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Research Article

Finite element modeling of crack propagation in RC beam by using energy approach

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Abstract

In present study interface element with nonlinear spring is used to simulate cohesive zone model (CZM) in reinforced concrete beam in Mod I of crack propagation. Modified crack closure integral method is implemented to model propagation of fracture process zone (FPZ). This model, energy can calculate energy release rate in concrete by using new method in energy approach. Energy dissipation rate by steel bars is obtained to affect on crack propagation criterion to implement in finite element method. The results show that proposed model does not depend on mesh size.

Keywords: Propagation, FPZ, stiffness, energy release rate

1. Introduction

Crack modeling in reinforced concrete (RC) beam is essential due to nonlinear behavior. One of the challenges is propagation of tensile crack in RC beam. There is a little knowledge on how to predict crack in RC beam. However many significant efforts have been made to study fracture mechanics in failure mode of RC beams [*1*,*2*].

Fracture mechanics was employed to model tensile crack in concrete with strain softening behaviour. It was first used to study crack propagation applying linear elastic fracture mechanics (LEFM) in warships in World War II [*3*]. Later some studies used LEFM in concrete propagation analysis, but Kaplan [*4*] found out that deploying LEFM is not acceptable to solve crack problems with normal concrete sizes. Hillerborg *et al.* [*5*] proposed the first model in concrete based on nonlinear fracture mechanics. Mentioned study introduces a region, often termed as fracture process zone (FPZ), ahead of real crack tip which leads to crack closure (Figure 1). This significant and large zone contains micro-cracks in matrix–aggregate, gel pores, shrinkage cracks, bridging, and branch of cracks that is located ahead of the macro-cracks. Since a significant amount of energy is stored in this region, a crack can have stable growth before peak load. In addition, the existence of the FPZ justifies the strain softening behaviour in the stress- crack opening curve after peak load. In this region, the interlocking crack surfaces after peak load contribute to a gradual decline in stress and prevent sudden failure [*3*]. The FPZ dimension depends on the size of structure, initial crack, loading and material properties of concrete. The length of the FPZ is of special interest as compared to its width. The effective modulus of elasticity is reduced when moving from undamaged regions into the FPZ.

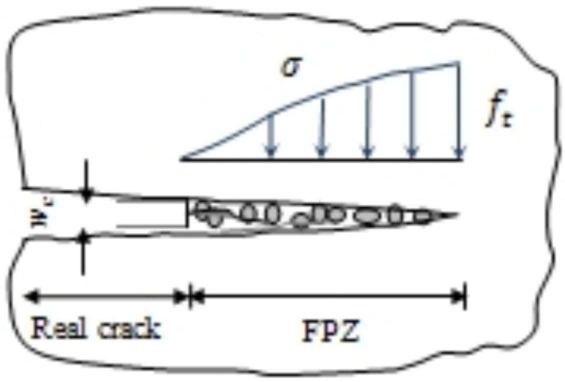

Fig. 1. FPZ in front of crack with normal stress

Energy approach can describe crack propagation criterion in fracture process at the crack-tip. The energy approach displays that the energy requisite, which stored in the FPZ, to form crack, it is called energy release rate, must be enough to overpower the critical fracture energy. A criterion for crack propagation can be defined in terms of energy release rate to study crack state. The energy approach criterion depends on stiffness matrix, displacement and crack geometry [*6*].

Different approaches have been investigated to model discrete crack as well as its propagation criteria. To simulate the FPZ, Hillerborg *et al.* [*5*] used cohesive stress which is a function of crack opening. Hillerborg's approach can be applied to any structure, even if no notch or fictitious crack

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exists [*7*]. In this model, as stress is a function of crack opening, it reaches tensile strength at the tip of the crack, and reduces to zero at its critical opening (w_c). The amount of the area under the stress-crack opening curve is equal to energy release rate. This model, often referred to as cohesive zone model (CZM), was deployed to simulate the FPZ in normal size structures, using either nodal force release method or interface element with zero initial thickness technique [*8*].

So far, the method suggested by Hillerborg et al. [*5*] has been applied more widely due its practically, accuracy and cost effectiveness. To model the CZM, two types of interface elements were deployed. One of the most widely used interface elements is continuum cohesive zone model (CCZM) (e.g. Xie and Gersle [*9*]). An alternative interface element is discrete cohesive zone model (DCZM) which is very simple to implement. The DCZM results are satisfactory compared to CCZM, especially for pre-cracking phase when stiffness is selected to have a very large value [*10*]. The DCZM is based on the basic idea that cohesive zone behaves like a spring. This point of view suggests that instead of using a 2-D interface element along the crack path, a spring element should be utilized between interfacial node pairs. In the present investigation DCZM is applied because this method reduces computational time and is compatible with the finite element method.

From finite element point of view, stiffness of FPZ should be properly chosen. In practice, this damage zone has a different stiffness due to micro-cracking, bridging, branching that undertake use of energy in crack growth. So it is significant to use more accurate stiffness element to simulate the FPZ in finite element method. Also, when the FPZ length is fully extended and arrived at the maximum rate, stress-free length is appears in front of notch or macro-crack, behind FPZ,[*11*] which was not considered by previous research [*12*,*13*,*9*,*1*,*8*,*14*,*15*,*16*]

Also, to predict crack propagation, correct estimation of energy release rate is important. As it is known, energy release rate is the basic idea of nonlinear fracture mechanics for crack propagation that depends on many parameters such as external load and element stiffness.

On the other hand, steel bars have usually been applied to improve flexural capacity and to rest crack growth in concrete. So, modeling the steel bars and its effect on propagation of tension cracks in RC beam is necessary.

In the present study, interface element boundaries are utilized to simulate cohesive cracks. This model justifies the softening behavior of normal stress in the FPZ in concrete. Modified crack closure integral method with nonlinear spring is applied. A nonlinear spring element is used to derive forces in nodes due to normal stress in the FPZ. Strain energy release rate is obtained by energy approach while effect of steel bars on propagation of tension crack criterion is considered. Results for RC beam with initial notch are presented and comparisons between computed and experimental recent results are made.

2. Numerical Model

2.1. Interface Element

Modified crack closure integral method is applied to model CZM [*17*]. As mentioned before, the FPZ has a softening action due to the interlock of aggregates and micro-cracks. Thus, a nonlinear spring is proposed to place between interfacial node pairs (Figure 2). In this figure, the node pairs '1' and '2' have initially the same coordinates. Spring softening is set at the crack tip between the nodes '1' and '2'. Node '3' is a dummy node and it is only used to illustrate the variation in the crack form.

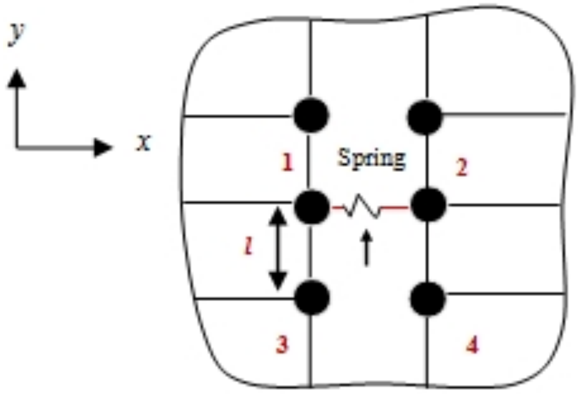


Fig. 2. Spring Interface Element Between Two Nodes

The local element stiffness matrix and the displacement vector related to nodes '1' and '2' are given by[*15*]:

$$\mathbf{K} = \begin{bmatrix} k_x & 0 & -k_x & 0 \\ 0 & k_y & 0 & -k_y \\ -k_x & 0 & k_x & 0 \\ 0 & -k_y & 0 & k_y \end{bmatrix}, \mathbf{u} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} \quad (1)$$

where k_x and k_y are the stiffness values corresponding to the local coordinates x and y, respectively. u_1 and u_2 are displacement components in x and y directions for node '1', u_3 and u_4 are displacement components in x and y directions for node '2', respectively. In this research, the value of the stiffness in x direction, k_x , is obtained based on the normal stress versus crack opening curve. Figure 3 illustrates the normal stress versus crack opening curve which can be explained by[*18*]:

$$\sigma = f_t \exp(-k\,w^{\lambda}) \quad (2)$$

where σ, f_t and w are normal stress, tensile strength of concrete, crack opening, respectively .The k , λ are constant. Thus stiffness in x direction, k_x , can be calculated as:

$$k_x = \frac{1}{k\lambda f_t \exp(-k\,w^{\lambda})} \quad (3)$$

Small displacements are assumed to model crack. Based on small displacements, the stiffness at ith iteration ($i=j+1$) in nonlinear solution can be written as:

$${k_x}^i = \frac{1}{k\lambda f_t \exp(-k\,{w^j}^{\lambda})} \quad (4)$$

Where the w^j is crack opening in jth iteration. For fast convergence on nonlinear solution, the initial stiffness is used as $w_c/30$ by small time step in nonlinear solution. The $w_c$is critical opening displacement[*19*].

Since only Mode I is considered and the crack path is known, the stiffness component in y direction can be calculated from shear modulus of concrete without any changes using previous study[*10*].

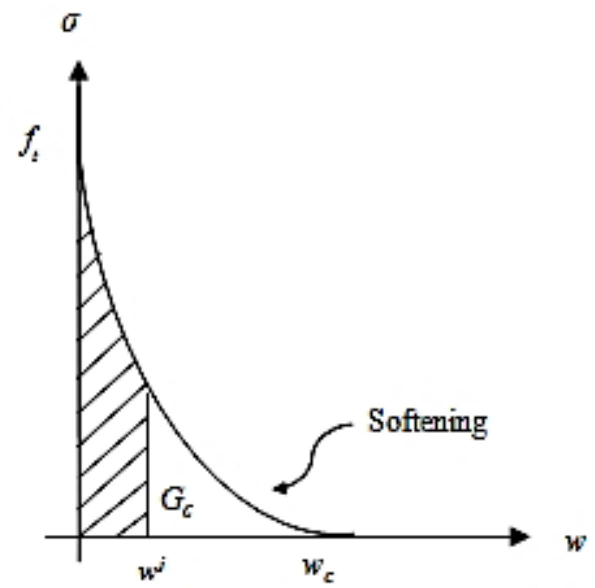


Fig. 3. Concrete σ –COD Curve

2.2. Energy Release Rate and Crack Propagation Criterion

The strain energy for mth element is the hatching area under σ –COD curve (Figure 3):

$$U_m = \int_0^{w^i} f_t \exp(-k\, w^{\lambda})\, d\, w \qquad (6)$$

that can be calculated by using Gaussian integration in finite element method. Strain energy release rate for Mode I, based on energy approach is:

$$G_I = \frac{\partial U}{\partial A} = \frac{U_m - U_{m-1}}{B\Delta} \qquad (7)$$

where A , B are crack surface area and thickness of the beam. The Δ is crack extension which in the present study is assumed[*3*]:

$$\Delta = 0.4\frac{n' E G_c}{{f_t}^2} \qquad (8)$$

where the n', E and G_c are number of elements whit changes their stiffness in behind the FPZ , modulus of elasticity of concrete and critical strain energy release rate ,respectively . At each step, more than one element may deal with crack extension and it propagates along other elements.

As for the steel bar to resist crack propagation, in the present study, energy dissipation rate based on energy approach, as surface force in [*6*], is obtained by:

$$R = \frac{\partial({u_9}^{\mathrm{m}} - {u_{11}}^{\mathrm{m}})F_x}{\partial A} \qquad (10)$$

where the F_x is nodal force due to existing steel bar in x direction. The ${u_9}^{\mathrm{m}}$ and ${u_{11}}^{\mathrm{m}}$ are displacements in x direction for node "5" and "6", respectively, in the vicinity of steel bar (Figure 4), for mth element due to propagation of the FPZ.The F_x due to propagation of the FPZ, for mth element is given by:

$$F_x = k_s({u_9}^m - {u_{11}}^m) \qquad (11)$$

where the k_s is elastic modulus of steel.

Substituting Eq. (11) into Eq. (10) and using finite deferencemethodyields

$$R = \frac{k_s[({u_9}^m - {u_{11}}^m)^2 - ({u_9}^{m-1} - {u_{11}}^{m-1})^2]}{(\frac{A_s}{B})\Delta} \qquad (12)$$

where the A_s is cross-section area of steel bars. Eq. (12) is used to estimate effect of steel bars as crack propagates in concrete. This relationship shows that initially when the FPZ length increases and crack opening mouth is small in concrete, effect of steel on preventing crack propagation is less. Finally, as the FPZ length reaches the constant value and crack opening mouth increases; steel bar role is to resist crack growth increases. So far, no model has presented a convincing equation to estimate effect of steel bar on crack propagation. The energy criterion of crack propagation will be:

$$G_I - R > G_{Ic} \qquad (13)$$

where G_c *is* critical strain energy release rate. It is noted that The R and G_I are not based on the size of the mesh.

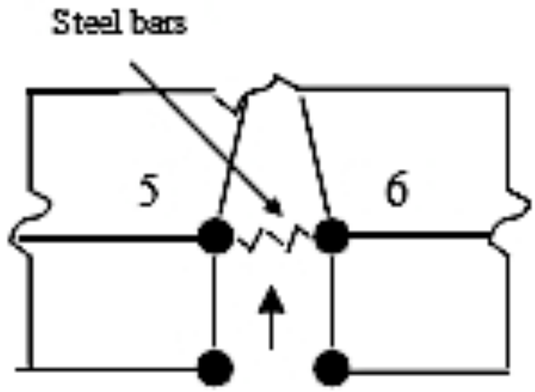


Fig. 4. Interface Element to model steel bars

2.3. Stress-Free Region

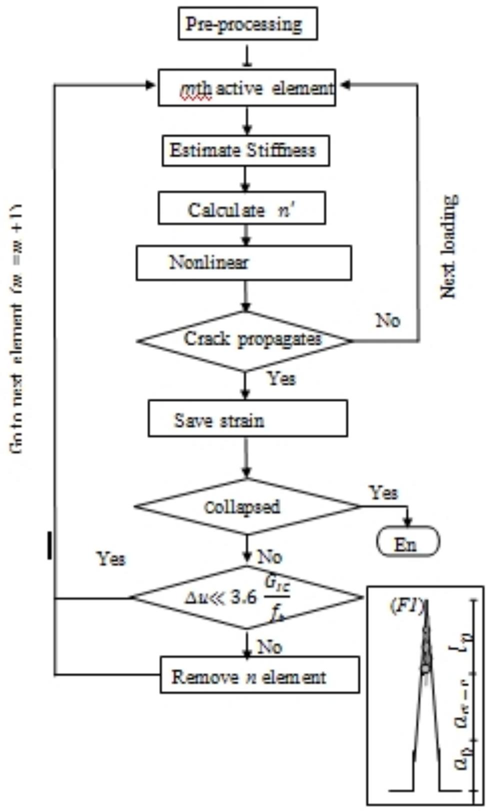
A more accurate explanation of propagation and crack formation must be considered in model such as stress-free region length. Wae *et al.* [*20*] shown as crack opening displacement reaches to 3.6 G_c/f_t, stress-free region appears in front of the notch tip while FPZ length increases linearly and fully develops. That means as crack length reaches about 0.91 times the ligament length, $h - a_0$, FPZ length increases linearly and fully develops.That is formulated in finite element methods by:

$$a_{\sigma=0} = n\, l \qquad (14)$$

where n and l are number of elements that have failed behind crack and length of mesh. When FPZ fully propagated, n element is set to zero behind crack as crack propagates.

2.4. Computer Implementation

FEAPpv program code is developed for analysis of 2-D plane stress in concrete [*21*]. Nonlinear spring is implemented for interface element in the User Subroutine FEAPpv Fortran programming while nonlinear dynamic relaxation method is used for interface element in the program[*9*]. Four-node isoparametric elements are used for bulk concrete as linear elastic. Two-node truss element is used to model steel bar with perfect plastic behaviour and the beam is not reinforced with stirrups. Bond-slip between longitudinal bars and concrete is modelled by Ingraffea *et al.* approach [*22*]. Crack does not propagate when the energy release rate, G_I is smaller than critical strain energy release rate (G_c). Algorithm showed the major step in the present numerical model to solve fracture in the beam.



Algorithm Flowchart of Fracture in *m* th element

3. Results and Discussion

The example is a reinforced concrete beam with simple supports (Fig. 5(a)) which experimental data were replicated by Prasad and Krishnamoorthy [*1*].

The geometry of the RC beam is 1220 mm length, 125 mm thickness. Material properties are 29270 MPa elastic modules, 0.18 Poisson ratios and 30.1 MPa compressive strength of concrete, 0.3 Poisson ratio, 100.48 mm^2 cross-section area and 395 MPa yield strength of steel. Tensile strength for concrete is 4.11 MPa, G_c=113 N/m and crack opening displacement critical is 0.15 mm. The k and λ are 1.01 and 0.063. The initial mesh is illustrated in Fig. 5(b). Load versus deflection at the mid-span of the beam in present study is compared with experimental result and previous model in Figure 7 [*1*]. Figure 6 shows the results are close to experimental data. It is seen that the stiffness of the beam in present study is slightly less than the previous model observation [*1*]. This difference may be acceptable as plastic deformation of steel and its effect crack propagation is considered in present model. It may be seen that the effect of steel bars on crack propagation has an important role on the crack behavior of the beam.

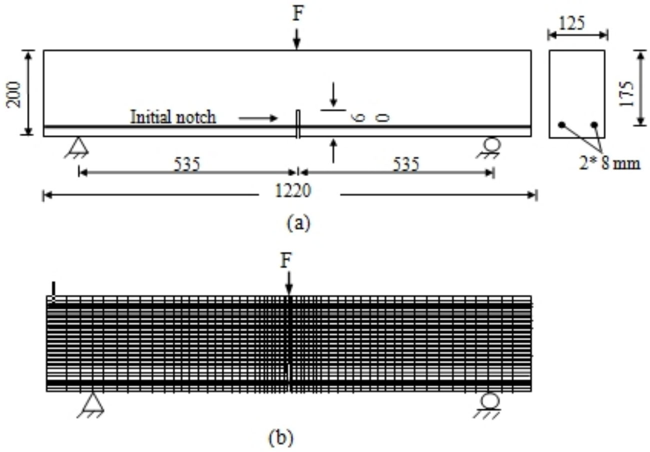


Fig. 5. (a) The notched RC beam (Unit: mm) (b) Initial mesh with 42 interface element

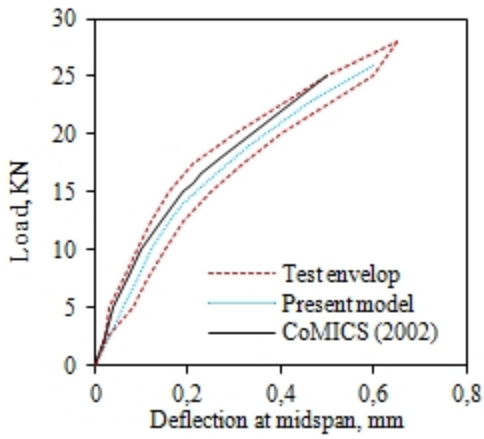


Fig. 6. Load-Deflection at the Mid-Span of the Model and Experimental [*1*]

Figure 7 shows crack patterns at load 26 KN in present study. Stress-free region length is 6.1 mm while FPZ length is 130.9 mm. The crack mouth opening is 0.481 mm while deflection at the mid-span is 0.534 mm at mentioned load. The FPZ propagation reaches almost three-fourth of the beam depth by fifth step of loading. At seventh step of loading the FPZ is fully propagates and stress-free region length appears. Initially the crack mouth opening increases gradually and then stays stabile due to effect of steel bars as load increases. At the final stage, crack mouth opening increases rapidly due to the bond-slip of the steel bars.

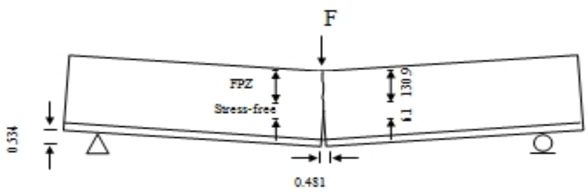


Fig. 7. Final Crack predicate scale=150 (Unit: mm)

Fig. 8 indicates load versus deflection at mid-span curve with three size meshes compared with experimental envelop[*1*]. Mesh (1) 102 interface elements, mesh (2) have 76 interface elements, and (c) have 42 interface elements. The aapproximate matching of the three curves demonstrates the independence of the model from mesh size and shows the model has fast convergence.

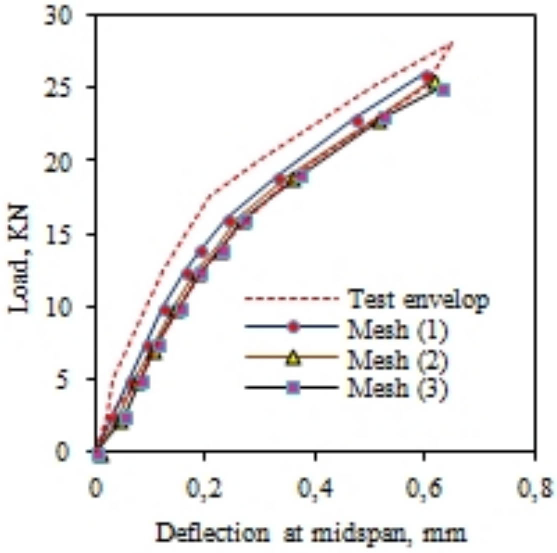


Fig. 8. Load-Deflection with three size meshes

Fig. 9 shows how the FPZ length is changed as the load increases by different cross-section area of steel bars.

It can be seen that the FPZ is increased linearly and then stay constant. It may be due to effect of steel bars or inherent behavior of FPZ [*23*]. As expected, when cross-section area increases, the load bearing capacity increases at the same FPZ length. It is also observed that effect of the increasing steel bars cross-section area is more at higher loads.

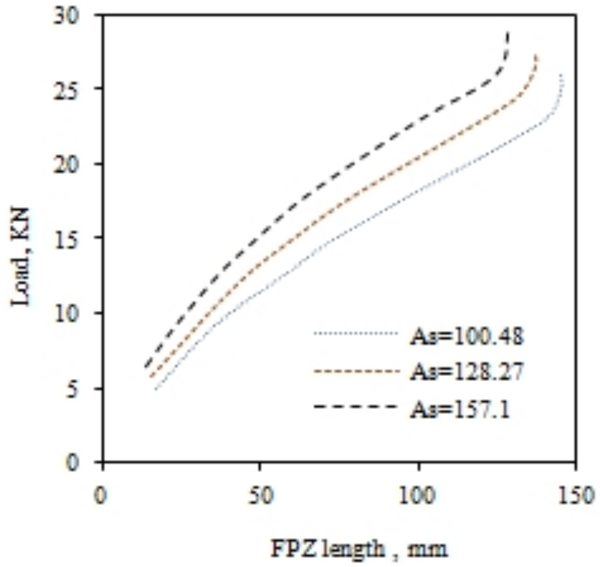


Fig. 9. Load versus the FPZ length with different cross-section area (mm^2) of steel bars

4. Conclusion

Present investigation proposes a simple approach to simulate cohesive crack in RC beam. In present study, interface element with nonlinear spring is used to simulate the CZM in beam to accurate explanation of Mode I crack propagation in RC beam. Modified crack closure integral method is implemented to simulate development of the FPZ and stress-free region length of fracture. An accurate element stiffness matrix is applied to derive forces in nodes due to normal stress in the FPZ. By using this model, energy release rate is calculated directly by a new method. The model is easy, accurate, efficient, with fast convergence and capable to model crack growth in RC beam. The model decreases computational time and complexity for discrete crack.

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