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SELF-TUNING REGULATOR DESIGN FOR ADAPTIVE CONTROL OF AIRCRAFT WING/STORE FLUTTER

by

Timothy L. Johnson
Charles A. Harvey
Gunter Stein

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TIMOTHY L. JOHNSON, MEMBER, IEEE, CHARLES A. HARVEY, MEMBER, IEEE, AND GUNTER STEIN, MEMBER, IEEE

Abstract—The application of the self-tuning regulator concept to adaptively control aircraft wing/store flutter instability is described. A simple design based on a reduced-order aircraft model has been successfully tested on a high-order simulation of an advanced aircraft, and performance was found to be comparable to another design using on-line maximum likelihood identification of plant parameters. The main advantage of the self-tuning regulator is its simplicity, while the main disadvantage is the inadequacy of prior performance guarantees.

I. Introduction

This account describes a preliminary but successful effort to adaptively control wing/store flutter instabilities using the self-tuning regulator (STR) concept proposed by Åstrom [1]. The key accomplishments have been to design and test a self-tuning regulator for a very simple reduced-order model of single-mode flutter on an advanced aircraft, and then to validate this design using the full-scale simulation of the aircraft dynamics which included rigid body and flexure modes. In tests with the reduced-order model, the self-tuning design successfully detected and controlled oscillatory instabilities in the following three cases: 1) constant plant parameters, representing level flight at and above flutter speed; 2) slowly-varying parameters, representing maneuvers which cause the aircraft to exceed its flutter speed gradually; and 3) abruptly changing parameters, representing the sudden onset of flutter due to release of stores, which causes the unloaded wing to be above its critical flutter speed.

On the reduced model the STR design achieved stability for parameter variations of up to 30 percent in these cases, indicating that it was both robust and adaptive. Then the same design was tested on a much larger aircraft simulation which included rigid body dynamics and other flutter modes; cases 1) and 2) were tested, and it was found that flutter could be stabilized. These initial results are thus quite encouraging, but they do not constitute either a complete flutter control design, or a thorough validation of the preliminary design. It would be premature to conclude from this study that the STR concept can be successfully applied to the design of aircraft control systems, but there is now a strong indication that control of wing/store flutter is feasible and that the STR concept warrants further development.

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T. L. Johnson was with the Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, MA 02139. He is now with Bolt Beranek and Newman, Cambridge, MA 02238.

C. A. Harvey and G. Stein are with Honeywell Systems and Research Center, Minneapolis, MN 55413.

II. Design of a Self-Tuning Regulator for Single-Mode Flutter

Introduction and Background

The reduced-order design model is an unstable oscillatory second-order system. Harvey, Stein, and Felt [5] describe in detail how this very simple model is distilled from a full aerodynamic and structural model of the aircraft. Only an outline of the main steps is given here: the flight condition involves level flight with a fixed stores configuration, at approximately the critical mode's flutter speed. Only longitudinal dynamics are considered. The linearized aircraft model includes rigid body modes, flexure modes, and gust aerodynamics. The disturbance inputs are due to vertical and head-on gusts, while the control input is an outboard aileron. A least-squares program is used to pick the best weights on a linear combination of several sensor signals so as to yield a synthetic flutter mode sensor which has a frequency response as close as possible to that of a second-order system in the flutter frequency band. Finally, low- and high-pass filters are used to "wash out" the rigid body and higher flexure mode signals from this modal sensor. The initial object of the self-tuning regulator study was to control this "modal" system. The values of the parameters of this second-order system provided from the full aircraft model can be used to furnish initial parameter estimates for the STR design; a second-order model with these parameter values was used for preliminary verification of the STR algorithm.

Derivation of Discrete Model

The continuous-time transfer-function model was given in the form

$$Y(s)=H_1(s)U(s)+H_2(s)V(s)+H_3(s)W(s) \quad (2.1)$$

where

$Y(s)$ Laplace transform of output, $y(t)$ (units of g's);

$U(s)$ Laplace transform of control input, $u(t)$ (units of radians);

$V(s)$ transform of gust input, $v(t)$ (defined so that a white noise of intensity 1 is equivalent to 6 ft/s rms gusts);

$W(s)$ transform of sensor noise, $w(t)$ (g-equivalent units)

and[1]

$$H_1(s)=\frac{26.2s}{s^2-0.059s+686.44} \qquad \text{(control to output)}$$

$$H_2(s)=\frac{26.2s}{s^2-0.059s+686.44} \qquad \text{(gust to output)}$$

$$H_3(s)=1 \qquad \text{(sensor noise to output)}.$$

The model was first transformed into a continuous-time state-space model, then to a sampled-data state-space model, then to an autoregressive model of the form

$$A(a^{-1})y_t=z^{-k}B(z^{-1})u_t+C(z^{-1})\xi_t \qquad (2.2)$$

where z^{-1} denotes the backward shift operator. For a sampling time $=0.01$ s, the polynomials $A(z^{-1})$ and $B(z^{-1})$ are found to be

$$\begin{aligned} A(z^{-1})&=a_0+a_1z^{-1}+a_2z^{-2}\\ &\cong 1-1.9662z^{-1}+1.0348z^{-2} \qquad (2.3)\\ B(z^{-1})&=b_0+b_1z^{-1}\\ &\cong 0.271(1-z^{-1}); \qquad (k=1). \end{aligned}$$

The roots of $A(z^{-1})$ are $z^{\pm}=0.9831\pm0.2614j$ which are unstable. There is a zero at $z=1$.

Finding the correct polynomial $C(z^{-1})$ is slightly more involved. A stochastic process $C(z^{-1})\xi_t$ is determined, where ξ_t is a discrete white noise process of intensity Ξ which is statistically equivalent [denoted $(=)$] to the sum of two other discrete white noise processes $C_1(z^{-1})v_t+C_2(z^{-1})w_t$:

$$C(z^{-1})\xi_t\ (=)\ C_1(z^{-1})v_t+C_2(z^{-1})w_t \qquad (2.4)$$

where $C_1(z^{-1})=0.271z^{-1}(1-z^{-1})$ and $C_2(z^{-1})=1$. For statistical equivalence of the discrete-time v_t, w_t with the continuous-time $v(t)$, $w(t)$ processes,

$$E\{v_t\}=0 \quad E\{v_tv_\tau\}=\tilde{V}\delta_t; \quad \tilde{V}=\left(\frac{1}{\Delta}\right)V$$

$$E\{w_t\}=0 \quad E\{w_tw_\tau\}=\tilde{W}\delta_{t\tau}; \quad \tilde{W}=\left(\frac{1}{\Delta}\right)W.$$

Then a spectral factorization problem is solved to obtain the polynomial $C(z^{-1})$ and Ξ which turn out to be

[1]The numbers in this section correspond to a preliminary sensor complement.

$$\begin{aligned} C(z^{-1})&=c_0+c_1z^{-1}+c_2z^{-2}\\ &\cong 1; \qquad \Xi=\tilde{W}+2(0.271)^2\tilde{V} \qquad (2.5) \end{aligned}$$

provided $\tilde{W}\geq\tilde{V}$.

Constant-Gain Case: Choice of Control Weighting

The control-weighting parameter can be found, the nominal control gains can be chosen, and a prior assessment of expected performance can be performed using the nominal parameter values. The nominal controller gains are also helpful in choosing initial conditions for the self-tuning regulator algorithm.

The results of Clarke and Gawthrop will now be applied to a second-order example with general coefficients; our notation follows [6]. In the design, the performance index

$$I_t=E\left(P(z^{-1})y_{t+k}\right)^2+\left(Q'(z^{-1})u_t\right)^2 \qquad (2.6)$$

was used, with

$$\begin{aligned} P(z^{-1})&=p_0; \qquad p_0=1;\\ Q'(z^{-1})&=q_0'; \qquad q_0'=\lambda'. \qquad (2.7) \end{aligned}$$

This choice yields a "short-sighted" controller because expected future inputs and responses are not penalized; it is to be expected that the resulting control action may be rougher and more vigorous than necessary. However, the main objective of this study is to establish feasibility rather than to fine tune a control law. The choice (2.7) also yields a simpler control law for which the design tradeoffs can be assessed intuitively.

The predictor gain calculations need only be carried out for one-step prediction ($j=1$), and it is found that

$$\begin{aligned} E_1(z^{-1})&=e_{1,0}=c_0/a_0=1\\ F_1'(z^{-1})&=f_{1,0}+f_{1,1}z^{-1};\\ &f_{1,0}=c_1-a_1, \qquad f_{1,1}=c_2-a_2\\ G_1'(z^{-1})&=g_{1,0}+g_{1,1}z^{-1};\\ &g_{1,0}=b_0, \qquad g_{1,1}=b_1. \qquad (2.8) \end{aligned}$$

The modified control-weighting polynomial is

$$Q(z^{-1})=q_0'Q'(z^{-1})/b_0=(\lambda')^2/b_0. \qquad (2.9)$$

Finally, the controller gains are

$$\begin{aligned} F(z^{-1})&=p_0F_1'(z^{-1})=f_0+f_1z^{-1};\\ &f_0=c_1-a_1, \qquad f_1=c_2-a_2\\ G(z^{-1})&=p_0G_1'+CQ=g_0+g_1z^{-1}+g_2z^{-2};\\ &g_0=b_0+\lambda'^2/b_0\\ &g_1=b_1+c_1\lambda'^2/b_0\\ &g_2=c_2\lambda'^2/b_0. \qquad (2.10) \end{aligned}$$

Note that in this case $F=z(C-A)$ and when $\lambda'=0$

(minimum-variance control), then $G = B$. The control law is then $Gu_t = -Fy_t$, which may be written as

$$u_t = -\left(\frac{f_0}{g_0}\right)y_t - \left(\frac{f_1}{g_0}\right)y_{t-1} - \left(\frac{g_1}{g_0}\right)u_{t-1} - \left(\frac{g_2}{g_0}\right)u_{t-2}. \tag{2.11}$$

Thus the compensator generally has one zero and two poles, but has only one pole when $c_2 = 0$. For a second-order plant this would normally imply a 4th order closed-loop system, but in this case pole-zero cancellation occurs (for the case of perfectly known parameters) and the closed-loop roots may be found from the second-order polynomial equation

$$PB + QA = 0$$

or

$$\left(b_0 + \frac{\lambda'^2}{b_0}\right)z^2 + \left(b_1 + \frac{\lambda'^2}{b_0}a_1\right)z + \left(\frac{\lambda'^2}{b_0}a_2\right) = 0. \tag{2.12}$$

This equation for the closed-loop poles is used as a design equation for choosing the best values of the control-weighting, λ'.[2] The root locus λ' begins with one root at zero and one at the zero of $B(z^{-1})$, and ends at the zeros of $A(z^{-1})$. Thus it is immediately clear that if $B(z^{-1})$ is nonminimum phase and $A(z^{-1})$ is unstable, then both extreme values of λ' will yield unstable closed-loop systems. Note that the performance index (2.6), *unlike* the performance index of the typical linear-quadratic regulator problem, does not guarantee closed-loop stability. Hence, it is necessary to choose an intermediate value of λ' which guarantees stability. This is facilitated if we define

$$\lambda = \lambda'/b_0 \quad \text{and} \quad \gamma = b_1/b_0 \tag{2.13}$$

in which case (2.12) has roots at

$$z = \frac{-\gamma + \lambda^2 a_1 \pm \sqrt{\gamma^2 + 2\lambda^2(a_1\gamma - 2a_2) + \lambda^4(a_1^2 - 4a_2)}}{2(1+\lambda^2)}. \tag{2.14}$$

In the case at hand, $0.1 \le \lambda \le 10$, yields reasonable closed-loop pole positions.

In addition to stability, it is desirable to have a crude estimate of the expected control and output variables of the closed-loop system. This is described by

$$\begin{aligned} Ay &= z^{-1}Bu + C\xi \\ Gu &= -Fy \end{aligned} \tag{2.15}$$

or

$$(AG + z^{-1}BF)y = CG\xi \tag{2.16}$$

and since $F = z(C - A)$, $G = B + CQ$, $Q = q_0 = \lambda'^2/b_0$,

$$(q_0A + B)Cy = C(B + q_0C)\xi. \tag{2.17}$$

[2]Note that the closed-loop poles are *independent* of $C(z^{-1})$.

Noting that C polynomial cancels in numerator and denominator, the estimates

$$E\{y^2\} = \frac{|q_0C + B|^2}{|q_0A + B|^2}E\{\xi^2\} \tag{2.18}$$

and

$$E\{u^2\} = \frac{|C - A|^2}{|q_0A + B|^2}E\{\xi^2\} \tag{2.19}$$

are obtained where $E\{\xi^2\} = E$ has been estimated in (2.5). For the numbers given previously and $\lambda = 1$ (i.e., $\lambda' = |b_0|$), (2.17)–(2.19) yield

$$\begin{aligned} z &= \{-0.0613, -0.4218\} \\ E\{y^2\} &= 0.3604\Xi \\ E\{u^2\} &= 4.846\Xi. \end{aligned} \tag{2.20}$$

The value of λ' should be chosen so that the rms control,

$$u_{\text{rms}} = \sqrt{E\{u^2\}}, \tag{2.21}$$

lies somewhat below the actuator limits [which were taken to be 0.1 (rads)].

The self-tuning regulator algorithm derived in the next paragraph is easily modified to run with constant gains. Running the simulation with constant gains provides a good verification of the numerical algorithm, since the prior estimates (2.20) are available. The results of such a run are shown in Fig. 1.

Self-Tuning Case: Evaluation Procedure

The results of [6] are now specialized for the second-order plant studied in the previous paragraph, with a minor modification. The case of a second-order plant with one-step delay in control is special in that one parameter of the closed-loop system is not identifiable; this is taken to be $b_0 = \bar{b}_0$. In this subsection, overbars will be used to denote input parameter data based on the nominal plant parameters.

The value of the performance index is the expected value of the square of the quantity

$$\begin{aligned} \phi_t &= Py_t + Qu_{t-k}; \qquad k = 1 \\ &= y_t + \left[(\lambda')^2/\bar{b}_0\right]u_{t-1}. \end{aligned} \tag{2.22}$$

The estimate of ϕ_t given data up to $(t-1)$ is

$$\begin{aligned} \phi^*_{t|t-1} &= [y_{t-1}, y_{t-2}, u_{t-1}, u_{t-2}, u_{t-3}] \begin{bmatrix} f_0 \\ f_1 \\ g_0 \\ g_1 \\ g_2 \end{bmatrix} \\ &\doteq x_{t-1}^T\theta. \end{aligned} \tag{2.23}$$

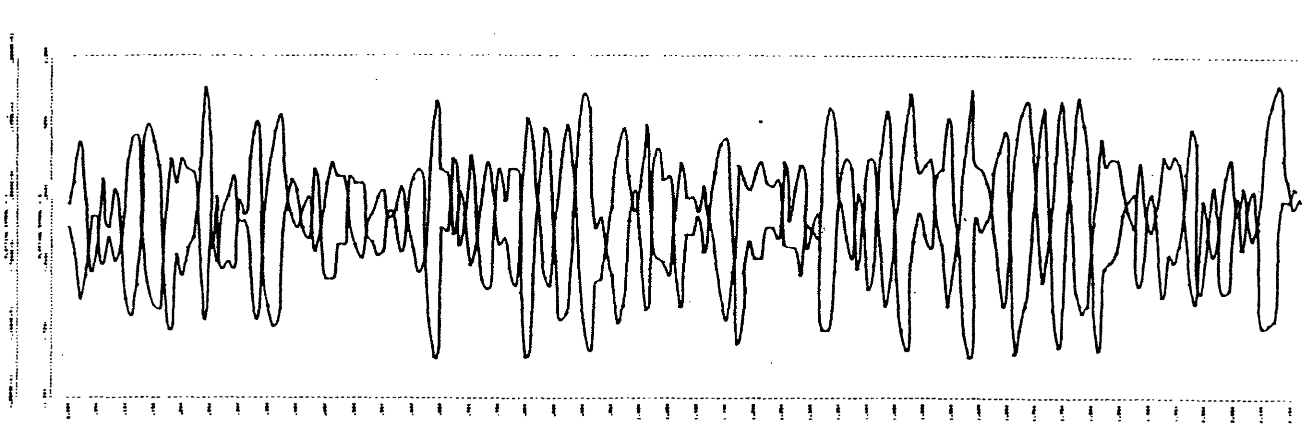

Fig. 1. Constant-gain minimum-variance regulator applied to reduced-order model described in Section II. Combined sensor output and control input shown.

The control u_{t-1} appears in both (2.22) and (2.23), which will eventually give rise to identifiability problems. This problem is readily resolved by subtracting $\bar{g}_0 u_{t-1}$ from both equations; hence the algorithm for a modified ϕ_t (denoted $\tilde{\phi}_t$) is implemented where

$$\begin{aligned}\tilde{\phi}_t &= \phi_t - \bar{g}_0 u_{t-1}\\ &= y_t + \left(-\bar{g}_0 + (\lambda')^2/\bar{b}_0\right) u_{t-1}\\ &= y_t - \bar{b}_0 u_{t-1} \quad (\text{see}\,(2.10)\,\text{for}\,\bar{g}_0)\end{aligned} \tag{2.24}$$

and

$$\begin{aligned}\tilde{\phi}^*_{t|t-1} &= [y_{t-1}, y_{t-2}, \bar{g}_0 u_{t-2}, \bar{g}_0 u_{t-3}] \begin{bmatrix} f_0 \\ f_1 \\ g_1/\bar{g}_0 \\ g_2/\bar{g}_0 \end{bmatrix}\\ &= \tilde{x}^T_{t-1}\tilde{\theta}.\end{aligned} \tag{2.25}$$

In order to improve numerical accuracy (the numbers in $\tilde{\theta}$ are of roughly equal magnitude) u_{t-2}, u_{t-3} in (2.25) are scaled by $\bar{g}_0$. Now the observation equation for $\tilde{\theta}$ is

$$\tilde{\phi}_t = \tilde{\phi}^*_{t|t-1} + \tilde{\theta}_t = \tilde{x}^T_{t-1}\tilde{\phi}_t + \tilde{\epsilon}_t \tag{2.26}$$

with consequent weighted recursive least-squares estimator[3]

$$\begin{aligned}\hat{\tilde{\theta}}_t &= \hat{\tilde{\theta}}_{t-1} + \tilde{K}_t\left(\tilde{\theta}_t - \tilde{x}^T_{t-1}\hat{\tilde{\theta}}_{t-1}\right)\\ \tilde{K}_t &= \tilde{P}_t\tilde{x}_{t-1}\left[\beta + \tilde{x}^T_{t-1}\tilde{P}_t\tilde{x}_{t-1}\right]^{-1}\\ \tilde{P}_t &= \frac{1}{\beta}\left[\tilde{P}_{t-1} - \tilde{K}_{t-1}\left(\beta + \tilde{x}^T_{t-2}\tilde{P}_{t-1}\tilde{x}_{t-2}\right)\tilde{K}^T_{t-1}\right]\end{aligned} \tag{2.27}$$

where $\tilde{K}_t$ is a 4-vector, $\tilde{P}_t$ is a symmetric 4×4 matrix, and $0<\beta\le1$ is a forgetting factor added so that estimation errors j steps in the past are discounted by β^j. The adaptive control is based on (2.11) as follows:

$$\begin{aligned}u_t &= -\left(\frac{\hat{f}_0}{\bar{g}_0}\right)y_t - \left(\frac{\hat{f}_1}{\bar{g}_0}\right)y_{t-1} - \left(\frac{\hat{g}_1}{\bar{g}_0}\right)u_{t-1} - \left(\frac{\hat{g}_2}{\bar{g}_0}\right)u_{t-2}\\ &= -\frac{1}{\bar{g}_0}\tilde{x}_t^T\hat{\tilde{\theta}}_t.\end{aligned} \tag{2.28}$$

In summary, the algorithm then consists of the following.

1) Calculate $\tilde{\phi}_t$ from (2.24) using y_t and u_{t-1}.

2) Update $\tilde{P}_t$ using the last equation of (2.27) based on $\tilde{x}_{t-2}$, $\tilde{P}_{t-1}$, $\tilde{K}_{t-1}$.

3) Form $\tilde{x}^T_{t-1}$ in (2.25) and update $\tilde{K}_t$ and $\hat{\tilde{\theta}}_t$.

4) Update $\tilde{x}^T_t$ using y_t, u_{t-1} and calculate the new control u_t using (2.28).

5) Generate a new output y_{t+1} from the plant (or plant simulation), represented by (2.2), increment t, and return to 1).

Numerically, the most demanding steps are 2) and 3) which involve roughly 30 multiplies and one scalar division. Data storage and an assembly-language program would only require on the order of 200–300 words of core, assuming hardware multiply. The real-time requirements are thus well within the capability of current computers.

Two issues have not been discussed to date: the initialization of the STR, and the use of forgetting factor, β. Of course, $\tilde{x}_0$ is initialized using the initial data (or it may be taken as the zero vector). $\hat{\tilde{\theta}}_0$ is initialized using the parameters of the constant-gain analysis in the previous subsection. Originally, $\tilde{P}_0$ was initialized as a diagonal matrix having entries $(0.1)^2\cdot(\bar{\tilde{\theta}}_i)^2$, i.e., a ±10 percent uncertainty in the nominal gains was assumed. However, $\tilde{P}_t$ cannot be interpreted this way in the RLS algorithm, and this value was found to be much too small, so that a very long transient run was required before $\tilde{P}_t$ reached steady state. This initialization was later replaced by an asymptotic analysis of the RLS equations assuming nominal plant parameter values and asymptotically optimum performance of the STR (see the Appendix).

One of the most interesting possibilities is the use of the forgetting factor, β, in the wing/stores flutter problem. Recall that $\alpha = 1/(1-\beta)$ is a measure of the "asymptotic sample length" of the time-scale of parameter variations.

[3]When $c_1 = c_2 = 0$, this estimator reduces to a third-order estimator.

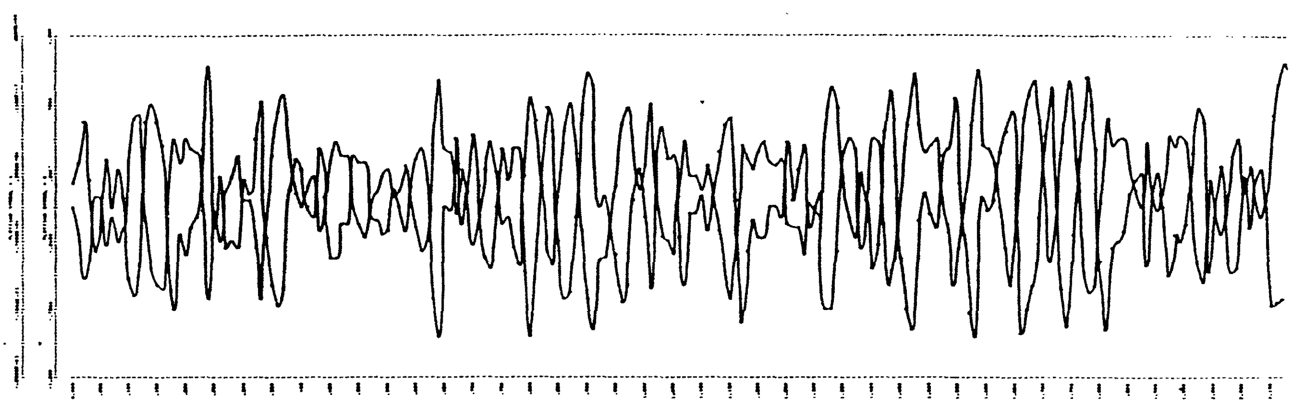

Fig. 2. Self-tuning regulator of Section II controlling reduced-order model (compare to Fig. 1); white noise sample function same as Fig. 1. Combined sensor output and control input shown.

For gradual parameter variations (no release of stores, aircraft gradually may fly over flutter speed), α was taken between 2 and 4 cycles (50–100 samples) or $\beta = 0.98$–0.99. When release of stores can be signaled to the STR, β may be transiently decreased to a small value and gradually restored to its nominal value, approximately as $\beta = 1 - (1/(p+1))$ where p is the number of time steps following release of stores. A sample run of the STR for the second-order stochastic model is shown in Fig. 2.

III. Performance Evaluation Using Full Aircraft Model

Analytical Model

Analytical models of a tactical fighter aircraft were developed for three wing/store configurations, chosen to provide a range of flutter modes, velocities, frequencies, and velocity-damping (V-g) characteristics. The first configuration, for which the results of Section II were derived, has a low flutter velocity and low flutter frequency. The second configuration has the highest flutter frequency of the three cases. The third configuration, obtained from the second by removing the tip store, has a lower flutter velocity than the second case.

The aircraft was modeled using 14 beam elements with lumped masses and inertias attached to a rigid fuselage. The stores were attached rigidly to the wing, and only symmetric modes were considered. Free–free vibration analyses were performed and 10 vibration modes were found for each store configuration. Unsteady strip theory was used to calculate generalized forces at sea level and Mach 0.95 for each vibration mode plus two rigid body modes. Forces due to control surface rotation and gusts were generated for eight control surfaces using doublet lattice unsteady aerodynamics. The full-order simulation model incorporates first-order (100 rad/s) actuator models and a first-order model of gust dynamics. This basic aircraft model was augmented with sensor outputs passed through second-order rolloff filters with break frequencies of approximately 200 rad/s. For each configuration a scalar output was formed from an optimal linear combination of sensor outputs. Defining the augmented state and noise vectors $\bar{x}$ and $\bar{\xi}$, the overall system may be written as

$$\dot{\bar{x}} = \bar{A}(V,k)\bar{x} + \bar{B}(V,k)u + \bar{D}(V,k)\bar{\xi} \tag{3.1}$$

where u denotes the actuator input, V denotes velocity, and $k = \omega b/V$ is the nondimensional reduced frequency (b is the moment reference length of the wing). To remove the frequency dependence in the matrices $\bar{A}$, $\bar{B}$, and $\bar{D}$, arising from the unsteady aerodynamics, the value of k was chosen to be $\omega_f b/V$ where ω_f denotes the frequency of the relevant flutter mode at velocity V. The coefficient matrices were then evaluated at five values of V, $V = 0.9V_f$, V_f, $1.1V_f$, $1.2V_f$, and $1.3V_f$, with V_f equal to the flutter speed, for each configuration.

Equation (3.1) was transformed into a block-diagonal form, and a sampled-data model was developed. A linear interpolation of the coefficients with respect to velocity was performed, so that the full-order simulation model took the form[4]

$$\bar{x}_{t+1} = \bar{\bar{A}}(V)\bar{x}_t + \bar{\bar{B}}_1(V)u_t + \bar{\bar{D}}_1(V)v_t \tag{3.2}$$

$$y_t = \bar{\bar{C}}(V)\bar{x}_t + \bar{\bar{B}}_2(V)u_t + \bar{\bar{D}}_2(V)v_t. \tag{3.3}$$

The effect of the sensor combination (carried out on the continuous-time model) procedure was that the scalar transfer function from u to y at $V = V_f$ could be represented by a second-order system of form (2.2). The speed V was varied with time according to the protocol described in the sequel.

Validation Procedure

The model (3.2), (3.3) was used to validate the self-tuning regulator design summarized by (2.26)–(2.30). For each of the three stores configurations, the initial regulator parameters were determined from the reduced second-order

[4]The presence of u_t and v_t in (3.3) arises from the fact that the accelerometers measure the state derivatives. Other details of the modeling procedure have been omitted in the interest of brevity.

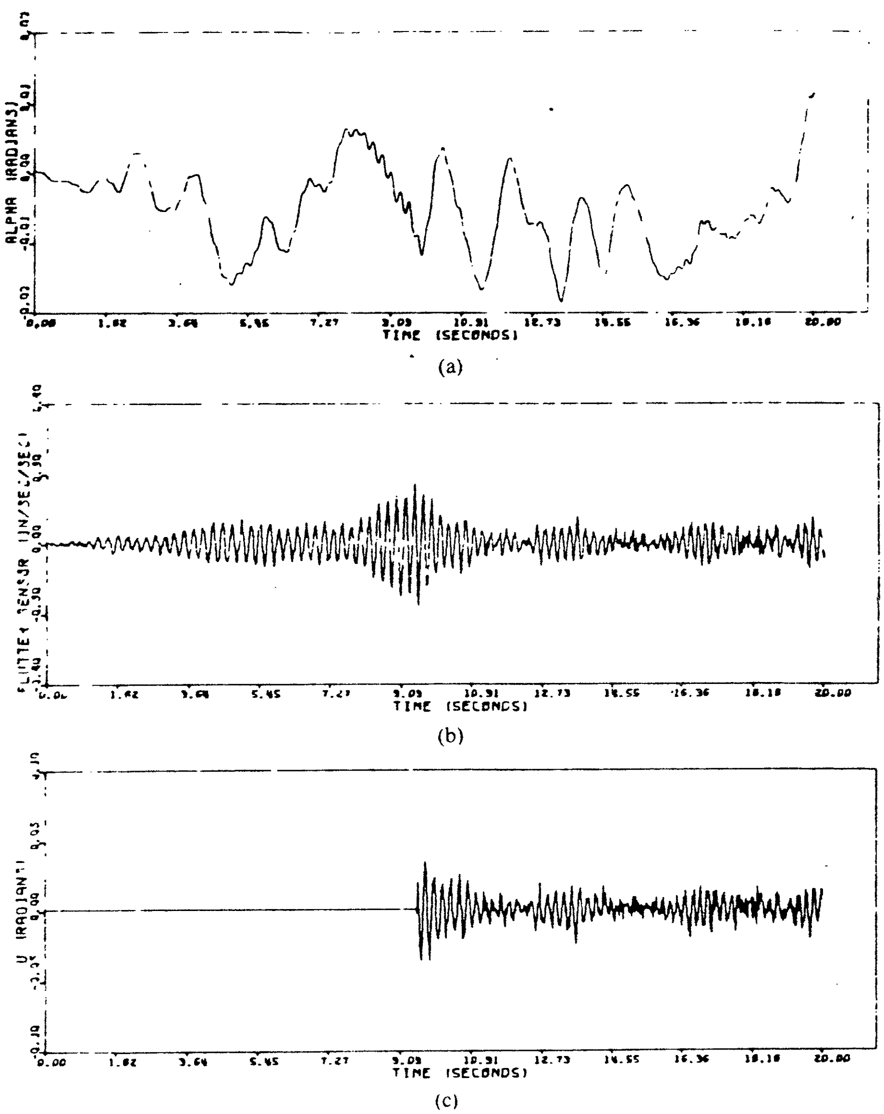

Fig. 3. Self-tuning regulator of Section II controlling full-order model. The velocity increases linearly over 20 s from 0.9 to 1.3 times the flutter velocity, V_f. The STR is turned on at the onset of flutter and successfully stabilizes the full-order model. (a) Pitch angle, (b) combined sensor output, (c) STR control input.

models found when $V = V_f$. The parameter values for the first configuration were given in Section II.

Two test cases were evaluated for each configuration. In the first case, the velocity was increased linearly with time from $0.9V_f$ to $1.3V_f$ and then held constant at $1.3V_f$; the regulator parameters were fixed at their initial values until approximately the time when $V = V_f$, and then the algorithms were made adaptive. In the second case, a longer duration test with $V = 1.3V_f$ was made to assess the asymptotic steady-state performance of the designs in the presence of noise. Only the results for the first configuration are shown here. Although the parameters of the other two configurations differed significantly, the qualitative features of the comparison are similar.

Fig. 3 illustrates the results of the first test case. The pitch rate, combined sensor output (which is essentially an estimate of the flutter mode velocity), and control input are shown.

Fig. 4 illustrates the rapid onset of instability for the open-loop plant in the second test case with $V = 1.3V_f$. Obviously, this application demands very rapid adaptation. Fig. 5 illustrates the results of the second test case. The parameter estimates for this case are shown in Fig. 6. The controller successfully stabilizes the plant and achieves qualitatively similar performance in terms of output variance and control energy requirements to a controller using on-line maximum likelihood estimation of parameters (see [5]).

IV. Conclusions

The self-tuning regulator concept is an adaptive control concept which has been successfully applied to a variety of single-input–single-output control problems including control of an ore crusher (Borisson and Syding [2]) and ship guidance and control (Åström and Kallstrom [3]). While

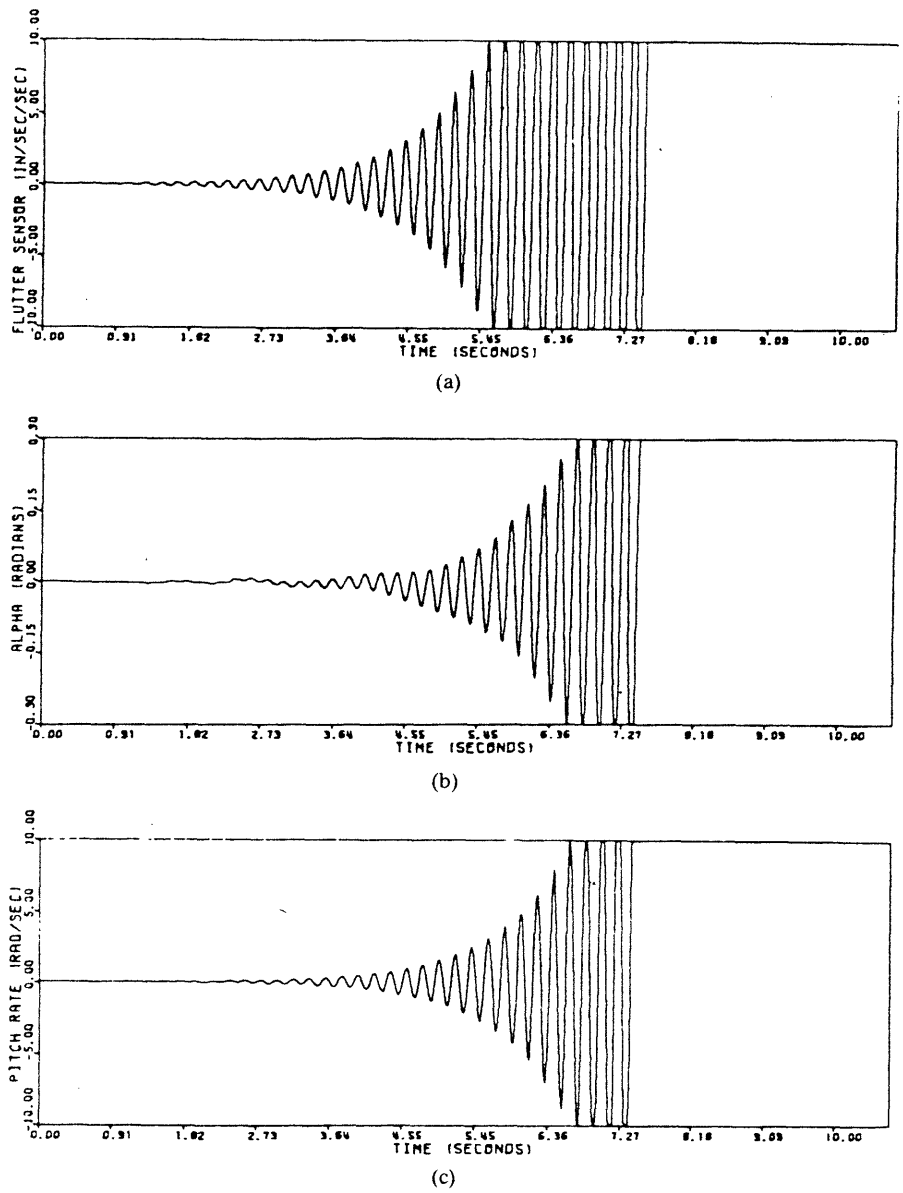

Fig. 4. Open-loop plant. Onset of flutter instability of full-order model when $V = 1.3V_f$. (a) Combined sensor output, (b) pitch angle, (c) pitch rate.

the simplicity of the self-tuning regulator is appealing both conceptually and computationally, the successful development of an actual design was found to require considerable attention to details of both theoretical and practical nature. To some extent, this is axiomatic for any design procedure, but the thorough validation of adaptive control algorithms is particularly critical because rigorous proofs of convergence are usually not available and because an algorithm may give the appearance of working successfully when in fact it is not. The wing/store flutter problem has certain features which have not been fully addressed in previous STR applications. First, there is the need to transform from continuous state-space or transfer-function models to the discrete-time autoregressive models required for STR design. Secondly, the modeling of uncertainty has to be done carefully in order to account for both gust disturbances and sensor noise; effects of time-correlated disturbances have not been fully documented in the literature. Third, the validity of the theoretical results for oscillatory, unstable, and nonminimum phase plants required further investigation, since previous applications did not exhibit all of these properties. Finally, the effects of using a reduced-order design model had to be assessed, particularly in the presence of additional noncritical flexure modes.

We found through experience that the pure minimum-variance controller used too much control energy and that, therefore, a derivation which allowed for control energy penalties was necessary. We found that recursive least-squares worked better than (extended) Kalman filtering for updating the compensator gains. For the cases we tested, we found it acceptable to neglect the effects of long-term noise correlation and model truncation errors. We found that after a sudden shift in plant parameters (or during initialization) the STR would produce undesirable control transients, which, however, had the fortuitous consequence of enhancing the identifiability of plant parameters.

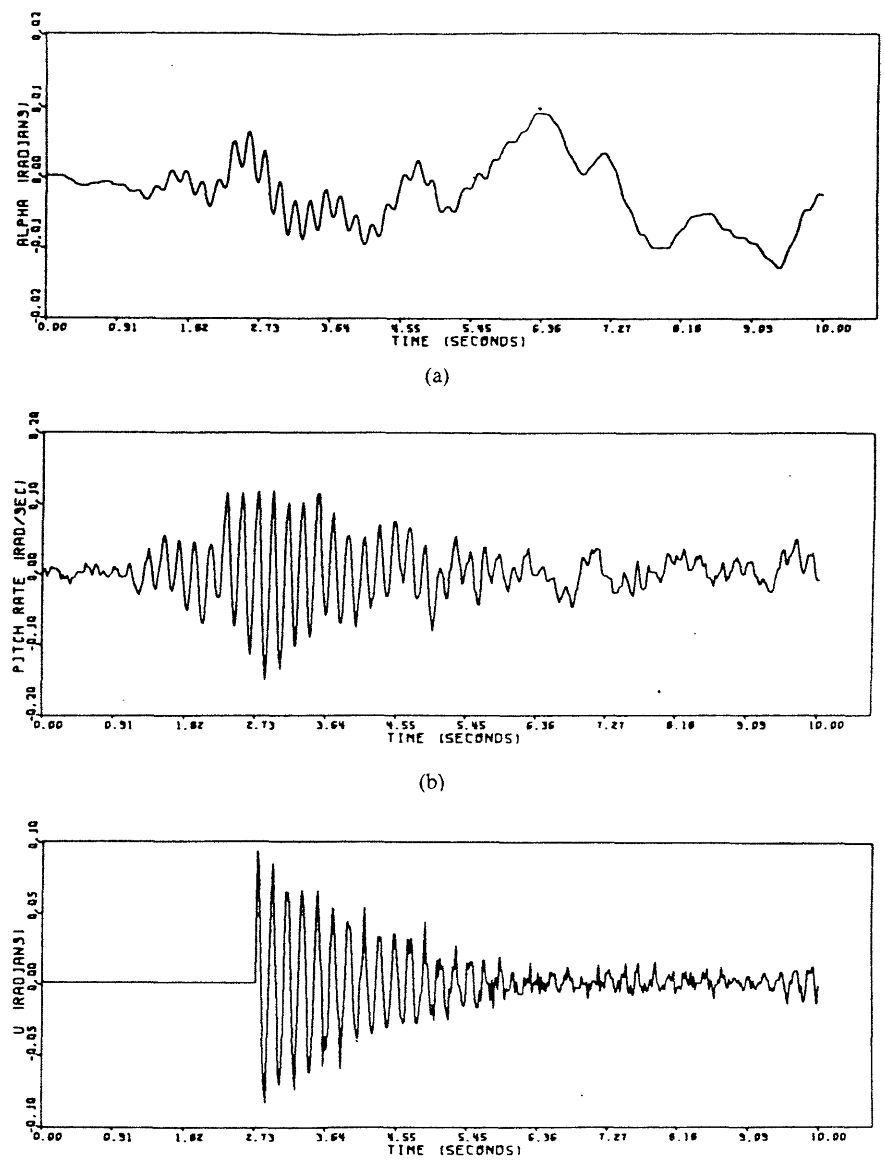

Fig. 5. Self-tuning regulator of Section II controlling full-order model, at a constant velocity of $1.3V_f$, 30 percent above the critical mode 1 flutter speed. (a) Pitch angle, (b) pitch rate, (c) STR control input.

We observed that one consequence of using a low-order STR design on a high-order aircraft model was a tendency of the compensator gain estimates to exhibit irregular long-term fluctuations, which did not, however, appear to significantly degrade control performance.

The performance of the self-tuning regulator and maximum-likelihood estimator/controller was comparable in all cases tested, and stability was always achieved. Thus, a comparison of the two approaches must be made on other grounds. The use of combined filtered sensor signals was critical to the success of both methods. The self-tuning regulator, in this application, would be much easier to implement and requires much less storage and computation time. The STR design procedure outlined in Section II is considerably simpler than the MLE design procedure, but it is not nearly as mature (see [10]). Identifiability and stability aspects of the STR have been studied by Ljung [4] and Gawthrop and Clark [17]. Furthermore, the extension of the STR design procedure to multiinput–multioutput plants has been reported only recently by Borisson [11], Koivo [12], and Keviczky [13], and has not reached the maturity of the MLE approach. It would also appear that some, but not all, of the design and implementation advantages are sacrificed in this more general case.

The importance of obtaining prior performance estimates in designing adaptive controllers cannot be overstressed. In this regard, the authors are of the opinion that the MLE approach currently offers more security than the STR approach. Prior performance estimates based on the asymptotic efficiency of the MLE approach, and sensitivity estimates of the effects of modeling errors have been made. Asymptotic stability can be guaranteed under specific controllability, observability, and identifiability hypotheses. By contrast, the available results for the STR approach are

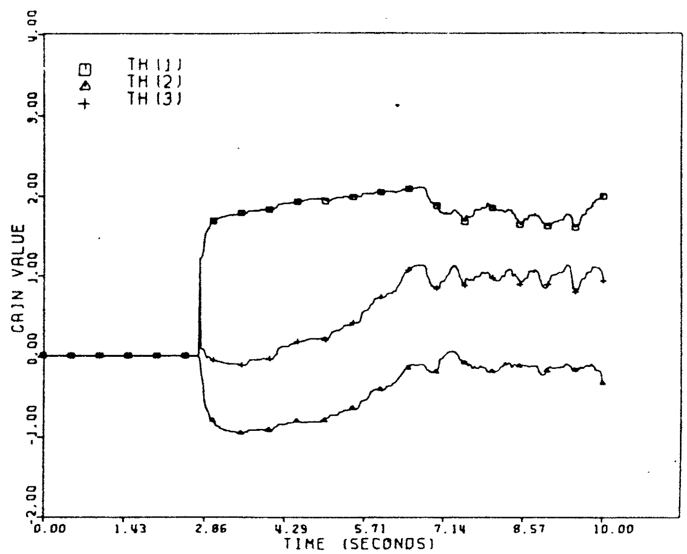

Fig. 6. Parameter estimates of self-tuning regulator for the trial shown in Fig. 5.

much less satisfactory. Of course, these do not reflect fundamental limitations of the STR method, but are rather a reflection of our current state of knowledge.

In closing, a number of prior works on flutter control and suppression should be mentioned [14]–[16]. To our knowledge, this is the first report of adaptive control of flutter instabilities in aircraft, and the first substantial application of the self-tuning regulator approach to an aircraft control problem containing flexure mode dynamics.

APPENDIX
STEADY-STATE ANALYSIS OF THE STR ALGORITHM

To initialize the self-tuning regulator it is desirable in (2.27) to set P_0 near its steady-state value. If x is the vector of inputs and outputs defined in (2.25), we show that the choice

$$\tilde{P}_0 = P = (1-\beta)E\{xx'\}/(E\{x'x\})^2 \qquad \text{(A.1)}$$

is a good initial guess, where the expectations can be estimated using the properties of the controller designed for nominal parameter values (see (2.18) *et seq.*).

To derive the estimate (A.1), suppose in (2.27) that $x_t \equiv x$ and $\tilde{P}_t \equiv P$ as $t \to \infty$. Then the last two equations of (3.27) may be combined to give

$$Pxx'P = \beta(1-\beta)P + (1-\beta)P(x'Px) \qquad \text{(A.2)}$$

or

$$[Pxx' - (1-\beta)(\beta + x'Px)I]P = 0. \qquad \text{(A.3)}$$

Evidently $P = 0$ is one solution of this equation, but it is not a "stable" one. If P were nonsingular, another solution could be obtained by putting the coefficient of P in (A.3) equal to zero; left multiplying this coefficient by x' yields the equations

$$x'Pxx' = x'(1-\beta)(\beta + x'Px)I$$

or

$$x' = x'(1-\beta)\left(1 + \beta(x'Px)^{-1}\right). \qquad \text{(A.4)}$$

By setting the coefficients of x' equal,

$$1 = (1-\beta)\left(1 + \beta(x'Px)^{-1}\right), \qquad \text{(A.5)}$$

we find

$$x'Px = 1 - \beta. \qquad \text{(A.6)}$$

This does not specify P, but it suggests the solution

$$P = (1-\beta)x(x'x)^{-2}x'. \qquad \text{(A.7)}$$

We can verify this to be a solution of (A.2) by direct substitution. We could go on to verify that it is the maximal and "stable" solution of (A.2).

The assumptions made in deriving (A.2) were based on the desire to find an estimate for the average asymptotic value $\lim_{t\to\infty} E\{P_t\}$; hence (A.7) is interpreted in the sense of (A.1), based on the fact that the second-order statistics of x are asymptotically stationary. Our formal procedure is, in effect, justified by the fact that the expression for P depends only on the products xx' and $x'x$ (note that $\lim_{t\to\infty} E\{x_t\} = 0$). From (A.7) one may expect that the asymptotic mean value of P_t will be nearly singular (rank 1, to be precise), and that $P \to 0$ and $\beta \to 1$; however, these statements apply only to the *mean* value of P_t, for P_t is a random variable due to the fact that x_t is random in (2.27).

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Timothy L. Johnson (S'69–M'73) received the B.S., M.S., and Ph.D. degrees in electrical engineering from the Massachusetts Institute of Technology, Cambridge, in 1968, 1969, and 1972, respectively.

From 1972 to 1980 he served as a faculty member of the M.I.T. Department of Electrical Engineering and Computer Science, specializing in automatic control theory and its applications. During 1976 he was a Visiting Scholar at I.R.I.A., Paris, France; L.A.A.S., Toulouse, France; and Imperial College, London, England. He continues to hold lectureships at the Department of Electrical Engineering and Computer Science, M.I.T., and at the Department of Neurology, Boston University School of Medicine. In 1980 he joined Bolt Beranek and Newman, Inc., as a Senior Scientist. His research interests include infinite-dimensional systems theory and applications, control of movement in humans and machines, and the theory of hybrid-state dynamic systems. He has developed applications of these ideas in active vibration control of air and spacecraft, bioelectric signal processing and neurophysiological modeling, and robotics and automated manufacturing. The author of more than 50 publications, he has served as an industrial consultant and as a reviewer for several technical journals and U. S. government agencies.

Dr. Johnson was the recipient of the 1974 Donald P. Eckman Award of the American Automatic Control Council and the 1973 Edgerton Professorship at M.I.T. He has served as Editor of the *Control Systems Society Newsletter* (1973–1976), Program Committee Member for three IEEE Decision and Control Conferences, Professional Activities Chairman for the Control Systems Society, and Division I IEEE Professional Activities Coordinator.

Charles A. Harvey (M'80) was born in Gering, NE, on August 14, 1929. He received the A.B. degree in physics from Nebraska Wesleyan University, Lincoln, in 1951, the M.A. degree in mathematics from the University of Nebraska, Lincoln, in 1953, and the Ph.D. degree in mathematics from the University of Minnesota, Minneapolis, in 1960.

He has been a member of the Control Sciences staff of the Honeywell Systems and Research Center, Minneapolis, MN, since 1960. In 1966 he was a Visiting Research Professor in the Control Sciences Center at the University of Minnesota, Minneapolis. He served as a National Research Council Senior Postdoctoral Research Associate at NASA's George C. Marshall Space Flight Center from August 1969 to August 1970. His principal research interest is in the development and application of modern control theory.

Dr. Harvey is a member of the American Mathematical Society, the Society for Industrial and Applied Mathematics, and Sigma Xi. He has served as the Chairman of the Control Systems Society Technical Committee on Applications (1979–1980) and is currently serving on the Board of Editors of the *Control Systems Magazine*.

Gunter Stein (S'67–M'69) received the B.S.E.E. degree from General Motors Institute, Flint, MI, and the M.S.E.E. and Ph.D. degrees from Purdue University, West Lafayette, IN, in 1966 and 1969, respectively.

Since 1979 he has been with the Honeywell Systems and Research Center, Minneapolis, MN. As of January 1977, he also has held an Adjunct Professor appointment with the Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge. His research interest is to make modern control theoretical methods work.

Dr. Stein is a member of AIAA, Eta Kappa Nu, and Sigma Xi. He has served as Chairman of the Technical Committee on Applications (1974–1975), IEEE TRANSACTIONS ON AUTOMATIC CONTROL, and as Chairman of the 14th Symposium on Adaptive Processes.