constant, and so on. No such dimensional constant has been identified as yet, therefore, some variables which have dimensions of time and information should be added to the relation. Moreover, the additional variables have to be relevant to the measurement of accuracy. Since the experiment is to investigate bounded rationality, that is, the maximum information processing rate, it is appropriate to introduce processing rate F into the equations. The equations describing accuracy and response time become

$$N_c = f(T_w, N, H, F) \qquad (5)$$

Each of the equations is dimensionally homogeneous. There are five dimensional variables in Eq. (5), that is, n = 5.

Step 2 Determine the number of dimensionless groups.

The number of dimensionless variables is equal to n-k, where k is the maximum number of dimensionally independent variables in Eq. (5). Dimensions of the variables are

$[N_c] = S, \qquad [N] = S \qquad [T_w] = T,$

$[H] = I, \qquad [F] = IT^{-1}$

The maximum number of dimensionally independent variables is three. Therefore, k is equal to three. Then, the number of dimensionless groups is:

$$n - k = 5 - 3 = 2.$$

There will be two dimensionless groups in the dimensionless equation corresponding to Eq.(5).

Step 3. Construct the dimensionless groups.

While the choice of primary variables is essentially arbitrary, consideration should be given that the dimensionless groups be meaningful. If $T_w$, N, and H are selected as the three (k=3) primary variables, two dimensionless groups are constructed on the basis of the remaining variables $N_c$ and F in Eq.(5). As an example, a dimensionless group $\pi_1$ is formed by combining $T_w$, N, H, and F. Using the power-product method, $\pi_1$ can be determined by the following procedure. Write $\pi_1$ as

$$\pi_1 = T_w{}^a \; N^b \; H^c \; F^d$$

where a, b, c, and d are constants that make the right hand side of the equation dimensionless, so that the equation is dimensionally homogeneous. In terms of the dimensions of $T_w$, N, H, and F we have

$$[\pi_1] = [S^0 I^0 T^0] = [T]^a [I]^b [S]^c [IT^{-1}]^d = T^{a-d} I^{b+d} S^c$$

By the Principle of Dimensional Homogeneity, the following set of simultaneous algebraic equations must be satisfied.

| | |
|---|---|
| For T: | a - d = 0 |
| For I: | b + d = 0 |
| For S: | c = 0 |

There are three equations, but four unknowns. The solution is not unique. In general, the choice of the solution depends on the particular interest in the subject. For our purpose, the secondary variables, in this example $N_c$ and F, are chosen to appear in the first power, that is, d is set equal to unity. Thus, by solving the set of algebraic equations, we obtain:

$$a = 1, \quad b = -1, \quad c = 0, \quad d = 1.$$

Then

$$\pi_1 = F/(H/T_w) \qquad (6)$$

Using the same power-product method, $\pi_2$ is found to be

$$\pi_2 = N_c / N \qquad (7)$$

Then, the dimensionless form of Eq. (5) is

$$\frac{N_c}{N} = \Psi\left(\frac{F}{\left(\frac{H}{T_w}\right)}\right) \qquad (8)$$

Looking at Eq.(8) carefully, we find the all variables except F are directly controllable or measurable. If the actual processing time $T_f$ is introduced, then the actual processing rate F can be expressed by

$$F = \frac{G}{T_f} \qquad (9)$$

where G is the workload associated with the task.
The actual processing time $T_f$ can be measured directly. Therefore, substitution of $G/T_f$ for F is necessary so that all variables in Eq.(8) are directly accessible. After the substitution, Eq.(8) becomes

$$\pi_2 = \frac{N_c}{N} = \Psi\left(\frac{\frac{G}{T_f}}{\frac{H}{T_w}}\right) = \Psi\left(\frac{G}{H}\frac{T_w}{T_f}\right) = \Psi(\pi_1) \qquad (10)$$

This introduction of $T_f$ will be very useful in developing a design procedure for new experiments later in this paper. This is the result obtained by the application of dimensional analysis. The functions $\Psi$ is unknown and need to be determined by experiments.

In Eq. (10), $\pi_2$ is the fraction of correct decisions; and $\pi_1$ represents the ratio of the actual processing rate and the average rate of input uncertainty. Equation (10) represents a model-driven experiment in which $\pi_1$ is the experimental variable to be controlled. The function $\Psi$ needs to be determined experimentally.

Comparing Eq.(5) and Eq.(10), one finds that the number of independent variables is reduced from four to one. This reduces the complexity of the equations and facilitates experiment design and analysis. Properly designed experiments using dimensional analysis provide similitude of experimental condition for different combinations of dimensional variables which result in the same value of $\pi$'s. Similitude reduces the number of trials needed to be run in order to define $\Psi$. This is a major advantage when the physical (dimensional) experimental variables cannot be set at arbitrary values.

## APPLICATION

One of the objectives of this paper is to illustrate the use of dimensional analysis for the design and analysis of model-driven experiments. Therefore, only new results from the earlier experiment, obtained using dimensional analysis, will be shown. The following procedure was used to analyze the data. Only three subjects are selected from the population of all subjects (25 subjects) for illustration.

Data for each trial

| Control variables | Measured variables | Computed variables |
|---|---|---|
| N, $T_w$ | $N_c$, $T_f$ | H, J, G |

In this analysis, N is fixed and its value is three (3). H and G are computed using Information theory. H is constant for the experiment, and G depends on the algorithm used by a subject to do the task; therefore, it varies across the subjects. The details of these computation can be found in Louvet (1988). J is the ratio of the number of correct decisions $N_c$ to the number of total decisions, N. The controlled variable in each trial is $T_w$. As stated previously, there are 12 values of $T_w$.

Step 1: Compute the input uncertainty H and the workload for all algorithms used by the subjects. The workload is denoted by $G^i$ for the $i^{th}$ algorithm.

The following steps are carried out for each subject.

Step 2: Let $I_j$ denote the set that indexes the trials with the $j^{th}$ value of $T_w$, denoted by $T_{wj}$: $I_j = \{1,2,3,\ldots, n_j\}$. The following average quantities are computed for each $T_{wj}$:

$$J_j = \frac{1}{n_j}\sum_{i}^{n_j}\left(\frac{N_{cji}}{N}\right)$$

$$T_{f_j} = \frac{1}{n_j}\sum_{i=1}^{n_j} T_{f_{ji}}$$

where j = 1,2,...,12.

Step 3: Compute the two dimensionless groups for each $T_{wj}$, j = 1 ,2,..., 12.

$$\pi_{1j} = \frac{G^k}{H}\frac{T_{wj}}{T_{fj}}$$

$$\pi_{2j} = J_j$$

where k is the index of the algorithm used by the subject. Since there are 12 values of $T_w$, $\pi_1$ and $\pi_2$ also have 12 values each.

Step 4: Find relations between $\pi_1$ and $\pi_2$. Equation (10) can be rewritten as:

$$J = \Psi(\pi_1)$$

since J is identical to $\pi_2$.

In order to determine the function $\Psi$, the mean value of J, as calculated in step 3, is plotted against the independent variable $\pi_1$. The resulting plot for one subject is shown on Fig.1.

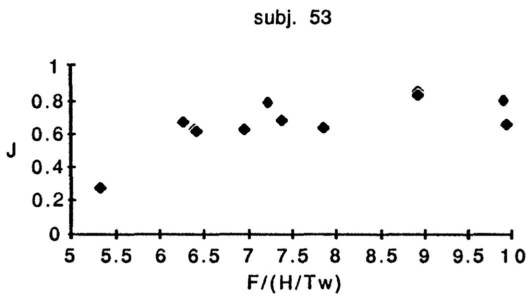


Fig. 1. Mean J vs. $\pi_1$ for subject 53

An exponential function is used to fit the curve. The exponential function is

$$y = a + b * \mathrm{Exp}.(-c*(\pi-\pi_0)) \quad \pi \geq \pi_0$$

where a, b, c, and $\pi_0$ are constants to be determined. By considering that the maximum possible value of J is one and the minimum value $J_0$, a and b can be determined as 1 and $(J_0-1)$ respectively. Then, we have

$$y = 1 + (J_0-1) * \mathrm{Exp}(-c*(\pi-\pi_0))$$

A Least Squares method is used to determine c and $\pi_0$ when assuming a value for $J_0$. The criterion is that $J_0$ is chosen so that the sum of the squares of residuals is minimized. The resulting function is plotted in Fig.2, along with the data. Figures 3 an 4 show comparable results for two other subjects.

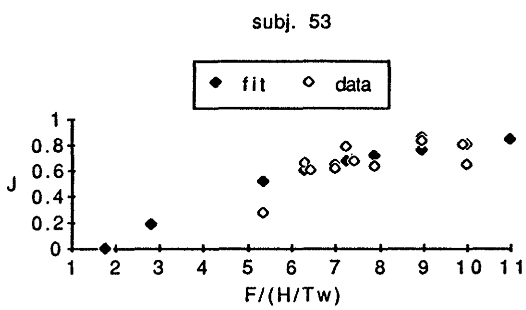


Fig. 2. Mean J vs. $\pi_1$ for subject 53

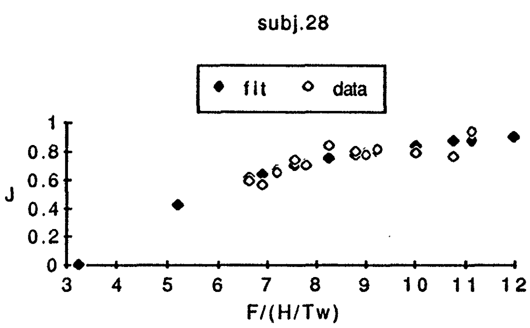


Fig. 3. Mean J vs. $\pi_1$ for subject 28

From these relations, the critical value of $\pi_1$, $\pi_1^*$, can be found for each subject according to some specific accuracy requirement. For example, the value of $\pi_1$ for which J is equal to J*, is given by:

$$\pi_1^* = \pi_0 + \frac{1}{c(1-J_0)}$$

J* is the accuracy corresponding to $\pi_1^*$ at which further decrease of $\pi_1$ can cause a significant drop of J.

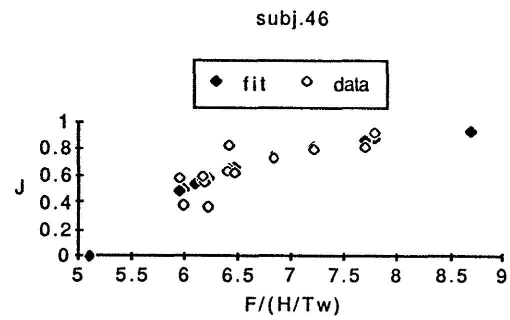


Fig. 4. Mean J vs. $\pi_1$ for subject 46

When the entire data set is processed, then the distribution of $\pi_1^*$ can be obtained, in a manner analogous to that used by Louvet et al. to determine the distribution of the critical response times. This information helps specify the range of some experimental parameters in the multi-person experiment.

The determination of the dimensionless groups and the estimation of the $\Psi$ completes the use of dimensional analysis.
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## RESULTS

One of the critical questions in any experiment design is the determination of the ranges of manipulated parameters. We want to choose the ranges of these parameters so that the measured variables will have significant differences and can be interpreted in a meaningful way. The results from above analysis are useful for this design purpose.

### Actual processing time $T_f$

The first result is obtained through the introduction of the actual processing time $T_f$ and the determination of a functional relation between $T_f$ and the allowed time, $T_w$. A proposition is formulated on the basis of the experimental data.

*Definition* A task is said to have been completed if and only if all necessary actions required by the task are taken before the allowed time expires, that is, $T_f \leq T_w$.

***Proposition 1*** When a task is completed, it is completed in less time than the allowed time, that is, $T_f < T_w$ . How much less depends on the length of the allowed time.

Proposition 1 confirms the functional relation between $T_f$ and $T_w$ and is used later in this paper to determine the range for the allowed time to assure that the operating point is close to the bounded rationality, but does not exceed it.

The functional relation between response time $T_f$ and allowed time $T_w$ has been found to be exponential, as shown in Fig.5 and Fig.6. This exponential relation implies that $T_f$ increases with $T_w$ quickly at the beginning until $T_w$ reaches a point beyond which $T_f$ does not change with $T_w$. The reason for this behavior is intuitive: when there is ample time to do a job, the effect of the time on the performance will become irrelevant. This result is useful in the multi-person experiment design in two ways.

First, $T_f$ leads to a better estimate of the processing rate because it is the time actually used in processing the information. Therefore, the bounded rationality can be expressed by

$$F_{max} = G / T_f^* \qquad (11)$$

where $T_f^*$ is the $T_f$ at which $J = J^*$.

Second, the functional relation between $T_f$ and $T_w$ allows us to predict the response time for a given $T_w$. The existence of the region in which further increases of $T_w$ do not result in any significant change in $T_f$ indicates the time pressure imposed by

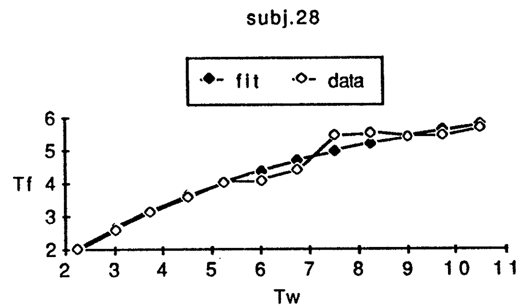


Fig.5 Response time $T_f$ vs. available time $T_w$ for subject 28

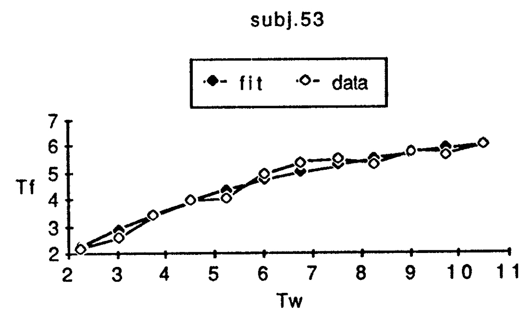


Fig.6. Response time $T_f$ vs. available time $T_w$ for subject 53

$T_w$ does not affect the performance any more. Thus, through $T_f$, we can avoid those values of $T_w$ which will not have any effect on performance.

Dimensionless parameter $\pi_1$

The second result obtained from dimensional analysis is the dimensionless parameter $\pi_1$ which provides knowledge on the values of the design variables H, G, and $T_w$. As stated previously, the critical value of $\pi_1$, denoted by $\pi_1$*, is the value at which any small decrease of $\pi_1$ will cause a significant decrease in accuracy. To determine how to use the information given by $\pi_1$*, we first consider how the subjects process information and make decisions respectively, then we describe how $\pi_1$* can lead to a design procedure.

When looking at the algorithms used by the subjects, we find that the subjects can be categorized into two different groups according to the ways they make decisions. The first group of subjects attempts to simplify the ratios as soon as the ratios are presented, then make a decision according to the simplified version of the data. They try to filter out an amount of information so that only a minimum amount of information necessary to make the decision is kept. On the other hand, the second group of subjects look at the raw data carefully before making any reduction. When processing the data, they retain a large amount of information which may be used in making a decision.

Wohl (1981) has described a model developed by Johnson (1978) in which individual styles of decision making are classified. In the model, Johnson identified two decision styles in gathering information, spontaneous and systematic. Table 2 list the traits of these two styles. By using Johnson's model of decision styles, we can describe the decision style of the first group as spontaneous, while of the second group as systematic. The division of the subjects into the two groups is shown in Table 3. The average values of workload, G, and the maximum processing rate, $F_{max}$, of different groups are shown in Table 4. These values indicate that the subjects in the systematic group have higher processing rate than those in the spontaneous group. One cautious observation is that the maximum rate is not proportional to the workload when decision styles vary.

Next, we discuss how to use $\pi_1$* to design new experiments.

TABLE 2 Characteristics of Decision Styles

Spontaneous

- A holistic reaction to events
  reacts to total experience
- Quick psychological commitment
  Personalize alternatives in order to evaluate them
- Flexible goal orientation

Systematic

- Collective reaction to situations/events.
  Breaks experience into segments and reacts separately to each one
- Cautious psychological commitment
- Methodical goal orientation

TABLE 3 Division of the Subjects in Different Groups

| Group | Number of Subjects |
|---|---|
| Spontaneous | 10 |
| Systematic | 15 |

TABLE 4 Average Values of Different Groups of the Subjects

| Group | Av.G | Av. $F_{max}$ |
|---|---|---|
| Spontaneous | 212.6 | 45.3 |
| Systematic | 257.6 | 86.0 |

The critical value of $\pi_1$, $\pi_1$*, of an individual subject characterizes his bounded rationality. The expression of $\pi_1$ in Eq.(10) can be rewritten as

$$\pi_1 = \left(\frac{G}{H}\right)\frac{1}{\left(\frac{T_f}{T_w}\right)}$$

Then, $\pi_1$* is

$$\pi_1^* = \left(\frac{G}{H}\right)\frac{1}{\left(\frac{T_f}{T_w}\right)^*}$$

$(T_f/T_w)^*$ is the ratio of critical response time and the allowed time corresponding to J*. According to Proposition 1, $T_f$ will become smaller and smaller if $T_w$ is decreasing. When the accuracy J is equal to J*, $T_f$ and $T_w$ become $T_f$* and $T_w$* respectively. Thus, $(T_f/T_w)^*$ characterizes the cognitive inertia of an individual in accelerating the processing rate to reach the maximum rate, $F_{max}$. On the other hand, the ratio of G/H depends on a particular task and protocols. Thus, when G/H changes, $(T_f/T_w)^*$ will vary accordingly to maintain $F_{max}$ to be the same. Consequently, $\pi_1$* will not change. Therefore, $\pi_1$* is the value which can be used in designing new experiments. Let us call the experiment described above as the calibration experiment on the bounded rationality of individuals. The following procedure is developed to design new experiments.

## EXPERIMENT DESIGN

The procedure for designing experiments to study the effects of organizational structures on performance is described as follows.

At the start of design, the average value of $\pi_1$* is given by the calibration experiment. And in accordance with Proposition 1, the exponential relation derived from the calibration experiment is adopted. The critical value for the design is

$$\pi_1^* = G/H(T_w/T_f)^*,$$

or
$$G/H = \pi_1^* (T_f/T_w)^* \quad (15)$$

where G, H, and $T_w$ are design parameters.
- G depends on H and the organizational structure, that is, the particular procedure and protocol;
- H can be controlled by designing the task;
- $T_w$ is the driving parameter for the tempo of the operations.
- $\pi_1$* is given.

The design steps are as follows.

Step 1 Design H.

Design a task according to the hypothesis being tested by the experiment. The input uncertainty H of the task can be computed.

Step 2 Design G.

Design an organizational structure which will perform the task. Then, the particular protocol and procedure can be specified for the organization. Workload associated with the protocol and procedure is computed.

Step 3 Determine $T_w$.

Determine the values of the allowed time $T_w$. Since H and G are known from steps 1 and 2, Equation (15) can be rewritten as

$$(T_f/T_w)^* = (G/H)/\pi_1^* \quad (16)$$

To decide the critical value of $T_w$, the functional relation between $T_f$ and $T_w$ is used. Substitute $T_f = f(T_w)$ into Eq.(16) to obtain

$$(f(T_w)/T_w)^* = (G/H)/\pi_1^* \quad (17)$$

From Eq. (17) the value of $T_w$ * can be computed. Then $T_f$* is estimated. $F_{max}$ is computed using Eq. (11).

Use of $T_f = f(T_w)$ and Eq.(16) permits the determination of the range of $T_w$ which satisfies the constraints specified by the designer. For example, the interval R in Fig. 7 is the interval from which the values of $T_w$ are taken so that the operating point will be in an appropriated range. $T_{wmax}$ is the value at which $T_f$ does not change significantly with $T_w$, or in terms of time pressure, the speed of the operation does not critically depend on the time. The values of $T_w$ outside the interval are either too small to allow the subject to carry out the task (the bounded rationality constraint) or too large to observe any variation of $\pi_2$ with $\pi_1$ (no effect on performance).

Step 4 Check all design and computed values.

List H, G, $T_w$*, $T_f$*, $F_{max}$, and create a table for $T_w$ and corresponding values of $T_f$ and F. If there is any undesired value, the designer can go back to step 1 to make modifications until he is satisfied.

According to Eq.(16), the critical values of $T_f$ and $T_w$ will change when either the task or the organizational structure changes. Therefore, the experiment designer can use the

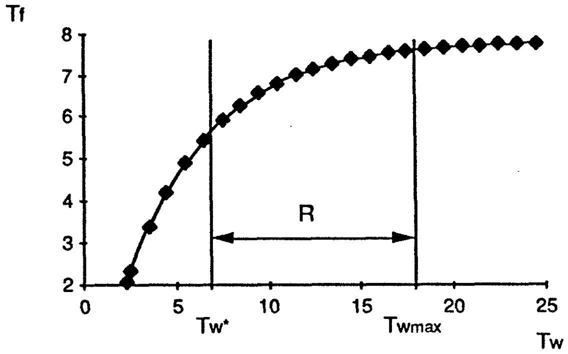


Fig. 7 An example of determining the values of $T_w$

freedom on the choice of tasks and organizational structures to design appropriate experiments for various hypotheses.

Step 5 Calibration of subjects.

Design a questionnaire to determine the decision style of a subject. This calibration helps the experimenter to make a preliminary assessment as to which group that subject may belong and to choose the parameter set for a particular set of trials.

## CONCLUSIONS

Dimensional analysis has been introduced to the design of experiments that have cognitive aspects. An extension has been presented that makes it possible to include variables such as cognitive workload and bounded rationality of human decision makers. An existing single person experiment has been used as an example to show how the methodology can be applied. A new result from the existing experiment has been presented to illustrate the possible advantages of using dimensional analysis. Note that dimensional analysis only determines possible relations between relevant variables; the actual functional expression has to be found from experimental data. Then it was shown how these results can be used to design model-driven experiments.

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