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Design and Analysis of a Smart Market For Industrial Procurement

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Abstract

This paper addresses the problem of designing multi-item procurement auctions in capacity-constrained environments. Using insights from classical auction theory, we construct an optimization-based auction mechanism ("smart market") relying on the dynamic resolution of a linear program minimizing the buyer's cost under the suppliers' capacity constraints. Suppliers can modify their offers in response to the optimal allocation corresponding to each set of bids, giving rise to a dynamic competitive bidding process. To assist suppliers, we also develop a bidding suggestion device based on a myopic best response (MBR) calculation that solves an associated optimization problem. Assuming linear costs for the suppliers, we study within a game-theoretic framework the sequence of bids arising in this smart market. Under a weak behavioral assumption and some symmetry requirements, an explicit upper bound for the winning bids is established. We then formulate a complete behavioral model and solution methodology based on the MBR rationale, and show that the bounds derived earlier continue to hold. We analytically derive some structural and convergence properties of the MBR dynamics in the simplest non-trivial market environment, which suggests further possible design improvements, and investigate bidding dynamics and incentive compatibility issues via numerical simulations. In particular, experiments suggest that suppliers can be relied upon to provide truthful capacity information when procurement contracts are properly designed.

1. Introduction

1.1. Background and Motivation. Online business-to-business auctions are becoming increasingly widespread, because their demand revelation properties are superior to that of fixed-price mechanisms, while the Internet keeps their transaction costs low: In a May 2000 study by Forrester Research, Inc. (Kafka et al. 2000), it was estimated that the total annual volume of transactions realized through online business-to-business auctions in the U.S. would reach $746 billion by 2004. However, online implementations of traditional open auction mechanisms are often poorly adapted to the selection of industrial suppliers in procurement markets, because this process typically involves capacity constraints, transportation costs, incumbent supplier switching costs, complex quality requirements, etc.: Transfer price, although the sole focus of these mechanisms, is only one consideration among several when it comes to choosing suppliers. Consequently, an important challenge faced by online procurement service providers is to design alternative auction mechanisms adapted to these complex allocation environments.

An example of such a mechanism is the bidding process used by FreeMarkets (Rangan 1999). One of the emerging leaders in electronic industrial auctions, FreeMarkets organizes "bidding events" for large industrial buyers typically seeking to auction off procurement contracts for specified quantities of several component types (or lots). In the mechanism currently used, the various component types are ordered, and each component is effectively auctioned off in a sequential manner. More precisely, binding bids on selling prices are solicited for each component type from competing suppliers, who can observe the value of the lowest bid submitted thus far. When a pre-specified time limit is reached for the first component type in the list (after perhaps some overtime period triggered by an activity-based criteria), bidding closes for that component type, competition then turns to the next component type, and this process repeats itself until the list of component types required by the buyer is exhausted. However, in contrast to traditional auctions, the actual allocation of procurement contracts is only decided by the buyer after the entire bidding event is completed, and the lowest bidder may not be awarded the contract for a component. The reason for postponing the allocation decision is to allow the buyer to take into account some of the non-price factors mentioned earlier.

Although this mechanism appears to represent current best practice as of the summer of 2000 (it is also used by FreeMarkets' competitor eBreviate), several sources suggest directions for improvement. In particular, a consequence of the delayed allocation decision is that suppliers' bids are based on very limited competitive information, which appears to be especially damaging in environments characterized by supplier capacity constraints: During our meeting with FreeMarkets co-founder Sam Kinney (Kinney 2000), he hypothesized that some suppliers grew nervous during the bidding event about receiving a total allocation exceeding their production capacity, especially if they had submitted relatively low bids on one or several of the components auctioned early on. As a result, these suppliers would withhold their bids on later components, thereby hampering competition. Indeed, some buyers also seem to be aware of this issue; for example, Jack Porter, a manager of central purchasing direct at Caterpillar, made the following comment during an interview on his experience as a FreeMarkets customer (Janhke 1998): "This concept only applies in those areas where there is a great deal of capacity."

Motivated by these observations, our objective is to design and analyze a stylized online multi-item procurement auction mechanism adapted to supply environments with production capacity constraints. Following a review of the relevant literature, we describe in §2 the market environment under study, the proposed auction mechanism, and the specifics of the game formulation. In §3, we discuss our solution methodology, derive some convergence bounds, and investigate bidding dynamics and incentive compatibility issues. Concluding remarks are provided in §4.

1.2. Literature Review. The study of multi-item auctions has been one of the most active research areas in microeconomics lately (see Klemperer 1999 for a recent survey). It is also becoming an increasingly popular topic in operations management, operations research and computer science. The crux of the problem lies in the interdependencies between the various objects being auctioned off, which arise in practice from factors such as capacity constraints, budget constraints, economies of scale/scope, and legal restrictions. In our model, capacity constraints are the only source of interdependencies across component types, and there are no interdependencies between different units of the same component type, since component production costs are assumed to be linear. Our discussion of the literature (restricted to papers studying multi-item auctions) is organized in two parts: The first is relevant to the design of our auction mechanism, while the second addresses outcome analysis.

The design of appropriate multi-item auction mechanisms is the object of impassioned debate, motivated in large part by the sales of broadband spectrum licenses by the Federal Communication Commission that began in the early nineties. A pivotal issue is whether bidders should be allowed to submit bids on packages or bundles of items (Bykowski et al. 1995), giving rise to so-called "combinatorial auctions" (Rothkopf et al. 1998, Kelly 1999, de Vries and Vohra 2000) that rely on optimization algorithms to determine the winners, in contrast with adaptations of more traditional auction formats where bids are only allowed for individual items (Milgrom 1999). A widespread concern about combinatorial auctions is the allocation complexity they induce (Rothkopf et al. 1998). While the market mechanism we study in this paper is also an optimization-based auction, the allocation complexity it addresses does not stem from package bidding, but rather from the production capacity constraint of each supplier across the multiple component types auctioned off: It relies on a simple linear program (LP), so that computational complexity is not an issue.

More generally, our auction format belongs to the family of market mechanisms known in the literature as *smart markets*, which are exchange institutions supported by a computer executing an optimization algorithm to solve the allocation problem associated with each given set of bids. This approach has been applied to a variety of market environments (see McCabe et al. 1991 for an early survey), including airport runway allocation (Rassenti et al. 1982), space shuttle/station payload allocation (Banks et al. 1989, Plott and Porter 1996), railroad track allocation (Brewer and Plott 1996), natural gas pipeline networks (McCabe et al. 1989), truckload allocation (Caplice 1996), project management (Ledyard et al. 1994, Walsh and Wellman 1998), general assignment problems (Olson and Porter 1994) and distributed scheduling (Kutanoglu and Wu 2000, Wellman et al. 1999). To our knowledge, Stanley et al. (1954) contains the earliest occurrence in the literature of using optimization to compute an allocation in a competitive bidding process. It is also the smart-market paper that is closest to our work from a contextual standpoint: They formulate a LP to determine the optimal allocation of procurement contracts for a fixed set of (sealed) bids submitted by army clothing manufacturers with capacity constraints. While the transportation problem they suggest is almost identical to the LP we use as the allocation engine of our mechanism, a distinguishing feature of our work is that we study analytically the dynamic competitive process that arises as the real-time solution to this LP generates competitive information feedback to the bidders, who can then modify their bids, thereby leading to a new instance of the LP. To our knowledge, the only other studies investigating analytically the properties of bidding sequences in an iterative optimization-based auction mechanism are Demange et al. (1986), Wellmann et al. (1999) and Parkes and Ungar (2000), which are discussed further below.

In designing the information structure of our mechanism, we found relevant the discussion by Cramton (1998) of how ascending auctions compare to sealed-bid auctions. More specifically, we have tried to take advantage of the design flexibility allowed by organizing the auction on a distributed network of computers such as the Internet in order to achieve a satisfactory balance between facilitating competition and preventing collusion. An important issue related to the former objective, which we feel is overlooked in the vast majority of the papers cited above, is how bidders confronted with a complex multi-item allocation mechanism can be assisted in making their bidding decisions: Rothkopf et al. (1998) show that the complexity of the bidder's "minimal winning bid" problem is tightly connected with that of the allocation (or "winner determination") problem, so that a human bidder's computational abilities are likely to be quickly overwhelmed in all smart markets of interest. Recognizing that this concern is not only relevant to bidders, but also to the design of an efficient auction mechanism, we investigate a particular form of competitive information feedback that is called *myopic best response* (MBR). This feature allows each supplier to enter his production costs into a private calculation device that will compute on his behalf the bids maximizing his potential payoff in the next round, under the assumption that his competitors' bids remain unchanged (Fudenberg and Tirole 1996); Note that the dynamic minimum ask price posting rules described in Demange et al. (1986) and Parkes and Ungar (2000) also address the limited computational abilities of human bidders. However, these rules require bidders to evaluate in real-time their utility over the set of all subsets of items being traded. Moreover, if the quantities of items to be allocated are continuous decision variables then these rules could, in theory, be adapted via discretization, but perhaps at the expense of computational intractability. The novelty of our MBR feedback is to fully account for the optimization problem that will be solved by the buyer in the next round when performing this calculation for the bidder.

We now turn to the relevant literature on outcome prediction. Because the computational requirements associated with the assumption of full rationality are particularly formidable in this game under uncertainty (Chapter 8 in Fudenberg and Tirole 1996, Samuelson 1997), we follow instead the approach developed in the theory of *tatonnement* and learning in games (Fudenberg and Levine 1998, Samuelson 1997); our rationale for using this approach is discussed in more detail in §3.1. That is, we explore the implications of less-than-fully rational behavioral models on the final outcome. Under a "weak" behavioral assumption defined in §3.2, some symmetry requirements, and a condition on the minimum level of competition, we derive an upper bound on the winning bids at the end of the auction, which to our knowledge is the first explicit outcome prediction as a function of the market environment parameters in the smart-market literature. We also propose a complete behavioral model (justified at some length in §3.3.1) that is a natural adaptation to our setting of the myopic best response bidding strategies assumed in Parkes and Ungar (2000) in a generic combinatorial auction, also referred to as "straightforward" bidding by Demange et al. (1986) in an assignment auction, by Wellman et al. (1999) in a decentralized scheduling mechanism and by Milgrom (1999) in the simultaneous ascending auction. These authors establish explicit relationships in their respective settings between the final bids and the minimum equilibrium bids, and between the final allocation and the surplus-maximizing (i.e. efficient) allocation.

In contrast, we consider the traditional perspective of a revenue-maximizing auctioneer; Although revenue-maximizing auctions are efficient if there is a resale market after the auction (Ausubel and Cramton, 1999), this is unlikely to occur in procurement settings. From the revenue-maximizing viewpoint, we derive some upper bounds on the final bids, and hence the buyer's final procurement cost, in terms of the specific data of the procurement problem under study (production costs, capacity constraints), and investigate how they are affected by the initial bids. We also provide a numerical study of some incentive compatibility issues, mostly related to the suppliers' revelation of their capacity constraints.

2. Mechanism Description and Game Definition

2.1. Market Environment. The initial situation giving rise to our market design problem is a monopsony where a buyer (typically a large manufacturing corporation) wants to purchase some specified quantity of m component types (namely q_j units of each component type j in $\{1, \ldots, m\}$). Facing this opportunity are n suppliers ready to compete on selling prices for the award of the corresponding procurement contracts. We assume that the only parameters left to specify in these contracts are the total quantity along with the corresponding selling price for each component type. In particular, the rules by which payment terms, delivery schedules, insurance provisions, etc. will be determined are clearly defined from the outset and accepted by suppliers. In addition to the buyer's quantity requirements and her low cost objective, the other consideration relevant to the allocation of components is that each supplier has a limited overall production capacity across all component types. In the present study, we assume that the capacity limitation of each supplier $i \in \{1, \ldots, n\}$ can be appropriately described by a linear model specifying a total amount c_i of production resource available (e.g., machine-hours or man-hours), and the amounts a_{ij} of this resource needed to produce one unit of each component type j. We also assume these capacity constraints to be exogenous, so that suppliers do not have the option of acquiring additional production capacity as part of the competitive interaction under study (see Budde and Göx 1999 for an analysis of a procurement auction with endogenous capacity decisions).

The remainder of this section is organized as follows: The core of the proposed optimization-based allocation mechanism is presented in §2.2, along with some practical implementation issues. Section 2.3 focuses on the suppliers' payoff function, which is subsequently used both to derive outcome predictions in §3, and to generate more efficient competitive information feedback. The design of this information feedback is discussed in §2.4, which more generally summarizes the information structure underlying this competitive interaction.

2.2. Proposed Mechanism. Bidding in our mechanism takes place through a discrete set of bidding rounds, denoted by $t \in \mathbb{N}$ ($t = 0$ is the initial round). More precisely, each supplier i is invited in each round t of the auction to simultaneously bid on his unit selling price $b_{ij}(t)$ for all component types j in $\{1, \ldots, m\}$.

Each bid $b_{ij}(t)$ corresponds to an offer in round t by supplier i to sell *any* quantity between 0 and q_j of component type j at a unit price given by the value of the bid (see §2.3 for a discussion of how this relates to the suppliers' objective). The first constraint on these bidding decisions is a classical *non-reneging* rule (i.e. $b_{ij}(t) \leq b_{ij}(t')$ for $t' \leq t$) imposed to ensure the efficiency of this mechanism: Increasing a bid submitted in some previous round would amount to reneging a prior offer to sell, and thus threaten the stability of the price formation process. The second constraint is a *common multiple* rule (i.e., $b_{ij}(t) \in \epsilon\mathbb{N}$ for some $\epsilon > 0$) that further restricts the set of admissible bids to a regular lattice with granularity ϵ, or "ϵ-grid". This rule both enforces a minimum bid decrement frequently imposed in practice (see Aeppel 1999), and prevents strategic collusion signals using the lower digits of the bids (reported in Kelly and Steinberg 1999 for example). In addition, the common multiple rule resolves a technical definition problem associated with the solution concept we introduce in §3.3.

The key feature of our auction format is that the overall capacity constraint of each supplier will always be satisfied by all possible allocation outcomes, which seems desirable from both the suppliers' and the buyer's perspective, as discussed in §1. This is achieved in two steps: First, the capacity constraints $\left((a_{ij})_{j\in\{1,\dots,m\}}, c_i\right)$ of each supplier i are estimated *before* the beginning of the auction. The parameters of these capacity constraints can be either assessed by an auditor during a pre-certification/qualification of supplier i prior to the auction, or merely provided by supplier i. In the latter case, the incentive compatibility issues linked to this capacity revelation become particularly important, and are discussed in §3.3.4. Second, once all the bids have been submitted for round t during the auction, the mechanism calculates the quantity allocation $x_{ij}(t)$ of component type j to supplier i minimizing the buyer's total procurement cost, subject to both the quantity requirement constraint for each component type and the capacity constraint of each supplier (estimated during the first step mentioned above). This allocation $\mathbf{x}(t) = (x_{ij}(t))_{\substack{i\in\{1,\dots,n\}\\ j\in\{1,\dots,m\}}}$ is obtained by solving in each round t the following LP, referred to in the remainder of this paper as

$AE[\mathbf{b}(t)]$ (for "allocation engine"):

$$\begin{array}{ll} \underset{x_{ij}}{\text{Min}} & \sum_{i=1}^{n}\sum_{j=1}^{m} b_{ij}(t)x_{ij} \\ \text{s.t.} & \sum_{j=1}^{m} a_{ij}x_{ij} \leq c_i \ \forall i; \\ & \sum_{i=1}^{n} x_{ij} = q_j \ \forall j; \\ & x_{ij} \geq 0 \ \forall (i,j). \end{array} \tag{1}$$

Note that the complete specification of the allocation mechanism must also include a selection rule discriminating among multiple optimal solutions to $AE[\mathbf{b}(t)]$ when needed. Although we do not impose in general any particular selection rule, the convergence results presented in §3.2 require the optimal solution $\mathbf{x}(t)$ to be an extreme point of the feasible set of $AE[\mathbf{b}(t)]$ (which seems both a desirable feature for the buyer, because solutions corresponding to extreme points tend to involve a smaller number of suppliers, and is easy to achieve in practice, since some optimization algorithms such as the simplex method naturally generate extreme point solutions). Finally, the allocation engine $AE[\cdot]$ along with this optima selection rule define a function which unambiguously associates a potential allocation $\mathbf{x}(t) = (\mathbf{x}_1(t), \ldots, \mathbf{x}_n(t))$ to any given set of bids $\mathbf{b}(t)$, which we can summarize with the notation $\mathbf{x}(t) = AE[\mathbf{b}(t)]$.

To encourage competition among suppliers across bidding rounds, our proposed mechanism displays on each supplier i's screen at the end of each round t his own private potential allocation $\mathbf{x}_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{im}(t))$; we refer to this process as *private potential allocation feedback*. Each supplier i can then respond to this competitive information in the following round by submitting a new set of bids $\mathbf{b}_i(t+1)$, and may in particular lower his bids on the component types for which his potential allocation $\mathbf{x}_i(t)$ is deemed unsatisfactory. We defer until §2.4 a more complete description of the information structure in this mechanism.

The relevance of the private potential allocation feedback as a competitive signal (and hence its ability to generate bidding competition) follows from the activity-based termination rule that we study. More precisely, the bidding round iterations described above terminate after the first round $T \geq 1$ where no supplier modifies any of his bids from the previous round, i.e., when $\mathbf{b}(T) = \mathbf{b}(T-1)$. At this point in time, the allocation $\mathbf{x}_i(T)$ calculated for each supplier i by the allocation engine becomes final (hence the adjective "potential" used to qualify $\mathbf{x}(t)$ for $t \leq T$, since the termination round of the auction is not known *a priori*), and the final unit prices paid by the buyer to supplier i for the component types for which he receives a positive allocation are given by $\mathbf{b}_i(T)$, so that the buyer's final procurement cost resulting from the auction is $\sum_{i=1}^{n}\sum_{j=1}^{m} b_{ij}(T)x_{ij}(T)$. Note that this objective function does not take into account the fixed cost of establishing and managing a vendor relationship, so that nothing prevents this auction mechanism from selecting a large number of suppliers and generating many fragmented orders for the same component type in the final allocation. More generally, while the stylized auction mechanism just described is well suited to an analytical study, the allocation environment it captures may be too simplistic in practical situations. Some possible generalizations, including a bid adjustment system allowing the buyer to incorporate relative preferences among suppliers (arising, for example, from incumbent switching costs), are briefly discussed in §4.

2.3. Bidders' Payoff Function. Our key assumption here is that each supplier i has a fixed unit production cost v_{ij} for each component type j, so that his payoff (or utility) function in this market mechanism is described by $\Pi_i(\mathbf{b}(T)) \equiv \sum_{j=1}^{m} (b_{ij}(T) - v_{ij})\, x_{ij}(T)$, where T is the terminal bidding round as previously defined, and $(b_{ij}(T), x_{ij}(T))$ are related through $\mathbf{x}(T) = AE[\mathbf{b}(T)]$. This expression can be interpreted as the sum of supplier i's final profits from each component type, in that $b_{ij}(T) - v_{ij}$ represents supplier i's final unit margin on component type j, and $x_{ij}(T)$ is his corresponding final allocation.

Note first that there is no need to model capacity constraints via overload costs in $\Pi_i(\cdot)$, because by construction all allocations generated by our mechanism will satisfy these constraints. More fundamentally, this payoff formulation implies a linear cost structure, where for all suppliers the unit production cost of each component type is independent of the quantity allocated. Consequently, our model does not capture the possible economies of scale that suppliers may experience when they receive large orders (set-up times, production learning curve, etc.). Nor does it capture the possible diseconomies of scale associated with, for example, the necessary purchase of additional production capacity. While this cost linearity is a strong assumption, we observe that it is consistent with the auction mechanism we propose to study, where the bids correspond to binding offers to sell at a specified unit price independent of the quantity received (see §2.2): Because capacity constraints are exogenous and will always be satisfied in this setting, the reason why suppliers may not be willing to submit such quantity-independent bids in some situations is precisely because their production costs may not be linear; see Gallien (2000) for a discussion on the design of improved smart auction mechanisms allowing suppliers to take (dis)economies of scales into account when they formulate their bids.

Finally, neither our payoff function nor our allocation mechanism capture the possible synergies (or incompatibilities) across component types, since each supplier's production cost for one component type is also independent of his allocation of the other component types. In procurement situations in which such (dis)economies of scope are particularly important, we believe that some alternate allocation mechanisms (e.g., a combinatorial auction) may be more appropriate (see references in §1.2).

2.4. Information Structure. In this section we describe the information structure underlying our proposed market mechanism, namely which entity knows what information. We also show how the suppliers' payoff function presented in the previous section can be used to complete the competitive information feedback provided to bidders during the auction and lead them to make more efficient bidding decisions.

There are three types of information depositories to consider: The suppliers, the buyer and the mechanism/auctioneer. We assume here that the auctioneer is trusted by the suppliers to be respectful of its confidentiality commitments. This is legitimized in our view by the current fierce competition between online procurement auctioneers (e.g., Freemarkets and eBreviate), where reputations for integrity standards seem to play a particularly important role: Even though procurement auctioneers primarily serving buyers are by no means neutral, their confidentiality commitment seems credible in that the long-term costs of losing the suppliers' trust will likely outweigh the short-term benefits of communicating to the buyer some private information obtained from suppliers under a non-disclosure agreement. This along with the technological medium used to support this auction (a distributed network of computers) effectively justify the decoupling between the buyer and the mechanism/auctioneer that is inherent in our mechanism. For studies of competition among auctioneers, see Burguet and Sákovics (1999) and references therein.

We first discuss the information structure relative to the market environment. Because the quantities $(q_j)_{j\in\{1,\dots,m\}}$ of each component type required by the buyer motivate in large part the suppliers' interest in the auction, it seems natural that they should be common knowledge to all participants. However, a key feature enabled by the use of a computer network to run the auction is that the suppliers need not know how many competitors they have, let alone their identities (for studies of traditional auctions with unknown or uncertain number of bidders, see Harstad et al. 1990 and references therein). Because this feature offers some protection against possible collusive behaviors, we have designed our auction mechanism to take advantage of it. In adherence to this design philosophy, suppliers are not informed by the mechanism of their competitors' capacity constraints $\left((a_{ij})_{j\in\{1,\dots,m\}}, c_i\right)$ either. However, this capacity information is required by the mechanism to ensure that all potential allocations generated satisfy these constraints (see §3.3.4 for a discussion of the incentive compatibility issues associated with the revelation of their capacity by the suppliers).

The only dynamic competitive information feedback proposed so far is the private potential allocation feedback $\mathbf{x}_i(t)$, communicated to each supplier i during the auction, but hidden to his competitors. In particular, because we seek to limit the possibilities of collusion and strategic behavior, our mechanism does not send any direct information to suppliers during the auction about their competitors' bids (Cramton 1998). Using private potential allocation feedback alone, however, would probably lead to an inefficient price formation process, because suppliers would lack sufficient information to rationally decide on which component types to lower their bids, and by how much. As a result, suppliers in this situation would most likely engage in exploratory bidding behaviors, whereby they would try to identify the minimal bid decrease leading to some improvement of their potential allocation through multiple rounds of minimum bid decrement trials.

The option that we have investigated to prevent this involves providing to each supplier in every round the possibility to have his *myopic best response bid* calculated by a local bidding suggestion device. In words, this suggested bid to supplier i is the one that maximizes his potential payoff function in the next round, assuming that all his competitors' bids remain unchanged from the current round; i.e., $\mathbf{b}_i^*(t+1) \equiv \sup \left[\arg \max_{\substack{0 \le \mathbf{w} \le \mathbf{b}_i(t) \\ \mathbf{w} \in (\epsilon \mathbb{N})^m}} \Pi_i(\mathbf{w}, \mathbf{b}_{-i}(t)) \right]$, where $\Pi_i(\cdot)$ is the payoff function defined in §2.3, $\mathbf{w}$ represents the decision variables associated with the bids of supplier i in the following round, and $\mathbf{b}_{-i}(t)$ is the standard game-theoretic notation for the bids of all supplier i's competitors in the current round. Note that the arg max operator acts upon a finite set because of both the non-reneging and the minimum decrement rules, so that the output of this operator is always non-empty even though $\Pi_i(\cdot)$ is not necessarily continuous. Moreover, the sup operation over the arg max set, provided it is associated with a total order over $(\epsilon \mathbb{N})^m$, ensures that the myopic best response bid is uniquely defined. With the lexicographic order, for example, suppliers will not be suggested to lower their bids unless their prospective potential payoff when doing so is strictly greater than their current potential payoff.

A key information requirement of the myopic best response calculation is that each supplier i willing to use this feature needs to provide the value of his own production costs $(v_{ij})_{j \in \{1,\ldots,m\}}$ via the auction software interface. Because the production costs of each supplier are sensitive and private information, it is essential for the auctioneer to credibly commit that these costs, if entered into the myopic best response calculation device, will not be disclosed to the buyer or to any other supplier. We also observe that in end-consumer online auction sites (Yahoo!, eBay, etc.), users routinely enter their maximum bid for an item into some software agent (known as proxy bidding, bidding agent, bidding elf, etc.) that then submits bids on their behalf whenever needed and up to this specified limit (Lucking-Reiley 1999). Because these maximum bid limits are the exact analogue of the production costs

		Information Available to:			
Information	*Notation*	Supplier i	Other Suppliers	Mechanism-Auctioneer	Buyer
number and identity of competing suppliers	n	no	no	yes	yes
component quantities required	$(q_j)_j$	yes	yes	yes	yes
capacity constraint of supplier i	c_i $(a_{ij})_j$	yes	no	yes	yes
production cost of supplier i	$(v_{ij})_j$	yes	no	yes	no
bids of supplier i	$\mathbf{b}_i(t)$	yes	no	yes	yes
potential allocation of supplier i	$\mathbf{x}_i(t)$	yes	no	yes	yes
best response suggestion of supplier i	$\mathbf{b}_i^*(t)$	yes	no	yes	no

Table 1: Summary of the information structure.

in our procurement auction setting (and more generally correspond to bidders' valuations in the traditional auction theory literature), we feel that the myopic best response bidding suggestion feature is likely to be implementable in practice.

To conclude this section, we summarize in Table 1 the information structure underlying our proposed auction mechanism. We are now ready to turn to the analysis of our proposed market mechanism.

3. Analysis

3.1. Methodology. The basic premise underlying our methodology is that bidders will be less-than-fully rational when confronted with the smart auction mechanism described earlier. While from a practical standpoint this assumption simplifies the analysis, we share with other researchers (Chapter 1 in Samuelson 1997) the belief that it is also motivated by behavioral considerations. In this dynamic game of incomplete information, the classical solution concepts associated with the requirement of full rationality predict that bidders will form probabilistic beliefs about all relevant uncertain information in the game, update these beliefs in a consistent manner during the auction, and derive and execute in every round a bidding strategy maximizing the expected value of their final payoff function accordingly (Chapter 8 in Fudenberg and Tirole 1996 and references therein). Because the uncertain information includes here not only the competitors' production costs, but also their capacity constraints, their bids and even their number, a rational Bayesian expected utility maximizer would have to solve an overwhelmingly complex stochastic dynamic program with imperfect information. The alternate methodology we have adopted consists of assuming a deterministic, less-than-fully rational behavioral model, ultimately specifying for each supplier a mechanistic relation between the information received in each round and his subsequent bidding decision. This type of model is known in the literature as a nonequilibrium adjustment or *tatonnement* process, and is the starting point to the theory of learning in games (see §1.2 for background references).

3.2. Weak Behavioral Assumption and Convergence Bounds. To establish some convergence bounds, we only rely initially on the *weak behavioral assumption* that a supplier receiving in some round a private allocation feedback of zero for all component types will lower at least one of his bids, unless doing so would generate a negative profit. Formally, this means that if $\mathbf{x}_i(t) = 0$ and there exists $j \in \{1, \ldots, m\}$ such that $b_{ij}(t) > v_{ij} + \epsilon$, then there exists $j \in \{1, \ldots m\}$ such that $b_{ij}(t+1) \leq b_{ij}(t) - \epsilon$.

We begin by stating an *ex post* result (i.e., relying on an observation of the final allocation made *a posteriori*) that relates the final winning bids and the order statistics of the production costs.

Proposition 1 (ex post **bound**) *Let T be the final round of the auction, let $v^j_{1:n}, \ldots, v^j_{n:n}$ denote the order statistics of $(v_{1j}, \ldots, v_{nj})$, $j \in \{1, \ldots, m\}$, and define $P^C \equiv \{i \in \{1, \ldots, n\}, \mathbf{x}_i(T) = 0\}$. Provided that $|P^C| \geq 1$ and under the weak behavioral assumption, we have*

$$x_{ij}(T) > 0 \Rightarrow b_{ij}(T) \leq v^j_{n-|P^C|+1:n} + \epsilon \; \forall j \in \{1, \ldots, m\}. \tag{2}$$

The proof of Proposition 1 (found in the Appendix along with all remaining proofs) exploits linear programming duality and complementary slackness; we strongly suspect that it can be extended to a more general class of optimization-based auctions. In words, Proposition 1 states that the highest selected bid on each component type at the end of the auction cannot be larger than some order statistic of the production costs for that component type (modulo the minimum bid decrement ϵ), and the greater the number of rejected suppliers, the lower the rank of this order statistic. Moreover, it seems natural that the upper bound in (2) involves the number of suppliers eventually receiving no allocation, because the only behavioral assumption required to derive this result concerns precisely the suppliers receiving a null private allocation feedback. Proposition 1 generalizes to the case of a multi-item and optimization-based procurement auction the well-known result (Vickrey 1961) that in a single-item English ascending auction, the price eventually obtained by the seller is equal to the second highest valuation among the bidders (modulo the minimum bid increment). Although the information structure is slightly different in our mechanism, the set of parameters $m = 1$, $q_1 = 1$, c_i unbounded and $a_{ij} = 1$ for all i and j, when the allocation selected is always an extreme point, corresponds to the classical auction of a single indivisible object with no capacity constraints. In this case, there are $|P^C| = n - 1$ losing suppliers, so that the bound provided by Proposition 1 is indeed $v^1_{2:n} + \epsilon$.

Although Proposition 1 is an *ex post* result, it can be used to prove an *ex ante* result (i.e., applicable before the auction outcome is observed) in the case where the suppliers have the *same production technology*, i.e., all capacity constraints satisfy $a_{ij} = a_{i'j} = a_j \ \forall (i, i') \in \{1, \ldots, n\}$. To state this result in a compact form, we first assume without loss of generality that the suppliers are numbered by increasing order of capacity, i.e., $c_1 \leq c_2 \leq \ldots \leq c_n$. This assumption allows us to conveniently define the *maximal load number* as the smallest integer p such that $\sum_{i=1}^{p+1} c_i > \sum_{j=1}^{m} a_j q_j$. Note that this definition is not appropriate when $\sum_{i=1}^{n} c_i \leq \sum_{j=1}^{m} a_j q_j$, but this case can be dismissed because it corresponds to either infeasible buyer requirements or, at equality, to the trivial situation in which all the available supply capacity is required. The observation that the maximal load number provides an upper bound on the number of suppliers loaded to their capacity in any feasible allocation is key to the following proposition.

Proposition 2 (ex ante **bound**) *Assume that the optimal solution selected by the allocation engine $AE[\cdot]$ is always an extreme point, that all the suppliers have the same production technology, and that the maximal load number p satisfies $n - p \geq m + 1$. Then under the weak behavioral assumption, we have*

$$x_{ij}(T) > 0 \Rightarrow b_{ij}(T) \leq v^j_{p+m+1:n} + \epsilon, \tag{3}$$

and the buyer's final procurement cost is bounded by

$$\sum_{i=1}^{n} \sum_{j=1}^{m} b_{ij}(T) x_{ij}(T) \leq \sum_{j=1}^{m} q_j \left(v^j_{p+m+1:n} + \epsilon \right). \tag{4}$$

This last result is noteworthy in that it provides a performance guarantee for the buyer *before* the auction takes place, even though the behavioral assumptions required are particularly mild. As in (2), the upper bound (4) generalizes in the setting of a multi-dimensional optimization-based procurement auction the relation between winning bids and valuations in simpler auctions. However, it shows much more explicitly how this bound for the final bids depends on the primary data of the problem, namely the suppliers' capacity constraints and production costs, and the buyer's requirements. In particular, the larger the buyer's required quantities in relation to the suppliers' available capacities, the looser the bound on the final bids; hence, this expression provides quantitative support for the intuition that the buyer's market power decreases as supply resources become more scarce. Likewise, the applicability condition $n - p \geq m + 1$ can be interpreted as a minimum requirement on the level of competition in the auction (via the number of participating suppliers) or on the buyer's market power. Finally, the upper bound (4) helps reveal the relationship between the buyer's final procurement cost and her required quantities; note that in this model with linear production costs, an increase in the component market size may potentially increase the cost per unit because of the competitive effects.

3.3. Myopic Best Response. We now introduce a fully specified behavioral model, which follows directly from the myopic best response (MBR) information feedback described in §2.4. Namely, we assume that every supplier uses in each round the MBR calculation device and follows exactly its bidding suggestions, i.e., $\mathbf{b}_i(t+1) = \mathbf{b}_i^*(t+1)$ for all $i \in \{1, \ldots, n\}$ and all $t \geq 0$, where $\mathbf{b}_i^*(t+1)$ is the expression defined in §2.4. Note that the MBR bidding model almost satisfies the weak behavioral assumption, the exception being the suppliers with no current potential allocation who may still not benefit from lowering any of their bids (e.g., when all competing bids are lower than his production costs), because in such cases the sup operator used in the definition of $\mathbf{b}_i^*(\cdot)$ will cause these suppliers' bids to remain constant. Nevertheless, the convergence bounds established in §3.2 remain valid under the MBR behavioral model, as can be quickly checked from their respective proofs.

From a mathematical standpoint, the MBR behavioral model amounts to a recursive relation between consecutive values of the bidding sequence $(\mathbf{b}(t))_{t \in N}$ of the form $\mathbf{b}(t+1) = \mathcal{F}[\mathbf{b}(t)]$, $t \geq 0$. Because $\mathcal{F}[\,\cdot\,]$ is a well-defined function (see §2.4), the entire bidding history of the auction is completely characterized by the initial bids $\mathbf{b}(0)$ and this recursive relation. A property of the MBR bidding sequences essential to the theoretical validity of our solution concept is that they always converge in finite time (they are non-decreasing by the non-reneging rule, bounded from below and may only take a finite set of values by the common multiple rule). This can be formalized by the following proposition.

Proposition 3 *Let* $(\mathbf{b}(t))_{t\in\mathbf{N}}$ *be a myopic best response bidding sequence defined by a set of initial bids* $\mathbf{b}(0)$ *and the recursive relation* $\mathbf{b}(t+1) = \mathcal{F}[\mathbf{b}(t)]$. *There exists an integer* $T \geq 0$ *such that* $\mathbf{b}(t) = \mathbf{b}(T)$ *for all* $t \geq T$.

By the recursive definition of the MBR bidding sequence $(\mathbf{b}(t))_{t\in\mathbf{N}}$, its limit $\mathbf{b}(T)$ is necessarily a fixed point of the relation $\mathcal{F}[\cdot]$, i.e., $\mathbf{b}(T) = \mathcal{F}[\mathbf{b}(T)]$. Conversely, for every fixed point $\mathbf{b}$ of $\mathcal{F}[\cdot]$, there obviously exists a MBR bidding sequence converging to $\mathbf{b}$ (e.g., $\mathbf{b}(0) = \mathbf{b}$). The solution concept we propose to predict the final outcome in our market mechanism given an initial set of bids $\mathbf{b}(0)$ is precisely the limit $\mathbf{b}(T)$ of the MBR bidding sequence $(\mathbf{b}(t))_{t\in\mathbf{N}}$. This is the natural adaptation to our setting of the auction-specific *local Nash equilibrium* defined in Bykowsky et al. (1995) in the context of the simultaneous ascending auction.

3.3.1. Model Discussion. Although the MBR behavioral model is close to the classical Cournot or best response *tatonnement* process (Chapter 1 in Fudenberg and Tirole 1996), there remains a few important differences between them. First, the information structure of the game to which they apply differ, since the duopoly competition studied by Cournot is a game of common knowledge/full information, whereas our auction format corresponds to a game under uncertainty. Moreover, payoffs in our mechanism are not obtained at the end of every round as in the Cournot game, but only at the end of the auction. Consequently, the objective function used in each supplier's MBR calculation only represents his potential payoff in the next round, and only coincides with the actual payoff in the round immediately preceding the end of the auction. The MBR behavioral model thus implicitly assumes the belief that the auction will terminate in the next round (see discussion two paragraphs below on why this is still a reasonable assumption in this setting), justifying the use of the adjective "myopic". Finally, because of the non-reneging bidding rule, decision spaces in different rounds are not independent from each other as in the Cournot adjustment. More precisely, each bid constrains the future bidding decisions of the supplier who submitted it, and this constraint is more stringent as the value of this bid is lowered.

These specific features provide what we feel is a strong justification for applying the MBR model to the game under study. In the classical case with no uncertainty, the best response adjustment process has been sharply criticized for the players' naiveness and/or reasoning inconsistencies it implies: While it relies in every round on the assumption that the competitors' actions will remain constant in the next round (strategy stability), this very assumption is observably proven wrong in every round until equilibrium is reached (Fudenberg and Levine 1998). However, in the information structure underlying the game we study, bidders cannot observe their competitors' bids directly, and the indirect process by which they could try to infer these bids – based on the history of their own bids, private allocation feedback and MBR bidding suggestions – seems particularly complex. As a consequence, the assumption of strategy stability is plausible, in sharp contrast to more classical games where past actions are common knowledge.

More fundamentally, suppliers in this auction will be particularly cautious not to submit low bids prematurely, because of the lock-in effect induced by the non-reneging rule. In this setting, strategy stability is likely to be perceived as a wise assumption on which to base bidding decisions, even when also perceived as probably wrong. This is because incorrectly assuming that the competition will be more aggressive in the next round may lead to lower bids than necessary, thus wasting some margin and restricting the space of future possible actions. On the other hand, the activity-based termination rule ensures that a supplier who underestimates his competitors' aggressiveness in a given round by assuming strategy stability will not be penalized: If this supplier ends up lowering one of his current bids then the auction will continue for at least one more round, giving him the opportunity to adapt his bids to the actual bidding strategy of his competitors. If this supplier does not modify any of his bids then the auction may terminate, proving the strategy stability assumption correct and therefore his bidding decision optimal, or it may continue, leaving him the option to further modify his bids.

3.3.2. Impact of Initial Bids. Although Propositions 1 and 2 show that the dependence of the auction equilibrium on the initial bids should be relatively limited when the level of competition among suppliers is sufficiently high, these results do not apply in market environments characterized by supply resource scarcity. To gain some insights into the impact of initial bids when the level of competition is low, we turn to the analysis of the MBR bidding dynamics in the simplest non-trivial market environment in which our smart-market mechanism can be applied: The 2×2 *symmetric* case, where $n = 2$, $m = 2$, $q_1 = q_2 = q$, $a_{ij} = 1 \; \forall(i, j)$ and $c_1 = c_2 = c$, but v_1 need not equal v_2. The only interesting choices of parameters in this environment satisfy $1 < \theta \leq 2$, where $\theta = \frac{2q}{c}$, which is the number of suppliers required to cover the total procurement needs[1]. For this range of θ, the buyer has no choice but to use a positive amount of capacity from both suppliers.

[1] In the case $\theta \leq 1$, the allocation of each component type is entirely independent from the other, and the case $\theta > 2$ is infeasible.

The following proposition shows that even with such scarce supply capacity, when the initial bids are sufficiently close to each other and are greater than the component-wise maximum of the production costs, they have relatively little impact on the final outcome. Moreover, it also shows that the particular information structure used in our mechanism may still enable some bidding competition.

Proposition 4 *Let* $d(\mathbf{b}_1, \mathbf{b}_2) = \max(|b_{21} - b_{11}|, |b_{22} - b_{12}|)$. *In the symmetric* 2×2 *auction and under the MBR bidding model, there exists a selection rule discriminating between multiple optimal solutions to the allocation engine* $AE[\cdot]$ *such that*

$$d(\mathbf{b}_1(0), \mathbf{b}_2(0)) \leq \epsilon \Rightarrow d(\mathbf{b}_i(T), \mathbf{v}_1 \vee \mathbf{v}_2) \leq (\frac{3q - c}{c - q} + 1)\epsilon, i \in \{1, 2\}.$$

A legitimate question at this point is to determine to what extent the final outcome is affected when the initial bids are further apart. Although we do not entirely resolve this issue here, it is revealing to plot for a particular choice of parameters $(c, q, \mathbf{v}_1, \mathbf{b}_1(0))$ the stability zones for supplier 1, i.e., the subset of supplier 2's bidding space such that supplier 1 would not move in the next round should $\mathbf{b}_2(0)$ belong to it (see Figure 1).

Figure 1 shows the existence of *premature equilibria,* arising when the initial bids are too far apart, so that the increased order volume the higher bidder may receive by outbidding his competitor does not compensate for the margin reduction that would be incurred. For example, if supplier 2's bid was to lie in the lower-left quadrant of the stability zone, supplier 1 would not move in the following round by definition. Depending on supplier 2's production costs, he will either not move (thereby achieving a premature equilibrium) or may try to switch the component type for which he received the larger volume. If these observations also apply to environments with more suppliers and component types, then there are implications for mechanism design: Whenever supply capacity is suspected to be scarce, it seems in the buyer's interest to have initial bids as close to each other as possible, which could be achieved,

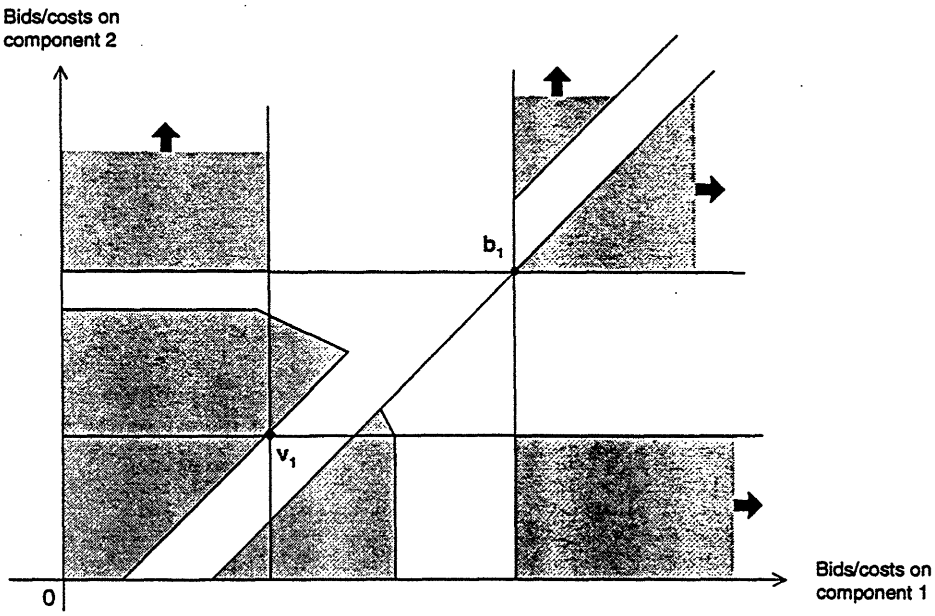

Figure 1: Supplier 1's stability zones (shaded areas).

for example, by enforcing a common minimum entry bid.

3.3.3. Bidding Dynamics. When the initial bids are sufficiently close, the proof of Proposition 4 also reveals the structure of the bidding dynamics in the symmetric 2×2 auction according to the MBR model. This structure is illustrated in Figure 2, which represents a typical MBR bidding sequence. Here, the initial bids are below the 45° lines going through the production costs of both suppliers, which we refer to as the *margin switching lines* for a reason that will soon become clear. As long as the bids remain in this area of the bidding plane, both suppliers try to outbid their competitor primarily on component type 1 (so as to receive the full order quantity of this component), because it is the one with the higher relative margin. Thus, the bidding sequence moves horizontally, until it hits supplier 2's margin switching line, at which point supplier 2 starts competing primarily for component type 2. The bidding sequence then moves downwards along this line, until it hits the horizontal line going through supplier 1's cost for component type 2, at which point this supplier stops competing altogether on this component type (his margin is then reduced to zero). The bidding sequence starts moving horizontally again, before it reaches an equilibrium in the neighborhood of $\mathbf{v}_1 \vee \mathbf{v}_2$, as predicted by Proposition 4.

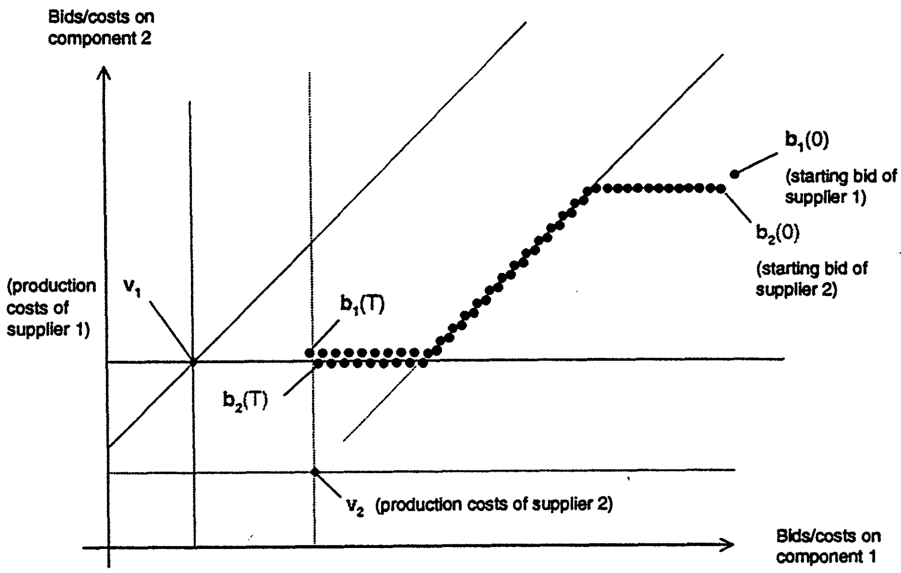

Figure 2: Bidding dynamics in the symmetric 2×2 auction.

Because of the combinatorial complexity involved, it seems hard to perform a similar analytical study of the MBR bidding dynamics in market environments with more suppliers and/or more component types. Instead, we undertook some numerical experiments simulating competitive interactions under the MBR behavioral model. The two experiments with eight suppliers and two component types represented in Figures 3 and 4 are fairly typical of the results we have obtained (see Gallien 2000 for more examples). These figures show each supplier's bidding sequence in the auction as a color-coded dotted line (a color version of these figures are available from the authors), and the corresponding production costs as diamonds. In addition, the set of connected squares is the sequence of dual prices associated in each round with the quantity requirement constraints in the formulation of the allocation engine $AE[\cdot]$ (referred to below as *market prices* according to the classical interpretation of LP duality theory). By design, these two experiments have the exact same parameters (capacity, required quantities, initial bids), with the exception of the production costs, which are more homogeneous across component types for the experiment[2] reported in Figure 4.

In Figure 3, all suppliers except one exhibit a specialized pattern of bidding, where they compete by decreasing their bids for only one of the two components. However, in Figure 4, there are many more "diagonal" bidding paths, i.e., competition takes place for more suppliers along the two component axes. The primary driving force behind these different bidding patterns seems to be the relative position of each supplier's production costs in each round with respect to the market prices. More specifically, as the auction evolves, the path formed by these market prices partitions the suppliers' production costs into three categories. Suppliers with production costs roughly proportional to the market prices tend to compete for both components, while suppliers with production costs clearly above or below the path of market prices typically only compete for the one component on which their current relative margin is the highest (as defined by the relative position of their production costs and the current market prices). This interpretation explains why more homogeneous production costs across components (as in Figure 4) lead to a competition pattern that is more mixed.

3.3.4. Incentive Compatibility. Although the incentive compatibility of suppliers' production costs input when using the MBR bid suggestion device is a legitimate question in theory, experiments described in Gallien (2000) tend to show that a truthful cost revelation is compatible with suppliers' incentives, so that we do not discuss this issue in further detail

[2] For each supplier, production costs in Figure 4 have been obtained from those of Figure 3 by a translation along the off-diagonal.

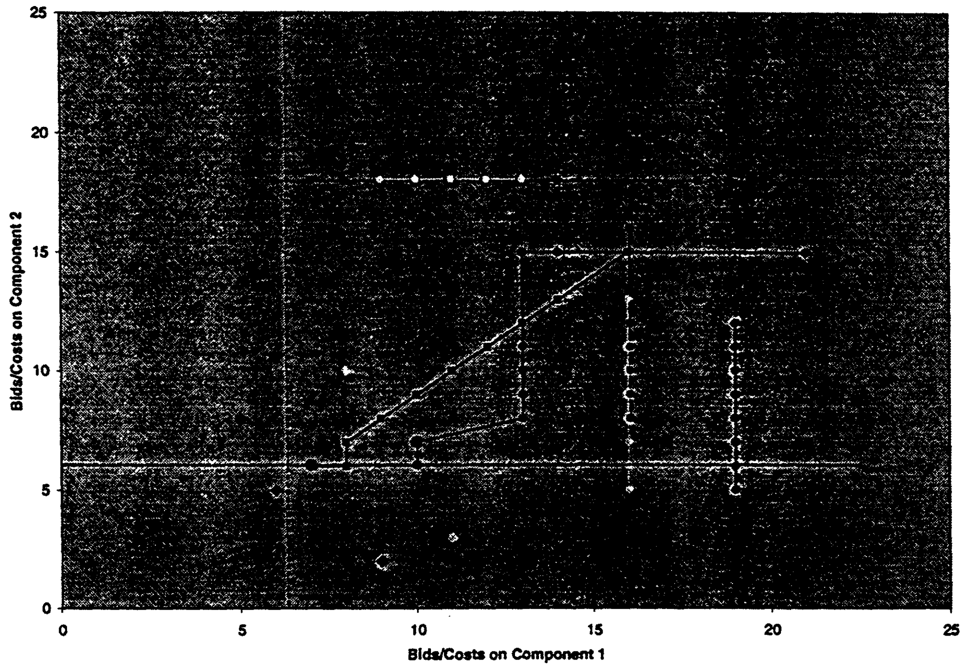

Figure 3: MBR dynamics in 8 × 2 auction with heterogeneous costs.

here. Instead, we focus on the incentive compatibility issues linked to capacity revelation.

A simple-minded analytical approach to this problem is to ignore the potential impact of a supplier's capacity input on the other suppliers' bidding behavior, and to perform a sensitivity analysis on this capacity input. That is, assuming that the bidding sequence has reached in round T an equilibrium $\mathbf{b}(T)$, what is the impact of a marginal change of supplier i's capacity c_i on his payoff function $\Pi_i(\mathbf{b}(T)) = \sum_{j=1}^{m} [b_{ij}(T) - v_{ij}]\, x_{ij}(T)$? By complementarity slackness, the dual variable associated with the capacity constraint of supplier i (which represents the marginal change in the buyer's objective resulting from a change in c_i) can only be positive if $\sum_{j=1}^{m} x_{ij}(T) = c_i$. Hence, a necessary condition for this marginal change to have any impact on supplier i's allocation is that his capacity constraint be binding. That is, a supplier cannot benefit by overstating his capacity without becoming overloaded and

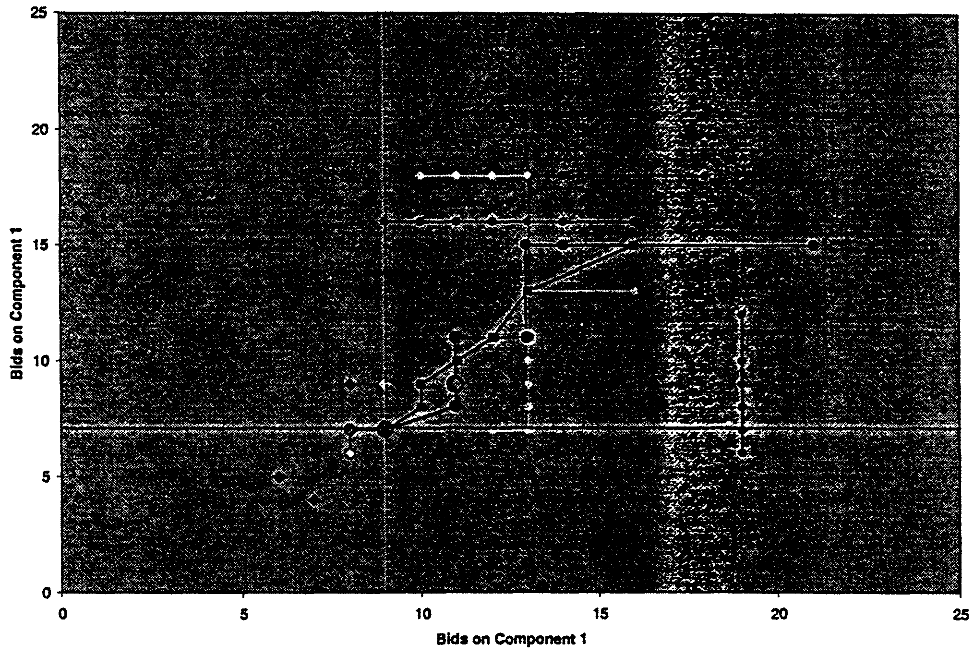

Figure 4: MBR dynamics in 8×2 auction with homogeneous costs.

incurring an observable default (i.e., refusing to accept all of the work he has been awarded).

However, this static sensitivity analysis does not take into account the strategic implications of a capacity statement on the other suppliers' bidding behavior. More specifically, it is conceivable that a supplier could earn higher margins by *understating* his capacity, because it may induce the other suppliers to bid less aggressively, which could compensate for the lower market share resulting from the capacity understatement. Likewise, this strategic effect could play against the incentive to overstate one's capacity, because it would induce the other suppliers to bid more aggressively.

Although it appears difficult to assess these strategic effects analytically, numerical experiments conducted under the MBR behavioral model and reported in Gallien (2000) confirm the abovementioned hypothesis that benefits derived from overstating one's capacity necessarily imply overloading (as opposed to strategic implications on other suppliers' behaviors). We also report in Gallien (2000) some "cooked-up" experiments demonstrating that it is possible for a supplier to increase his profits by understating his capacity (which can also be interpreted as a disincentive for capacity overstatement), thus illustrating the strategic effect hypothesized earlier.

In summary, it appears that there is no incentive for a supplier to overstate his capacity, and there are examples, which require more information (e.g., the other suppliers' capacity constraints), rationality and risk-taking than would typically be observed in practice, in which a supplier can raise his profits by understating his capacity. Hence, if suppliers are penalized for capacity overloading, then our smart-market mechanism should be able to induce truthful revelations of suppliers' production capacity. Regarding penalties, it is interesting to note that Freemarkets keeps instances of capacity overloading to a minimum by suspending from subsequent bidding events overloaded suppliers who default after an auction (Wnorowski 2000); it is this punitive action that led to the observed impact of overload fear on suppliers' bidding behavior, which in turn motivated the design of our smart-market approach (see §1). Alternatively, a procurement contract specifying an appropriately high fixed penalty in the event of misdeliveries and/or quality problems may also ensure a relatively truthful capacity input by the suppliers.

4. Conclusion

Partly motivated by the observed inefficiencies of current online procurement auction mechanisms when suppliers' capacity constraints are stringent, this paper investigates an alternative mechanism designed specifically for such environments. It relies on an estimation of these capacity constraints prior to the bidding event, which enables the use of an optimization-based allocation engine (justifying the term "smart market"). For every set of bids submitted during the auction, this engine dynamically computes the allocation of procurement contracts minimizing the buyer's total cost under both the buyer's quantity requirement and the suppliers' capacity constraints. The bidders' information sets in this mechanism are designed to achieve a middle ground between facilitating competition and preventing collusion, exploiting some of the possibilities offered by the use of a distributed network of computers to support the auction. They include in particular an original *myopic best response* bidding suggestion device, taking as an input a supplier's production costs, and delivering as an output the set of bids maximizing his potential profit in the next round under the assumption that his competitors' bids remain constant.

The solution concept we propose for this auction mechanism is based on *tatonnement* theory, whereby we first formulate less-than-fully rational behavioral models for the bidders, and then examine the implications of these models on the dynamics and the convergence properties of the resulting bidding sequences. Under a *weak behavioral assumption* loosely characterizing the reactions of potentially rejected suppliers, some symmetry requirements and a minimum competition level condition, we constructed upper bounds for the bids of the winning suppliers at the end of the auction, providing in turn an upper bound on the buyer's final procurement cost. These bounds were shown to carry over to the complete behavioral model, in which the suppliers followed the myopic best response bid suggestions. We then derived some additional structural properties of the myopic best response bidding sequences in a symmetric (except for production costs) market with two bidders and two component types, which suggested that when supplier capacity is scarce (i.e., competition is low), the buyer may find it beneficial to impose a common maximum entry bid to discourage premature equilibria that are generated by initial bids that are far apart. Finally, by performing numerical simulations of myopic best response sequences in more complex market configurations, we inferred that the negative incentive compatibility effects linked to the revelation by the suppliers of their private capacity and cost information should be relatively limited.

We believe this work constitutes a useful basic framework towards the development of electronic trade systems enabling real-time complex industrial transactions. Within the scope of our model, natural steps in this direction include more complex capacity constraints, nonlinear production costs and bid structures, requirements on the supply base size, and incumbent switching costs; this last generalization is particularly important, because these switching costs (which could, more generally, incorporate the bidder's perceptions of the suppliers' reputations) are the primary reason why Freemarkets makes its allocation decisions after – rather than during – their bidding events. In the framework of our model, one way for the buyer to account for her relative preferences across suppliers is to use a bid adjustment mechanism. In this method, the actual bids b_{ij} submitted by suppliers are automatically adjusted for the purpose of computing potential allocations by an additive factor $\alpha_{ij} > 0$ reflecting the buyer's relative preferences. That is, while the selling prices resulting from the auction still correspond to the actual bids b_{ij} submitted by the winners, the potential and final allocations of components are now calculated by solving $\mathbf{x}(t) = AE[\mathbf{b}(t) - \boldsymbol{\alpha}]$; the higher α_{ij}, the more supplier i is advantaged relative to his competitors for the allocation of component type j. Some of our results can be generalized to include these adjusted bids. For example, the right side of the ex-ante bound in Proposition 2 becomes $[\mathbf{v} - \boldsymbol{\alpha}]^j_{m+p+1:n} + \epsilon +$

α_{ij}, where $[\mathbf{v} - \boldsymbol{\alpha}]^j_{k:n}$ denotes the k-th order statistic of $(v_{1j} - \alpha_{1j}, \ldots, v_{nj} - \alpha_{nj})$. In words, modifying the bids for allocation purposes through the mechanism just described roughly amounts to modying the production costs by the same adjustment factors, as one would intuitively expect.

Unfortunately, this adjustment model does not allow us to capture fixed switching costs (i.e. independent of the quantity allocated). From a practical standpoint, this particular extension along with most of the others previously mentioned can be incorporated by using more sophisticated integer programming formulations for the allocation engine $AE[\cdot]$ in (1) (see Gallien 2000 for details), although we have not derived theoretical results corresponding to Propositions 1-4 in these more complex settings. Another relevant challenge would be to model into our allocation engine more criteria specific to industrial procurement, such as delivery performance, quality, and insurance provisions, leading perhaps to the design of a multi-dimensional bidding mechanism. In such complex and dynamic trading environments, efficiency will require the design of decision support tools available to bidders, for example by generalizing our myopic best response bid computational device. From a methodological perspective, we note that formulating behavioral models associated with these decision support tools seems to be a fruitful approach when trying to predict various mechanism outcomes.

Appendix

A.1. Proof of Proposition 1. Let $\mathbf{z}(T)$ denote the value in round T of the optimal dual variables associated with the quantity constraints in $AE[\mathbf{b}(T)]$, and $\mathbf{e}_m$ an m-dimensional vector with all its components equal to 1. Then we have

$$i \in P^C \Rightarrow \mathbf{v}_i \geq \mathbf{z}(T) - \epsilon \mathbf{e}_m. \tag{5}$$

This is because $\mathbf{b}_i(T) \geq \mathbf{z}(T)$ for any supplier $i \in P^C$, so that if we had $v_{ij} < z_j(T) - \epsilon$ for some j, then supplier i could increase his profit (currently equal to zero) by bidding $b_{ij}(T+1) = z_j(T) - \epsilon < b_{ij}(T)$. But $b_{ij}(T+1) = b_{ij}(T)$ by definition of T, and therefore $z_j(T) - \epsilon \leq v_{ij}\ \forall j$. This last statement applies to at least $|P^C|$ suppliers, so that

$$z_j(T) \leq v^j_{n-|P^C|+1:n} + \epsilon. \tag{6}$$

Finally, $x_{ij}(T)$ is the dual variable associated with the constraint $z_j(T) \leq b_{ij}(T) + a_{ij}y_i$ of the dual to $AE[\mathbf{b}(T)]$, where $y_i \geq 0$ is the dual variable associated with the capacity constraint of supplier i in $AE[\mathbf{b}(T)]$. Therefore, $x_{ij}(T) > 0$ implies $b_{ij}(T) \leq z_j(T)$, which combined with (6) completes the proof.

A.2. Proof of Proposition 2. The proof amounts to finding a lower bound for $|P^C|$ (or equivalently, an upper bound for $|P|$), and then invoking Proposition 1 in the cases where this bound allows us to claim that $|P^C| \geq 1$. In any feasible allocation, the number of suppliers who are fully loaded is clearly smaller than the maximal load factor p, because of the quantity requirement in (1). We now claim that in any extreme point of the polytope of feasible solutions to (1), the number of suppliers that are not fully loaded but still receive a positive allocation is smaller than m. This is because the allocation would otherwise include at least two non-fully loaded suppliers receiving a positive allocation of the same product, and would therefore not be an extreme point (as it could be written as a convex combination with positive weights of two distinct feasible solutions). Therefore, among all the suppliers receiving a positive allocation of each product, at most one can be non-fully loaded, and the total number of non-fully loaded suppliers is smaller than m. This completes the proof of $|P| \leq p + m$, from which (3) follows immediately by Proposition 1. The bound on the buyer's final procurement cost is a direct consequence of (3).

A.3. Proof of Proposition 4.

A.3.1. Notation. In this market environment, $AE[\cdot]$ has six extreme points, namely $(\mathbf{x}_1, \mathbf{x}_2) \in \{(A, \tilde{C}), (\tilde{C}, A), (B, \tilde{B}), (\tilde{B}, B), (\tilde{A}, C), (C, \tilde{A})\}$, where $A = (2q - c, 0)^T$, $B = (q, 0)^T$, $C = (q, c - q)^T$ and $\sim$ (tilde) denotes an exchange of the two elements in the vector (i.e., $\tilde{A} = (0, 2q - c)^T$). In this proof, we refer to $h(t)$ and $h(t + 1)$ as h and $\bar{h}$, respectively, where h denotes any variable or vector of interest (allocation, bids). Also, to describe a player's strategy for the next round, we use the notation $\mathbf{b}_Y$ where $Y \in \{A, B, C, \tilde{A}, \tilde{B}, \tilde{C}\}$ with $Y = \left(AE[\bar{\mathbf{b}}_i = \mathbf{b}_Y, \mathbf{b}_{-i}]\right)_i$ (i.e. under the MBR assumption that $\bar{\mathbf{b}}_{-i} = \mathbf{b}_{-i}$, player i will obtain an allocation $\bar{\mathbf{x}}_i = Y$ if he plays $\mathbf{b}_Y$). As shown in Gallien (2000), for a given selection rule, $\mathbf{b}_Y$ can be uniquely defined among all bids yielding allocation Y under strategy profile stability as the one requiring the smallest decrease in margins (so that each player's strategy space at each round is practically discrete). When comparing the impact of two different bid choices on a player's payoff function, we refer to the case $\Pi_i(\bar{\mathbf{b}}_i = \mathbf{b}_Y, \mathbf{b}_{-i}) \geq \Pi_i(\bar{\mathbf{b}}_i = \mathbf{b}_Z, \mathbf{b}_{-i})$ as $\mathbf{b}_Y \overset{i}{\geq} \mathbf{b}_Z$. Finally, we use in the bidding space the distance metric defined by $d(\mathbf{b}_1, \mathbf{b}_2) = \max(|b_{21} - b_{11}|, |b_{22} - b_{12}|)$.

A.3.2. Selection Rule. We consider a multiple optima selection rule symmetric across bidders and component types, where the selected allocation is always an extreme point, and that is characterized by

$\cdot \left\{ \begin{array}{l} \bar{b}_{11} = \bar{b}_{21} \\ \bar{b}_{12} > \bar{b}_{22} \end{array} \right. \Rightarrow (\bar{\mathbf{x}}_1, \bar{\mathbf{x}}_2) = (A, \tilde{C});$

$\cdot \left\{ \begin{array}{l} \bar{b}_{11} = \bar{b}_{21} \\ \bar{b}_{12} > \bar{b}_{22} \end{array} \right. \Rightarrow (\bar{\mathbf{x}}_1, \bar{\mathbf{x}}_2) = (A, \tilde{C});$

$\cdot \left\{ \begin{array}{l} \bar{b}_{11} - \bar{b}_{21} = \bar{b}_{12} - \bar{b}_{22} \\ \bar{b}_{11} > \bar{b}_{21} \end{array} \right. \Rightarrow (\bar{\mathbf{x}}_1, \bar{\mathbf{x}}_2) = \left\{ \begin{array}{l} (A, \tilde{C}) \text{ if } (\mathbf{x}_1, \mathbf{x}_2) \in \{(\tilde{C}, A), (\tilde{B}, B), (\tilde{A}, C)\} \\ (\tilde{A}, C) \text{ otherwise} \end{array} \right. ;$

$\cdot\ \bar{\mathbf{b}}_1 = \bar{\mathbf{b}}_2 \Rightarrow (\bar{\mathbf{x}}_1, \bar{\mathbf{x}}_2) = (\mathbf{x}_1, \mathbf{x}_2).$

Note that all missing cases in the definition above can be resolved by symmetry, and a random selection rule can be used at the first round if necessary.

A.3.3. Strategy Space Restrictions. While at every round the strategy space of each player is a priori $\{\mathbf{b}_A, \mathbf{b}_B, \mathbf{b}_C, \mathbf{b}_{\tilde{A}}, \mathbf{b}_{\tilde{B}}, \mathbf{b}_{\tilde{C}}\}$, it can typically be reduced to a smaller set by considering both the non-reneging rule (e.g., if $\mathbf{x}_1 = B$ then $\bar{\mathbf{b}}_1 \in \{b_B, b_C, b_{C'}\}$) and an adaptation of Lemma 4 in Part I of Gallien (2000) to the selection rule described earlier, which shows that under the MBR rationale:

1. If $b_{22} - v_{12} \geq b_{21} - v_{11}$ (player 2's bid is above player 1's margin switching line), then $\mathbf{b}_{\tilde{A}} \overset{1}{\geq} \mathbf{b}_A$;

2. If $b_{22} - v_{12} \geq b_{21} - v_{11}$ and $\mathbf{x}_1 \in \{A, \tilde{A}, \tilde{B}, \tilde{C}\}$, then $\mathbf{b}_{\tilde{C}} \overset{1}{\geq} \mathbf{b}_C$;

3. $b_{22} \geq v_{12} \Leftrightarrow \mathbf{b}_C \overset{1}{\geq} \mathbf{b}_B$ and $b_{21} \geq v_{11} \Leftrightarrow \mathbf{b}_{\tilde{C}} \overset{1}{\geq} \mathbf{b}_{\tilde{B}}$.

Note here again that many more such statements follow by symmetry. In summary, the set of possible bid choices $\bar{\mathbf{b}}_1$ for player 1 can be obtained from the following table (in order to avoid a long and uninteresting discussion of the tie cases, we assume that the production costs $\mathbf{v}_1$ and $\mathbf{v}_2$ do not belong to the ϵ -grid):

	$\mathbf{x}_1$					
	A	$\tilde{A}$	B	$\tilde{B}$	C	$\tilde{C}$
Case:						
$b_{22}-v_{12} > b_{21}-v_{11}$ and $\mathbf{b}_2 > \mathbf{v}_1$	$\{\mathbf{b}_{\tilde{A}}, \mathbf{b}_{\tilde{C}}\}$	$\{\mathbf{b}_{\tilde{A}}, \mathbf{b}_{\tilde{C}}\}$	$\{\mathbf{b}_C, \mathbf{b}_{\tilde{C}}\}$	$\{\mathbf{b}_{\tilde{C}}\}$	$\{\mathbf{b}_C, \mathbf{b}_{\tilde{C}}\}$	$\{\mathbf{b}_{\tilde{C}}\}$
$b_{22}-v_{12} > b_{21}-v_{11}$ and $\left\{\begin{array}{l} b_{21} < v_{11} \\ b_{22} > v_{12} \end{array}\right.$	$\{\mathbf{b}_{\tilde{A}}, \mathbf{b}_{\tilde{B}}\}$	$\{\mathbf{b}_{\tilde{A}}, \mathbf{b}_{\tilde{B}}\}$	N/A	$\{\mathbf{b}_{\tilde{B}}\}$	N/A	N/A

A.3.4. Neighborhood Stability and Convergence. Let us now assume that $d(\mathbf{b}_1, \mathbf{b}_2) \leq \epsilon$. We can use the above table to specify the bid chosen by player 1 under the MBR rationale (results for player 2 and for missing cases follow by symmetry):

Case 1: $b_{22} - v_{12} > b_{21} - v_{11}$ and $\mathbf{b}_2 > \mathbf{v}_1$

- When $\mathbf{x}_1 \in \{A, \tilde{A}\}$, we have

$$\mathbf{b}_{\tilde{A}} \overset{1}{\geq} \mathbf{b}_{\tilde{C}} \quad \begin{array}{l} \Leftrightarrow (2q-c)(b_{22}+\epsilon-v_{12}) \geq q(b_{22}-\epsilon-v_{12}) + (c-q)(b_{21}-v_{11}) \\ \Leftrightarrow b_{22}-v_{12}+b_{21}-v_{11} \leq \frac{3q-c}{c-q}\epsilon \end{array} ;$$

- When $\mathbf{x}_1 = B$,

$$\mathbf{b}_C \overset{1}{\geq} \mathbf{b}_{\tilde{C}} \quad \begin{array}{l} \Leftrightarrow (c-q)(b_{22}-v_{12}) + q(b_{11}-v_{11}) \geq q(b_{22}-\epsilon-v_{12}) + (c-q)(b_{11}-v_{11}) \\ \Leftrightarrow b_{22}-v_{12}-b_{21}+v_{11} \leq \frac{c-q}{2q-c}\epsilon \end{array} ;$$

- When $\mathbf{x}_1 = C$,

$$\mathbf{b}_C \overset{1}{\geq} \mathbf{b}_{\tilde{C}} \quad \begin{array}{l} \Leftrightarrow q(b_{11}-v_{11}) + (c-q)(b_{12}-v_{12}) \geq (c-q)(b_{11}-v_{11}) + q(b_{12}-\epsilon-v_{12}) \\ \Leftrightarrow b_{12}-v_{12}-b_{11}+v_{11} \leq \frac{q}{2q-c}\epsilon \end{array} .$$

Case 2: $b_{22} - v_{12} > b_{21} - v_{11}$ and $\left\{\begin{array}{l} b_{21} < v_{11} \\ b_{22} > v_{12} \end{array}\right.$

- When $\mathbf{x}_1 \in \{A, \tilde{A}\}$, we have

$$\begin{array}{l} \mathbf{b}_{\tilde{A}} \overset{1}{\geq} \mathbf{b}_{\tilde{B}} \text{ if } (2q-c)(b_{22}-v_{12}) \geq q(b_{22}-\epsilon-v_{12}) \\ \Leftrightarrow b_{22}-v_{12} \leq \frac{q}{c-q}\epsilon \end{array} .$$

In words, supplier 1 prefers $\mathbf{b}_{\tilde{C}}$ to $\mathbf{b}_{\tilde{A}}$ as long as supplier 2's bid $\mathbf{b}_2$ remains sufficiently greater than $\mathbf{v}_1$ (i.e., when $b_{22} - v_{12} + b_{21} - v_{11} > \frac{3q-c}{c-q}\epsilon$), and prefers $\mathbf{b}_{\tilde{C}}$ to $\mathbf{b}_C$ when $\mathbf{b}_2$ is far enough above his margin switching line (i.e., when $b_{12} - v_{12} - b_{11} + v_{11} > \frac{q}{2q-c}\epsilon$). When b_{21} drops below v_{11}, supplier 1 plays $\mathbf{b}_{\tilde{B}}$ if b_{22} is sufficiently greater than v_{12} (i.e., when $b_{22} - v_{12} > \frac{q}{c-q}\epsilon$), and $\mathbf{b}_{\tilde{A}}$ otherwise. Applying the same reasoning to player 2, we can construct for all relevant cases the tables describing the joint bidding strategies and resulting allocation for both players, thus characterizing the dynamics of the bidding sequence. For example, the table corresponding to the case $\left\{\begin{array}{l} b_{22}-v_{12} > b_{21}-v_{11} \text{ and } \mathbf{b}_2 > \mathbf{v}_1 \\ b_{12}-v_{22} > b_{11}-v_{21} \text{ and } \mathbf{b}_1 > \mathbf{v}_2 \end{array}\right.$ is (missing allocation cases can be derived by symmetry)

	$A:$		$\tilde{A}:$		$B:$	
	$d(b_2,v_1) > \frac{3q-c}{c-q}\epsilon$	$d(b_2,v_1) \leq \frac{3q-c}{c-q}\epsilon$	$d(b_2,v_1) > \frac{3q-c}{c-q}\epsilon$	$d(b_2,v_1) \leq \frac{3q-c}{c-q}\epsilon$	$b_{22}-v_{12}-b_{21}+v_{11} > \frac{c-q}{2q-c}\epsilon$	$b_{22}-v_{12}-b_{21}+v_{11} \leq \frac{c-q}{2q-c}\epsilon$
$\tilde{C}$	$(b_{\tilde{C}},b_{\tilde{C}}) \rightarrow (A,\tilde{C})$	$(b_{\tilde{C}},b_{\tilde{A}}) \rightarrow (C,\tilde{A})$				
$C: b_{12}-v_{12}-b_{11}+v_{11} > \frac{q}{2q-c}\epsilon$			$(b_{\tilde{C}},b_{\tilde{C}}) \rightarrow (C,\tilde{A})$	$(b_{\tilde{C}},b_{\tilde{A}}) \rightarrow (\tilde{C},A)$		
$C: b_{12}-v_{12}-b_{11}+v_{11} \leq \frac{q}{2q-c}\epsilon$			$(b_C,b_{\tilde{C}}) \rightarrow (A,\tilde{C})$	$(b_C,b_{\tilde{A}}) \rightarrow (C,\tilde{A})$		
$\tilde{B}$					$(b_{\tilde{C}},b_{\tilde{C}}) \rightarrow (A,\tilde{C})$	$(b_{\tilde{C}},b_C) \rightarrow (\tilde{B},B)$

where the rows correspond to player 2's allocation $\mathbf{x}_2$, the columns to $\mathbf{x}_1$, and each entry (I,J) in this table provides $(\bar{\mathbf{b}}_2,\bar{\mathbf{b}}_1) \rightarrow (\bar{\mathbf{x}}_2,\bar{\mathbf{x}}_1)$ when $(\mathbf{x}_2,\mathbf{x}_1) = (I,J)$. Constructing the tables corresponding to cases

$$\left\{\begin{array}{l} b_{22}-v_{12} > b_{21}-v_{11} \text{ and } \mathbf{b}_2 > \mathbf{v}_1 \\ b_{12}-v_{22} < b_{11}-v_{21} \text{ and } \mathbf{b}_1 > \mathbf{v}_2 \end{array}\right. \quad \text{and} \quad \left\{\begin{array}{c} b_{22}-v_{12} > b_{21}-v_{11},\ b_{21} < v_{11} \text{ and } b_{22} > v_{12} \\ b_{12}-v_{22} > b_{11}-v_{21} \text{ and } \mathbf{b}_1 > \mathbf{v}_2 \end{array}\right.$$

(tables for other cases follow by symmetry), we can observe that

$$\left\{\begin{array}{l} d(\mathbf{b}_1,\mathbf{b}_2) \leq \epsilon; \\ b_{i2}-v_{-i2}+b_{i1}-v_{-i1} > \frac{3q-c}{c-q}\epsilon \text{ for } i \in \{1,2\}; \text{ and} \\ b_{ij}-v_{-ij} > \frac{q}{c-q}\epsilon \text{ for } i,j \in \{1,2\} \end{array}\right.$$

implies

$$\left\{\begin{array}{l} d(\bar{\mathbf{b}}_1,\bar{\mathbf{b}}_2) \leq \epsilon; \text{ and} \\ \bar{b}_{11} < b_{11}, \bar{b}_{12} < b_{12}, \bar{b}_{21} < b_{21} \text{ or } \bar{b}_{22} < b_{22}. \end{array}\right.$$

In words, if at one given round the bids of the two suppliers are within a distance ϵ of each other and sufficiently far away from the production costs, then under the MBR rationale the bids at the next round will still be within ϵ of each other, and at least one of the bids will have been strictly decreased. Applying this reasoning iteratively, observing that $\frac{3q-c}{c-q}\epsilon > \frac{q}{c-q}\epsilon$ since $2q - c > 0$, and that $b_{i2} - v_{-i2} + b_{i1} - v_{-i1} \leq \frac{3q-c}{c-q}\epsilon$ for $i = 1$ or 2 implies $d(\mathbf{b}_i, \mathbf{v}_1 \vee \mathbf{v}_2) \leq \frac{3q-c}{c-q}\epsilon$, we can conclude that $d(\mathbf{b}_i(T), \mathbf{v}_1 \vee \mathbf{v}_2) \leq (\frac{3q-c}{c-q} + 1)\epsilon$ for $i \in \{1, 2\}$, which completes the proof.

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