FEEDBACK SYSTEM DESIGN WITH AN UNCERTAIN PLANT

D. Milich. L. Valavani. and M. Athans
Laboratory for Information and Decision Systems
Massachusetts Institute of Technology
Cambridge. MA 02139 U.S.A.

Abstract

A method is developed to design a fixed-parameter compensator for a linear, time-invariant, SISO plant model characterized by significant structured, as well as unstructured, uncertainty. The controller minimizes the H^∞ norm of the worst-case sensitivity function over the operating band and the resulting feedback system exhibits robust stability and robust performance. It is conjectured that such a robust nonadaptive control design technique can be used on-line in an adaptive control system.

1. Introduction

The mathematical description of a physical plant (i.e. the nominal model) is always characterized by uncertainty, or modeling error. The plant/model mismatch caused by neglecting high-frequency phenomena (i.e. unmodeled dynamics) is known as unstructured uncertainty because only a frequency dependent magnitude bound on the error is available. Differences between the actual values and the nominal values of the parameters in the finite-dimensional, low-frequency model are the source of structured uncertainty. The goal of the robust design method presented here is to maintain closed-loop stability and performance in the presence of both types of uncertainty.

The most popular modern design methodologies (LQG/LTR, H^∞) only deal with plant models which contain unstructured uncertainty [1,2]. The effect of plant parameter variations may be incorporated into the unstructured uncertainty; however, the directional (phase) information associated with the structured uncertainty is lost and the result is an overly conservative design. Although these methodologies

guarantee nominal closed-loop stability, stability-robustness with respect to the unstructured uncertainty, and nominal performance, the issue of robust performance is ignored.

Techniques which deal directly with structured uncertainty have been developed. Horowitz [3-5] has proposed the so-called Horowitz templates which represent, at a particular frequency, the gain and phase changes associated with parameter variations as a region on a Nichols chart. A loop transfer function is derived from the templates which ensures closed-loop stability and a certain amount of performance over the possible range of parameters. This procedure is extremely tedious for more than a few frequency points and a great deal of judgment is required on the part of the designer.

Sideris and Safonov [6] approach the problem of structured uncertainty by examining a plant template in the complex plane. A series of transformations at each frequency is performed which maps the irregularly shaped region in the complex plane onto the unit disk. The directional properties of the uncertainty are eliminated and the transformed problem is essentially a design with unstructured uncertainty. Nevanlinna-Pick interpolation is used to find the compensator which is then transformed back in such a way that it is a solution to the original problem. This approach may be promising, however it appears that a great deal of computation is required at each frequency point.

The problem of robust stabilization in the presence of parameter uncertainty is addressed by Khargonekar and Tannenbaum [7]. The authors do not deal with performance issues directly and simultaneous variations in the poles and zeros cannot be considered within the present framework. Doyle [8-10] has developed a new mathematical quantity μ, the structured singular value, to handle structured uncertainty and the problems of robust stability and robust performance. While it may be a valuable analysis tool [11,12], a feedback synthesis methodology based on μ is not yet available.

It is the purpose of this paper to develop techniques for analyzing the stability and performance of a SISO feedback system which contains a plant model with

parameter uncertainty and unmodeled dynamics. Based on the analysis, a robust control design method which uses concepts from LQG/LTR and H^∞ theory is outlined. The compensator guarantees nominal closed-loop stability and stability-robustness with respect to structured and unstructured uncertainty. In addition, the H^∞ norm of the worst-case sensitivity function over the operating band is minimized. Since this norm provides an upper bound on all possible sensitivity functions, the closed-loop system satisfies the condition of robust performance.

2. Problem Formulation

Consider the feedback system in Figure 1 with plant model g(s) and compensator k(s). The physical plant to be controlled is single-input, single-output, linear, and time-invariant. The transfer function $g_t(s)$, which may be of infinite order, fully describes the true plant. The low-frequency behavior of $g_t(s)$ is captured in a parameterized n^{th} order transfer function $g(s,\underline{\theta})$, where $\underline{\theta}$ represents the parameter vector. The actual value of $\underline{\theta}$ is $\underline{\theta}^*$. Unstructured uncertainty results from the fact that the nth order transfer function $g(s,\underline{\theta}^*)$ cannot completely describe the (possibly) infinite-dimensional plant $g_t(s)$. Using the multiplicative form of the modeling error as in [1], the frequency-domain description of the unstructured uncertainty is represented by the stable transfer function $[1+\Delta(s)]$. The true plant can be described as follows.

$$g_t(s) = g(s,\underline{\theta}^*)[1 + \Delta(s)] \qquad (1)$$

It is assumed that a frequency-dependent magnitude bound on the unstructured uncertainty is available, but the phase of $\Delta(j\omega)$ is completely unknown.

$$\left| \Delta(j\omega) \right| \leq \Delta_0(\omega) \qquad (2)$$

In any practical situation $\underline{\theta}^*$ is unknown; however, the true parameter values are bounded by the known set Θ (i.e. $\underline{\theta}^* \in \Theta$). For design purposes a nominal parameter vector $\underline{\hat{\theta}}$ is chosen from the set Θ. Structured uncertainty, which arises when $\underline{\hat{\theta}}$ and $\underline{\theta}^*$ are not identical, can be described in the frequency-domain using the multiplicative model error representation as in Equation (1).

$$g(s,\underline{\theta}^*) = g(s,\underline{\hat{\theta}})[1 + \delta(s,\underline{\theta}^*,\underline{\hat{\theta}})] \qquad (3)$$

From (1) and (3), the true plant $g_t(s)$ can be represented in terms of the low-frequency model, the parametric uncertainty, and the unmodeled dynamics.

$$g_t(s) = g(s,\underline{\hat{\theta}})[1 + \delta(s,\underline{\theta}^*,\underline{\hat{\theta}})][1 + \Delta(s)] \qquad (4)$$

In order to design the compensator k(s) and to analyze the feedback system containing the true plant, information about the nominal model $g(s,\underline{\hat{\theta}})$, the structured uncertainty $\delta(s,\underline{\theta}^*,\underline{\hat{\theta}})$, and the unmodeled dynamics $\Delta(s)$ is required. The nominal model and the magnitude bound on the unstructured uncertainty $\Delta_0(\omega)$ are known a priori. The exact value of $\delta(s,\underline{\theta}^*,\underline{\hat{\theta}})$ is unknown, but the set containing all possible values may be constructed from Θ. Using Equation (3) and replacing $\underline{\theta}^*$ with $\underline{\theta}$, $\delta(s,\underline{\theta},\underline{\hat{\theta}})$ may be computed for a given $\underline{\hat{\theta}}$.

$$\delta(s,\underline{\theta},\underline{\hat{\theta}}) = [g(s,\underline{\theta})/g(s,\underline{\hat{\theta}})] - 1 \quad \text{for } \underline{\theta}, \underline{\hat{\theta}} \in \Theta \qquad (5)$$

The above equation defines the set of structured uncertainty U_s.

$$U_s(s,\underline{\hat{\theta}}) = \{ \delta(s,\underline{\theta},\underline{\hat{\theta}}) \mid \underline{\theta} \in \Theta \} \quad \text{for } \underline{\hat{\theta}} \in \Theta \qquad (6)$$

U_s and the set of transfer functions $\Delta(s)$ satisfying Equation (2) lead to the definition of the set of possible plant transfer functions $G(s)$ which contains the true plant $g_t(s)$.

$$G(s) = \{ g(s) \mid \delta(s,\underline{\theta},\underline{\hat{\theta}}) \in U_s(s,\underline{\hat{\theta}}) , \left| \Delta(j\omega) \right| \leq \Delta_0(\omega) \} \quad (7)$$

where $g(s) = g(s,\underline{\hat{\theta}})[1 + \delta(s,\underline{\theta},\underline{\hat{\theta}})][1 + \Delta(s)]$.

The design method in section 4 will find a compensator k(s) such that the closed-loop system is stable for all $g(s) \in G(s)$. In addition to robust stabilization, the compensator must achieve closed-loop performance and performance-robustness. The sensitivity transfer function $s(s) = [1+g(s)k(s)]^{-1}$ evaluated along the $j\omega$-axis governs the command-following and disturbance-rejection performance of the feedback system in Figure 1. The performance objective of the design method is to minimize the H^∞ norm of the worst-case sensitivity function over the frequency range where the commands and disturbances have energy (i.e. the operating band Ω_0), subject to closed-loop stability. By definition, the magnitude of the worst-case sensitivity function is an upper bound on the magnitude of the sensitivity function for all $g(s) \in G(s)$ and for all ω. Thus the feedback system satisfies the condition of robust performance with respect to the worst-case sensitivity.

3. Analysis

This section covers the stability and performance analysis of the feedback system in Figure 1. The compensator $k(s)$ has been designed based on a plant $g(s) \in \mathbf{G(s)}$. The loop transfer function $t(s)=g(s)k(s)$ and the sensitivity $s(s)$ are the functions of interest. Let $t(s,\hat{\theta}) = g(s,\hat{\theta})k(s)$ be the nominal loop transfer function. Then,

$$t(s) = t(s,\hat{\theta})[1 + \delta(s,\theta,\hat{\theta})][1 + \Delta(s)] \qquad (8a)$$

$$= t(s,\hat{\theta}) + t(s,\hat{\theta})\delta(s,\theta,\hat{\theta}) + t(s,\hat{\theta})[1+\delta(s,\theta,\hat{\theta})]\Delta(s) \qquad (8b)$$

3.1 Stability

At the very least, the compensator $k(s)$ ensures the stability of the nominal closed-loop system. From the Nyquist criterion the number of encirclements of the critical point in the Nyquist diagram of $t(j\omega,\hat{\theta})$ is known. At this point it must be assumed that $g(s,\theta)$ has the same number of unstable poles for all $\theta \in \Theta$. Since the structured uncertainty does not change the number of unstable poles and $\Delta(s)$ is stable, the Nyquist criterion requires the number of encirclements of the critical point to remain unchanged for the loop transfer function $t(j\omega)$.

A stability-robustness condition may be derived by examining the loop transfer function $t(j\omega)$ in the complex plane at a particular frequency ω (Figure 2). From Equation (8b), there are two perturbation terms added to $t(j\omega,\hat{\theta})$ which may alter the stability of the nominal loop (i.e. change the number of encirclements of the critical point). The perturbation caused by the structured uncertainty is $t(j\omega,\hat{\theta})\delta(j\omega,\theta,\hat{\theta})$ and the second term, $t(j\omega,\hat{\theta})[1+\delta(j\omega,\theta,\hat{\theta})]\Delta(j\omega)$, is a result of the combined effect of structured and unstructured uncertainty. This perturbation is represented as a circle in the complex plane because the phase of $\Delta(j\omega)$ is unknown.

Before a stability-robustness test can be derived, it must be assumed that the structured uncertainty does not alter the stability of the nominal loop (i.e. the perturbation $t(j\omega,\hat{\theta})\delta(j\omega,\theta,\hat{\theta})$ does not change the number of critical point encirclements of the nominal loop $t(j\omega,\hat{\theta})$). Note that this is a very restrictive assumption; however, it is only used to derive a stability-robustness condition for design purposes and it does not limit the classes of uncertainty which may be considered. Under this assumption the loop transfer function $t(j\omega)$ will be closed-loop stable if the distance to the critical point (i.e. $|1+t(j\omega)|$) is greater than zero over all frequencies. If this is the case it is impossible for the perturbations due to uncertainty to change the number of critical point encirclements. From Figure 2, the distance to the critical point d_c of the loop $t(j\omega)$ may be computed. The closed-loop system may be unstable if the critical point is located in the interior of the circular region in Figure 2 (i.e. d_c is negative).

$$d_c(\omega,\theta,\hat{\theta}) = | 1+t(j\omega,\hat{\theta})[1+\delta(j\omega,\theta,\hat{\theta})]| - | t(j\omega,\hat{\theta})[1+\delta(j\omega,\theta,\hat{\theta})]\Delta(j\omega)| \qquad (9)$$

where $\delta(j\omega,\theta,\hat{\theta}) \in \mathbf{U_s}(j\omega,\hat{\theta})$, $|\Delta(j\omega)| \leq \Delta_0(\omega)$

To perform a worst-case stability analysis the minimum value of $d_c(\omega,\theta,\hat{\theta})$ must be found by searching over the set $\mathbf{U_s}(j\omega,\hat{\theta})$ and by replacing $|\Delta(j\omega)|$ by its bound $\Delta_0(\omega)$.

$$\tilde{d}_c(\omega,\hat{\theta}) = \min_{\delta \in U_s} \{ | 1+t(j\omega,\hat{\theta})[1+\delta(j\omega,\theta,\hat{\theta})]| - | t(j\omega,\hat{\theta})[1+\delta(j\omega,\theta,\hat{\theta})]|\Delta_0(\omega) \} \qquad (10)$$

<u>Theorem 1:</u> Sufficient Condition for Robust Stability

If $\tilde{d}_c(\omega,\hat{\theta}) > 0$ for all ω, then the true closed-loop system is stable.

<u>Proof:</u>   From Figure 2 and the above assumption, the number of encirclements of the critical point cannot be altered by the uncertainty if the inequality in Theorem 1 is satisfied.

Note the fundamental difference between structured and unstructured uncertainty. In the case of structured uncertainty, directional (phase) information is exploited. That is, only structured perturbations which decrease the distance to the critical point are of concern, and perturbations away from the critical point increase stability-robustness and can be ignored. No phase information is available for the unstructured uncertainty. Therefore it must be assumed that the unstructured perturbations are always in the direction of the critical point.

3.2 Performance

From the definition of the sensitivity function, $s(s)=[1+t(s)]^{-1}$, performance can be interpreted from the Nyquist diagram in terms of the distance to the critical point. That is,

$$|s(j\omega)| = 1/d_c(\omega,\theta,\hat{\theta}) \qquad (11)$$

The worst-case sensitivity function magnitude is found as an immediate consequence of Equation (11).

$$|\tilde{s}(j\omega)| = 1/\tilde{d}_c(\omega,\hat{\theta}) \qquad (12)$$

Define a scalar measure of performance ϵ as the H^∞ norm of the worst-case sensitivity over the operating band.

$$\epsilon = \max_{\omega \epsilon \Omega_0} |\tilde{s}(j\omega)| \qquad (13)$$

The objective of the robust design method is to minimize ϵ (i.e. maximize performance), subject to the closed-loop stability condition in Theorem 1. Since $|s(j\omega)| \leq |\tilde{s}(j\omega)|$ for all $g(s) \epsilon G(s)$ and for all ω, the requirement of robust performance is satisfied.

4. Robust Design Method

A method is proposed to design a compensator which guarantees closed-loop stability and performance in the presence of uncertainty (i.e. robust stability and robust performance). The controller minimizes the H^∞ norm of the worst-case sensitivity function over the operating band.

The concept of Nyquist-shaping is introduced as an integral part of the robust design process. While loop-shaping techniques are concerned with tailoring the magnitude of the loop transfer function, Nyquist-shaping requires magnitude and phase information to construct a target Nyquist diagram. The target satisfies all stability and performance conditions and represents the desired nominal loop transfer function. For the given plant, the robust design method produces a compensator which yields a nominal loop transfer function $t(j\omega,\hat{\theta})$ whose Nyquist diagram approaches the target to any required degree of accuracy.

It is assumed that the low-frequency, nominal plant model $g(s,\hat{\theta})$, the parameter set Θ (and hence U_s), and the magnitude bound on the unstructured uncertainty $\Delta_0(\omega)$ are known to the control system designer. Then, the first step in the robust design method consists of finding the target Nyquist diagram via the Nyquist-shaping procedure. The target curve cannot have an arbitrary shape in the complex plane. Stability and analytic function theory require the target Nyquist diagram to satisfy the following constraints.

(1) Nyquist stability criterion
(2) Bode's Integral Theorem [13]
(3) Theorem 1

Subject to the above constraints, the target must minimize the H^∞ norm of the worst-case sensitivity function over the operating band. The task of incorporating these constraints into a standard optimization problem is a subject of current research. While not all of the details of the Nyquist-shaping algorithm have been rigorously formalized, the basic idea is to start with a Nyquist diagram which corresponds to a loop transfer function which is known to be closed-loop stable with respect to the structured and unstructured uncertainty in the given plant model (i.e. satisfies Theorem 1). This initial curve in the complex plane is continuously deformed in such a way that the above constraints are met and the H^∞ norm of the worst-case sensitivity function over $\omega \epsilon \Omega_0$ is monotonically decreasing. The procedure terminates when acceptable performance has been achieved, or when it is no longer possible to improve worst-case performance without violating the stability-robustness condition in Theorem 1.

Finding the initial curve for the Nyquist-shaping algorithm is relatively straight-forward. Since current methodologies can handle unstructured uncertainty, the idea is to transform the original problem (with structured and unstructured uncertainty) into one with just unstructured uncertainty. This must be done in a conservative manner so that the resulting compensator guarantees a stable closed-loop system.

A new multiplicative uncertainty may be defined by lumping the structured and unstructured uncertainty together.

$$\Delta'(s,\theta,\hat{\theta}) = \delta(s,\theta,\hat{\theta}) + [1+\delta(s,\theta,\hat{\theta})]\Delta(s) \qquad (14)$$

The plant model $g(s)$ is represented as follows.

$$g(s) = g(s,\hat{\theta})[1 + \Delta'(s,\theta,\hat{\theta})] \qquad (15)$$

By ignoring the directional properties of the structured uncertainty a frequency-dependent magnitude bound on $\Delta'(s,\theta,\hat{\theta})$ is computed at each frequency.

$$\Delta'_0(\omega,\hat{\theta}) = \max_{\delta \epsilon U_s} \{ |\delta(j\omega,\theta,\hat{\theta})| + |1+\delta(j\omega,\theta,\hat{\theta})|\Delta_0(\omega) \} \quad (16)$$

$$|\Delta'(j\omega,\theta,\hat{\theta})| \leq \Delta'_0(\omega,\hat{\theta}) \quad \text{for } \theta,\hat{\theta} \epsilon \Theta, \text{ for all } \omega \qquad (17)$$

With the nominal plant model $g(s,\hat{\theta})$ and the bound on the unstructured uncertainty $\Delta'_0(\omega,\hat{\theta})$, LQG/LTR [14] or another suitable control design methodology is used to find a compensator $k_0(s)$ which guarantees a stable closed-loop system. The Nyquist diagram of the loop transfer function $t_0(j\omega)=g(j\omega,\hat{\theta})k_0(j\omega)$ is the starting

point for the Nyquist-shaping procedure. Note that this is an overly conservative control system design because the directional nature of the structured uncertainty has not been used. The Nyquist-shaping procedure results in a target Nyquist diagram which should remove this conservatism and improve performance.

The target curve will typically correspond to a high-order system. A finite-dimensional, parameterized loop transfer function must be obtained from the target Nyquist diagram. This can be accomplished by a least-squares fit to the magnitude and phase data at specific frequency points. The stability of the finite-dimensional target should be checked by Theorem 1. If the robustness condition is not satisfied, a frequency-weighted least-squares procedure is used to improve the transfer function fit in the frequency range where Theorem 1 was violated. Alternatively, a higher order transfer function can be used to fit to the target Nyquist diagram.

The finite-dimensional target loop is used in the Formal Loop Shaping LQG/LTR methodology to arrive at a compensator k(s). In [15], Stein and Athans outline the framework for using LQG/LTR to recover arbitrary (stable, minimum phase) target loop transfer functions. That is, the LQG/LTR methodology can be used to find a compensator k(s) such that the magnitude of the loop transfer function g(s,$\hat{\theta}$)k(s) matches the magnitude of the target loop. Unfortunately, this application of LQG/LTR with Formal Loop Shaping requires the plant model g(s,θ) to be stable and minimum phase. Research is being conducted to remove this restriction.

5. Implications For Adaptive Control

A very active search for a robust adaptive control methodology is being conducted and a trend in the literature is developing. Many researchers now believe that a robust adaptive control system must consist of a robust system identification algorithm coupled with a robust control design method [16-20]. This philosophy has been referred to as adaptive robust control, in contrast to robust adaptive control. Compensator redesign takes place infrequently compared to the system sample period and only when more accurate information about the system can be provided by the identification algorithm. While the robust design method presented here will be useful in its own right for fixed-parameter compensator design, the goal is to develop an on-line algorithm as part of a practical (i.e. robust) adaptive control system. In addition, the design method will provide the initial guess for the adaptive compensator.

Over the years a great deal of attention has been paid to the development of specific adaptive algorithms; however, very little consideration has been given to an issue at the heart of the adaptive control problem: what are the performance benefits of adaptive control? In theory an adaptive control system provides better performance with respect to a fixed-parameter compensator because more information about the physical plant is incorporated into the design process (on-line).

However, robust adaptive control systems rely upon some combination of external persistently exciting signals (to ensure good identification), slow sampling (to provide stability robustness with respect to unmodeled high-frequency dynamics, [21]), and extensive real-time computation (to provide safety nets which turn off the adaptation when it exhibits instability, [22]). These robustifying measures degrade command-following and disturbance-rejection performance and tend to neutralize the anticipated benefits of an adaptive compensator. In light of these circumstances it is imperative that the decision to use adaptive control, for a real engineering application, is based upon a quantitative assessment of the costs and the benefits of an adaptive system. The robust design method proposed here produces the nonadaptive feedback system which minimizes the H^{∞} norm of the worst-case sensitivity function over the operating band. This system may serve as a performance benchmark to which an adaptive control system is compared.

6. Conclusions

A frequency-domain analysis of the stability and performance of a SISO feedback system with structured and unstructured uncertainty has been performed. The crucial analysis parameter is the distance to the critical point in the Nyquist diagram. Directional information (in the complex plane) associated with the structured uncertainty is exploited to reduce conservatism. A new method is outlined to design linear, time-invariant compensators for SISO plant models characterized by parameter uncertainty and unmodeled dynamics. The resulting feedback system minimizes the H^{∞} norm of the worst-case sensitivity function over the operating band and is guaranteed to be closed-loop stable. The concept of Nyquist-shaping was introduced as an integral part of the design process.

It is conjectured that the robust design method can be used on-line in an adaptive control context. However before the decision to use adaptive control is made, the control designer must have a quantitative measure of the performance improvement over the "best" nonadaptive system. The robust design method provides a benchmark for the performance comparison.

References

[1] J.C. Doyle and G. Stein, "Multivariable Feedback Design Concepts for a Classical/Modern Synthesis", IEEE TAC, Vol. AC-26, February 1981.

[2] S.D. O'Young and B.A. Francis, "Optimal Performance and Robust Stabilization", Automatica, Vol. 22, No. 2, March 1986.

[3] I.M. Horowitz, Synthesis of Feedback Systems, Academic Press, New York, 1963.

[4] I.M. Horowitz, "Synthesis of Feedback Systems with Large Plant Ignorance for Prescribed Time-Domain Tolerances", Int. J. of Control, Vol. 16, No. 2, 1972.

[5] I.M. Horowitz, "Quantitative Feedback Theory", Proc. IEE, Part D, Vol. 129, No. 6, November 1982.

[6] A. Sideris and M.G. Safonov, "A Design Algorithm for the Robust Synthesis of SISO Feedback Control Systems Using Conformal Maps and H^∞-Theory", Proc. ACC, 1986.

[7] P.P. Khargonekar and A. Tannenbaum, "Non-Euclidean Metrics and the Robust Stabilization of Systems with Parameter Uncertainty", IEEE TAC, Vol. AC-30, October 1985.

[8] J.C. Doyle, "Analysis of Feedback Systems with Structured Uncertainty", Proc. IEE, Part D, Vol. 129, No. 6, November 1982.

[9] J.C. Doyle, "Synthesis of Robust Controllers and Filters", Proc. IEEE CDC, 1983.

[10] J.C. Doyle, "Structured Uncertainty in Control System Design", Proc. IEEE CDC, 1985.

[11] B.G. Morton and R.M. McAfoos, "A Mu-Test for Real-Parameter Variations", Proc. ACC, 1985.

[12] B.G. Morton, "New Applications of Mu to Real-Parameter Variation Problems", Proc. IEEE CDC, 1985.

[13] J.S. Freudenberg and D.P. Looze, "Right Half Plane Poles and Zeros and Design Tradeoffs in Feedback Systems", IEEE TAC, Vol. AC-30, June 1985.

[14] M. Athans, "A Tutorial on the LQG/LTR Method", Proc. ACC, 1986.

[15] G. Stein and M. Athans, "The LQG/LTR Procedure for Multivariable Control Design", Report LIDS-P-1384, M.I.T., Cambridge, MA, May 1984 (accepted for publication in the IEEE TAC).

[16] K.J. Astrom, "Analysis of Rohrs' Counterexamples to Adaptive Control", Proc. IEEE CDC, 1983.

[17] B. Wittenmark and K.J. Astrom, "Practical Issues in the Implementation of Self-Tuning Controllers", Automatica, Vol. 20, No. 5, September 1985.

[18] G.C. Goodwin, D.J. Hill, M. Palaniswami, "A Perspective on Convergence of Adaptive Control Algorithms", Automatica, Vol. 20, 1982.

[19] G.C. Goodwin, D.J. Hill, M. Palaniswami, "Towards an Adaptive Robust Controller", 7th IFAC Symposium on Identification and System Parameter Estimation, York, 1985.

[20] G.C. Goodwin, D.J. Hill, D.Q. Mayne, and R.H. Middleton, "Adaptive Robust Control (Convergence, Stability, and Performance)", Technical Report EE8544, Dept. of Electrical and Computer Engineering, University of Newcastle, Australia, N.S.W. 2308, December 1985.

[21] C.E. Rohrs and J. Sisianni, "Examples of Robust Adaptive Control in Sampled Data Systems", Proc. ACC, 1986.

[22] D.M. Orlicki, "Model Reference Adaptive Control Systems Using a Dead Zone Non-Linearity", Ph.D. Thesis, Dept. of E.E., M.I.T., Cambridge, MA, April 1985.
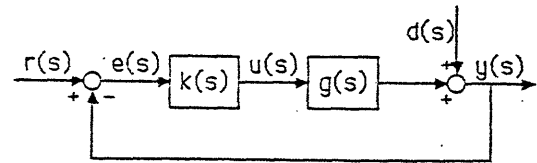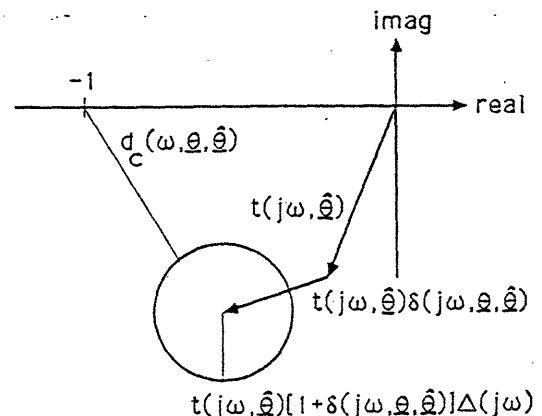
Figure 1. Feedback System.

Figure 2. Representation of $t(j\omega)$ in the complex plane.