ABSTRACT

M. Sc. Thesis

LIE RINGS WITH DERIVATION

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In this thesis, some properties from works which have been done up till now about structure of the Lie ring of derivations of an associative ring R, denoted Der (R), are given.

This thesis consists of five chapters.

In the first chapter, subject of the thesis is introduced and a short summary of works which are related this subject is shortly given. In the second chapter, some basic definitions and properties are mentioned to make easy understanding of the thesis.

In the third chapter, the first section of Herstein [13]'s book which is published in 1969 is taken essentially to understand the structure of the Lie ring and works which are related with this subject and some of works about Lie and Jordan structures in simple rings are gathered.

In the fourth chapter, the properties of Lie ring Der (R) of derivations of R are investigated for the cases where R is commutative and non-commutative

seperately, the theorems which are expressed the Lie ring $\mathrm{Der}\,(R)$ under which conditions is prime are given. Unless R is commutative, the primeness of $\mathrm{Der}\,(R)$ is investigated by approach is via the study of the structure of $I\,(R)$, the Lie ring of inner derivations of the prime ring R. In the commutative case, the approach is via the Lie structure of the Lie ring $R\delta$ of all derivations of the form $r\delta$ where $r \in R$ and δ is a given derivation of R. The properties of $R\delta$ are worked in detail and it's seen that $R\delta$ and $I\,(R)$ have similar properties. Furthermore, the hypothesis of primeness on R which is a Noetherian ring of characteristic not 2 with identity is weakened to δ-primeness and it is proved that $R\delta = \{r\delta \mid r \in R\}$ is a prime Lie ring.

In the fifth chapter, a theorem is presented in the light of the theorems given in the previous chapter by taking R only as a commutative ring. In this theorem, the primeness of the Lie ring $R\delta$ is proved without assuming any further condition except from the δ-primeness of R which is a 2-torsion free ring with identity.

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Key Words:

Simple ring, Lie ideal, derivation, Lie ring, δ-prime.