Models and Algorithms for Mailing Catalogs

by

Gabriel R. Bitran
Susana V. Mondschein

WP #3737-94 November 1994

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Gabriel R. Bitran
MIT Sloan School of Management
Susana V. Mondschein
Universidad de Chile

September, 1994

Abstract

The catalog sales industry is one of the fastest growing business in the U.S. The most important asset a company in this industry has is its list of customers, called the *house list.* Building a house list is expensive, since the response rate of names from rental lists is low. Therefore, cash management plays a central role in this capital intensive business.

This paper studies optimal mailing policies in the catalog sales industry when there is limited access to capital. We consider a stochastic environment given by the random responses of customers and a dynamic evolution of the house list. Given the size of real problems, it is not possible to compute the optimal solutions. Therefore, we develop heuristics based on the optimal solutions of simplified versions of the problem studied. The performance of these heuristics is evaluated by comparing their outcome with upper bounds derived for the original problem. Computational experiments show that these heuristics behave satisfactorily.

1 INTRODUCTION

One of the fastest growing segments within American business is the catalog sales industry. Companies that use catalogs as their sole marketing strategy sent a daily average of 30 million catalogs during 1989, selling 50 billion US dollars during that year (Holtz, 1990).

We define the *catalog sales industry* as those companies doing business by pursuing prospective customers by means of a catalog. The catalog recipient can order a product by mail, telephone, or any other alternative.

Initially, catalog companies were oriented toward rural areas, at a time when access to stores was difficult and expensive. Nowadays, people continue buying via catalogs for several reasons; for example, the convenience of not having to leave the home or office to buy a product, the popular notion that catalogs offer items more cheaply than retail stores, and the exclusivity of the items offered. On the other hand, customers often hesitate to buy by catalogs because they perceive disadvantages; for example, the delay experienced in the delivery of the product, the impossibility of examining the product directly before buying it, and the inconvenience of returning an item that, for some reason, is unsatisfactory.

Companies in the catalog sales industry divide the planning horizon into campaigns, which usually coincide with the seasons of the year. During a campaign, all the marketing effort is oriented to sell a specific set of products, which have common characteristics that justify promoting all of them in a same catalog. In what follows, the terms *campaign* and *season* will be used interchangeably.

Companies in the catalog sales industry define a customer as a person that has already bought products from the company; thus her name can be used – in the sense of sending her new catalogs – as often as the company wants (unless the customer explicitly demands the opposite). The list of customers is called the *house list*. The typical behavior of a customer is that after buying a number of times from the company, she stops doing so because her taste changes, she switches to the competition, etc. Therefore a company selling by catalog must constantly add new customers to its house list.

A *rental list* is a set of names with certain characteristics in common, such as age, sex and income. These lists can be rented by the company, usually for a one-time use, paying a price that depends on the number of names rented. These names correspond to potential customers, their response rate — i.e., the percentage of people that respond with a sale — is usually much lower than that of customers on the house list. It follows that the company must use rental lists to obtain new customers, which it then may add to its house list. When a person from a rental list responds with a mail order, she can be incorporated into the house list and from that time onwards the name can be used without paying for doing so.

Within the major characteristics that have been used to measure purchase behavior, the primary bases of segmentation are **R**ecency, **F**requency, and **M**onetary value, known as the **RFM** classification. Many people consider these three characteristics to be the most important in determining the likely profitability of mailing to an individual (Roberts and Berger (1989)). Thus, usually the house list is divided into segments or states according to the RFM classification. Each company has its own definition of recency, frequency, and monetary value. In what follows we present the most common definitions used in the industry. Recency of the last purchase is defined by the number of periods since the last purchase. Frequency of purchase is usually defined by the proportion of mailings that the customer has responded with an order or by the total number of orders placed over a period of time. Even though the latter definition is used more often, the former is more useful because it discriminates between a customer that has bought three times out of three mailings and a customer that has bought the same three times but out of 10 mailings. The definition of frequency is often times simplified further to consider only two states: customers that have bought once and more than once (Fleischmann, 1992). Finally, monetary value is defined by the average dollar amount per purchase or by the dollar value that the customer spent in the last purchase or all purchases to date.

Each RFM segment in the house list is characterized by a response rate, which is the probability that a customer in that segment will place an order when she receives one or more catalogs. Additionally, the size of the order is also random with probabilities that depend on the segment classification. Empirical data from a jewelry company that sells inexpensive items shows that for a given frequency and monetary value, customers with smaller recency (defined as the time since the last purchase) are more likely to respond to a mailing. This behavior is usually observed in companies that sell products that are regularly consumed; for example, clothes, food, and office supplies. The following dynamic in the customer's behavior explains, to some extent, this phenomenon. When considering products that are constantly "consumed", the fact that a customer does not buy in a period usually means that she is buying elsewhere. The longer the time the customer has not purchased from the company's catalogs, the harder it is to "reactivate" that customer. Thus, the probability that she responds with an order gets lower as the recency increases. There is another factor that contributes to the phenomenon explained above. After responding to a mailing, some customers "leave" the house list; they change address, die, etc. Therefore, a customer that has not responded the mailings for several periods is more likely to have left the house list. There are other situations where this negative correlation between recency and response rate does not hold. For example, when companies sell durable goods that last for several periods. In these cases, the response rate could increase with the recency, for certain set of recencies.

The RFM classification allows us to distinguish between casual and loyal buyers; even though both have recency equal to one after a purchase, frequency discriminates between customers that have bought several times in the past from those that have bought occasionally.

Rental lists are also characterized by their response rates. Usually, mailing companies do not have reliable estimates of the rental lists' response rates. These rental lists have been previously used by other companies that sell different products. Hence, since it is too risky to rely on their estimates to decide whether or not to use these names in a new campaign, mailing companies are constantly testing new rental lists. With this purpose, a fraction of their budgets is regularly assigned for testing the behavior of new potential customers that come from lists that have not been previously explored. These tests provide better estimates of the rental lists' response rates.

Given cash flow constraints several trade-offs must be considered when making the mailing decisions; for example long run profits from new potential customers vs profits from old customers, and mailing to customers with large response rate and low probability of a big purchase vs customers with low response rate and high probability of a big purchase. Therefore, companies face the problem of assigning limited resources for mailings to customers with heterogeneous behaviors. The uncertainty in the responses leads to a stochastic demand that depends directly on the number of catalogs mailed during the season to the different classes of customers.

A common practice is the *multiple mailing* of catalogs to the same customers within a season, thereby increasing the response rate. According to managers, two main reasons explain this increment in the response rate: (*i*) some catalogs get lost before they reach their destination. Thus, multiple mailing reduces the probability that a "good" customer will not receive a catalog, and (*ii*) a second or third catalog can encourage a customer that was willing to buy from the first one but did not for a variety of reasons (the catalog was misplaced, she needs a remainder, etc.)

A large house list is one of the main assets a company in the catalog sales industry can have. This is expensive to build, since the response rate of names from rental lists is low. Additionally, these names must be rented. Thus, firms face a high demand for cash during their first years of existence. Catalog companies usually lose money for two or three years until their house list reaches proportions which contribute more than the costs of acquiring new customers and overhead. When a company starts in the catalog business, it must invest an important fraction of its budget in acquiring customers. We illustrate this fact with the following example: suppose that the company rents names from a rental list with an average response rate of 1%, and an average size of the order of \$80. If we consider a cost of goods and fulfillment of \$40 and a marketing cost (including the catalog, the mail, and the name's rent) of \$0.70 per customer, the company spends an average of \$30 $(80 - 40 - 0.70 * 100)$ in adding a new customer into its house list. However, people from rental lists that respond with a sale become "good customers" that are expected to generate profits over their lifetime in the house list. It is not unusual to find companies that do not understand the dynamic of the problem and are surprised by the capital intensive phenomenon described above. Recently, a newspaper published a case of a software company that failed in the catalog sales industry because, among several reasons, it was not prepared for a long period of investment to build the house list. The company was sending 120 pieces of mail just to produce the sale of one product. It cost the company 70 cents to generate $1 in sales - a "totally unacceptable" expense ratio, as one of its executives pointed out. "We went too fast. None of us had the direct mail experience," he added. A similar situation was reported by an executive of a hairdresser chain, that was selling sophisticated hair products by mail. The mailing operations stopped after two periods.

One of the most important decisions that a manager faces in the catalog sales industry is to define the mailing policy, i.e., from which rental lists names will be rented and the fraction of the people in those rental lists and in the states of the house list that receive catalogs. The manager also has to decide on the number of catalogs that a customer receives during the season if multiple mailings are allowed.

The purpose of this paper is to determine the optimal mailing policy considering cash flow constraints. We study two tactical models. In the first we consider a catalog company that is part of a major retail operation, where all the inventory management decisions are made by the retail store division. We also assume that catalog requests are always satisfied because they correspond to a fraction of the total retail sales. In the second model we consider a catalog company that manages the cash flow constraints incorporating the financial impact of carrying inventory. Hence, the model determines the optimal aggregate reordering policy together with the optimal mailing policy. Because the number of people that respond with a sale is a random variable, the manager faces a non-trivial decision when deciding how many products to have in store: having few products may lead to lost sales while having too many products implies spending money that can otherwise be utilized to send additional mailings.

Most of the literature related to this topic gives a qualitative analysis describing the main characteristics of the industry, see for example Hill (1989). A good review of the catalog sales industry can be found in Holtz (1990). Bitran and Ramalho (1992) determine the optimal mailing policy in a deterministic environment. In this paper we study optimal mailing and reordering policies in a stochastic environment, where uncertainty originates from customers' random responses.

The remainder of the paper is organized as follows. In Section 2 we introduce two dynamic programming formulations for the problems described above. We also present some properties of the optimal solutions. In Section 3 we develop a methodology to calculate the discounted net profit corresponding to a customer: *Lifetime value of a customer.* In Section 4 we describe various heuristics to solve the optimization problems. In Section 5 we find upper bounds for the dynamic and stochastic programming formulations, which are useful to measure the performance of the heuristics developed in the previous section. Section 6 contains the computational experiments that show the characteristics of the optimal policies and the performance of the heuristics. Finally, Section 7 presents extensions and conclusions.

2 Mathematical Models

In this section we describe two planning models to maximize the total expected profit in the catalog sales industry. The first model, the **Catalog Mailing Problem**, determines the optimal policy for sending catalogs, considering that the catalog company is part of a major retail store and that the catalog sales are only a fraction of the total sales. Therefore, as a good approximation of reality, we assume that all the mail orders received are satisfied. We also assume that the inventory management is carried out by the retail store. The second model, the **Catalog Mailing Problem with Aggregate Inventory Costs**, corresponds to the case where companies must manage the cash flow incorporating the financial impact of carrying inventory. This model determines the optimal mailing policy together with the optimal aggregate reordering policy. We assume that the company can order only at the beginning of the season. Therefore, it must determine the optimal reordering amount taking into account the costs of having unsatisfied customers and carrying inventory from one season to another.

2.1 Description of the Dynamics in the House List

The house list is divided into states according to the RFM classification. Each of the states has associated a response rate, i.e., a probability that a customer in that state will place an order when she receives one or more catalogs. When the customer places an order her recency becomes equal to one and increases by one otherwise. Another source of uncertainty is given by the order size when the customer responds with an order. Therefore, the new frequency and monetary value of the customer depend on whether or not she places an order, the size of the order, the current state, and the definition of frequency and monetary value that the company has adopted.

For example, if we define frequency by the number of times the person has bought from the company and monetary value by the size of the last purchase, then the customer's current state, (r, f, m) is updated as follows:

- Customer places an order: $(1, f+1, m_i)$, where m_i is the current order size

- Customer does not place an order: $(r+1, f, m)$.

More information is needed when companies adopt a more complex definition for frequency and monetary value. For example, if we define frequency as the number of times the person has bought in the last 4 seasons, then it is necessary to keep track of the complete pattern of purchases in the last four periods in order to update the frequency.

When customers do not respond a certain number of consecutive mailings, they are eliminated from the house list, i.e., there is a upper limit for the customer's recency. This limit usually depends on the frequency and monetary value associated with the customer; customers that buy large orders are more likely to be kept in the house list for a longer period of time compared to customers that buy small orders. Therefore, for each frequency, and monetary value there is a "trapping state" in the house list which is determined by the maximum admissible recency.

In the remainder of this paper we consider a Markov chain representation for the customers' behavior within the house list, where only some well defined transitions are allowed. Figure 1 shows the feasible transitions in the Markov chain representation. When the customer places an order her recency is equal to 1, her frequency equals $f(s_i)$ (where $f(s_i)$ is defined by the measure chosen), and her monetary value takes a random value according to her order size. If the customer does not place an order then her recency increases by one, and her frequency and monetary value equal $\tilde{\mathrm{f}}(s_i)$ and $\tilde{\mathrm{m}}(s_i)$ respectively ($\tilde{\mathrm{f}}(s_i)$ and $\tilde{\mathrm{m}}(s_i)$ are computed according to the measures adopted).

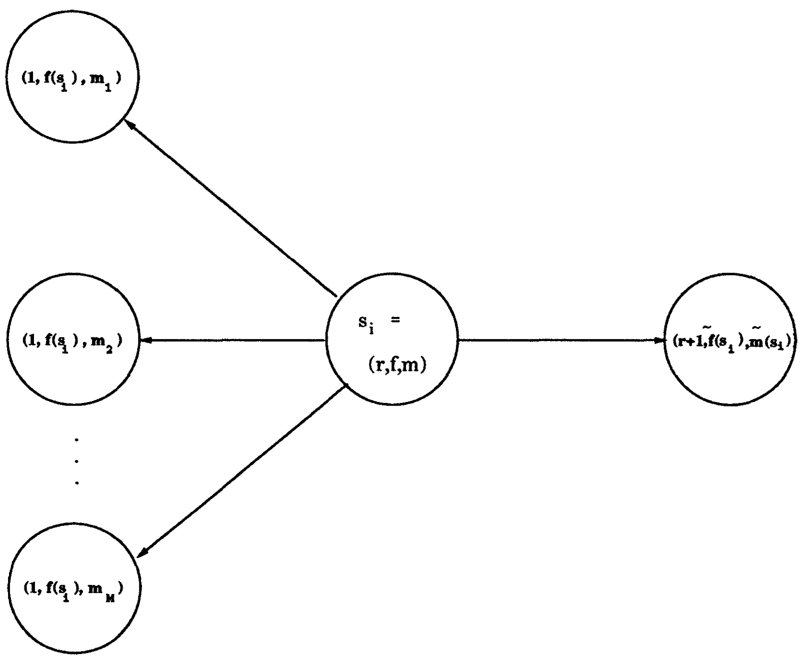

Figure 1: Markov chain representation

2.2 Catalog Mailing Problem

The mathematical model is a stochastic and dynamic programming formulation where the objective function is to maximize the total expected profit during the planning horizon. The planning horizon is divided into seasons, with, generally, four seasons per year (some companies might consider five seasons including the Christmas sale).

One of the most important decisions that a manager faces in the catalog sales industry is to define the mailing policy, i.e., the fraction of the people in the rental lists and in the states of the house list that receive catalogs. The number of people in the house list that respond with an order is a random variable that depends on the response rate of the corresponding state (the response rate increases with the number of mailings), and on the number of people that receive catalogs. Responses from rental lists are also random. In this case, only a single mailing is usually made because names are typically rented for one-time use. Therefore, the company faces a stochastic demand that depends directly on the marketing effort, i.e., the number of catalogs mailed in the season. We make no assumptions regarding the value of the response rates for the states in the house list and rental lists.

It is not unusual that catalog companies are part of major retail operations and that the inventory levels are managed by the retail stores. The model in this subsection assumes that the requests originated from catalogs are always satisfied because they represent only a fraction of the total sales. The model also assumes that all the costs of managing the inventory are considered in the global planning model for the retail stores.

In what follows we introduce the notation for the parameters, decision variables, and random variables.

Parameters

s_i = state s_i which is defined by the vector (r_i, f_i, m_i), where r_i corresponds to the recency, f_i to the frequency and m_i to the monetary value associated with the state of the customer.

S = set of states.

S_0 = set of states that have recency equal to 1. Customers that place an order move to a state in S_0.

$\bar{S}_0$ = complement set of S_0. Thus, $\bar{S}_0 = S - S_0$.

$p_{s_i,s_j,k}$ = probability that a customer in state s_i in the house list moves to state s_j when she receives k catalogs during a season. If the customer places an order she will move to a state with recency equal to 1. The frequency and monetary value are updated according to the definition for those parameters that the company has adopted. Otherwise, her recency will increase by one and the frequency and monetary value will be updated accordingly. For all the transitions that are not feasible this probability is equal to zero.

p_{j,s_i} = probability that a customer in rental list j moves to state s_i if she receives a catalog. These probabilities will be equal to zero for all $s_i \in \bar{S}_0$. Adding over all $s_i \in S_0$ we obtain the response rate of rental list j.

d_{s_i,s_j} = average order size in dollars when moving from state s_i to s_j. Hence, d_{s_i,s_j} is equal to zero when $s_j \in \bar{S}_0$, because no order was placed.

d_{j,s_i} = average order size in dollars when moving from rental list j to state s_i in the house list.

c_h = variable marketing cost per customer in the house list (it consists of printing and mailing costs).

c_m = variable marketing cost per customer in the rental lists (it consists of printing, mailing and name renting costs). We have ignored the quantity discounts that can take place when renting names from rental lists. It is possible to incorporate this feature in the models that we present in this section. however, this modification has only second order effects in the decision process.

g = average cost of goods as a fraction of the price .

c_1 = average variable cost of an order.

c_2 = penalty for failing an order.

c_3 = holding inventory cost per dollar in inventory.

a_t = exogenous money for investment that is available at the beginning of season t.

$L_{j,t}$ = number of available names in rental list j at the beginning of season t.

β = discount rate per season.

K = maximum number of mailings within a season.

J = total number of available rental lists.

Decision Variables

$H_{s_i,k,t}$ = total number of customers in state s_i in the house list that receive k catalogs during season t.

$W_{j,t}$ = number of people in rental list j that receive a catalog during season t.

Random Variables

$X_{s_j,s_i,k,t}$ = total number of customers that moved from state s_j to s_i that receive k catalogs during season t. The distribution of this random variable is the marginal distribution of the multinomial distribution with probabilities equal to $p_{s_j,s_i,k}$ $\forall s_i$ and total number of trials equal to $H_{s_j,k,t}$.

$X_{j,s_i,t}$ = number of customers from rental list j that moved to state s_i during season t. The distribution of this random variable is the marginal distribution of the multinomial distribution with probabilities equal to p_{j,s_i} $\forall s_i$ and total number of trials equal to $W_{j,t}$.

Y_t = total amount of money available for investment at the beginning of season t.

N_t = number of customers in the house list at the beginning of season t. N_t is a vector with as many elements as the number of states in the house list. Therefore, the element $N_{s_i,t}$ of N_t corresponds to the number of customers in state s_i in the house list during season t.

Finally, we define the function $F_t(Y_t, N_t)$ as the maximum discounted expected profit from season t onwards if the company starts with Y_t dollars for investment, and N_t customers in the house list at the beginning of season t.

<u>The Model</u>: the optimization model at the beginning of time t is given by the following stochastic and dynamic programming formulation:

$$F_t(Y_t, N_t) = \max_{H_{s_i,k,t}, W_{j,t} \ \forall s_i,k,j} \{ -\sum_{k=1}^{K} \sum_{s_i \in S} k c_h H_{s_i,k,t} - \sum_{j=1}^{J} c_m W_{j,t} +$$

$$E_{X_{s_i,s_j,k,t}, X_{j,s_i,t}} [\sum_{s_i \in S} \sum_{s_j \in S_0} \sum_{k=1}^{K} (d_{s_i,s_j}(1-g)-c_1) X_{s_i,s_j,k,t} + \sum_{j=1}^{J} \sum_{s_i \in S_0} (d_{j,s_i}(1-g)-c_1) X_{j,s_i,t} + \beta F_{t+1}(Y_{t+1}, N_{t+1})]$$

$\forall s_i, s_j, k, j$

s.t.

CASH FLOW CONSTRAINT.

$$\sum_{k=1}^{K} \sum_{s_i \in S} k c_h H_{s_i,k,t} + \sum_{j=1}^{J} c_m W_{j,t} \leq Y_t + a_t. \tag{1}$$

UPPER BOUND FOR THE NUMBER OF PEOPLE IN EACH STATE OF THE HOUSE LIST.

$$\sum_{k=1}^{K} H_{s_i,k,t} \leq N_{s_i,t} \qquad \forall s_i \in S. \tag{2}$$

UPPER BOUND FOR THE NUMBER OF AVAILABLE NAMES IN THE RENTAL LISTS.

$$W_{j,t} \leq L_{j,t} \qquad j = 1, \ldots, J. \tag{3}$$

UPDATING THE NUMBER OF CUSTOMERS IN EACH HOUSE LIST SEGMENT.

$$N_{s_i,t+1} = \sum_{s_j \in S} \sum_{k=1}^{K} X_{s_j,s_i,k,t} + \sum_{j=1}^{J} X_{j,s_i,t} \qquad \forall s_i \in S. \tag{4}$$

CASH FLOW BALANCE EQUATION.

$$Y_{t+1} = Y_t + a_t - \left(\sum_{k=1}^{K} \sum_{s_i \in S} k c_h H_{s_i,k,t} + \sum_{j=1}^{J} c_m W_{j,t}\right) +$$

$$\left(\sum_{s_j \in S} \sum_{s_i \in S_0} \sum_{k=1}^{K} (d_{s_j,s_i}(1-g) - c_1) X_{s_j,s_i,k,t} + \sum_{j=1}^{J} \sum_{s_i \in S_0} (d_{j,s_i}(1-g) - c_1) X_{j,s_i,t}\right). \tag{5}$$

BOUNDARY CONDITION:

$$F_{T+1}(Y_{T+1}, N_{T+1}) = \sum_{s_i \in S} LF(s_i) N_{s_i,T+1}$$

The objective function at period t is equal to the immediate expected profit plus the total expected discounted profit from period $t+1$ onwards. The immediate expected profit is computed as the total expected profit of sales minus marketing costs (catalog, mailing, and renting costs) in the current period.

The first constraint corresponds to the fact that the total marketing cost must be less than or equal to the initial budget plus the exogenous investment. This constraint considers that when the optimal decision is to send k catalogs to a group of customers, all of them receive k mailings even though some customers respond after the first, second or $(k-1)^{th}$ mailing. Since the average number of responses is low, companies usually do not incur the cost of purging from the second mailing list those customers that respond after the first mailing, but send a second catalog to the complete segment. The same situation happens with a larger number of mailings.

The second and the third constraints correspond to the upper bounds for the number of people in the house list and in the rental lists, respectively. In constraint (4) we update the number of people in the house list at the end of the season. This procedure has implicit the Markov chain representation for the behavior of the customers described in subsection 2.1. Thus, if a customer receives a catalog, she either decreases her recency to one (if she places an order) or increases her recency by one (if she does not respond with an order). The frequency and monetary values are updated accordingly. The customer automatically increases her recency if she does not receive a catalog. In constraint (5), we update the budget at the end of the season.

2.3 Catalog Mailing Problem with Aggregate Inventory Costs

In this subsection we present a **planning model** where companies manage the cash flow incorporating the financial impact of carrying inventory. In our analysis the inventory level is defined as the dollar value of the products in inventory at the selling price and the inventory decision variables are the total amount of money that will be assigned to inventory at each season. Thus, in addition to the mailing policies we have to define the optimal aggregate resource assignment for inventory.

Alternatively, the inventory decision variables in this model can be formulated in terms of physical units. In such situations, for planning purposes, it is useful to define a unit equivalent which has each item in the proportion that has been ordered in the recent past. Furthermore, if we are interested in an operational model to manage the inventory, then the problem can be formulated considering constraints for each individual item. A discussion on how to formulate the model in these two cases is included in Appendix 1. It is important to mention that the computational complexity of the model remains the same whether monetary or physical units are used. Naturally, the number of inventory related constraints and inventory decision variables increases linearly with the number of items in the catalog for the operational model.

In Bitran and Mondschein (1994), the authors have made an analysis of the impact of product aggregation on the inventory and mailing decisions. The main problem that arises from product aggregation is the presence of infeasibilities at the operational level, where disaggregate inventory decisions are made, due to inventory imbalances. For example, a large amount of inventory of an individual item can lead to zero reorders for an aggregate product while other items of the same group cannot satisfy their demand. The authors propose a one-time feedback process, between the operational and tactical levels to avoid these infeasibilities, which leads to excellent results when compared with an upper bound for the problem. We remark that this hierarchical approach to solve the problem is commonly used in the catalog sales industry.

In practice companies can order a limited number of times from their suppliers. Frequently, they order one month before the beginning of the season and they reorder just once within the first three weeks of the season. Therefore, it is reasonable to assume that the company orders only once

during every season, and that this order takes place at the beginning of it.
The model relies on the following three assumptions:

- If a customer places an order her recency decreases to one independently of whether or not the request is satisfied.
- There is a monetary cost for not satisfying an order, which represents the company's loss of reputation.
- There is a holding cost for carrying inventory from one season to another.

We introduce the following additional notation:

1. I_t = total amount of money in inventory valued at selling price at the beginning of season t.
2. Z_t = amount of money that is invested in inventory valued at the selling price at the beginning of season t.
3. $V_{s_i,t}$ = total monetary value of orders sold to customers in state s_i at season t.
4. $V_{j,t}$ = total monetary value of orders sold to customers from rental list j at season t.
5. $F_t(Y_t, I_t, N_t)$ = maximum discounted expected profit from season t onwards if the company starts with Y_t dollars for investment, I_t dollars in inventory and N_t customers in the house list at the beginning of season t.

__The Model__: the objective function at time t is given by the immediate expected profit during the current season plus the expected profit from the next season onwards. During season t, the manager has to decide the optimal mailing and reordering policies satisfying the cash flow constraints. The set of constraints at season t is given by:

Cash flow constraint.

$$gZ_t + \sum_{k=1}^{K} \sum_{s_i \in S} kc_h H_{s_i,k,t} \; + \; \sum_{j=1}^{J} c_m W_{j,t} \; \leq \; Y_t \; + \; a_t. \tag{6}$$

Upper bound for the number of people in each state of the house list

$$\sum_{k=1}^{K} H_{s_i,k,t} \; \leq \; N_{s_i,t} \qquad \qquad \forall s_i \in S. \tag{7}$$

Upper bound for the number of available names in the rental lists.

$$W_{j,t} \ \leq \ L_{j,t} \qquad\qquad j = 1, \ldots, J. \tag{8}$$

Updating the number of customers in each house list segment.

$$N_{s_i,t+1} = \sum_{s_j \in S} \sum_{k=1}^{K} X_{s_j,s_i,k,t} \ + \ \sum_{j=1}^{J} X_{j,s_i,t} \qquad \forall s_i \in S. \tag{9}$$

Cash flow balance equation.

$$Y_{t+1} = Y_t \ + \ a_t \ - \ gZ_t \ - \ (\sum_{k=1}^{K} \sum_{s_i \in S} kc_h H_{s_i,k,t} + \sum_{j=1}^{J} c_m W_{j,t}) \ -$$

$$c_1(\sum_{s_j \in S} \sum_{s_i \in S_0} \sum_{k=1}^{K} X_{s_j,s_i,k,t} + \sum_{j=1}^{J} \sum_{s_i \in S_0} X_{j,s_i,t}) \ + \ \sum_{s_i \in S} V_{s_i,t} + \sum_{j=1}^{J} V_{j,t}. \tag{10}$$

Upper bound for the number of sales.

$$\sum_{s_i \in S} V_{s_i,t} \ + \ \sum_{j=1}^{J} V_{j,t} \ \leq \ I_t \ + \ Z_t, \tag{11}$$

where,

$$V_{s_i,t} \ \leq \ \sum_{s_j \in S_0} \sum_{k=1}^{K} d_{s_i,s_j} X_{s_i,s_j,k,t} \qquad\qquad \forall s_i \in S. \tag{12}$$

and,

$$V_{j,t} \ \leq \ \sum_{s_i \in S_0} d_{j,s_i} X_{j,s_i,t} \qquad\qquad \forall j. \tag{13}$$

Inventory balance equation.

$$I_{t+1} = I_t \ + \ Z_t \ - \ \sum_{s_i \in S} V_{s_i,t} \ - \ \sum_{j=1}^{J} V_{j,t}. \tag{14}$$

The optimization model at time t is given by the following stochastic and dynamic programming formulation:

$$F_t(Y_t, I_t, N_t) = \max_{H_{s_i,k,t}, W_{j,t}, Z_t \forall s_i,k,j} \{-gZ_t - \sum_{k=1}^{K} \sum_{s_i \in S} kc_h H_{s_i,k,t} - \sum_{j=1}^{J} c_m W_{j,t} +$$

$$E_{X_{s_j,s_i,k,t}, X_{j,s_i,t}, \forall s_j,s_i,k,j}[G_t(Y_t, I_t, N_t, Z_t, X, H, W)]\}$$

$$\text{s.t.} \ \ (6), \ (7), \ (8)$$

and,

$$G_t(Y_t, I_t, N_t, Z_t, X, H, W) = \max_{V_{s_i,t}, V_{j,t}, \forall s_i, j} \sum_{s_i \in S} V_{s_i,t} + \sum_{j=1}^{J} V_{j,t} - c_1 \Big(\sum_{k=1}^{K} \sum_{s_j \in S} \sum_{s_i \in S_0} X_{s_j,s_i,k,t} + \sum_{j=1}^{J} \sum_{s_i \in S_0} X_{j,s_i,t} \Big)$$

$$-c_2 l_t - c_3 I_{t+1} + \beta F_{t+1}(Y_{t+1}, I_{t+1}, N_{t+1}).$$

$$\text{s.t. } (9),\ (10),\ (11),\ (12),\ (13),\ (14)$$

where l_t is equal to the amount of unsatisfied demand:

$$l_t = \sum_{s_i \in S} \sum_{s_j \in S_0} \sum_{k=1}^{K} d_{s_i,s_j} X_{s_i,s_j,k,t} + \sum_{j=1}^{J} \sum_{s_i \in S_0} d_{j,s_i} X_{j,s_i,t} - \sum_{s_i \in S} V_{s_i,t} - \sum_{j=1}^{J} V_{j,t}.$$

Boundary condition: at the end of the planning horizon the names in state s_i in the house list have a residual value, $LF(s_i)$, and the inventory has a residual value equal to v. Therefore, the boundary condition is given by:

$$F_{T+1}(Y_{T+1}, I_{T+1}, N_{T+1}) = \sum_{s_i \in S} LF(s_i) N_{s_i,T+1} + v I_{T+1}$$

Constraints (6) to (10) are similar to those in the catalog mailing problem. Constraint (11) corresponds to the upper bound for the total sales; the dollar value of the total sale must be less than or equal to the total dollar value in inventory. Constraints (12) and (13) correspond to the upper bound in the total sales in dollars in each state of the house list and in each rental list respectively, with respect to the total dollar value of products ordered by customers. Finally, constraint (14) updates the total dollar value in inventory at the end of the season.

In most applications, the dimensions of the dynamic programming model do not allow the model to be solved optimally. Usually, the house list consists of several states with hundreds of customers each. Additionally, the initial inventory and money for investment can vary over a broad range of values. Therefore, in real cases, the dimension of the state space in the mathematical formulation is large. This motivates the development of heuristics to solve the problems described above.

The optimal solutions of the optimization problems described above do not always match the intuition. For instance, it is not always true that the company should spend resources currently available on sending catalogs as long as there remain profitable customers. We have constructed examples where it is better to save part of the current season's budget to spend during the next season on customers whose profitability is larger than that of current customers. We have also found examples where, under the capital constraint, it is better to send a catalog to a customer with larger recency even though there is an "available customer" with a larger response rate. This case happens when the company is going to lose the customer if she is not activated by means of a catalog (this customer is in the last admissible state). Bitran and Ramalho (1992) state that a common practice of the catalog industry for the mailing strategy is to focus on short term performance, i.e., it understates the long-term impact of the current campaign. Hence, the two counterintuitive situations above show that a formal approach can lead to better decisions.

The following proposition shows that the expected number of customers in the house list converges to a constant when the available number of people in the rental lists is constant from a certain period in the planning horizon onwards. The average size of the house list monotonically increases until it reaches a point where the average arrival rate of new customers from rental lists gets equal to the average rate of customers that leave the house list. This result has an important practical implication; continuous growth requires a constant search for new markets. Nowadays, we observe that several American catalog companies are extending their business overseas. For example, L.L. Bean opened a branch in Japan only few years ago.

Proposition 1 *The number of customers in each state of the house list converges to a constant as time increases, assuming that the number of new names from rental lists is constant in time.*

$$\lim_{t \to \infty} N_{s_i,t} = \bar{N}_{s_i}$$

PROOF: See Appendix 2 . ∎

3 Lifetime Value of a Customer

In what follows we describe a methodology to calculate the *lifetime value of a customer*, i.e., the total discounted net profit that a customer generates during her life in the house list. We estimate the value of the company as the sum of its customers' lifetime values. For this purpose, we assume that there is no capital constraint. The lifetime value of a customer plays a central role in the heuristics that are described in the following section.

We consider the Markov chain representation described in subsection 2.1 to model the behavior of the customers in the house list, where the states are defined by their recency, frequency, and monetary value. The transition probabilities in the Markov chain depend on the mailing policy; the response rate in a state of the house list depends on the number of catalogs that are sent to the customers within a season. Therefore, the problem of calculating the lifetime value of a customer is equivalent to determining the optimal policy for sending catalogs to each state of the house list when there are unlimited resources.

We define the additional notation:

1. $LF(s_i)$ = lifetime value corresponding to a customer whose current state is s_i.

2. $\bar{b}_{s_i,k}$ = immediate expected profit if a customer in state s_i receives k catalogs during a season.

$$\bar{b}_{s_i,k} = \sum_{s_j \in S_0} (d_{s_i,s_j}(1-g) - c_1)p_{s_i,s_j,k} - kc_h$$

Hence, the mathematical formulation to determine the lifetime value of a customer in state s_i is given by:

$$LF(s_i) = \max \begin{cases} \text{Send k catalogs:} & \bar{b}_{s_i,k} + \beta \sum_{s_j \in S} p_{s_i,s_j,k} LF(s_j), \quad \forall k \\ \text{Do not send a catalog:} & \beta \sum_{s_j \in S} p_{s_i,s_j,0} LF(s_j), \end{cases} \tag{15}$$

where:

$$p_{s_i,s_j,0} = \begin{cases} 1 \text{ for } s_j \text{ admissible from } s_i \text{ if the customer does not receive a catalog} \\ 0 \text{ otherwise} \end{cases}$$

The optimality equation (15) has a unique solution, $LF(s_i)$, $\forall s_i$, that corresponds to the expected discounted profit when the optimal stationary policy is implemented (see, e.g., Ross, 1983, Chapter II). In order to find the optimal policy and the value of the maximum expected profit it is neccesary to solve the optimality equation (15). This can be done using one of several methods; for example the Policy Improvement method and the Linear Programming method. In particular, in the computational experiments, we use the Policy Improvement method which is based on successive approximations to the optimal solution. The algorithm starts with a feasible policy and in iteration n computes the left hand side of equation (15) using the solution of iteration $n-1$ in the right hand side (see, e.g., Ross (1983, p. 38) for a complete description of this algorithm). The algorithm converges to the optimal solution under the following assumptions: (i) bounded rewards (which correspond to the immediate expected profits, $\bar{b}_{s_i,k}$,in the current formulation), (ii) discount factor less than one, and (iii) finite state space; all of them are satisfied in our formulation.

For the case of monotonically decreasing response rates as a function of recency, it is possible to show that, for a given frequency and monetary value, if it is optimal not to send catalogs to a state with recency r then it is also optimal not to send catalogs to a state with recency $r+1$. The monotonicity of the response rate is defined as follows:

$$p_{s_i,s_j,k} \geq p_{\bar{s}_i,s_j,k} \qquad \forall s_j \in S_0$$

where, $s_i = (r, f, m)$ and $\bar{s}_i = (r+1, f, m)$. See Bitran and Mondschein (1993), unpublished, for a more complete discussion.

In the case of limited budget, it is possible to show that the lifetime value of a customer is an increasing and concave function of the initial budget. It is possible to show that there is a budget, B^*, such that the lifetime value of a customer starting with a budget greater or equal than B^* is equal to the lifetime value with unlimited budget.

4 HEURISTICS

It is not uncommon to find companies that base their mailing decisions on the RFM approach. This method consists of sending catalogs to those states with the largest lifetime values, until the budget constraint is binding. Usually, the lifetime values are calculated as the discounted present value of the expected profits that the customer will generate in future purchases. For this purpose, a profit profile is assumed according to some given mailing strategy. As we will see in proposition 2 this method works well as long as the cash flow constraints do not play a central role. However, in the presence of budget constraints, there are several trade-offs that must be incorporated in the mailing decisions. Moreover, inventory and mailing decisions cannot be analyzed independently of each other, because the total budget must be optimally distributed for these two purposes.

The heuristics that we describe in what follows incorporate explicitly the effects of limited resources for investment at each period. Initially, we describe a heuristic for the catalog mailing problem; later we introduce a heuristic for the general problem that includes the financial impact of inventory.

4.1 Heuristic for the Catalog Mailing Problem

We propose an enhanced RFM method for the catalog mailing problem which consists of sorting the states in the house list and the rental lists in decreasing order of their lifetime values (the rental lists are considered as particular states where the customer leaves the house list immediately if she does not respond with a purchase). Catalogs are then sent according to this order until either the budget constraint is reached or there are no more available customers with positive lifetime value. The number of mailings for every customer that receives a catalog is determined by the corresponding lifetime value (only single mailing is allowed for the rental lists). We called this approach enhanced RFM method because the lifetime values are computed considering the stochastic behavior of customers. Additionally, the algorithm to compute the lifetime values described in section 3 determines the optimal unconstrained mailing policy for each state, i.e., the optimal number of catalogs that should be sent to a customer during a season, for each state in the house list, when there is no cash flow constraint.

Computational experiments show a good performance of this heuristic in a variety of realistic scenarios (Bitran and Mondschein (1993), unpublished) which will be discussed in the computational results section. However, when cash flow constraint plays a central role, this heuristic does not take into account the following effect. Suppose that the optimal unconstrained decision, given by the lifetime value, is to send 2 catalogs to every customer in state s_i. If in the current period, we only send one catalog to every customer in state s_i, the lifetime value is smaller than it would be if the optimal policy is implemented, but we spend half of the money we would spend on marketing under the optimal unconstrained policy. This additional money can be spent on sending catalogs to additional (twice as many) customers. Therefore, there is a trade-off between the decrement in the lifetime value when the optimal unconstrained decision is not implemented and the additional profit corresponding to more customers. This modification is only relevant when there is limited amount of money for investment.

We present the following example to clarify the trade-off above. Suppose there are two states in the house list; the second one is the trapping state. Let us consider a probability of 0.15 that a customer in state 1 responds with a sale if she receives one catalog, and a probability equal to 0.2 if she receives two catalogs, a marketing cost of \$1.0, an average size of the order equal to \$50, a variable cost per sale (including the cost of the good) equal to \$25, and a discount rate of 1. In this case the optimal decision (when computing the lifetime value) is to send two catalogs to state 1 with a lifetime value equal to \$3.75. In what follows assume that we have money to send only two catalogs and there are two customers in state 1. One alternative is to send two catalogs to only one customer with an expected profit equal to \$3.75 or to send one catalog to each customer with a expected profit of \$3.31 for each one (assuming that from the next period onwards we have unlimited budget for investment). Therefore, given the capital constraint, it is better in this case to send catalogs to both customers. The following heuristic captures this effect.

<u>HEURISTIC 1.1</u>: HEUR 1.1

To simplify the notation, we assume that a maximum of two mailings can be sent in every season. We introduce the following additional notation:

1. $n(s_i)$ = optimal number of mailings to customers in state s_i given by the lifetime value, when there is no cash flow constraints.

2. $LF(j)$ = lifetime value corresponding to rental list j.

$$LF(j) = \bar{b}_j + \beta \sum_{s_i \in S_0} p_{j,s_i} LF(s_i) \qquad \text{and} \quad \bar{b}_j = \sum_{s_i \in S_0} [d_{j,s_i}(1-g) - c_1] p_{j,s_i} - c_m$$

3. $LF(s_i, k)$ = lifetime value corresponding to state s_i if we send k catalogs in the current season (from the next season onwards the optimal decision is implemented).

4. $d(s_i, k)$ = number of people in state s_i that receive k catalogs.

The purpose of redefining the lifetime values is to have a measure of how much they deteriorate if the optimal unconstrained policy is not implemented. Thus, if the optimal unconstrained policy is to send no catalogs to a state in the house list, then the redefined lifetime values when sending one or two catalogs are equal to a negative number. When the optimal unconstrained policy is to send one catalog to a state, the redefined lifetime value when sending one catalog remains the same and when sending two catalogs it takes a negative value. Finally, when the optimal unconstrained policy is to send two catalogs, the redefined lifetime value when sending two catalogs remains the same. However, the redefined lifetime value when sending only one catalog is calculated as the immediate expected profit plus the unconstrained expected lifetime values from the next period onwards. The transition probabilities are those probabilities associated with a single mailing.

If $(n(s_i) = 0) LF(s_i, 1) = LF(s_i, 2) = -\infty$
If $(n(s_i) = 1) LF(s_i, 1) = LF(s_i)$, and $LF(s_i, 2) = -\infty$
If $(n(s_i) = 2)$ then
$LF(s_i, 1) = -c_h + \sum_{s_j \in S_0} (d_{s_i,s_j}(1-g) - c_1) p_{s_i,s_j,1} + \beta \sum_{s_j \in S} p_{s_i,s_j,1} LF(s_j)$
$LF(s_i, 2) = LF(s_i)$.

The value of the decision variables $d(s_i, k)$, which correspond to the mailing strategy, is given by the solution of the following linear programming problem:

$$\max \sum_{s_i \in S} \sum_{k=1}^{2} LF(s_i, k) d(s_i, k) + \sum_{j=1}^{J} LF(j) d(j) \tag{16}$$

s.t.

$$\sum_{s_i \in S} \sum_{k=1}^{2} k c_h d(s_i, k) + \sum_{j=1}^{J} c_m d(j) \le Y_t + a_t$$

$$\sum_{k=1}^{2} d(s_i,k) \le N_{s_i,t} \qquad \forall s_i \in S,$$

$$d(j) \le L_j, t \qquad \forall j = 1,\ldots,J,$$

$$d(i,k) \ge 0, d(j) \ge 0 \qquad \forall i,k,j.$$

In the linear programming above, the objective function is to maximize the total lifetime values of the customers in the house list and rental lists. In the absence of the first constraint, this linear problem is equivalent to the enhanced RFM method (without cash flow constraint) described previously, i.e., catalogs are sent to all customers in the house list and rental lists for which the unconstrained lifetime value is positive.

The first constraint in the linear problem takes into account the costs of the different mailing strategies. The solution to this problem will differ from the enhanced RFM method, specially in the cases where multiple mailings are allowed and limited amount of money is available for mailings.

The following proposition shows that heuristic 1.1 is optimal when there is no budget constraint.

Proposition 2 *Heuristic 1.1 is optimal when there is unlimited amount of money for investment in every season.*

PROOF: In the Catalog Mailing Problem only the cash flow constraint involves more than one (in fact, all) clients. This restriction becomes redundant when an unlimited amount of money is available for investment. Therefore, the optimization problem separates by customers; these problems are equal to the optimization problems for calculating lifetime values. Thus, the optimal solution for the stochastic, dynamic formulation is to send catalogs to all that have positive lifetime values. Moreover, the solution of the linear programming problem in the heuristic becomes trivial; catalogs are sent to all customers that have a positive lifetime value. ∎

4.2 Heuristics for the Catalog Mailing Problem with Aggregate Inventory Costs

In what follows we describe a heuristic for the problem with cash flow constraints and inventory level constraints. In this heuristic we have to decide simultaneously the mailing and reordering policies.

HEURISTIC 2.1: HEUR 2.1

This heuristic is based on the Newsboy problem for a one period horizon. We first determine the probability distribution for the total demand. The random variable that describes the order size of a customer in state s_i that receives k catalogs takes value d_{s_i,s_j} with probability $p_{s_i,s_j,k}$ $\forall s_j \in S_0$, and value equal to zero otherwise. Hence, its expected value and variance are given by:

$$\mu_{s_i,k} = \sum_{s_j \in S_0} p_{s_i,s_j,k} d_{s_i,s_j},$$

and,

$$\sigma^2_{s_i,k} = \sum_{s_j \in S_0} p_{s_i,s_j,k}(d_{s_i,s_j} - \mu_{s_i,k})^2$$

Similar expressions can be obtained for the expected value, μ_j, and variance, σ_j^2 of the order size of a customer from a rental list j.

Usually, more than a hundred people receive catalogs in each state of the house list and in each rental list. Hence, a good approximation for the distribution of the total demand in dollars is the normal distribution with mean, μ_t, and variance, σ_t^2, equal to:

$$\mu_t = \sum_{k=1}^{K} \sum_{s_i \in S} \mu_{s_i,k} H_{s_i,k,t} + \sum_{j=1}^{J} \mu_j W_{j,t},$$

and,

$$\sigma_t^2 = \sum_{k=1}^{K} \sum_{s_i \in S} \sigma^2_{s_i,k} H_{s_i,k,t} + \sum_{j=1}^{J} \sigma_j^2 W_{j,t},$$

In what follows we determine the optimal amount of money to invest in inventory in a one period problem, considering a normal distribution for the demand. We also consider a residual value for the unsold products equal to their discounted cost (these products can be used to satisfy the demand in the next period). We define $\varphi(Z_t, I_t)$ as the total expected profit for a one period problem if we start with I_t dollars in inventory and order Z_t dollars in additional goods. Therefore, the function $\varphi(Z_t, I_t)$ is equal to:

$$\varphi(Z_t, I_t) = -gZ_t + \int_0^{Z_t+I_t} (x - c_3(Z_t + I_t - x) + \beta g(Z_t + I_t - x))f(x)dx$$
$$+ \int_{Z_t+I_t}^{\infty} ((Z_t + I_t) - c_2(x - Z_t - I_t))f(x)dx - c_1 \int_0^{\infty} x(\bar{d})^{-1} f(x)dx.$$

where the demand has a probability density function equal to $f(x)$, and $\bar{d}$ is the average size of an order. Hence, to obtain the "optimal reordering amount" we set the derivative of $\varphi(Z_t, I_t)$ with respect to Z_t equal to zero, which implies:

$$F(Z_t + I_t) = \frac{1 + c_2 - g}{1 + c_2 + c_3 - \beta g},$$

where $F(x)$ denotes the cumulative distribution function for the demand. Hence, using the normal distribution for the demand, the expression for the "optimal reordering amount" is given by:

$$Z_t = \sigma_t a + \mu_t - I_t,$$

and $F_y(a) = (1 + c_2 - g)/(1 + c_2 + c_3 - \beta g)$ where y has a standard normal distribution.

We note that the reordering amount is a function of the number of catalogs to be sent in the current season, because the mean and the variance of the normal distribution are a function of the mailing policy. The reordering amount in the cash flow constraint in the original problem is replaced by this "optimal reordering amount". The rest of the heuristic works as follows: we send catalogs according to the decreasing order of the lifetime values until either the cash flow constraint is binding or there are no more available customers with positive lifetime values.

This heuristic takes into account the same effect described in heuristic 1.1: the trade-off between the decrement in the lifetime value when the optimal unconstrained mailing policy is not implemented and the additional profit corresponding to an extra customer that receives a catalog. In this case, before calculating the optimal mailing policy in step 2 of the heuristic, we solve the same linear problem described in heuristic 1.1 to determine in which cases we send less catalogs than the optimal number of catalogs determined by the lifetime values, when there is no cash flow constraints.

Description of heuristic 2.1 at the beginning of season t.

We introduce the following additional notation:

1. U = total number of rental lists and states in the house list ($U = |S| + J$), where $|S|$ is the cardinality of S or equivalently the number of states in the house list.

2. $n(u)$ = number of mailings to each customer in the u^{th} state.

3. $N(u)$ = number of people in the u^{th} state.

4. $c(u)$ = marketing cost associated with the u^{th} state.

5. $d(u)$ = number of people in the u^{th} state that receive catalogs (u can correspond to a rental list).

Step 0: Sorting.

$s(u)$ = state of the u^{th} largest life time value.

Step 1: Initialization.

$u = 1$,
$d(u) = 0 \qquad \forall u$,
I_t=initial inventory at the beginning of season t,

$Z_t = 0$.

Solve the LP (16) to determine the value of $n(u), \forall u$.

Step 2: determining the number of catalogs to send to state $s(u)$.

d^* = maximum value that satisfies the following 2 inequalities:

$g[a\sqrt{(}\sum_{l=1}^{u-1} \sigma^2_{s(l),n(s(l))} d(s(l)) + \sigma^2_{s(u),n(s(u))} d^*) +$
$\sum_{l=1}^{u-1} \mu_{s(l),n(s(l))} d(s(l)) + \mu_{s(u),n(d(u))} d^* - I_t] + \sum_{l=1}^{u-1} n(s(l))c(s(l))d(s(l)) +$
$n(s(u))c(s(u))d^* \leq Y_t + a_t \qquad (i),$

$\sum_{l=1}^{u-1} n(s(l))c(s(l))d(s(l)) + n(s(u))c(s(u))d^* \leq Y_t + a_t \qquad (ii).$

$d(s(u)) = \min\{d^*, N(s(u))\}$.

Step 3: Stopping criterium.

$u = u + 1$
if $(u > U)$ then
  GOTO Step 4
else
  GOTO Step 2.

Step 4: Calculating the reordering amount.

$$Z_t = \max\{0, a\sqrt{(}\sum_{l=1}^{U} \sigma^2_{s(l),n(s(l))} d(s(l))) + \sum_{l=1}^{U} \mu_{s(l),n(s(l))} d(s(l)) - I_t\}$$

Step 5: STOP.

5 Upper Bound

In this section we describe an upper bound for the optimization model, which is useful to determine the performance of the heuristics described in the previous section.

Proposition 3 *An upper bound for the optimization problems described in Section 2 is the solution of the deterministic versions of the stochastic models, where the random variables are replaced by their expected values.*

Proof: The proof is straightforward and the details will be omitted. It is based on successive applications of Jensen's inequality and the concavity of the maximization of a linear programming problem as a function of the right hand side. ∎

Therefore, the upper bound for the catalog mailing problem with aggregate inventory costs is given by:

$$UB = \max\{-g\sum_{t=1}^{T}\beta^{t-1}Z_t - c_h\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{s_i\in S}\beta^{t-1}kH_{s_i,k,t} - c_m\sum_{t=1}^{T}\sum_{j=1}^{J}\beta^{t-1}W_{j,t} + \sum_{t=1}^{T}\sum_{s_i\in S}\beta^{t-1}V_{s_i,t} +$$

$$\sum_{t=1}^{T}\sum_{j=1}^{J}\beta^{t-1}V_{j,t} - c_1\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{s_i\in S}\sum_{s_j\in S_0}\beta^{t-1}p_{s_i,s_j,k}H_{s_i,k,t} - c_1\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{s_j\in S_0}\beta^{t-1}p_{j,s_j}W_{j,t} -$$

$$c_2\sum_{t=1}^{T}\beta^{t-1}l_t - c_3\sum_{t=1}^{T}\beta^{t-1}I_{t+1} + \sum_{s_i\in S}\beta^{T}LF(s_i)N_{s_i,T+1} + v\beta^{T}I_{T+1}\}$$

s.t.

$$gZ_t + \sum_{k=1}^{K}\sum_{s_i\in S}kc_hH_{s_i,k,t} + \sum_{j=1}^{J}c_mW_{j,t} \le Y_t + a_t \qquad \forall t. \tag{17}$$

$$\sum_{k=1}^{K}H_{s_i,k,t} - N_{s_i,t} \le 0 \qquad \forall s_i \in S, t. \tag{18}$$

$$W_{j,t} \le L_{j,t} \qquad \forall j, t. \tag{19}$$

$$N_{s_i,t+1} = \sum_{s_j\in S}\sum_{k=1}^{K}p_{s_j,s_i,k}H_{s_j,k,t} + \sum_{j=1}^{J}p_{j,s_i}W_{j,t} \qquad \forall s_i \in S, \forall t. \tag{20}$$

$$Y_{t+1} - Y_t - a_t + gZ_t + (\sum_{k=1}^{K}\sum_{s_i\in S}kc_hH_{s_i,k,t} + \sum_{j=1}^{J}c_mW_{j,t}) +$$

$$+c_1(\sum_{k=1}^{K}\sum_{s_i\in S}\sum_{s_j\in S_0}p_{s_i,s_j,k}H_{s_i,k,t} + \sum_{j}^{J}\sum_{s_i\in S_0}p_{j,s_i}W_{j,t}) - (\sum_{s_i\in S}V_{s_i,t} + \sum_{j=1}^{J}V_{j,t}) = 0 \qquad \forall t. \tag{21}$$

$$\sum_{s_i\in S}V_{s_i,t} + \sum_{j=1}^{J}V_{j,t} - I_t - Z_t \le 0 \qquad \forall t. \tag{22}$$

$$V_{s_i,t} - \sum_{k=1}^{K}\sum_{s_j\in S_0}d_{s_i,s_j}p_{s_i,s_j,k}H_{s_i,k,t} \le 0 \qquad \forall s_i, t. \tag{23}$$

$$V_{j,t} - \sum_{s_i\in S_0}d_{j,s_i}p_{j,s_i}W_{j,t} \le 0 \qquad \forall j, t. \tag{24}$$

$$I_{t+1} - I_t - Z_t + \sum_{s_i\in S}V_{s_i,t} + \sum_{j=1}^{J}V_{j,t} = 0 \qquad \forall t. \tag{25}$$

The objective function is equal to the discounted expected profits during the planning horizon. The random variables that represent the number of responses from each rental list and each state of the house list have been replaced by their expected values. Constraint (17) corresponds to the cash flow constraint. Constraints (18) and (19) are the upper bounds for the number of people in the states of the house list and in the rental lists respectively. Constraints (20) updates the number of people in the states of the house list. The cash flow balance equation is given by constraint (21). Constraints (22), (23), and (24) determine the sales in each state of the house list and each rental list. Finally, constraint (25) is the inventory balance equation.

6 Computational Experiments

In this section we study the performance of the heuristics in several computational experiments. We use Monte Carlo simulations to estimate the company's expected profit over the planning horizon for the different heuristics. For the catalog mailing problem the heuristic described in Section 4 generates the mailing policies. We use a multinomial distribution to represent the number of people that move from a given state s_i to all accessible states s_j, $\forall s_j \in S$ when a multiple mailing of k catalogs is sent; the number of trials is equal to the total number of people that receive k catalogs in state s_i and the probabilities are equal to $p_{s_i,s_j,k}$, $\forall s_j \in S$. A similar probability distribution is used for the rental lists. For the catalog mailing model with aggregate inventory costs, the heuristic described in Section 4 generates additionally the reordering policies, i.e., the amount of stock to buy at the beginning of every season.

In the experiments, we implement the mailing and reordering policies given by the heuristics, we simulate the corresponding number of responses, and compute the profit for each particular outcome of the random variables during the planning horizon. By averaging the profits given by repeated simulations we obtain an estimate of the expected profit. We stop the simulations when the coefficient of variation (standard deviation over expected profit) is less than 0.1%. The heuristics are compared with the corresponding upper bounds described in Section 5.

The simulations are based on real data provided by the largest custom jewelry catalog sales company in the U.S. We also use the company's definition of recency, frequency, and monetary value. Recency is defined by the number of periods since the last purchase. Frequency is defined equal to 1 if the customer has bought only once and equal to 2 if the customer has bought more than once from the company. Finally, monetary value is defined by the dollar value that the customer spent in the last purchase.

The initial budget Y_0 is utilized as a reference point. Y_0 is equal to the minimum budget such that from that value onwards the upper bound given by the optimal value of the linear program in section 5 does not change.

We use the following set of data in the computational experiments:

Planning horizon: 8 seasons.

Number of states in the house list: 28 (7 recencies, 2 frequencies, and 2 monetary values.)

One rental list with 3000000 available names each season.

Marketing cost for the house list: \$0.6

Marketing cost for the rental list: \$0.7

Monetary values: \$55 and \$80

Cost of the goods: 35% of the average monetary value.

Variable cost per request: \$1

Penalty cost for rejecting an order: 10% of the order's monetary value.

Holding inventory cost: 12% of the product's selling price per year.

Residual value of inventory = 10% of the product's selling price.

Initial inventory: 0

Exogenous Investment $a_t = 0 \qquad t \in \{2, \ldots, T\}$

Discount rate: 0.975

Response rate of the rental list: 1.5%

Response rates for the states in the house list in increasing order of recency:

Response rate with one mailing and frequency equal to one:

Monetary value \$55: .041 .035 .030 .025 .021 .018 0.0

Monetary value \$80: .045 .038 .033 .028 .024 .020 0.0

Response rate with one mailing and frequency equal to two:

Monetary value \$55: .057 .049 .041 .035 .030 .025 0.0

Monetary value \$80: .059 .050 .043 .037 .031 .026 0.0

Probability of response with two mailings and frequency equal to one:

Monetary value \$55: .062 .053 .049 .038 .032 .028 0.0

Monetary value \$80: .064 .055 .051 .039 .034 .030 0.0

Probability of response with two mailings and frequency equal to two:

Monetary value \$55: .085 .072 .061 .052 .044 .038 0.0

Monetary value \$80: .087 .074 .063 .054 .046 .040 0.0

Probability that a customer will make a purchase of \$55 in the current period, conditional on making a purchase this period and having purchased \$55 in the last period = 0.75.

Probability that a customer will make a purchase of \$80 in the current period, conditional on making a purchase this period and having purchased \$55 in the last period = 0.25.

Probability that a customer will make a purchase of \$55 in the current period, conditional on making a purchase this period and having purchased \$80 in the last period = 0.18.

Probability that a customer will make a purchase of \$80 in the current period, conditional on making a purchase this period and having purchased \$80 in the last period = 0.82.

6.1 Catalog Mailing Problem

This case assumes that all the demand is satisfied and there are no costs of managing the inventory. In the data set, we use a maximum of two mailings per season for customers in the house list and a single mailing for customers in the rental list. The performance of heuristic HEUR 1.1 is shown in Table 1.

The first column is the initial budget as a percentage of the reference budget, Y_0. The second column is the initial budget in dollars. Column 3 contains the performance of heuristic HEUR 1.1 with respect to the upper bound. We observe that the behavior of the heuristic improves as the initial budget increases, having an excellent performance with an initial budget greater than or equal to 1% of Y_0. The expected profit corresponding to each experiment can be found in Table 4 in Appendix 3. As we showed in proposition 2, in the limit, when there is an unlimited amount of money for investment, the heuristic leads to the optimal solution. Moreover, it is possible to show in that case that the stochastic formulation for the catalog mailing problem is equal to the corresponding deterministic model. Therefore, in that case, the upper bound is also equal to the solution of the catalog mailing problem. The proof is straightforward and is not presented in this paper.

Table 1: Catalog Mailing Model with Two Mailings

$(A_1/Y_0) * 100$	A_1=Initial Budget (US$)	HEUR 1.1 w/r UB
1%	25500	99.7%
5%	127500	99.7 %
10%	255000	99.8%
25%	637500	99.9%
50%	1275000	99.9%
75%	1912500	99.9%
100%	2550000	99.9%

The heuristic is easy to implement. It is based on a set of indices (lifetime values) that are computed once at the beginning of the planning horizon, and on the resolution of a small linear programming problem. The number of constraints of this linear problem is equal to the number of states in the house list plus the number of rental lists plus 1. The number of decision variables is equal to the number of states in the house list times the multiple mailings allowed plus the number of rental lists. Therefore, in the computational experiments shown in this section, the linear problem has 30 constraints and 57 decision variables.

Computational experiments were performed to compare this heuristic with the enhanced RFM method. When multiple mailings are allowed, heuristic HEUR 1.1 has a performance that is on average 3% better than the enhanced RFM method. This improvement is explained by the trade-offs described in section 4; in the presence of cash flow constraints it is not always optimal to mail the optimal number of catalogs, given by the unconstrained lifetime values, to the most profitable customers. It can be better to send fewer catalogs to each customer and reach more of them.

6.2 Catalog Mailing Problem with Aggregate Inventory Costs

For the catalog mailing problem with aggregate inventory costs, we present two sets of experiments. In the first set we consider the single mailing case. In the second group of experiments, we allow a maximum of two mailings for the customers in the house list.

For the single mailing case, the performance of heuristic 2.1 is shown in Table 2.

The first column is the initial budget as a percentage of the reference budget Y_0. The second column is the initial budget in dollars. Column 3 shows the behavior of heuristic 2.1 with respect

Table 2: Catalog Mailing Model with Aggregate Inventory Costs: Single Mailing Case

$(A_1/Y_0)*100$	A_1=Initial Budget $	HEUR 2.1 w/r UB
1%	37500	86.9%
5%	187500	94.0%
10%	375000	95.6%
25%	937500	97.5%
50%	1875000	98.2%
75%	2812500	98.6%
100%	3750000	98.8%

to the upper bound. We observe that the heuristic has a good performance with an achievement of more than 95% of the upper bound when the budget is greater than 10% of Y_0. The detailed data about the expected profits can be found in Appendix 3, Table 5.

In the second set of experiments, we allow a maximum of two mailings within a season for the house list. The performance of heuristic 2.1 is shown in Table 3.

Table 3: Catalog Mailing Model with Aggregate Inventory Costs: Two Mailings.

$(A_1/Y_0)*100$	A_1=Initial Budget (US$)	HEUR 2.1 w/r UB
1%	38500	91.5%
5%	192500	95.3 %
10%	385000	96.4%
25%	962500	96.8%
50%	1925000	96.9%
75%	2887500	98.8%
100%	3850000	99.4%

In this case we observe the same pattern of behavior as in the previous set of experiments. When we increase the initial amount of money that is available for investment at the beginning of the planning horizon, the performance of the heuristic improves significantly. However, in this model we cannot guarantee that the heuristic leads to the optimal solution when there is an unlimited amount of money for investment. Although, it is still true that with unlimited budget it is optimal to send catalogs to all the profitable customers (all customers that have a positive lifetime value), the reordering policy given by the heuristic is determined considering a single period problem instead of the multiperiod problem. The impact of considering a single period problem is significantly reduced in heuristic 2.1 by incorporating a residual value for the products equal to their discounted value in the next period.

Finally, we remark that the heuristic has a very good performance. The quality of the solutions improves as the initial budget increases. With a budget greater than or equal to 10% with respect to the reference budget, the heuristic attains more than 96% of the upper bound. It is reasonable to assume that in practice companies have access to loans when they have a profitable business. Therefore, they can finance at least a 10% of the total possible cash needed.

It is important to notice that the difference between the solutions given by the heuristic and the upper bound can partly be explained because the upper bound is not necessarily tight. The upper bound is the solution to the deterministic version of the problem, and therefore, with a deterministic demand, holding and shortage costs are never incurred (there are no fixed costs associated with reorderings). On the other hand, in the stochastic problem there are always outcomes for the random demand for which the optimal solution leads to shortage or holding costs. To illustrate this effect we consider a one period problem with unlimited amount of money for investment. It is straightforward to prove that, in this particular case, heuristic 2.1 leads to the optimal solution. For the same set of parameters used in the previous experiments, heuristic 2.1 has a performance of 99.4% of the upper bound. Therefore, the 0.6% difference is due to the error in the upper bound. This difference tends to be larger when considering several periods and limited budget.

In what follows, we present a set of computational experiments to compare the effect of multiple mailings versus a single mailing. We use the same parameters as in the previous experiments, with the exception of the planning horizon and the number of names available in the rental list. In this case we use 100 seasons and 100000 names in each period. We use a long planning horizon with the purpose of illustrating the asymptotic behavior of the system. The initial budget is $35000.

We compare the accumulated cash at the beginning of every season, the total number of people in the house list at the end of each season, and the number of catalogs mailed every season. The results are shown in figures 2, 3 and 4, respectively. We observe that during the first three seasons the company loses money with the single mailing strategy; only after the tenth season it recovers the initial investment. However, with the two mailings strategy, the company recovers its investment in only seven seasons, facing losses during the first two campaigns. The acceleration in the investment recovery period is due to the increment in the customer response rates produced by the multiple mailings. We remark that when considering the multiple mailing strategy, the company can still send a single mailing to some (or all) states during the planning horizon. Therefore, the multiple mailing strategy is at least as good as the single mailing one. We also observe that the number of people in the house list converges to a constant in the multiple mailing case after approximately season seventy. In the single mailing case the convergency occurs towards the end of the planning horizon. Finally, the number of catalogs mailed every season also converges to a constant, because the same happens with the number of profitable customers.

Informal evidence suggests that firms in the catalog sales industry often go bankrupt. The methodology developed in this paper allows to assess the risk of the company running out of cash at the beginning of a period. With this purpose, after each simulation, we could compute an index equal to one if the company runs out of cash for that specific outcome of the random demand or equal to zero otherwise. Averaging the indices given by repeated simulations we can obtain an estimate of the probability of running out of cash and eventually going bankrupt.

Catalog companies usually have good estimates for the response rate of customers in the house list. Large data bases allow them to keep track of detailed information which is used to estimate the house list response rate. In contrast, companies have little information to estimate the response rates of rental lists. As we mentioned earlier, they assign a fraction of their budget for market tests in order to improve the quality of their estimations. The more they spend on market tests, the more precise is the estimation of the response rates. With the purpose of understanding the impact of changes in the response rates of rental lists on the optimal expected profit over the planning horizon we perform several computational experiments. For example, with the set of parameters used in the previos experiments we have obtained the following results: (*i*) if the initial budget is \$35000 and the initial house list is empty, then a variation of $\pm 5\%$ in the response rate of the rental list leads to an increment of 110% and a reduction of 92% in the objective function respectively, (*ii*) if the initial budget is \$35000 and the house list has the steady state number of customers in each state, then a variation of $\pm 5\%$ in the response rate leads to an increment of 25% and a reduction of 22% in the objective function respectively, (*iii*) finally, if the initial budget is \$250000 and the house list has the steady state number of customers in each state, then a variation of $\pm 5\%$ in the response rate leads to an increment of 41% and a reduction of 41% in the objective function respectively. The same pattern of behavior has been observed in several other experiments. We observe that the results of the company are very sensitive to changes in the response rate of rental lists when the operation starts; catalog companies are building the house list, and therefore, they must rely almost completely on rental lists. On the other hand, when the catalog company has a mature house list, the results of its operations are less sensitive to changes in the response rate of rental lists. If the initial budget is relatively small in comparison to the number of profitable customers, then most of the targeted customers will be customers in the house list (these are the customers with larger lifetime values). In this case, the impact of changes in the response rates is considerably smaller. A more significative impact is observed when the initial budget is large, because the optimal mailing strategy is to send to all profitable customers either from rental lists or from the house list. These results suggest that the value of perfect information is high, and therefore, catalog companies should be willing to spend a significant amount of resources in market tests, especially when they are building the house list.

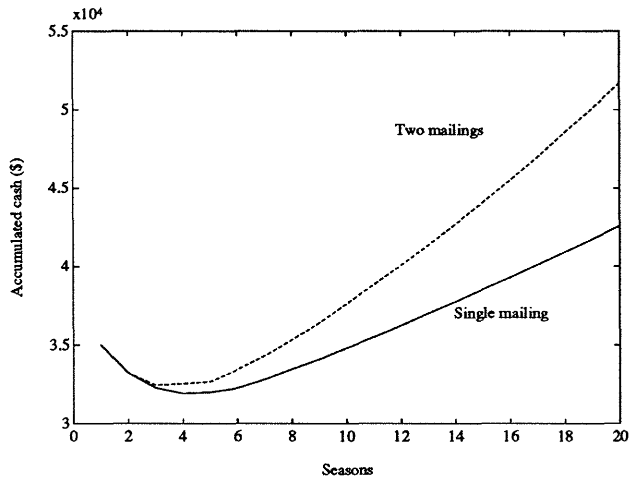

Figure 2: Accumulated Cash at the End of Every Season

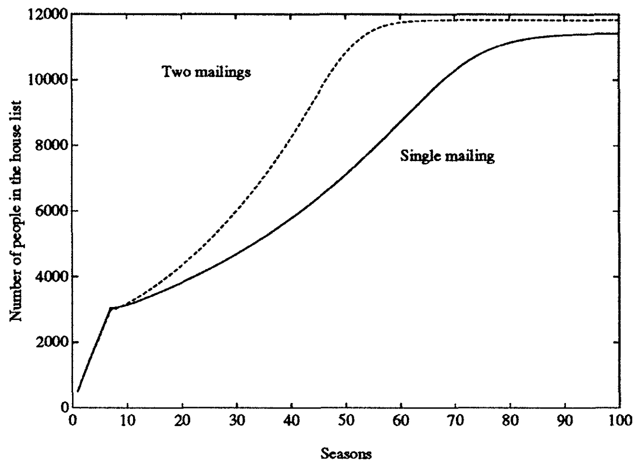

Figure 3: Total Number of People in the House List

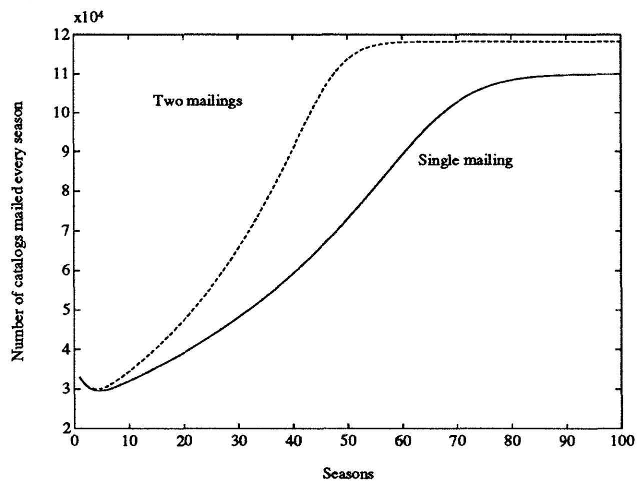

Figure 4: Number of Catalogs Mailed Every Season

7 Conclusions and Extensions

This paper has presented two mathematical models, which vary in the way suppliers interact with a catalog company, to determine the mailing and reordering policies that maximize the expected profit. Stochastic demand and dynamic evolution of the customers within the house list were considered. Optimal solutions are hard to compute for real size problems. Therefore, ad-hoc heuristics were implemented based on the solutions of simplified versions of these problems. Computational experiments showed that these heuristics give satisfactory results. For the computational experiments a particular definition for recency, frequency, and monetary value was adopted. The models can also be used to study how different definitions of these parameters affect the performance of the company.

The models proposed in this paper allow us to study how the optimal solution changes with market conditions; for example price, cost of goods, response rates, and mailing costs. In particular, we have performed computational experiments to understand the impact of changes in the response rate of rental lists in the results of the catalog companies. We have concluded that, especially when companies are building the house list, the value of perfect information is very high. Therefore, companies should spend an important fraction of their budgets to estimate these response rates. This suggests that the design of experiments is an interesting research question to pursue in future research.

The models can also be used to do risk analysis, i.e. to study what fraction of the times the company runs out of cash. Informal evidence indicates that firms in the catalog industry often go bankrupt. Running simulations, we can easily compute if the company runs out of cash for every outcome of the stochastic demand during the planning horizon. With this information, we can estimate the probability of running out of cash and eventually going bankrupt. This suggests that it would be interesting to incorporate the probability of bankruptcy into the firm's objective function; we leave this topic for future research.

The tactical model presented in this paper establishes the optimal aggregated levels of reordering and the optimal number of catalogs to be sent to each segment in the house list and to each rental list at the beginning of every season. As a topic of future research, a hierarchical approach could be pursued to disaggregate the total reordering amount into individual items and to schedule the mailings during the corresponding season.

The current formulation considers a penalty cost for the unsatisfied demand. An alternative approach that could be studied is to replace the penalty cost by a service level constraint that assures the demand is satisfied with a given probability.

Acknowledgments

The authors thank Jonathan Fleischmann, the associate editor, and three anonymous referees for their excellent comments and suggestions.

Appendix 1

The inventory can also be defined in terms of physical units as an alternative to the monetary value definition. Usually, a single catalog promotes several products. Therefore, there are different levels of inventory disaggregation at which the model can be formulated: inventory management for each individual item, for families of products, or for a single "average product". For the purpose of illustration we show how to formulate the problem in terms of a single equivalent unit.

In general, an order consists of several items. We define a *unit equivalent* as a unit that has the average ordered fraction of each item. We introduce the following notation to show how to compute the average size of the order, the average cost of the goods and the proportion of each item in the unit equivalent. Using the company's relevant data for the past seasons we define:

Q = total number of orders.

Q_u = total number of units of item u ordered.

d_u = price of item u.

g_u = cost of item u as a fraction of its price.

$\bar{d}$ = average size of the order in dollars (defined below).

$\bar{g}$ = average cost of goods as a fraction of the average sale per customer (defined below).

Hence, a unit equivalent consists of Q_u/Q units of item u, for all u. We also define the average size of the order and the average costs of goods as follows:

$$\bar{d} = \frac{\sum_u d_u Q_u}{Q} \qquad\qquad \bar{g} = \frac{\sum_u g_u d_u Q_u}{\bar{d}Q}$$

Finally, we define e_{s_i,s_j} as the number of unit equivalents that correspond to an order of a customer that moves from state s_i to s_j ($e_{s_i,s_j} = \frac{d_{s_i,s_j}}{\bar{d}}$). Hence, the decision variables Z_t and I_t correspond to the total number of unit equivalents to order and in inventory respectively, at the beginning of period t. The equation to update the inventory can be written as:

$$I_{t+1} = I_t + Z_t - \sum_{s_i \in S}^{I} V_{s_i,t} - \sum_{j=1}^{J} V_{j,t},$$

where,

$$\sum_{s_i \in S} V_{s_i,t} + \sum_{j=1}^{J} V_{j,t} \leq I_t + Z_t,$$

$$V_{s_i,t} \leq \sum_{s_j \in S_0} \sum_{k=1}^{K} e_{s_i,s_j} X_{s_i,s_j,k,t} \qquad \forall s_i \in S,$$

and,

$$V_{j,t} \leq \sum_{s_i \in S_0} e_{j,s_i} X_{j,s_i,t} \qquad \forall j.$$

This approach is suitable when the aggregate composition of orders in all the monetary value segments are approximately the same. This usually happens when catalogs promote products in a homogeneous range of prices. If this assumption does not hold, then a less aggregate approach must be considered. In this case, the inventory must be defined for families of items or even for each individual item. In the latter case we define $e_{s_i,s_j}(u)$ as the average proportion of item u that is ordered by a customer that moves from state s_i to s_j. Therefore, the inventory equations described above are replaced by as many constraints as the number of items in the catalog, where $Z_{u,t}$ and $I_{u,t}$ correspond to the number of item u units to order and in inventory at the beginning of season t respectively.

Appendix 2

Proof of Proposition 1

A new customer can enter the house at different states in S_0 depending on the size of her order. Using well known results of Markov chains with rewards, the average time that she will spend in the house list before reaching the trapping state is a bounded time, t_{s_i}, if the entering state is equal to $s_i, \forall s_i \in S_0$. This time is independent of the evolution of the other customers in the house list.

The Markov chain system can also be seen as a queuing network, where each state corresponds to an activity with infinite number of servers (the presence of other customers in the state do not interfere with the transitions of the customer under consideration). The service time is deterministic and equal to one period. With this representation and the fact that the average time per customer until reaching the trapping state is bounded, Little's law leads to:

$$\lambda_{s_i} t_{s_i} = C_{s_i} \qquad \forall s_i \in S_0$$

where,

$\lambda_{s_i} = \sum_{j=1}^{J} p_{j,s_i} L_j$, which corresponds to the arrival rate of customers that enter the house list from state s_i.

C_{s_i} is equal to the average number of customers in the house list whose first state was s_i.

Therefore, the total average number of customers in the house list is equal to

$$\bar{N} = \sum_{s_i \in S_0} C_{s_i}$$

A similar proof holds for each state in the house list individually. The actual arrival rate to state s_j is equal to $\lambda_{s_i} v_{s_i,s_j}$ where v_{s_i,s_j} corresponds to the average number of visits to state s_j of customers that entered the system through state s_i (v_{s_i,s_j} is bounded). The time in state s_j is deterministic and equal to one. Hence, applying Little's law we obtain:

$$\lambda_{s_i} v_{s_i,s_j} 1 = \bar{N}_{s_j} \qquad \forall s_j \in S,$$

where $\bar{N}_{s_j}$ is the average number of people in state s_j in the house list. ∎

The equilibrium condition is guaranteed by the fact that the system has an "infinite number of servers" at each state and the average time in the system is bounded.

Appendix 3

Table 4: Expected Profits for the Catalog Mailing Problem

$(A_1/Y_0) * 100$	Initial Budget A_1(\$)	HEUR 1.1 \$	BOUND \$
1%	25500	10806	10835
5%	127500	54003	54173
10%	255000	108082	108346
25%	637500	270576	270864
50%	1275000	541502	541728
75%	1912500	812276	812592
100%	2550000	1008500	1008708

Table 5: Expected Profits for the Catalog Mailing Model with Aggregate Inventory Costs.(Single Mailing Case)

$(A_1/Y_0) * 100$	Initial Budget A_1($)	HEUR 2.1 $	BOUND $
1%	37500	5177	5955
5%	187500	27979	29776
10%	375000	56943	59553
25%	937500	145080	148882
50%	1875000	292461	297764
75%	2812500	440424	446646
100%	3750000	561096	568152

Table 6: Expected Profits for the Catalog Mailing Model with Aggregate Inventory Costs. (Two Mailings Case)

$(A_1/Y_0) * 100$	Initial Budget A_1($)	HEUR 2.1 $	BOUND $
1%	38500	9777	10680
5%	192500	50896	53402
10%	385000	102997	106805
25%	962500	258436	267012
50%	1925000	517285	534024
75%	2887500	791646	801036
100%	3850000	1002350	1008709

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