PRELAUNCH FORECASTING OF NEW CONSUMER DURABLES:
IDEAS ON A CONSUMER VALUE - PRIORITY MODEL

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ABSTRACT

This paper derives a consumer model that can be used to forecast sales of new durable products. We begin with the neoclassical utility maximization economic model and show that the standard complex dynamic optimization consumer model can be approximated with a very simple consumer model. The advantage of this model, which we call the value-priority model, is that measurement and estimation requires reasonable data. Furthermore, if one considers search and thinking costs, the simple model is likely to provide the consumer with the highest net utility.

We show that the model retains its structure for borrowing (saving), depreciation (appreciation), operating costs, price expectations, and trade-ins. We also suggest approximations for more complex situations.

We close by suggesting measurement and estimation procedures and future research.

1. INTRODUCTION

A number of models have been proposed for the evaluation of frequently purchased consumer products prior to test marketing (See Urban and Hauser (1980) for a summary). Evidence indicates that these models display good predictive capabilities. For example, see Urban and Katz (1982) for one systematic evaluation. However, no comparable procedures exist for pre-market forecasting of consumer durables. Our objective is to develop a model, measurement, and estimation system to predict the sales of a new durable consumer good before market introduction.

If such a system were available, management would find it valuable as a method to control risk. This is particularly true for durable products because the costs and risks of new product development are very high for durable products. In addition, the usual procedure of risk reduction by test marketing is not commonly conducted because the fixed production costs incurred to produce test marketing quantitites are almost the same as for a full national launch. A pre-marketing forecasting system is needed to avoid the risk of lost investment in production facilities and introductory marketing costs.

Building such a forecasting system poses many difficulties. There are many complex phenomena that affect the sales of a new consumer durable product. Among these phenomena are the diffusion of information, consumer beliefs about social norms, cost reductions and quality improvements made possible by production experience, the order and strength of competitive entry over the life cycle, potential saturation of the market, depreciation, interest rates, disposable income, general economic conditions, the availability of competitive products and alternative products that supply similar benefits, the consumer's current and expected portfolio decisions with

respect to all durable purchases, search costs, evaluation costs, and marketing actions by the innovating firm and its competitors.

Ideally, we would like to model all of these phenomena. Our long run research goal is to strive for such comprehensiveness. However, we begin by (1) selecting the phenomena that appear to dominate the durable purchase decision, (2) developing a practical measurement, modeling and forecasting system based on these phenomena, (3) testing the forecasting system, and (4) continuing with evolutionary development to improve the managerial utility of the forecasting system. Underlying this forecasting system we need a flexible, theory-based model of consumer response. The purpose of this paper is to outline initial model ideas on how consumers make choices among durable goods. Since the development of measurement instruments is proceeding in parallel with the development of theory, we fully expect modifications and improvements as our research progresses.

We begin in section 2 with the basic model to illustrate the key concepts underlying the development. Section 3 considers non-linear, non-durable utility, and section 4 extends the model to a multiple period decision environment and includes the effects of interest rates, borrowing, and depreciation. Our final extensions represent product aging, replacement, and complementarity. We briefly consider some measurement and estimation issues associated with the model and the paper closes with an indication of these future research tasks.

2. BASIC CONSUMER MODEL

Neoclassical Economic Theory

The basic economic paradigm is the axiom that a consumer maximizes utility subject to a constraint on his budget. Let x_j for $j = 1$ to J represent the amount of durable good j that the consumer purchases, let y be a composite for

all other goods (services and frequently purchased consumer goods), let p_j be the price of the jth durable good, and let K be the consumer's budget. Let $\underline{x}$ and $\underline{p}$ be the goods and price vectors respectively. Following convention, e.g., Rosen (1974), we measure y in dollars so that its price, p_y, automatically equals 1.0. If $U(x_1, x_2, x_3, \ldots, y)$ is the consumers utility function, then the consumer's decision problem is to select $x_1, x_2, x_3, \ldots$, and y to solve the following mathematical program:

$$\text{maximize } U(x_1, x_2, x_3, \ldots, y) \qquad \text{(MP1)}$$

$$\text{subject to } \Sigma_j \, p_j \, x_j + y \leq K$$

If the x_j are continuous, $U(\underline{x}, y)$ is increasing in each of its arguments but marginally decreasing, and second partials exist, then the solution to MP1 is obtained via Lagrange multipliers and is given by the condition:

$$\frac{\partial U (\underline{x}, y)}{\partial x_j} \Big/ p_j = \frac{\partial U (\underline{x}, y)}{\partial x_k} \Big/ p_k = \frac{\partial U (\underline{x}, y)}{\partial y} = \lambda \qquad (1)$$

where λ is a Lagrange multiplier.

The intuitive interpretation of the condition in equation (1) is quite simple; the consumer keeps buying good j as long as its "value", i.e., marginal utility obtained per dollar spent, exceeds some "cutoff", λ. The cutoff is implicit in the solution to MP1.

We present the traditional model because the concepts of "value" and "cutoff" will continue to be important throughout our generalizations.

Properties of Durable Goods

There are three generic properties of durable goods which affect our

model: (1) they are sufficiently expensive that their price, p_j, is a noticeable fraction of the consumer's budget, K, (2) they are discrete, e.g., it is hard to purchase a fractional automobile, and (3) they last for many purchase periods. Conditions (1) and (2) imply that the x_j cannot be treated as continuous variables in MP1. Condition 3 implies the need for a multiperiod model. We extend the model for the first two conditions and later in this paper consider condition 3. Based on conditions (1) and (2), the conditional utilities of the durable goods, e.g., the utility of x_j holding all other purchases fixed, are given by figure 1a. The composite good, y, has a conditional utility function as in figure 1b. u_{j1} is the utility obtained by purchasing 1 unit of good j, u_{j2} is the utility obtained by purchasing a second unit of good j, etc. $u_y(y)$ is the utility obtained by purchasing y units of the composite good.

In general, $u_{j\ell}$ and $u_y(y)$ depend upon the amount of all other goods purchased. However, if we assume that preference tradeoffs among any two goods do not depend upon the levels of a third good, then $U(\underline{x}, y)$ is separable and $u_{j\ell}$, $u_y(y)$ are not dependent upon all other goods. See Blackorby, Primont, and Russell (1975, p. 26). Furthermore, we can write $U(\underline{x}, y)$ as the sum of the conditional utility functions, i.e.,

$$U(\underline{x}, y) = \Sigma_j u_j(x_j) + u_y(y) \qquad (2)$$

where $u_j(x_j) = u_{j1}$ if $x_j = 1$, $u_j(x_j) = u_{j2}$ if $x_j = 2$, etc. We note that equation 2 is a standard measurement model used in marketing. See Green and Wind (1975), Pekelman and Sen (1979), and Srinivasan and Shocker (1973).
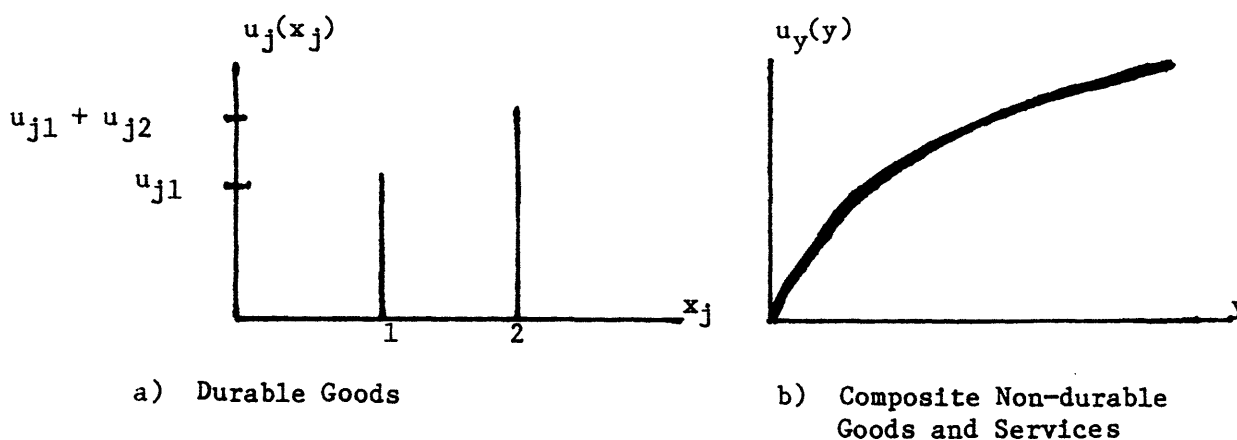
a)   Durable Goods

b)   Composite Non-durable
     Goods and Services

Figure 1:   Conditional Utilities, i.e., Utility of x_j (or y)
            Holding All Other Purchases Fixed

Separability does represent a first-order approximation which may need to
be relaxed later.   For example, the purchase of an automobile may be dependent
upon whether or not the consumer builds a garage.   However, separability is
more general than it might appear.   While preferences may be independent,
purchases will definitely not be independent because of the budget
constraint.   Thus, the purchase of an automobile may depend upon or affect the
purchase of a home video center because such purchases represent alternative
uses for the consumer's dollar.   Furthermore, since utility is a monotonic
measure, equation (2) represents a general class of utility functions
including multiplicative functions such as those used by Johnson (1974) and
Keeney (1974).   For example, log $U(\underline{x}, y)$ will be additive if $U(\underline{x}, y)$ is
multiplicative.   Log $U(\underline{x}, y)$ is permissible since it does not change any
preference ordering.

Based on equation 2 and conditions (1) and (2), the neoclassical
mathematical program, MP1, becomes the durable purchase mathematical program,
MP2.

$$\max \quad \Sigma_j(\Sigma_\ell \, u_{j\ell} \, \delta_{j\ell}) + u_y(y)$$

$$\text{s.t.} \quad \Sigma_j p_j(\Sigma_\ell \delta_{j\ell}) + y \leq K \qquad\qquad (MP2)$$

$$\delta_{j\ell} = 0,1$$

where $\delta_{j\ell} = 1.0$ if and only if the consumer purchases ℓ or more units of good j.

If $u_y(y)$ were not included, MP2 would be an integer linear program called a "knapsack program". Its solution is non-trivial although efficient computer algorithms exist. The presence of $u_y(y)$ modifies the solution technique but does not necessarily simplify the solution technique. Thus, even with separable utility, the neo-classical solution requires that the consumer solve a non-trivial mathematical programming problem in order to choose his portfolio of durable goods. Furthermore, because of the integer nature of the problem, we can construct examples, where $\delta_{j\ell} = 1$ even if the marginal utility, $u_{j\ell}$, is small and the price, p_j, large.

Intuitively, we do not expect the consumer to solve a non-trivial mathematical program in his head for every durable purchase, and empirically we observe that consumers do not obtain information on all durables before making a purchase decision. We would like to reconcile MP2 with these observations.

The Concept of Value - An Approximation to Optimality

There is a cost to thinking and there is a cost to search. In a recent article, Shugan (1980) has shown that even in the presence of perfect information, it is rational for a consumer to simplify his decision rule if there is a non-zero thinking cost. Since Shugan's theory is operant for even

a single product category, we expect it to be operant for MP2 which involves all durable purchases. Thus, if the consumer can approximate optimality with an extremely simple decision algorithm, then it is reasonable to posit that the simpler algorithm may be the empirically observed decision rule.

By the same token, MP2 requires maximal search cost. Because of the integer nature of MP2, the consumer must know most if not all of the $u_{j\ell}$'s and p_j's before he is sure he has identified his optimal portfolio. For example, two products, say products 1 and 2, may have the largest and second largest utility per dollar spent, but they may not be included in the optimal solution to MP2. Consider table 1:

$$u_{11} = 10.0 \qquad p_1 = .51 \qquad u_{11}/p_1 = 19.61$$
$$u_{21} = 9.8 \qquad p_2 = .50 \qquad u_{21}/p_2 = 19.60$$
$$u_{31} = 6.8 \qquad p_3 = .35 \qquad u_{31}/p_3 = 19.43$$
$$u_{41} = 6.6 \qquad p_4 = .34 \qquad u_{41}/p_4 = 19.41$$
$$u_{51} = 6.0 \qquad p_5 = .31 \qquad u_{51}/p_5 = 19.35$$

$$K = 1.0 \qquad u_y(y) = 18y$$

TABLE 1: EXAMPLE ONE

The optimal solution to this problem is for the consumer to set $y = 0$ and choose products 3, 4, and 5 giving an optimal utility of 19.4. In general, the consumer must know the exact values of all of the $u_{j\ell}$'s and p_j's before he can be sure he has identified the optimal solution. Considering the search cost for automobiles alone, such knowledge on the part of most consumers is an unrealistic requirement.

Suppose the consumer is willing to rank order the products in terms of value, that is, in terms of marginal utility per dollar, $u_{j\ell}/p_j$. Suppose further that $u_y(y) = \lambda y$. This represents the utility of non-durables as a linear function with the slope equal to the marginal utility

per dollar (λ) obtained by the last dollar available to be spent within the budget constraint. (In the next section we relax this assumption). Now consider the following algorithm, A1:

1. Choose the highest value product affordable within the budget if its value, u_{11}/p_1, exceeds the cutoff, λ.

2. Continue choosing products in order of value as long as their value exceeds the cutoff. Skip any unaffordable products, that is, any products whose price is greater than the remaining budget.

3. Spend any leftover budget on non-durables, y.

For example, if we apply this algorithm to the above problem, we obtain the solution of setting y = 0.14 and choosing products 1 and 3 which gives a total utility of 19.32, 4/10's of 1 percent less.[1] However, by using A1 the consumer greatly simplifies his decision process and avoids full evaluation (and hence search cost) for products 4 and 5. He only needs to know the relative value ranking and the fact that the price of product 4 exceeds .14. If search cost and thinking cost are included, it is highly likely that the net utility from A1 exceeds the net utility from the optimal solution to MP2.

Clearly, one can construct arbitrary examples where the utility of A1 is significantly less than the utility obtained from MP2, but in most realistic cases we expect that the utility obtained from A1 will be within a few percent of that obtained from MP2. We based this hypothesis on the following arguments.

First, if the durable market is reasonably efficient in the economic sense, then the <u>decrease</u> in $u_{j\ell}/p_j$ as the consumer buys down the value ranking should be a small fraction of value. (A walk through a department

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[1] For the sake of exposition we treat utility as a cardinal measure. The same qualitative ideas apply for ordinal utility.

store indicates the many durables available as options in the $u_{j\ell}/p_j$ array.) Furthermore, durables that are significant fractions of a budget (housing, automobiles) are available at a wide range of prices, say $4000 for a new Chevette to over $30,000 for a new Mercedes. Finally, an upper bound on MP2 is the linear programming (LP) solution in which we relax the integer constraint. A1 is the exact LP solution except for the skipping of products and the allocation of the remainder of the budget to y (in A1) instead of to the lowest value product in the chosen portfolio (in the LP solution). Other goods, y, may or may not be in the chosen portfolio. The skipping of products in A1 should not have a large impact based on the efficiency arguments. The allocation from the least value within the portfolio (in the LP) to y (in A1) should not have large impact (1) if the least value product represents a small fraction of the budget as in most durables other than automobiles and housing or (2) if similar value, lower cost options are available as in the case of automobiles and housing. Based on these arguments we select A1 as the basic consumer model that we will use to approximate consumer behavior in our forecasting methodology. The reasonableness of this approximation is an empirical question subject to future testing.

3. NON-LINEAR NON-DURABLE UTILITY

In the preceeding section we assumed $u_y(y) = \lambda y$ because it simplified exposition, but we do not need this assumption. Suppose we approximate $u_y(y)$ with the piecewise linear curve in figure 2 where the λ_k are the slopes of the line segments. Since $u_y(y)$ is marginally decreasing $\lambda_k > \lambda_{k+1}$ for all k.
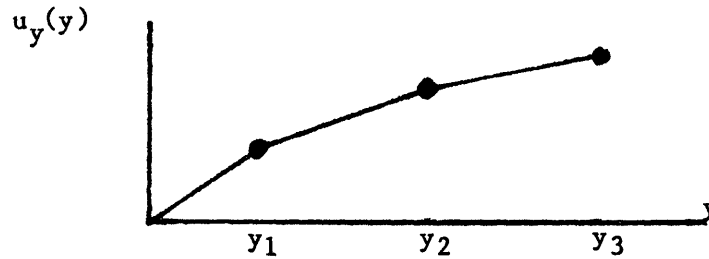
$u_y(y)$

Figure 2:   Piecewise Linear Approximation to the Conditional
             Utility of the Composite Good

The consumer's decision algorithm is similar to A1 except that non-

durables are also bought in blocks. The modified algorithm, which we call

A1', is given as follows:

1. Choose the highest value product affordable within the budget if
    its value exceeds λ_1.

2. Continue choosing products in order of value as long as their
    value exceeds λ_1; skip unaffordable products.

3. Spend y_1 dollars on non-durables if possible within the
    budget, otherwise spend only the remaining budget on
    non-durables.

4. If the budget is not exhausted, return to step 2, replace λ_k
    by λ_{k+1}, and continue.

Algorithm A1' retains all of the advantages of A1 despite the fact that the

operant cutoff, λ_c, is not known a priori. We are not interested

managerially in the total dollars allocated to non-durables, only its

implications for the choice of durables. Thus, if we can measure λ_c, we

can ignore λ_k for k < c.

For purposes of forecasting new product sales we must recognize that

λ_c may increase. If the new product has high value, it will be chosen in

the new market and the consumer will not buy as far down the buying priority.

He may not reach λ_c. The index of the new cutoff, n, will be less than or

equal to c, hence $\lambda_n \geq \lambda_c$. The magnitude increase in the cutoff is

a forecasting issue, but a priori we expect $\lambda_n - \lambda_c$ to be small due to efficiency arguments. We will examine this assumption with future empirical testing.

If $u_y(y)$ is continuous (figure 1b) rather than piecewise linear (figure 2), the basic algorithm is modified, but only slightly. The priority buying order of durables remains, but within this priority buying order the consumer first allocates dollars to non-durables (up to y_1 such that $\partial u_y(y_1)/\partial y = u_{11}/p_1$), then purchases the highest value durable, then allocates dollars to non-durables (up to y_2 such that $\partial u_y(y_2)/\partial y = u_{21}/p_2$), then purchases the second highest value durable within the budget, etc. He continues alternating between durables and marginal increases in non-durables until the budget is exhausted. Again we observe a budget cutoff, λ_c, and again $\lambda_n \geq \lambda_c$.

Thus, the basic priority buying model remains when the non-durable utility is marginally decreasing. For the remainder of this paper we assume $u_y(y)$ is as given in figure 1b. We call the consumer's buying algorithm the value-priority algorithm, and we will retain the notation λ_c and λ_n.

An Example

Suppose a consumer has already made his housing decision and that his remaining disposable income is $10,000 for the year. Suppose that he has examined potential durable purchases, identified the following items as relevant for his situation and rank ordered them in terms of value, $u_{j\ell}/p_j$.

(Prices are rounded for expositional purposes):

BEDROOM SET ($1500)
DISHWASHER ($ 500)
STORM WINDOWS ($2000)
VIDEO CASSETTE RECORDER ($1000)

——————————————————— BUDGET CUTOFF

SEWING MACHINE ($500)
LANDSCAPING ($500)
HOME SECURITY SYSTEM ($1000)
VIDEO DISK ($500)
NEW KITCHEN FLOOR ($500)
MICROWAVE OVEN ($500)

In following the value-priority algorithm the consumer would purchase the first four items on his list that are above the budget cutoff. These are a bedroom set, a dishwasher, storm windows, and a video cassette recorder. The specific brand would be the highest value affordable brand in the category. The consumer does not allocate all of his budget to durables because a significant fraction, in this case $5000, is allocated to high-value, non-durable goods and services such as food, clothing, and household expenses. In our new product forecasting problem we are not interested in the details of that allocation, only its effect on the cutoff. A new durable, perhaps a water purifying system, would be purchased only if its value exceeds the cutoff. However, if it were purchased, it might displace the last item in the portfolio, in this case the video cassette recorder.

To model the choice of a new automobile, whose average cost of approximately $10,000 may exceed the budget cutoff, we must extend our model to include borrowing.

4. MULTIPERIOD MODEL INCLUDING BORROWING, INTEREST, AND DEPRECIATION

An automobile is an expensive item. Most consumers do not pay cash for a new car but rather borrow against future expenditures to do so. Any realistic model must include this phenomena.

Review the example in table 1. The optimal solution to MP2 was to set y=0

and purchase products 3, 4, and 5. This gave an optimal utility of 19.4.

Suppose we increase the budget by 1% to K = 1.01, then the optimal solution is

to purchase products 1 and 2 giving an optimal utility of 19.8. In other

words, if the consumer could borrow 1% of his budget from future expenditures,

he could increase his utility by 2.1%. The contribution of the "borrowed"

budget is even more significant if we assume that repayment of the loan is by

reducing expenditures for y where the marginal opportunity loss is $u_y(y) =$

$18(.01) = .18$, which is only 0.9% of his utility. Thus he gets an increase of

2.1% in this period at a cost of 0.9% next period. Finally, note that the

application of the value-priority algorithm with a relaxation of the budget

would give products 1 and 2 as the optimal solution.

Borrowing (Saving) and Interest Payments

Suppose that the consumer has an expected budget, K_t, for each time

period t. If that budget is not sufficient, then he can borrow b_t dollars

in period t to enable him to purchase more durables and non-durables. If b_t

is negative, it represents savings.

However, the decision to borrow or save affects not only period t, but all

subsequent periods in which the loan must be repaid (or in which savings are

available for spending). Let r be the interest rate and let D_t be the debt

at time t, then we can relate the consumer's debt to borrowing (savings) by

the following recursive equation:

$$D_t = D_{t-1} (1 + r) + b_t \qquad\qquad (3)$$

Note that debt will decrease when b_t is sufficiently negative to offset the

interest payment, rD_{t-1}, and pay back part of the principal. Negative debt

represents savings.[2]

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If there were no penalty for debt, the consumer would simply let D_t become arbitrarily large. Thus, we constrain the consumer to have no net debt over some planning horizon, τ. In other words, we add the following constraint to the consumer's decision problem:

$$D_\tau = 0 \qquad (4)$$

Recognizing that equation 4 is a boundary condition for the recursive relation in equation 3, we can combine equations 3 and 4 into the following constraint:

$$D_\tau = \sum_{s=0}^{\tau-1} (1 + r)^s b_{\tau-s} = 0 \qquad (5)$$

We can now construct the consumer's decision problem by recognizing that once he purchases a durable, he receives utility from it for all subsequent decision periods. His decision problem in the absense of depreciation is then:

$$\max \sum_{t=1}^{t} \Sigma_j \Sigma_\ell [(\tau-t)\, u_{j\ell} \delta_{j\ell t}] + \sum_{t=1}^{\tau} u_y(y_t) \qquad \text{(MP3)}$$

s.t. $\quad \Sigma_j p_j (\Sigma_\ell \delta_{j\ell t}) + y_t - b_t \leq K_t$

$$\sum_{s=0}^{\tau-1} (1+r)^s b_{\tau-s} = 0$$

$$\delta_{j\ell t} = 0,1$$

$$\delta_{j\ell T} = 0 \text{ if } \delta_{j\ell t} = 1 \text{ and } T > t$$

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[2] For tractability we have assumed that the savings interest rate is the same as the borrowing interest rate and that both are stationary. These assumptions may need to be relaxed at a later time.

where we have added t subscripts to the purchase decision, $\delta_{j\ell t}$ and y_t. $u_{j\ell}$ is the utility _per period_ for ℓ units of item j. The multiplier, $(\tau - t)$, in the objective function incorporates the fact that a durable can be enjoyed for all periods following its purchase. Non-durables, y_t, are used up in the period in which they are purchased. The last constraint is definitional to ensure that the consumer "remembers" his previous purchases.

Rather than analyzing MP3 in detail, we simply state the result that as a model of consumer decision making it tends to overstate borrowing. Intuitively the mathematical result reflects the assumption of no depreciation which in turn leads the consumer to purchase a durable sooner than he would if durables depreciated in value.

Depreciation (and Appreciation)

With few exceptions, an automobile purchased in 1982 gives more utility to the consumer in 1982 than it does in 1992. We model depreciation as an exponential decay, i.e., if the consumer gets $u_{j\ell}$ utility from a purchase at time t, he gets $\gamma_j u_{j\ell}$ utility at time $t + 1$ and $\gamma_j^q u_{j\ell}$ utility at time $t + q$. Over the planning horizon, the total utility a consumer gets from a purchase made at time t is given by:

$$\text{TOTAL UTILITY} = \sum_{q=0}^{\tau-t} \gamma_j^q \, u_{j\ell} \qquad (6)$$

If $\gamma_j < 1$, then durable j depreciates over time. If $\gamma_j > 1$, then durable j appreciates over time. (In most cases, except maybe housing, we would expect γ_j to be less than $1 + r$).

Incorporating equation 6 into the consumers' problem yields the following mixed-integer mathematical program as our model of the consumer's decision making problem:

$$\max \sum_{t=1}^{\tau} \Sigma_j \Sigma_\ell \, [\sum_{q=0}^{\tau-t} \gamma_j^q \, u_{j\ell} \delta_{j\ell t}] + \sum_{t=1}^{\tau} u_y \, (y_t) \qquad \text{(MP4)}$$

$$\text{s.t.} \quad \Sigma_j p_j \, (\Sigma_\ell \delta_{j\ell t}) + y_t - b_t \leq K_t$$

$$\sum_{s=0}^{\tau-1} (1 + r)^s b_{\tau-s} = 0$$

$$\delta_{j\ell t} = 0,1; \quad \delta_{j\ell T} = 0 \text{ if } \delta_{j\ell t} = 1 \text{ and } T > t$$

We next show that MP4 can be approximated with only a slight modification in the value-priority algorithm.

Analyses

Suppose that the $\delta_{j\ell t}$'s were unconstrained continuous variables, then we could solve MP4 with Lagrange multipliers. Let λ_t be the multiplier for the budget constraint in time period t and let μ be the multiplier for the debt payback constraint, $D_\tau = 0$. Taking derivatives we obtain the following first-order conditions for the optimization of MP4:

$$\delta_{j\ell t}: \, u_{j\ell} [\sum_{q=0}^{\tau-t} \gamma_j^q] - \lambda_t \, p_j = 0 \qquad \text{for all } j, \ell, t$$

$$y_t \quad : \partial u_y / \partial y_t - \lambda_t = 0 \qquad \text{for all } t \qquad (7)$$

$$b_t \quad : \lambda_t - (1 + r)^{\tau-t} \mu = 0 \qquad \text{for all } t.$$

Algebraic solution of conditions (7) yield:

$$\frac{u_{j\ell}}{P_j} \left[\sum_{q=0}^{\tau-t} \gamma_j^q / (1 + r)^{\tau-t} \right] = \mu \qquad (8a)$$

$$\frac{\partial u_y(y_t)}{\partial y_t} = \mu(1 + r)^{\tau-t} = \lambda_t \qquad (8b)$$

The final result, conditions 8, is quite simple and exactly parallels conditions 1 of the neoclassical economic model.

It should not surprise the reader, therefore, if we follow the arguments of sections 2 and 3, modifying them for conditions 8 rather than conditions 1, we get a value-priority algorithm in which the quantity on the left-hand side of condition 8a is used rank durables rather than simply $u_{j\ell}/p_j$.[3]

(We can obtain the same result without resorting to a continuous model. The budget constraints can be incorporated into the debt payback constraint yielding a knapsack problem when $y_t = 0$. We follow the same arguments with an LP bound on the integer solution to obtain a modified value-priority algorithm.)

The value-priority algorithm is now used to rank all durables available during the planning horizon.

First, the adjusted value priority from the left side of equation 8a is calculated for each t. If a product is purchased at all it is purchased in the period when this value is greatest. The adjusted value and borrowing constraint will determine if it is purchased at all. In the first period products with their maximum adjusted value are purchased and borrowing is determined. The requirement to pay back combined with interest on borrowing will reduce subsequent available funds. In each period the products whose adjusted maximum is greatest will be purchased given that all borrowing can be paid back over the planning period. If it cannot be paid back, products with low maximum adjusted

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[3] One obvious modification for non-stationary interest rates is to substitute r_t for r in condition 8a. However, this is an approximation, not an exact solution.

values are elminated until this constraint is met.

Consider, the illustrative two period example ($\tau = 2$) with an interest rate (r) of .25 and $K_t = 1.0$. See table 2.

| | | | | Adjusted Values | |
| | | | | Period t = 1 | Period t = 2 |
Product	u_{j1}	p_j	γ_j	$(u_{j1}/p_j)(1.0+\gamma_j)/(1+.25)$	(u_{j1}/p_j)
1	10.0	.51	.8	28.23	19.61
2	9.8	.50	.2	18.82	19.60
3	6.8	.35	.8	27.98	19.43
4	6.6	.34	.2	18.64	19.41
5	6.0	.31	.6	24.77	19.35
$y_1(\leq.05)$	25	1		25	25
$y_2(>.05)$	18	1		18	18

TABLE 2: EXAMPLE TWO

The first column shows the utility (taken from table 1) with the utility for nondurables shown as a piecewise function of $u_y(y) = 25y$ for y less than or equal to .05 and $u_y(y) = 18y$ for y greater than .05. The prices (p_j) are shown in column two. One minus the depreciation rate or the retention rate (γ_j) is shown in column three. The adjusted values are calculated by the left side of equation 8a. Products 1, 3, and 5 are a maximum in period one. If they are purchased along with .05 of the nondurables, the expenditure is 1.28 (.51 + .35 + .37 + .05) this implies borrowing of .28. In period two this must be paid back plus interest. The period two budget is reduced by .35, i.e., (.28(1 + .25)). That is, .65 is available for expenditure (1 - .35). In period two, product 2 would be purchased at a price of .50 along with .15 of nondurables. Product 4 would not be purchased.

Extensions (Operating Cost, Price Expectation, Quality Improvement Expectations)

Operating costs of durables can be considered in MP4 by modifying the

budget constraint. Let c_{jn} be the cost of operating and maintaining durable j, n periods after purchase. The total outlay for cost is the price plus the operating costs. The budget must now be less than the cost of current period (t) acquisitions and the operating costs of previously purchased durables. The constraint is:

$$\Sigma p_j (\Sigma_\ell \delta_{j\ell t}) + \sum_{n=1}^{\tau-t} c_{jn} \, \delta'_{j\ell t} + y_t - b_t \leq K_j$$

where

$$\delta'_{j\ell T} = 1 \text{ if } \delta_{j\ell t} = 1 \text{ and } T > t$$

This constraint modifies the value-priority algorithm by replacing p_j in the value ratio with $p_j + \sum_{n=1}^{\tau-t} c_{jn}$. With this extension we can represent phenomena such as a new car with high operating costs not being purchased before a projection TV system with low operating costs even though their utilities and prices are the same $(u_{j\ell}, p_j)$.

Price expectations can be included in MP4 by adding a time subscript to prices (p_{jt}). This modification affects the adjusted value-priority (equation 8a). In computing the maximum adjusted value (over t), we replace p_j by p_{jt}. This would represent the situation in which a consumer expects major price reductions for video disk machines and hence delays purchase. The same phenomena may be important for auto rebates. In MP4 the lower p_{jt} could lead to a maximum value priority in a later period than the case of constant prices. Quality improvement expectations would be handled in an analogous way by replacing $u_{j\ell}$ by $u_{j\ell t}$.

Similarly interest rates (r) can be subscripted by time to represent expected future conditions. The model in MP4 and the associated value-priority algorithms capture many important phenomena, but they all assume that products are independent. In the next section we deal with these issues.

5. INTERRELATIONSHIPS AMONG PRODUCTS

There are at least four ways in which interrelationships among products can affect the consumer model:

(1) Budget substitution, e.g., a more expensive automobile is not purchased because the consumer bought a new projection television set and a video cassette recorder.

(2) Decreasing marginal returns, e.g., a consumer is less willing to sacrifice a home entertainment system for a second automobile than he was for the first automobile in the family.

(3) Replacement, e.g., as the 1975 automobile gets older (depreciates) the consumer is more willing to buy a new automobile. I.e., the utility of trading in the old car and buying a new car is greater in 1982 than it was in 1981.

(4) Complementarity or substitutability. For example, complementarity might imply that the marginal value of a cartridge electronic game increases if the consumer now owns a projection television set. Substitutability might imply that the value of a video disk machine goes down if a videotape machine has been purchased.

Our consumer model handles budget substitutability by the spending constraint (K_t) and it explicitly models decreasing marginal returns with the discrete utility functions in figure 1a. This section discusses how the other effects might be incorporated into the model.

Replacement

If buying a durable is associated with replacing an older good that has some value, the price for the new good should be reduced and the incremental utility calculated. For example, a trade-in automobile reduces the cost of obtaining the new automobile. The new automobile generates an incremental utility equal to the difference between the utility of the new automobile and the residual utility of the old automobile. The depreciation relationships described above provide the key to modeling depreciation. The incremental price would be p_J less the trade-in value of the old car (J). Assuming the market values depreciate at the same rate as the consumer's utility the value is $p_J \gamma_J^n$ where n = the number of periods since purchase of auto J.

The incremental utility is the new auto utility $(\sum_{q=0}^{\tau-t} \gamma_j u_j^q)$ less the old

car residual utility $(\sum_{q=0}^{\tau-t} \gamma_J [u_J^q \gamma_J^n])$. If $\gamma_j = \gamma_J$ this simplifies

further.

Complementarity or Substitutability

If two (or more) products are strongly interrelated (complements or substitutes)

then the utility of one will be a function of whether or not the other is

purchased. Notationally, $u_{k\ell}$ is a function of δ_{j1} (or δ_{j1t}) if product

k is interrelated to product j. Thus, to model complementarity or substitutability,

we must replace $u_{k\ell}$ by $u_{k\ell|j}$ where

$$u_{k\ell|j} = \begin{cases} u_{k\ell} & \text{if } \delta_{j1} = 0 \\ u'_{k\ell j} & \text{if } \delta_{j1} = 1 \end{cases} \qquad (9)$$

where $u'_{k\ell j} > u_{k\ell}$ if the products are complements and $u'_{k\ell j} < u_{k\ell}$ if

they are substitutes. We can assume that the consumer continues to use the

value-priority algorithm in the face of complementarity or substitutability.

He begins by using the algorithm myopically based on the $u_{k\ell}$, but once a

product is purchased, say product j, he updates all subsequent utilities,

(i.e., $u_{k\ell|j}$) and reorders the remaining products according to

value-priority. For example, if he finds a projection television to be a high

value item and purchases it, he updates his assessed utility for a video

cassette recorder (VCR) and for a cartridge video game once he purchases the

television. This may increase the position of the VCR in the buying priority

rank order.

This myopic algorithm is clearly a strong assumption which will need be

tested if it is used for forecasting. It is further complicated if we wish to

model interdependencies that are of orders greater than pairwise. If

measurement is to feasible and the value-priority algorithm is to be a

reasonable approximation to behavior, the number of interdependent product

groups must be small.

SUMMARY OF CONSUMER MODEL

This completes our discussion of the value-priority algorithm. We feel that the model represents a reasonable compromise between the neoclassical economic model (optimization of utility subject to a budget constraint) and the marketing concepts of thinking cost and search cost.

The basic concept is simple. The consumer evaluates each product in terms of overall value. Value is shown to be utility divided by price adjusted for any borrowing costs and for depreciation over the product's lifetime. The consumer chooses durable goods in a priority order determined by adjusted value in each period. He continues until he reaches a value cutoff. The value cutoff is a function of the parameters of the overall decision problem (including non-durables) and decreases with time.

We have argued that the value-priority algorithm approximates the math programming solution. If one includes thinking and search costs, is likely to be a very good solution in terms of net utility.

Non-linear utility for non-durable products and optimization over a multiperiod time horizon are considered. Budget substitutability and decreasing marginal returns are explicitly incorporated and replacement phenomena can be approximated quite well. One major phenomena that is not fully modeled is product complementarity and substitutability, although we do propose a first-order approximation.

Since we are still developing measurement procedures for the consumer model we can only speculate on their feasibility. The next section briefly outlines some initial thoughts on measurement and estimation. We then close with an outline of future research.

6. ESTIMATION OF MODEL PARAMETERS

We have identified at least three ways to potentially estimate the basic parameters of the model. If we assume that the consumer can make a judgment of the likelihood, $L_{j\ell}$, that he will purchase the ℓth unit of product j,

then a logit model can be used to estimate utilities and the value cutoff. If we assume that he can only specify whether or not he will purchase product j, then a linear program can be formulated to estimate the utilities and the value cutoff. Finally, intensity measures of preference can be used to directly infer utilities. We discuss each in turn.

For simplicity of exposition, for the remainder of this section we deal with the single period model. Extensions to the multiperiod model present no conceptual difficulties, but are notationally cumbersome. Furthermore, we present the basic ideas for logit and linear programming estimation recognizing that there are extensions which may be necessary for empirical applications. For example, see Currim and Sarin (1981) and Srinivasan (1981).

<u>Logit Estimation</u>

According to the basic value-priority algorithm the consumer will purchase the ℓth unit of product j if its value, $u_{j\ell}/p_j$, is above the cutoff, λ. Mathematically,

$$\text{Prob [purchase product j]} = \text{Prob } [u_{j\ell}/p_j \geq \lambda]$$
$$= \text{Prob } [u_{j\ell} - p_j\lambda \geq 0] \tag{10}$$

If we assume that net measurement error can be modeled as a Weibull random variable, then equation 10 can be transformed into a standard logit model. (For example, see derivation in McFadden, 1973.) In other words, we have:

$$\hat{L}_\ell = \frac{\exp [\beta(u_{j\ell} - \lambda p_j)]}{1 + \exp[\beta(u_{j\ell} - \lambda p_j)]} \tag{11}$$

where $\hat{L}_{j\ell}$ is an estimate of $L_{j\ell}$ and β is a fitting parameter to be determined. If, as in our case, $L_{j\ell}$ is observed and $u_{j\ell}$, λ are to be estimated we can use general maximum likelihood techniques (McFadden 1973) or use the following regression equation as an approximation:

$$\log[L_{j\ell}/(1-L_{j\ell})] = \beta(u_{j\ell} - \lambda p_j) + \text{error} \tag{12}$$

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We estimate β, $u_{j\ell}$ and λ by (1) specifying β, (2) obtaining

conditional estimates, $\hat{u}_{j\ell}(\beta)$ and $\hat{\lambda}(\beta)$, of the utilities and cutoff, and

(3) searching on β.

Unfortunately, equation 12 is overspecified having only $J \times L$ observations

for $J \times L + 2$ parameters. We obtain sufficient degrees of freedom by (1)

obtaining representative utilities and possibly cutoffs, i.e., running the

logit model across consumers, or (2) having the consumer specify $L_{j\ell}$ for

more than one economic or environmental situation, s. In the latter case we

have $S \times J \times L$ observations for only $J \times L + S + 1$ parameters because only λ

varies by situation, s. The appropriate choice of data collection is an

empirically issue to be resolved through experience. We must also face the

empirical issue of whether or not there is enough variation across situations

in consumers' intent to enable us to obtain reasonable estimates of the

unknown parameters.

Linear Programming Estimation

Consider equation 10 and suppose that we observe only whether or not the

consumer has previously purchased ℓ units of product j. (In this case, the

value-priority parameters are estimated for a purchase history.) Then if

$\delta_{j\ell} = 1$ we expect that $u_{j\ell} - \lambda p_j > 0$, and if $\delta_{j\ell} = 0$ we

expect that $u_{j\ell} - \lambda p_j < 0$. This problem is similar in structure to

the conjoint estimation problem faced by Srinivasan and Shocker (1973) when

they developed LINMAP.

We can obtain parameter estimates of $u_{j\ell}$ and λ by the following

linear program:

$$\min \Sigma_{j,\ell} \ z_{j\ell} \qquad\qquad\qquad (13)$$

$$\text{s.t.} \ \ u_{j\ell} - \lambda p_j - z_{j\ell} \leq 0 \qquad\qquad \text{if } \delta_{j\ell} = 1$$

$$u_{j\ell} - \lambda p_j + z_{j\ell} \geq 0 \qquad\qquad \text{if } \delta_{j\ell} = 0$$

$$z_{j\ell} \geq 0$$

As in the Srinivasan-Shocker algorithm, the parameters of interest, $u_{j\ell}$ and λ, are unique only to a scaling constant and therefore must be constrained to avoid degenerate solutions. See Srinivasan and Shocker (1973). Similarly, recent advances to ensure strict inequalitites can readily be incorporated into the linear program in 13. See Srinivasan (1981).

Equation 13 faces the same overspecification problem as does equation 12. Estimation must be handled accordingly through grouping of consumers or through observations for multiple situations.

Constant Sum Paired Comparisons

A final means to estimate utilities $(u_{j\ell})$ is through direct scaling by the consumer. Since the utilities must be ratio-scaled, we recommend constant sum paired comparisons (CSPC) in which the consumer is asked to allocate 100 "chips" between pairs of products. See Hauser and Shugan (1980) for detailed discussion and statistical tests to evaluate data integrity. This method has been shown to work well for non-durable products when all products are in the same category (Silk and Urban 1978, Urban and Katz 1982), but there is no guarantee that it will work as well when comparisons are made among durable products chosen from a variety of categories. This remains an empirical question.

Estimation is based on regression or linear programming and is discussed in Hauser and Shugan (1980).

Multiple Time Periods

To extend any of the three methods to multiple time periods if depreciation and interest rates are known, we simply multiply $u_{j\ell}$ by its adjustment factor, $\sum_{q=0}^{\tau-t} \gamma_j^q / (1 + r)^{\tau-t}$, in equations 12 and 13 and proceed as discussed above.

Considering other model extensions such as replacement and product complementarity and substitutability presents new estimation issues that will be addressed in future research. Empirical experience may cause us to modify the initial concepts suggested here.

7. CLINIC MEASURES OF PURCHASE

The estimations of section 6 are based on data collected directly from the consumer before he is exposed to the new durable product of interest.

Logit requires the consumer to estimate his purchase likelihoods for alternative economic or environmental scenarios. Linear programming requires the consumer to simply indicate whether or not he will purchase a product under different scenarios or to provide a purchase history. Constant sum paired comparisons require the consumer to evaluate the relative utilities of many pairs of products.

Once this information is collected, the consumer is exposed to the new durable products of interest and asked to evaluate them. For example in the case of automobiles this evaluation is called a clinic. The consumer sees a variety of new automobiles in a showroom setting.

For forecasting purposes we collect measures of utility for the new products. However, we propose to collect measures with which to confirm the model prediction. These include:

(1) buying priority, i.e., the consumer is asked to provide his new buying priority now that the new durable product is available;

(2) purchase intent, i.e., the consumer is asked to estimate his likelihood of purchasing each of the new products; and

(3) lottery measures, i.e., the consumer is asked to select a prize (from among the products available) which he will actually receive if he wins a lottery. The drawing will be made at the end of a series of interviews.

The measurement instruments that reflect these rough ideas are yet to be developed, and will require major research efforts.

8. CONVERGENT MEASURES

The discussion above proposes three alternative means to estimate the parameters of the model and proposes three alternative confirmation measures. Our experience, e.g., Silk and Urban (1978), with forecasting models suggests that accuracy is greatly enhanced if one uses multiple measures and methods to obtain convergent estimates of the model's constructs. We propose to apply the same concepts to the value-priority model. See figures 3a and 3b.

The logit and linear programming (LP) estimates are derived from alternative measures of purchase. The first step is to examine the convergence of the estimates of $u_{j\ell}$ and λ_s. In the second step we examine the convergence of the derived logit-LP estimates and the direct paired comparison estimates of utility. The net convergence provides us with pre-clinic estimates of $u_{j\ell}$, λ_s, and β before exposure to the new product.

On the clinic side of figure 3, we use the three alternative purchase measures to get a convergent forecast of choice which is then compared to the estimates obtained from the calibrated value-priority model. Finally, we use the pre-clinic measures and the clinic "purchase" observations to obtain our final forecasts of purchase probability.

The details of this convergent testing have yet to be developed, but they should follow the procedures suggested by Bagozzi (1980), Campbell and Fiske (1959), and Silk and Urban (1978).
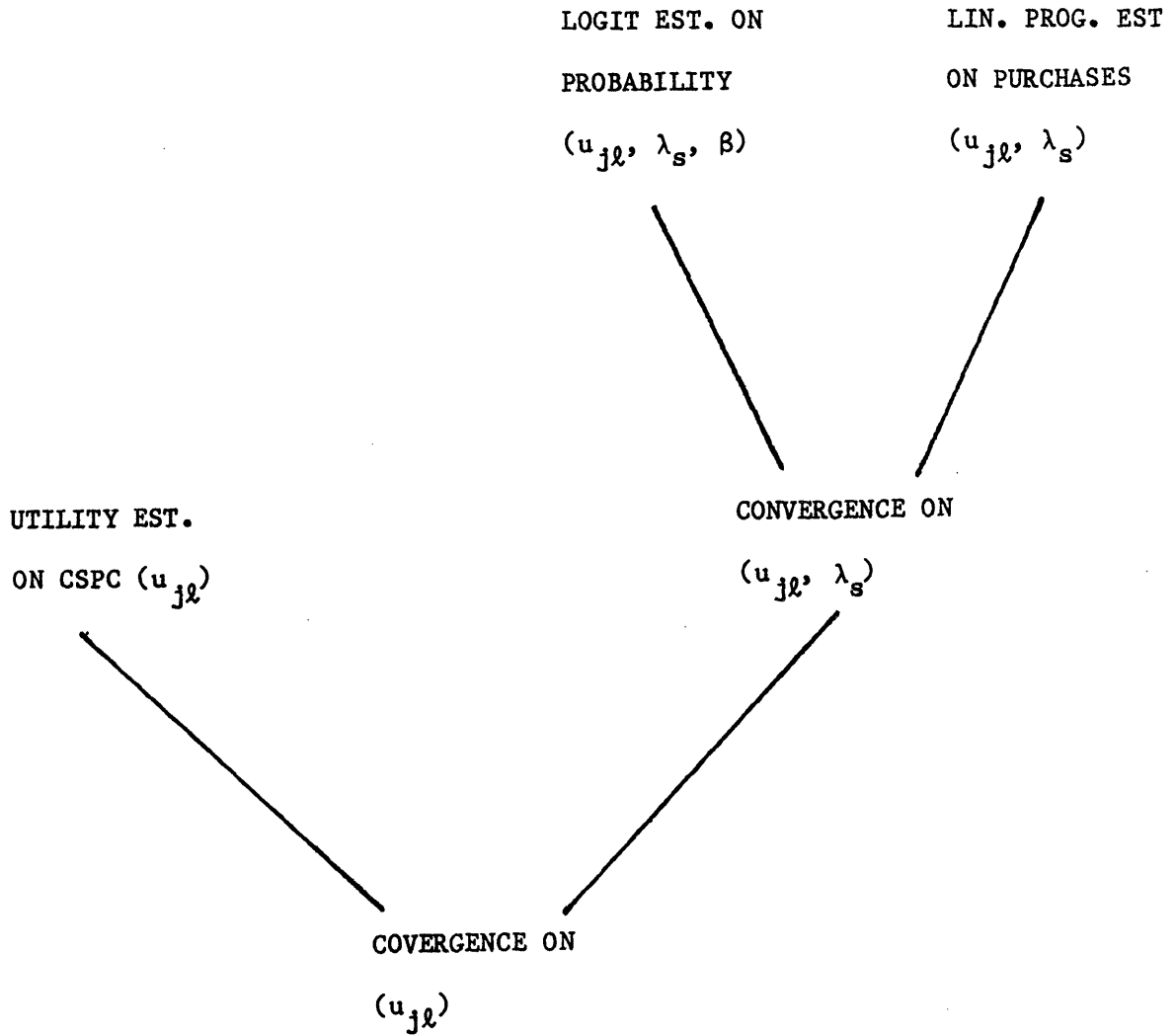
BEFORE EXPOSURE TO

NEW PRODUCT MEASURES

LOGIT EST. ON

PROBABILITY

$(u_{j\ell}, \lambda_s, \beta)$

LIN. PROG. EST

ON PURCHASES

$(u_{j\ell}, \lambda_s)$

UTILITY EST.

ON CSPC $(u_{j\ell})$

CONVERGENCE ON

$(u_{j\ell}, \lambda_s)$

COVERGENCE ON

$(u_{j\ell})$

Figure 3a: Covergent Relationships in Empirical Measurement for Value-Priority
Consumer Model—Before Exposure to New Product

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AFTER EXPOSURE TO

NEW PRODUCT

PROBABILITY   PAIRED

OF PURCHASE   COMPARISONS

VALUE-PRIORITY   BUYING

MODEL $(u_{j\ell}, \lambda_s, \beta)$ PRIORITY   INTENT   LOTTERY

CHOICE   CONVERGENCE

PREDICTION   ON CHOICE
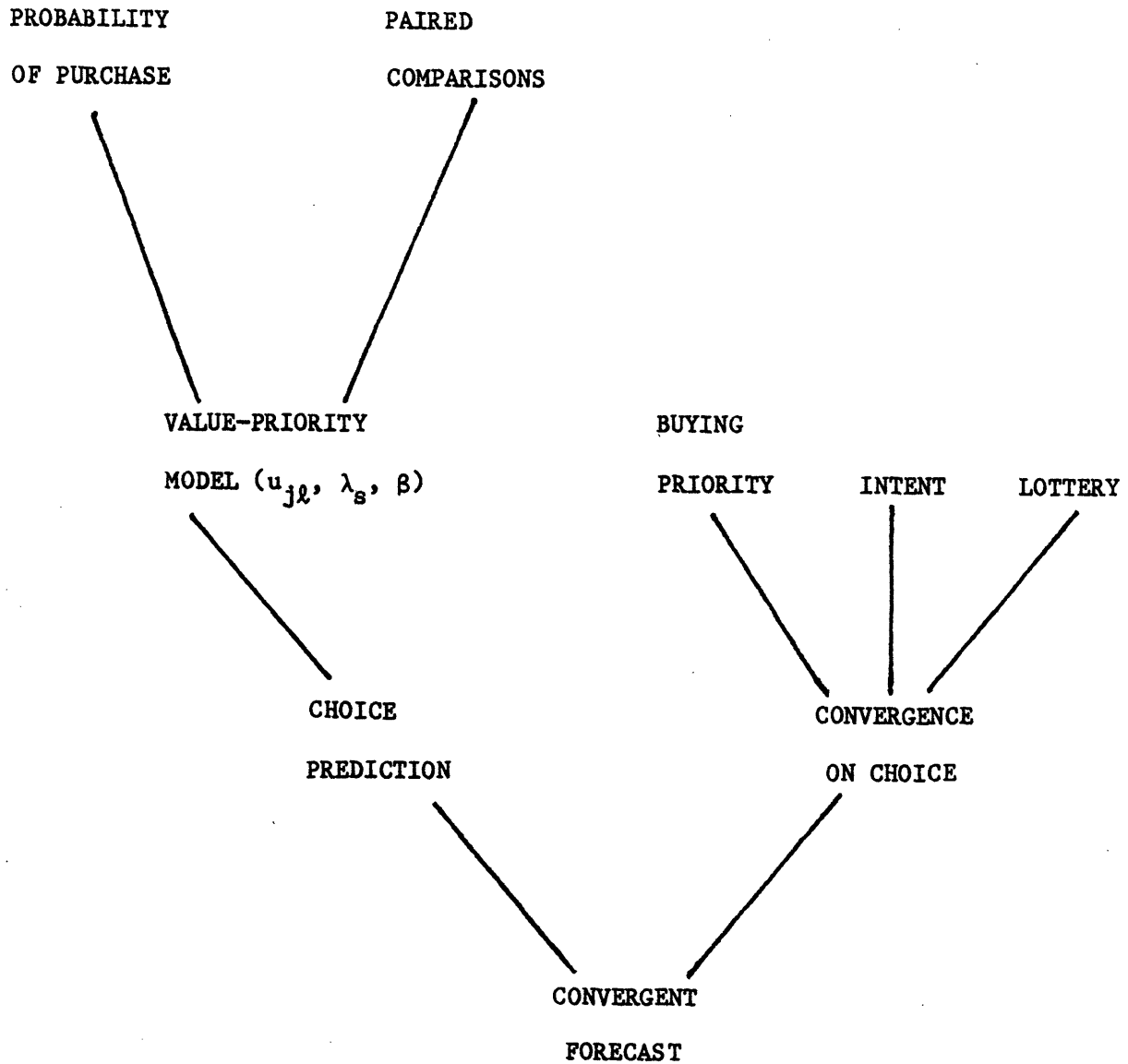
CONVERGENT

FORECAST

Figure 3b: Convergent Relationships after Exposure to New Product

CONCLUSION

This paper reflects preliminary thinking about modeling consumer durable purchasing and procedures to forecast purchase of a new durable good. We hope this work is provocative and precipitates comments, criticisms and suggestions so we can revise more effectively this initial set of ideas and carry out the many necessary future research tasks.

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