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TANDEM PACKET-RADIO QUEUEING NETWORKS*

by

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Abstract

Most research in the area of packet-radio networks has been confined to one-hop hearing topologies. This paper investigates tandem networks which are multihop networks. We assume that the nodes of the tandem have infinite buffers and they share a common radio channel for transmission of data packets. We also assume that each node transmits whenever it has a packet ready for transmission. We treat two tandem networks here. The first is a tandem having arbitrary number of nodes and packets arrive to the network only at the "top" node. The second is a four-node tandem network where packets arrive to all the nodes. The arrival processes to the nodes here might, in general, be dependent.

For these networks we derive the joint generating functions of the contents of the queues at the nodes in steady-state. From the generating functions, any moment of the queue lengths can, in principle, be derived as well as average time delays in the network. We also give an example for independent Bernoulli arrival processes.

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1. Introduction

In packet-radio networks, many contending devices share a common radio channel in a given locality. It is well-known that within such an environment the outcome of the transmission of a packet by a node depends on both the states and the actions of neighboring nodes. This dependence in general inhibits any attempt to obtain explicit analytic results for general networks.

As an initial attempt to understand the behavior of multi-hop packet-radio networks, there is need to first analyze accurately typical configurations. Such configurations are tandem networks that are the issue of this paper. Packet-radio nodes in tandem have recently received some attention in the literature [1]. A schematic figure of such a network is depicted in Fig. 1. The network consists of N nodes having infinite buffers. The nodes transmit data over a common shared radio channel. Fixed-length data packets enter the system at the nodes from corresponding sources. Time is divided into slots of size corresponding to the transmission time of a packet and transmissions are started only at the beginning of a slot.

The nodes of the tandem have radio transmitters with omnidirectional antennas and their transmission range is such that a transmission at node i ($2 \le i \le N-1$) can be heard only by nodes (i-1) and (i+1). Nodes 1 and N can be heard at nodes 2 and N-1 respectively.

The final destination of all packets is a station (Fig. 1) that receives the packets transmitted by node 1, and packets entering the network at any node i are forwarded via nodes i-1, i-2,...,1 till they finally reach the station. We assume that a node can transmit only one packet in a given slot

Two main features characterize packet radio nodes that use a common shared channel for transmission of packets: (a) if two or more nodes transmit packets simultaneously, then a node that hears both these transmissions cannot receive any of them successfully; (b) a node cannot transmit and receive a packet simultaneously.

In tandem packet-radio networks these two features imply several facts: (i) The transmissions of node 1 are always successful since no other node can interfere; (ii) The transmissions of node 2 are successful only if node 1 is not transmitting at the same time (because of feature (b)); (iii) The transmissions of node i ($3 \leq i \leq N$) are successful only if both node i-1 (because of feature (b)) and node i-2 (because of feature (a)) are not transmitting at the same time. From this discussion it is clear that the status of a node i (empty or not) affects the behavior of the queues at other nearby nodes, a fact that implies statistical dependence between the nodes.

We note that there is a distinct difference between tandem networks with up to three nodes and networks with more than three nodes. In the former no two nodes can successfully transmit a packet simultaneously, while in the latter, nodes i, i+3, i+6... can both transmit successfully at the same time, though they share the same channel. This phenomena is known as spatial reuse of the radio channel. In [2] a three-node tandem packet radio network with general arrival processes has been analyzed and the joint generating function of the queue lengths distribution has been obtained.

A tandem packet-radio network in which packets arrive only at the "top" node (node N) has been considered in [1]. The nodes in [1] use a random access scheme in which a nonempty node i transmits a packet with probability p_i, and the probabilities p_i obey certain constrains (polite scheme and fair scheme). For such access schemes it has been shown in [1] that the maximal throughput is 4/27. However, it has been also indicated that if each node transmits whenever it has a packet ready for transmission i.e. $p_i = 1 \quad 1 \leq i \leq N$, then the maximal throughput is 1/3. The only parameter that has been considered in [1] is the maximal throughput and not the detailed statistical behavior of the network.

In the present paper we shall consider the statistical behavior analysis of tandem networks where each node transmits whenever its queue is nonempty. We shall first analyze a tandem network where packets arrive only at node N (the "top" node) and shall give explicit expressions for the generating function of the queue length distribution at each node, for the average queue length, for the average time delay at each node and for the total average delay in the network. Then, we shall analyze in detail a tandem network with four nodes where packets arrive to all the four nodes from corresponding sources. Here the analysis is much harder due to the possible reuse of the channel by nodes 1 and 4, but we can still obtain expressions for the joint generating function of the queue length distribution at the nodes for general arrival processes, as well as the average quantities.

2. N-Node Tandem: Packets Arrive Only at the Top Node.

In this section we analyze in detail the statistical behavior of an N-node tandem packet-radio network in which packets arrive only at node N. As said before, we assume that each node transmits whenever its queue is nonempty. In Fig. 2 we show how packets advance through the network. It is easy to see that the only node that can have more than one packet at any instant is node N. All the other nodes (1,2,...,N-1) can have at most one packet at a time.

Let a(i) i=0,1,2,... be the probability that i packets arrive at node N during a slot, and let F(z) be the generating function of this arrival process, i.e.

$$F(z) = \sum_{i=0}^{\infty} a(i) z^{i}. \qquad (1)$$

In steady-state the average arrival rate into each node is the same as the departure rate from this node and therefore, in our case, is the same as the average arrival rate into the next node in the tandem. Let r be the average arrival rate at node N, i.e.

$$r = \sum_{i=1}^{\infty} i a(i). \qquad (2)$$

Then r is the average arrival rate into each node of the tandem. Let L_i be a random variable denoting the number of packets at node i ($1 \le i \le N$) at an arbitrary slot. Since L_i is either 0 or 1 for $1 \le i \le N-1$ we immediately obtain:

$$L_i = \begin{cases} 0 \text{ with probability } 1-r \\ 1 \text{ with probability } r \end{cases} \qquad (3)$$

If we denote the generating function of the queue length distribution in steady-state at node i by $G_i(z)$ $1 \le i \le N$ we have:

$$G_i(z) = rz + 1-r \qquad 1 \le i \le N-1. \qquad (4)$$

In addition, from the description of the network we notice that the activity at nodes 1,2,...,N-3 does not affect node N at all. Therefore, in order to obtain $G_N(z)$ we only have to consider a three-node tandem network in which packets arrive only at the "top" node. A three-node tandem network has been analyzed in [2] for general arrival processes. Applying the results of [2] to our case we obtain:

$$G_N(z) = F(z) \frac{(1-3r)(1-z)}{F^3(z)-z} \tag{5}$$

and the condition for steady-state is that $r < \frac{1}{3}$. Alternatively, $G_N(z)$ in (5) can be obtained by analyzing node N as a single discrete-time queue with arrival process with generating function F(z) and a packet leaves the node (if any) every third slot.

From (3) we see that the average number of packets at node i $(1 \leq i \leq N-1)$ is given by:

$$\bar{L}_i = r \tag{6}$$

and the average time delay at these nodes is one slot (no queue at the nodes).

$\bar{L}_N$ - the average number of packets at node N is given from (5) by:

$$\bar{L}_N = r + \frac{6r^2 + 3\sigma}{2(1-3r)} \tag{7}$$

where

$$\sigma = \left. \frac{d^2F(z)}{dz^2} \right|_{z=1} \tag{8}$$

Applying Little's law [4] to the whole network we obtain the average time delay in the network - T:

$$T = N + \frac{6r + 3\sigma/r}{2(1-3r)} \tag{9}$$

The first term in (9) expresses the total time to transverse the tandem and the second term expresses the average waiting time at node N.

If we try to extend the model of this section and assume that packets arrive only to nodes N, N-1,...,N-k ($N \geq k+3$) from their corresponding sources, then it is clear that in order to analyze such a network we have first to analyze a tandem network with k+3 nodes and packets arrive to all nodes. The reason is that in such a tandem network, nodes 1,2,...,N-(k+3) do not affect at all the k+3 nodes at the top of the tandem. In the following section we shall analyze a four-node tandem network. This analysis is, as said above, equivalent to the analysis of N-node tandem network in which packets arrive only to nodes N and N-1.

3. Four-Node Tandem: General Arrival Processes

In this section we analyze a four-node tandem packet radio network when the arrival processes into the nodes are arbitrary. Let $A_i(t)$ $1 \leq i \leq 4$ $t = 0,1,2,\ldots$ be the number of packets entering node i from its corresponding source in the interval (t,t+1). The input process $\{A_i(t)\}_{i=1}^{4}$ is assumed to be a sequence of independent and identically distributed random vectors with integer-valued elements and let the corresponding probability distribution and generating function be:

$$a(i_4,i_3,i_2,i_1) = \mathrm{Prob}\{A_4(t) = i_4,\ A_3(t) = i_3,\ A_2(t) = i_2,\ A_1(t) = i_1\} \tag{10}$$

$$F(\underline{z}) = E\left\{\prod_{i=1}^{4} z_i^{A_i(t)}\right\} \tag{11}$$

where here $\underline{z} = (z_4, z_3, z_2, z_1)$.

In the following we shall assume that packets do arrive at node 4, otherwise the network degenerates to a three-node network.

3.1 Steady-State Analysis

To describe the evolution of the system we need several notations. Let $L_i(t)$ $1 \leq i \leq 4$, $t=0,1,2,\ldots$ be the number of packets at node i at time t. Let $U_i(t)$ $1 \leq i \leq 4$, $t=0,1,2,\ldots$ be binary valued random variables defined as follows:

$$U_1(t) = \begin{cases} 1 & L_1(t) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{12a}$$

$$U_2(t) = \begin{cases} 1 & L_1(t) = 0,\ L_2(t) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{12b}$$

$$U_3(t) = \begin{cases} 1 & L_1(t) = L_2(t) = 0,\ L_3(t) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{12c}$$

$$U_4(t) = \begin{cases} 1 & L_2(t) = L_3(t) = 0,\ L_4(t) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{12d}$$

From the description of tandem networks it is easy to see that for $t = 0,1,2,\ldots$ the system evolves according to the following equations:

$$L_i(t+1) = A_i(t) + L_i(t) - U_i(t) + U_{i+1}(t) \qquad 1 \leq i \leq 4 \tag{13}$$

where $U_5(t) \equiv 0$.

Consider now the steady-state joint generating function of the queue lengths:

$$G(\underline{z}) = \lim_{t\to\infty} E\{\prod_{i=1}^{4} z_i^{L_i(t)}\} \tag{14}$$

Here we assume that the Markov chain $\{L_i(t)\}_{i=1}^{4}$ is ergodic, i.e., $G(\underline{z})|_{z_1=z_2=z_3=z_4=0} > 0$.

From (13), using a standard technique we obtain:

$$\begin{aligned} G(z_4,z_3,z_2,z_1) &= F(z_4,z_3,z_2,z_1)\{G(0,0,0,0) + \\ &+ [G(z_4,0,0,0)-G(0,0,0,0)]z_4^{-1}z_3+[G(z_4,z_3,0,0)-G(z_4,0,0,0)]z_3^{-1}z_2+ \\ &+ [G(z_4,0,0,z_1)-G(z_4,0,0,0)-G(0,0,0,z_1)+G(0,0,0,0)](z_4^{-1}z_3z_1^{-1}-z_1^{-1})+ \\ &+ [G(z_4,z_3,z_2,0)-G(z_4,z_3,0,0)]z_2^{-1}z_1 + \\ &+ [G(z_4,z_3,z_2,z_1)-G(z_4,z_3,z_2,0)]z_1^{-1}\}. \end{aligned} \tag{15}$$

The complexity of the problem lies in the fact that in order to determine $G(\underline{z})$ uniquely we have to determine the five boundary functions $G(z_4,0,0,0)$, $G(z_4,z_3,0,0)$, $G(z_4,z_3,z_2,0)$, $G(z_4,0,0,z_1)$ $G(0,0,0,z_1)$ and the constant $G(0,0,0,0)$.

Determination of $G(z_4,0,0,0)$ and $G(0,0,0,z_1)$

In order to determine $G(z_4,0,0,0)$ and $G(0,0,0,z_1)$ let $z_2 \to 0$ and $z_3 \to 0$ in (15). Then

$$G(z_4,0,0,z_1) = F(z_4,0,0,z_1)\{G(0,0,0,0)+[G(0,0,0,z_1)-G(0,0,0,0)]z_1^{-1} + G_z'(z_4,0,0,0)z_1\} \tag{16}$$

where here we use $G_i'(\underline{z})$ to denote the partial derivative of $G(\underline{z})$ with respect to z_i.

Now let $z_1 \to 0$ in (16). Then,

$$G(z_4,0,0,0) = F(z_4,0,0,0)[G(0,0,0,0) + G_1(0,0,0,0)]. \tag{17}$$

By substituting $z_4 = 0$ in (17) we immediately obtain:

$$G(z_4,0,0,0) = \frac{F(z_4,0,0,0)G(0,0,0,0)}{F(0,0,0,0)} \tag{18}$$

(Note: It is easy to see that for steady-state to exist we must have $F(0,0,0,0) > 0$, i.e., the probability that no packet will arrive to the system in a given slot is strictly positive.)

From (18) we see that the boundary function $G(z_4,0,0,0)$ is determined up to the constant $G(0,0,0,0)$.

To obtain $G(0,0,0,z_1)$ let $z_4 \to 0$ in (16). Then we obtain:

$$G(0,0,0,z_1) = F(0,0,0,z_1) \frac{G(0,0,0,0)(1-z_1^{-1})+G_2'(0,0,0,0)z_1}{1-F(0,0,0,z_1)z_1^{-1}} \tag{19}$$

By applying Rouche's Theorem [3] we can show that the equation $z_1 = F(0,0,0,z_1)$ has a unique solution in the unit circle $|z_1| < 1$. Let $\hat{z}_1$ be this solution.

Then since $G(0,0,0,z_1)$ is an analytic function for $|z_1| < 1$ we obtain from (18) that

$$G_2'(0,0,0,0) = \frac{\hat{z}_1^{-1} - 1}{\hat{z}_1} G(0,0,0,0) \tag{20}$$

Substitution of (20) in (19) yields $G(0,0,0,z_1)$ in terms of the constant $G(0,0,0,0)$.

Now, from (14) using (15) we obtain:

$$G(\underline{z}) = F(\underline{z}) \frac{H_1(\underline{z})+H_2(\underline{z})G_2'(z_4,0,0,0)+H_3(\underline{z})G(z_4,z_3,0,0)+H_4(\underline{z})G(z_4,z_3,z_2,0)}{1 - F(\underline{z})z_1^{-1}} \tag{21a}$$

where

$$\begin{aligned} H_1(\underline{z}) = {} & G(0,0,0,0)(1-z_1^{-1})[1-z_4^{-1}z_3z_1^{-1}+F(z_4,0,0,z_1)(z_4^{-1}z_3z_1^{-1}-z_1^{-1}] + \\ & + G(z_4,0,0,0)[z_4^{-1}z_3-z_3^{-1}z_2+z_4^{-1}z_3z_1^{-1}-z_1^{-1}] + \\ & + G(0,0,0,z_1)(z_4^{-1}z_3z_1^{-1}-z_1^{-1})[1+F(z_4,0,0,z_1)z_1^{-1}]. \end{aligned} \tag{21b}$$

$$H_2(\underline{z}) = F(z_4,0,0,z_1)(z_4^{-1}z_3-1) \tag{21c}$$

$$H_3(\underline{z}) = z_3^{-1}z_2-z_2^{-1}z_1 \tag{21d}$$

$$H_4(\underline{z}) = z_2^{-1}z_1-z_1^{-1} \tag{21e}$$

From (21b)-(21e) we see that the functions $H_i(\underline{z})$ $2 \leq i \leq 4$ are known and the function $H_1(\underline{z})$ is known up to the constant $G(0,0,0,0)$. We still have to determine the three boundary functions $G_2'(z_4,0,0,0)$, $G(z_4,z_3,0,0)$ and

$G(z_4,z_3,z_2,0)$.

Determination of $G_2'(z_4,0,0,0)$

To determine $G_2'(z_4,0,0,0)$ let us use the following substitution in (21):

$$z_2 = z_1^2; \quad z_3 = z_1^3 \tag{22}$$

With the notation $\underline{z}^{(3)} = (z_4,z_1^3,z_1^2,z_1)$ we have from (21) and (22) that:

$$H_3(\underline{z}^{(3)}) = H_4(\underline{z}^{(3)}) = 0, \tag{23}$$

and therefore

$$G(\underline{z}^{(3)}) = F(\underline{z}^{(3)}) \frac{H_1(\underline{z}^{(3)}) + H_2(\underline{z}^{(3)})G_2'(z_4,0,0,0)}{1 - F(\underline{z}^{(3)})z_1^{-1}} \tag{24}$$

Applying Rouche's Theorem we can show that for $|z_4| < 1$ the equation $z_1 = F(z_4,z_1^3,z_1^2,z_1)$ has a unique solution in the unit circle $|z_1| < 1$. Let us denote this unique solution by $f_3(z_4)$. Then since the function $G(\underline{z}^{(3)})$ is analytic in the polydisk $|z_1| < 1$, $|z_4| < 1$ we obtain from (24) that:

$$G_2'(z_4,0,0,0) = -\frac{H_1(z_4,f_3^3,f_3^2,f_3)}{H_2(z_4,f_3^3,f_3^2,f_3)} \tag{25}$$

Thus, $G_2'(z_4,0,0,0)$ is determined up to the constant $G(0,0,0,0)$.

Determination of $G(z_4,z_3,0,0)$

To determine $G(z_4,z_3,0,0)$ let $z_2 = z_1^2$ in (21). Then:

$$G(\underline{z}^{(2)}) = F(\underline{z}^{(2)}) \frac{H_1(\underline{z}^{(2)})+H_2(\underline{z}^{(2)})G_2'(z_4,0,0,0)+H_3(\underline{z}^{(2)})G(z_4,z_3,0,0)}{1 - F(\underline{z}^{(2)})z_1^{-1}} \tag{26}$$

where $\underline{z}^{(2)} = (z_4,z_3,z_1^2,z_1)$. Applying Rouche's Theorem we can show that when $|z_3| < 1$, $|z_4| < 1$, the equation $z_1 = F(z_4,z_3,z_1^2,z_1)$ has a unique solution in the unit circle $|z_1| < 1$. Denote this unique solution by $f_2(z_4,z_3)$. Then, since $G(\underline{z}^{(2)})$ is analytic in the polydisk $|z_1| < 1$, $|z_3| < 1$, $|z_4| < 1$ we obtain from (26):

$$G(z_4,z_3,0,0) = - \frac{H_1(z_4,z_3,f_2^2,f_2)+H_2(z_4,z_3,f_2^2,f_2)G_2'(z_4,0,0,0)}{H_3(z_4,z_3,f_2^2,f_2)} \tag{27}$$

so using (25) we see that $G(z_4,z_3,0,0)$ is determined up to the constant $G(0,0,0,0)$.

Determination of $G(z_4,z_3,z_2,0)$

Using Rouche's Theorem, we can show that when $|z_i| < 1$, $2 \leq i \leq 4$, the equation $z_1 = F(z_4,z_3,z_2,z_1)$ has a unique solution in the unit circle $|z_1| < 1$. Denote this solution by $f_1(z_4,z_3,z_2)$. Since $G(\underline{z})$ is analytic in in the polydisk $|z_i| < 1$, $1 \leq i \leq 4$ we obtain from (21):

$$G(z_4,z_3,z_2,0) =$$

$$-\frac{H_1(\underline{z})\Big|_{z_1=f_1} + G_2'(z_4,0,0,0)H_2(\underline{z})\Big|_{z_1=f_1} + \dot{G}(z_4,z_3,0,0)H_3(\underline{z})\Big|_{z_1=f_1}}{H_4(\underline{z})\Big|_{z_1=f_1}} \quad (28)$$

so the function $G(z_4,z_3,z_2,0)$ is determined up to the constant $G(0,0,0,0)$.

Determination of G(0,0,0,0)

The constant $G(0,0,0,0)$ is determined from the normalization condition $G(1,1,1,1) = 1$. For $j=1,2,3,4$ we substitute $z_i = 1 \quad 1 \leq i \leq 4,\ i \neq j$ and let $z_j \to 1$ in (15) and use the notation $r_i = \frac{\partial F(\underline{z})}{\partial z_i}\Big|_{z_1=z_2=z_3=z_4=1}$. Then for $j=1$ we obtain:

$$1-r_1 = -G(1,1,0,0) + 2G(1,1,1,0). \quad (29)$$

For $j=2$ we obtain:

$$-r_2 = -G(1,0,0,0) + 2G(1,1,0,0) - G(1,1,1,0). \quad (30)$$

For $j=3$ we obtain:

$$-r_3 = G(1,0,0,0) - G(1,1,0,0) + G(1,0,0,1) - G(0,0,0,1) \quad (31)$$

Finally, for $j=4$ we obtain:

$$r_4 = G(1,0,0,1) - G(0,0,0,1). \quad (32)$$

From (29)-(32) we obtain:

$$G(1,0,0,0) = 1-3r_4-3r_3-2r_2-r_1. \quad (33)$$

Therefore, from (18) we finally obtain:

$$G(0,0,0,0) = \frac{F(0,0,0,0)}{F(1,0,0,0)}(1-r_1-2r_2-3r_3-3r_4). \tag{34}$$

Consequently, the condition for steady-state is:

$$r_1 + 2r_2 + 3r_3 + 3r_4 < 1. \tag{35}$$

This concludes the analysis of a four-node tandem packet radio network for general arrival processes. As explained before, this analysis can be used to analyze a general tandem network where packets arrive only at nodes N and N-1.

From the joint generating function $G(\underline{z})$ any moment of the queue lengths at the various nodes can, in principle, be derived. Specifically, the average number of packets at node i $(1 \leq i \leq 4)$ is given by:

$$\overline{L}_i = \left.\frac{\partial G(\underline{z})}{\partial z_i}\right|_{z_1=z_2=z_3=z_4=1} \tag{36}$$

In addition, applying Little's law [4], we can also obtain the average time delay at node i $(1 \leq i \leq 4)$ which is given by:

$$T_i = \frac{\overline{L}_i}{\sum_{j=i}^{4} r_j} \tag{37}$$

since the total arrival rate at node i is given by $\sum_{j=i}^{4} r_j$. Finally, the the total average delay in the network is given by:

$$T = \frac{\sum_{i=1}^{4} \bar{L}_i}{\sum_{i=1}^{4} r_i} \tag{38}$$

3.2 Independent Bernoulli Arrival Processes

In order to give some numerical results we assume that packets arrive at the nodes according to independent Bernoulli arrival processes, i.e.,:

$$F(\underline{z}) = \prod_{i=1}^{4} (z_i r_i + \bar{r}_i) \tag{39}$$

where $\bar{r}_i = 1-r_i$.

The explicit expressions for the generating functions, for the average queue lengths and for the average time delays are very complex. In the appendix we provide such expressions for the latter quantities only.

As an example we plot in Fig. 3 T_i $1 \le i \le 4$, the average time delays at node i $1 \le i \le 4$ respectively and the total average delay T versus γ --the total throughput of the system when $r_i = r$ $1 \le i \le 4$ (i.e. $\gamma = 4r$).

4. Discussion

Two multi-hop tandem packet-radio networks have been analyzed in this paper. The first is a tandem having arbitrary number of nodes and packets arrive to the network only at the "top" node. Since we assume that each node transmits whenever it is non-empty we have shown that the tandem topology imposes perfect scheduling within the network. As a result, node N - the "top" node transmits successfully once every three slots if it has packets and the transmissions of all other nodes are always successful. This regular behavior of the network enabled simple analysis.

The second tandem packet-radio network considered, consists of four nodes that packets may arrive to all of them. We have shown that there is strict difference between this network and a three-node tandem network, due to the possible simultaneous use of the radio channel by nodes one and four. The analysis of this network was shown to be very complex because of the need to determine six boundary terms (five boundary functions and one boundary constant) in order to obtain the joint generating function of the queue lengths at the nodes. Clearly the joint generating function does not possess a product-form, therefore no decompositions are possible. It is also clear that the analysis will become much more complex (if possible at all), as the number of nodes grow, since the number of boundary terms to be determined will also grow.

Regarding the steady-state condition in a tandem network: we have a conjecture that in an N-node network we need that $r_1+2r_2+\sum_{i=3}^{N} 3r_i < 1$ where r_i is the arrival rate of packets to node i from its corresponding source. We have proved this conjecture for up to five nodes.

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APPENDIX

For a four-node tandem packet radio network with independent Bernoulli arrival processes we obtain after very tedious algebra:

$$T_i = \frac{r_i + A_i}{\sum\limits_{j \neq i}^{4} r_j} \qquad 1 \leqslant i \leqslant 4 \tag{A1}$$

where

$$A_1 = \frac{r_2 + r_3 + r_4}{1 - r_1} \tag{A2}$$

$$A_2 = \frac{\bar{r}_1(r_1 r_2 + r_3 + r_4) + r_2(r_2 + r_3 + r_4)}{\bar{r}_1(1 - r_1 - 2r_2)} \tag{A3}$$

$$A_3 = \frac{A_3' + A_2(r_2 + r_3) - (A_1 + A_2)(1 - r_1 - r_2 - r_3)}{1 - r_1 - 2r_2 - 3r_3} \tag{A4}$$

$$A_4 = \frac{A_4' + r_2 + 2r_3 + 2r_4 + r_4 G(\underline{0})/\bar{r}_4}{1 - r_1 - 2r_2 - 3r_3 - 3r_4} \quad . \tag{A5}$$

The constants A_3' and A_4' are given by:

$$A_3' = r_2+2r_3+3r_4+r_1r_2+r_1r_3+2r_2r_3-r_4(r_1+2r_2+3r_3+3r_4) + $$
$$+ G_1'(1,0,0,1) - G_1'(0,0,0,1) \tag{A6}$$

$$A_4' = r_1r_2+r_1r_3+r_1r_4+2r_2r_3+2r_2r_4+3r_3r_4 - A_1(1-r_1-r_2-r_3-r_4) -$$
$$- A_2(1-r_1-2r_2-2r_3-2r_4) - A_3(1-r_1-2r_2-3r_3-3r_4) \tag{A7}$$

The constants $G_1'(0,0,0,1)$ and $G_1'(1,0,0,1)$ are given by:

$$G_1'(0,0,0,1) = \bar{r}_2\bar{r}_3\bar{r}_4 G(\underline{0}) \frac{(1+\frac{t^{-1}-1}{t})(1-\bar{r}_2\bar{r}_3\bar{r}_4) - \bar{r}_1\bar{r}_2\bar{r}_3\bar{r}_4 \frac{t^{-1}-1}{t}}{(1-\bar{r}_2\bar{r}_3\bar{r}_4)^2} +$$

$$+ r_1 G(0,0,0,1) \tag{A8}$$

$$G_1'(1,0,0,1) = r_1\bar{r}_2\bar{r}_3[G(0,0,0,1) + G_2'(1,0,0,0)] +$$

$$+ \bar{r}_3\bar{r}_2[G(\underline{0}) - G(0,0,0,1) + G_2'(1,0,0,0)] + G_1'(0,0,0,1). \tag{A9}$$

Finally t, $G(0,0,0,1)$, $G_2'(1,0,0,0)$ and $G(0,0,0,0)$ are given by

$$t = \frac{\bar{r}_1\bar{r}_2\bar{r}_3\bar{r}_4}{1-r_1\bar{r}_2\bar{r}_3\bar{r}_4} \tag{A10}$$

$$G(0,0,0,1) = \bar{r}_2\bar{r}_3\bar{r}_4 G(\underline{0}) \frac{\frac{t^{-1}-1}{t}}{1-\bar{r}_2\bar{r}_3\bar{r}_4} \tag{A11}$$

$$G_2'(1,0,0,0) = \frac{r_4 + (1-\bar{r}_2\bar{r}_3)G(0,0,0,1)}{\bar{r}_2\bar{r}_3} \tag{A12}$$

$$G(\underline{0}) = (1-r_4)(1-r_1-2r_2-3r_3-3r_4). \tag{A13}$$

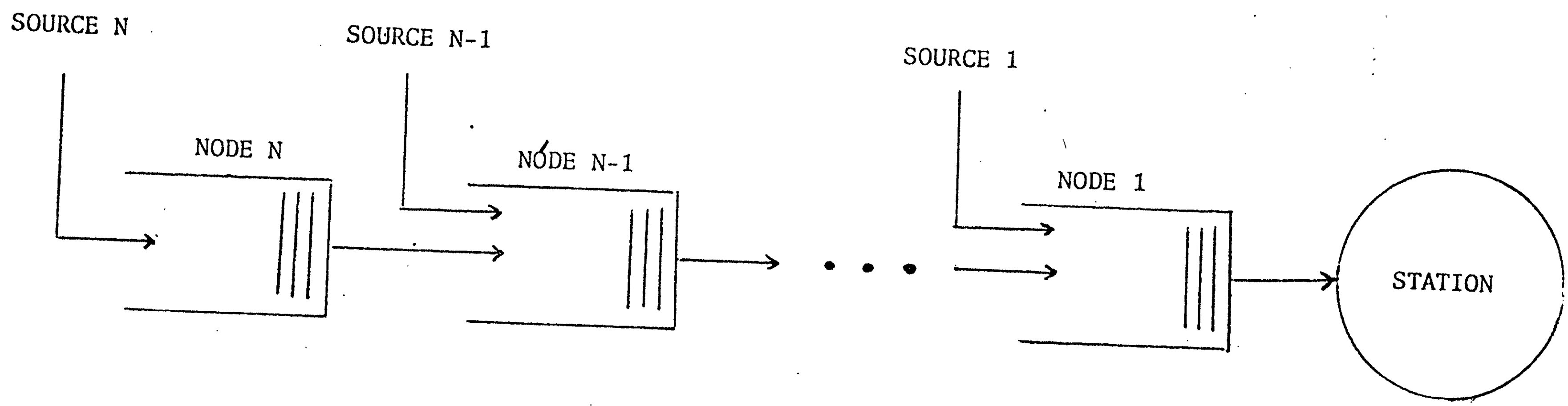

Fig. 1 - Tandem packet radio network with N nodes.

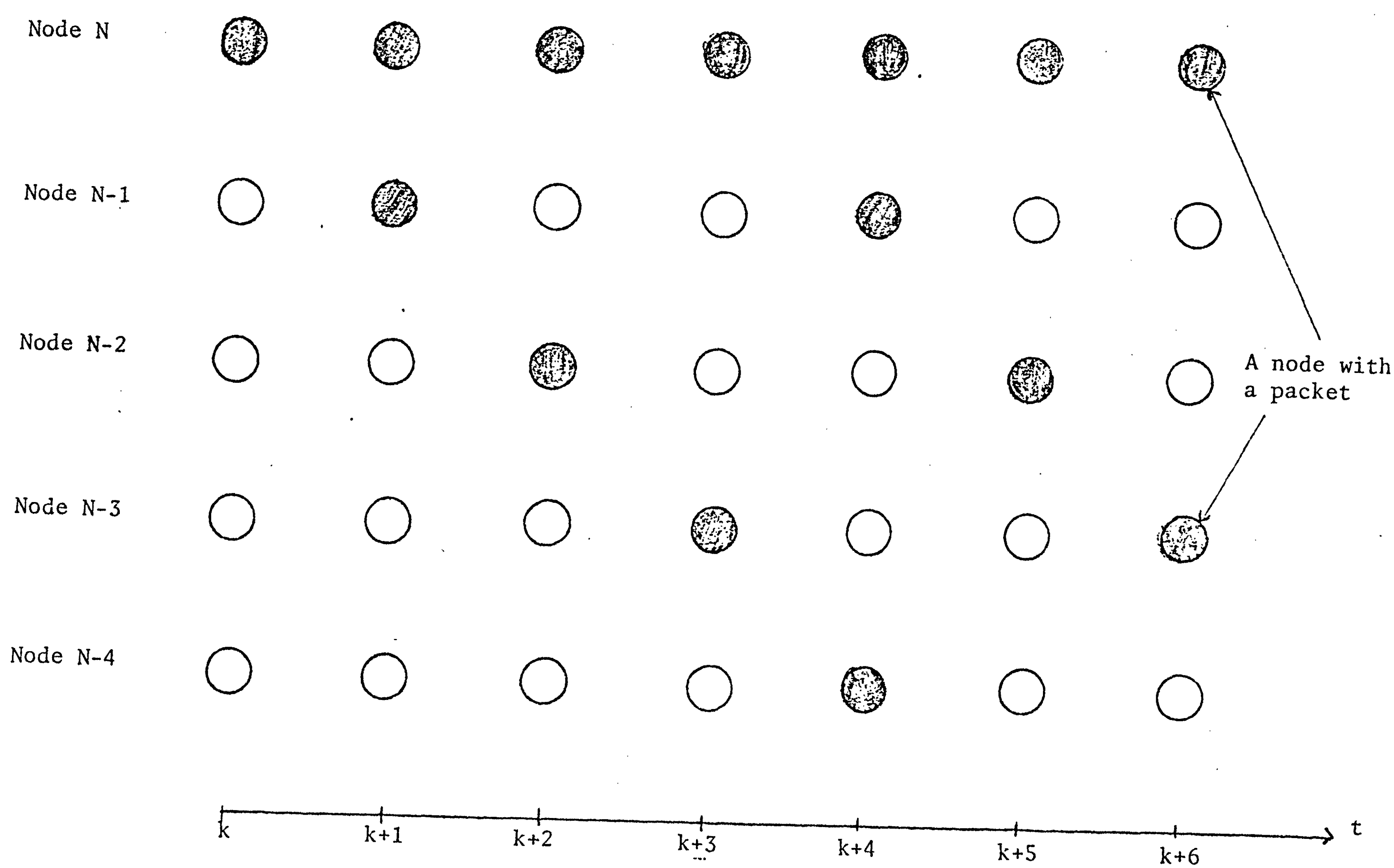

Fig. 2 - Packets forwarding in a tandem network (Packets arrive only at node N).

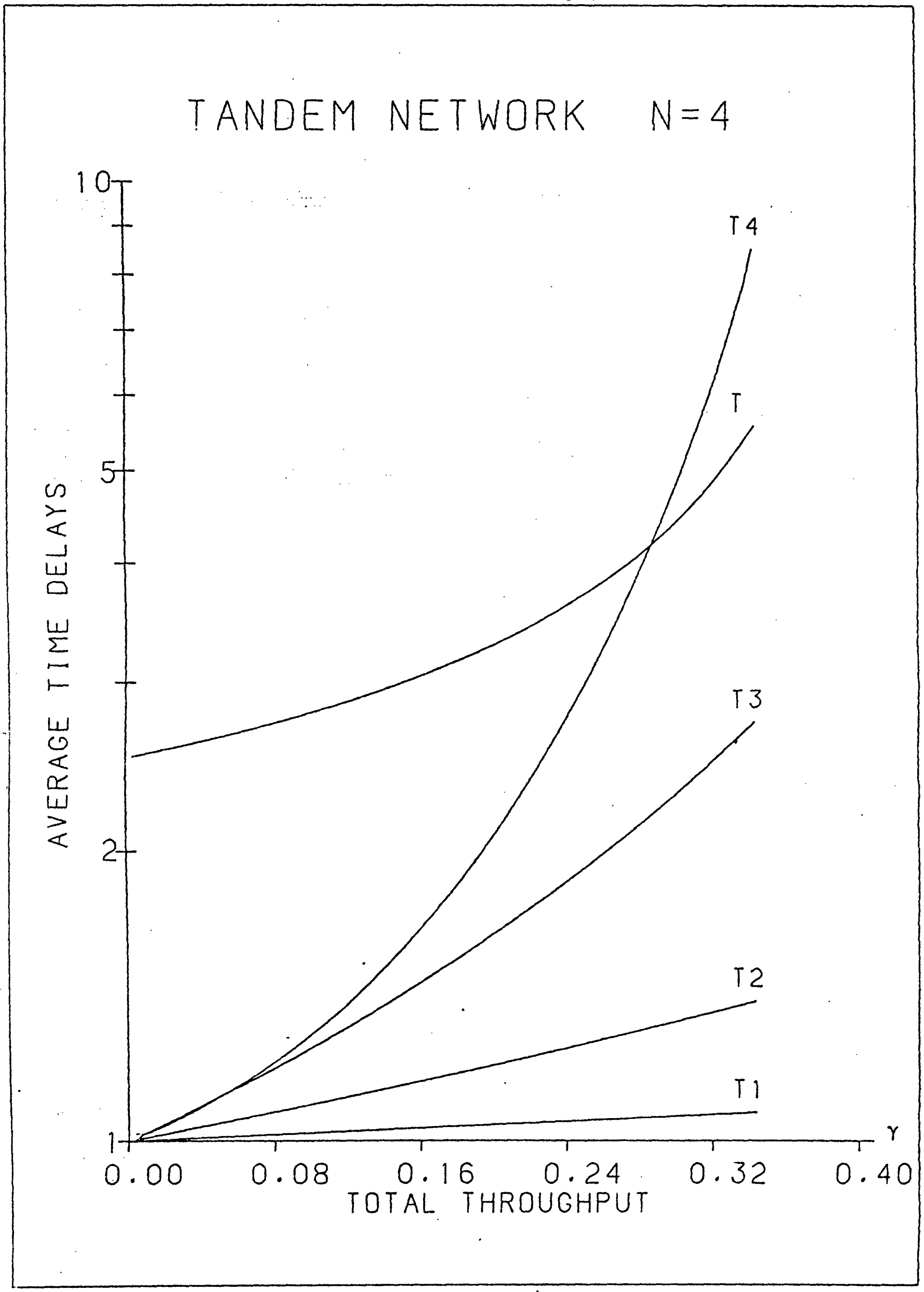

Fig. 3 - Four-node tandem packet-radio network: delays versus γ.