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INFORMATION PROCESSING AND DECISIONMAKING ORGANIZATIONS: A MATHEMATICAL DESCRIPTION*

by

ALEXANDER H. LEVIS**

ABSTRACT

An overview of an analytical approach to the modeling and evaluation of information processing and decisionmaking organizations is presented. The mathematical framework used in modeling the individual decisionmakers as well as the organization is that of n-dimensional information theory. The data flow formalism is used to model in a precise manner the various types of interactions between decisionmakers as well as interactions between humans and the command, control, and communication system that supports the organization. Comparison and evaluation of alternative organizational forms is accomplished by considering organizational performance, individual workload, and the sets of satisficing decision strategies.

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**Laboratory for Information and Decision Systems, Massachusetts Institute of Technology, Cambridge, MA 02139.

1. INTRODUCTION

In considering organizational structures for teams of decisionmakers, a designer must address the questions of who receives what information and who is assigned to make which decisions. The resolution of these questions specifies the organizational form. The designer's problem is the selection of a form so that the resulting organization meets its performance specifications and the individual members are not overloaded, i.e., the task requirements do not exceed their individual processing limitations.

While the role of the human decisionmakers is central to the design problem, the latter cannot be decoupled from the consideration of the information system that supports the organization. Consider, for example, a tactical military organization supported by a command, control, and communications (C^3) system. Information is collected from many sources, distributed to appropriate units in the organization for processing, and used by the commanders and their staff to make decisions. These decisions are then passed to the units responsible for carrying them out. Thus, a given organization design implies the existence of a C^3 system that supports it. Conversely, the presence of a C^3 system in support of an organization modifies the latter's operations; it may create operational modes not foreseen during the organizational design phase. Therefore, if a quantitative description of the organization design problem is to be developed, it must take into account not only the organization members, but also the collection of equipment and procedures that constitute the organization's C^3 system.

In order to develop a quantitative methodology for the analysis and evaluation of information processing and decisionmaking organizations, it is necessary that a set of compatible models be obtained that describe the organization and its environment. This modeling effort has been divided in three steps. The first one is the modeling of the tasks the organization is to execute and the definition of the boundary between the organization and its environment. The second step is the selection of mathematical models that describe the members of the organization. The third step is the modeling of organizational form, i.e., the specification of the information and decision structures that characterize the organization. This step includes the specification of the protocols for information exchange and the modeling of the communication systems, the data bases, and the decision aids that the organization uses to perform its tasks.

The methodology itself consists of two main parts. In the first one, the analysis of the organization, the models are used to describe the organization in terms of a locus defined on a generalized performance - workload space. This locus is obtained by computing an index of performance for the organization and measures of the workload for each individual member of the organization as functions of the admissible decision strategies used by the decisionmakers. The second part of the methodology addresses the question of evaluating organizational designs and comparing alternative structures.

The analytical framework used for modeling the tasks, the individual organization members, the C^3 system, and the organization as a whole is that of n-dimensional information theory [13]. A brief description of the key quantities and of the partition law of information [5] is presented in the next section.

2. INFORMATION THEORETIC FRAMEWORK

Information theory was first developed as an application in communication theory [15]. But, as Khinchin [9] showed, it is also a valid mathematical theory in its own right, and it is useful for applications in many disciplines, including the modeling of simple human decisionmaking processes [16] and the analysis of information-processing systems.

There are two quantities of primary interest in information theory. The first of these is **entropy**: given a variable x, which is an element of the alphabet X, and occurs with probability p(x), the entropy of x, H(x), is defined to be

$$H(x) \equiv -\sum_{x} p(x) \quad \log p(x) \tag{2.1}$$

and is measured in bits when the base of the algorithm is two. The other quantity of interest is **average mutual information** or **transmission:** given two variables x and y, elements of the alphabets X and Y, and given p(x), p(y), and p(x|y) (the conditional probability of x, given the value of y), the transmission between x and y, T(x:y) is defined to be

$$T(x:y) \equiv H(x) - H_y(x) \tag{2.2}$$

where

$$H_y(x) \equiv -\sum_{y} p(y) \sum_{x} p(x|y) \log p(x|y) \tag{2.3}$$

is the conditional uncertainty in the variable x, given full knowledge of the value of the variable y.

McGill [13] generalized this basic two-variable input-output theory to N dimensions by extending Eq. (2.2):

$$T(x_1:x_2:\ldots:x_N) \equiv \sum_{i=1}^{N} H(x_i) - H(x_1,x_2,\ldots,x_N) \tag{2.4}$$

For the modeling of memory and of sequential inputs which are dependent on each other, the use of the entropy rate, $\bar{H}(x)$, which describes the average entropy of x per unit time, is appropriate:

$$\bar{H}(x) \equiv \lim_{m\rightarrow\infty} \frac{1}{m} H[x(t),\ x(t+1),\ldots,x(t+m-1)] \tag{2.5}$$

Transmission rates, $\bar{T}(x:y)$, are defined exactly like transmission, but using entropy rates in the definition rather than entropies.

The Partition Law of Information [5] is defined for a system with N-1 internal variables, w_1 through w_{N-1}, and an output variable, y, also called w_N. The law states

$$\sum_{i=1}^{N} H(w_i) = T(x:y) + T_y(x:w_1,w_2,\ldots,w_{N-1})$$

$$+ T(w_1:w_2:\ldots:w_{N-1}:y) + H_x(w_1,w_2,\ldots w_{N-1},y) \quad (2.6)$$

and is easily derived using information theoretic identities. The left-hand side of (2.6) refers to the total activity of the system, also designated by G. Each of the quantities on the right-hand side has its own interpretation. The first term, T(x:y), is called **throughput** and is designated G_t. It measures the amount by which the output of the system is related to the input. The second quantity,

$$T_y(x:w_1,w_2,\ldots,w_{N-1}) = T(x:w_1,w_2,\ldots,w_{n-1},y) - T(x:y) \quad (2.7)$$

is called **blockage** and is designated G_b. Blockage may be thought of as the amount of information in the input to the system that is not included in the output. The third term, $T(w_1:w_2:\ldots:w_{N-1}:y)$ is called **coordination** and designated G_c. It is the N-dimensional transmission of the system, i.e., the amount by which all of the internal variables in the system constrain each other. The last term, $H_x(w_1,w_2,\ldots,w_{N-1},y)$, designated by G_n represents the uncertainty that remains in the system variables when the input is completely known. This noise should not be construed to be necessarily undesirable, as it is in communication theory: it may also be thought of as **internally-generated information** supplied by the system to supplement the input and facilitate the decisionmaking process. The partition law may be abbreviated:

$$G = G_t + G_b + G_c + G_n \tag{2.8}$$

A statement completely analogous to (2.8) can be made about information rates by substituting entropy rate and transmission rates in (2.6).

3. TASK MODEL [8,18]

The organization, perceived as an open system [10], interacts with its environment; it receives signals or messages in various forms that contain information relevant to the organization's tasks. These messages must be identified, analyzed, and transmitted to their appropriate destinations within the organization. From this perspective, the organization acts as an information user.

Let the organization receive data from one or more sources external to it. Every τ_n units of time on the average, each source n generates symbols, signals, or messages x_{ni} from its associated alphabet X_n, with probability p_{ni}, i.e.,

$$p_{ni} = p(x_n=x_{ni}) \quad ; \quad x_{ni} \varepsilon X_n \qquad i = 1,2,,\ldots\gamma_n \tag{3.1}$$

$$\sum_{i=1}^{\gamma_n} p_{ni} = 1 \quad ; \quad n = 1,2,\ldots,N' \tag{3.2}$$

where γ_n is the dimension of x_n. Therefore, $1/\tau_n$ is the mean frequency of symbol generation from source n.

The organization's task is defined as the processing of the input symbols x_n to produce output symbols. This definition implies that the organization designer knows a priori the set of desired responses Y and, furthermore, has a function or table $L(x_n)$ that associates a desired response or a set of desired responses, elements of Y, to each input $x_n \varepsilon X_n$.

It is assumed that a specific complex task that must be performed can be modeled by N' sources of data. Rather than considering these sources separately, one supersource composed of these N' sources is created. The input symbol $\underline{x}'$ may be represented by an N'-dimensional vector with each of the sources represented by a component of this vector; i.e.,

$$\underline{x}' \equiv (x_1, x_2, \ldots, x_N) \quad ; \quad \underline{x}' \varepsilon X \tag{3.3}$$

To determine the probability that symbol $\underline{x}_j'$ is generated, the independence between components must be considered. If all components are mutually independent, then p_j is the product of the probabilities that each component of $\underline{x}_j'$ takes on its respective value from its associated alphabet:

$$p_j = \prod_{n=1}^{N'} p_{nj} \tag{3.4}$$

If two or more components are probabilistically dependent on each other, but as a group are mutually independent from all other components of the input vector, then these dependent components can be treated as one supercomponent, with a new alphabet. Then a new input vector, $\underline{x}$, is defined, composed of the mutually independent components and these super-components.

This model of the sources implies synchronization between the generation of the individual source elements so that they may, in fact, be treated as one input symbol. Specifically, it is assumed that the mean interarrival time for each component τ_n is equal to τ. It is also assumed that the generation of a particular input vector, $\underline{x}_j$, is independent of the symbols generated prior to or after it.

The last assumption can be weakened, if the source is a discrete stationary ergodic one with constant interarrival time τ that could be approximated by a Markov source. Then the information theoretic framework can be retained [8].

The vector output of the source is partitioned into groups of components that are assigned to different organization members. The j-th partition is denoted by $\underline{x}^j$ and is derived from the corresponding partition matrix $\underline{\pi}^j$ which has dimension n_j x N and rank n_j, i.e.,

$$\underline{x}^j = \underline{\pi}^j \, \underline{x}. \tag{3.5}$$

Each column of $\underline{\pi}^j$ has at most one non-zero element. The resulting vectors $\underline{x}^j$ may have some, all, or no components in common.

The set of partitioning matrices $\{\underline{\pi}^1, \underline{\pi}^2, \ldots, \underline{\pi}^n\}$ shown in Figure 1 specify the components of the input vector received by each member of the subset of decisionmakers that interact directly with the organization's environment. These assignments can be time invariant or time varying. In the latter case, the partition matrix can be expressed as

$$\pi^j(t) = \begin{cases} \pi_o^j & \text{for } t \in T \\ 0 & \text{for } t \in T \end{cases} \tag{3.6}$$

The times at which a decisionmaker receives inputs for processing can be obtained either through a deterministic (e.g., periodic) or a stochastic rule. The question of how to select the set of partition matrices, i.e., design the information structure between the environment and the organization, has been addressed by Stabile [17,18].

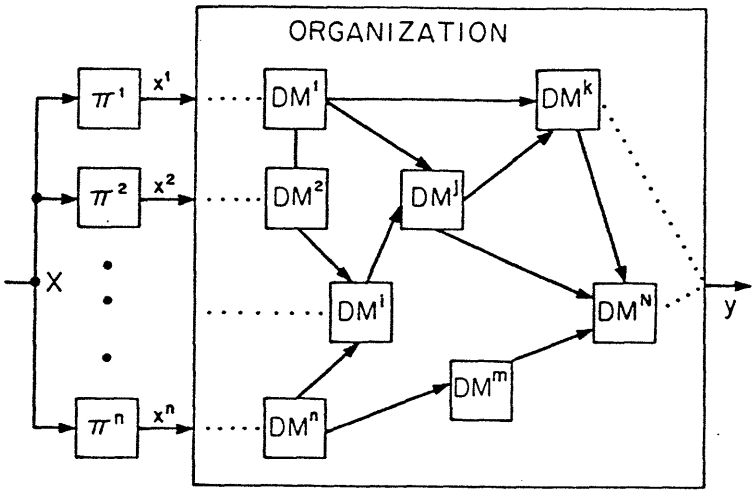


Figure 1. Information Structures for Organizations

4. MODEL OF THE ORGANIZATION MEMBER [2,3,11]

The complete realization of the model of the decisionmaker (DM) who is interacting with other organization members and with the environment is shown schematically in Figure 2.

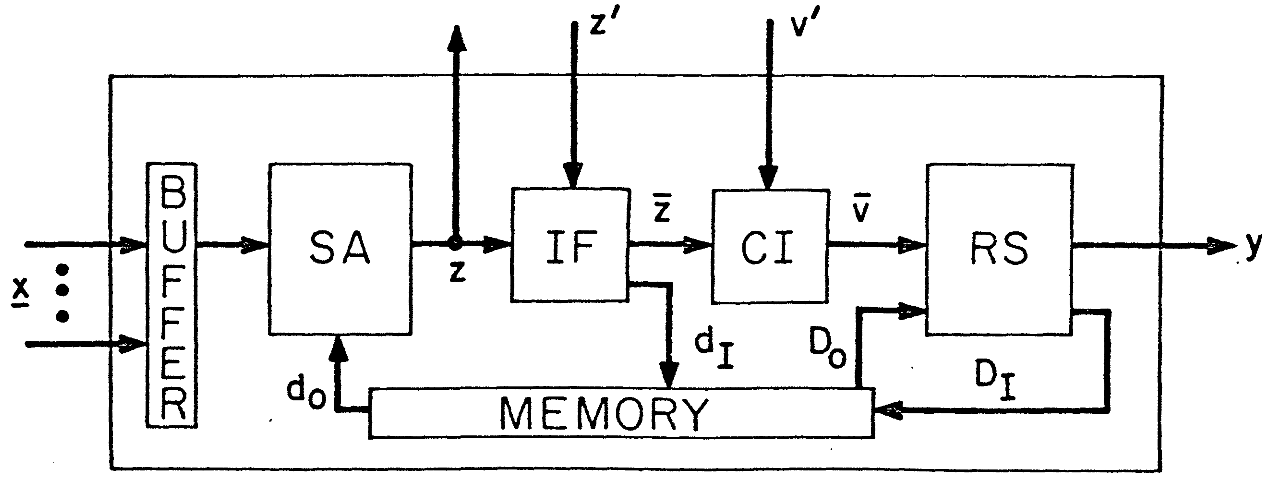


Figure 2. The Interacting Decisionmaker with Memory

The DM receives singals $\underline{x} \varepsilon X$ from the environment with interarrival time τ. A string of signals may be stored first in a buffer so that they can be processed together in the situation assessment (SA) stage. The SA stage contains algorithms that process the incoming signals to obtain the assessed situation $\underline{z}$. The SA stage may access the memory or internal data base to obtain a set of values d_o. The assessed situation z may be shared with other organization members; concurrently, the DM may receive the supplementary situation assessment z' from other parts of the organization; the two sets z and z' are combined in the information fusion (IF) processing stage to obtain $\bar{z}$. Some of the data (d_I) from the IF process may be stored in memory.

The possibility of receiving commands from other organization members is modeled by the variable v' and a command interpretation (CI) stage of processing is necessary to combine the situation assessment $\bar{z}$ and v' to arrive at the choice $\bar{v}$ of the appropriate strategy to use in the response selection (RS) stage. The RS stage contains algorithms that produce outputs y in response to the situation assessment $\bar{z}$ and the command inputs. The RS stage may access data from or store data in memory [7,8].

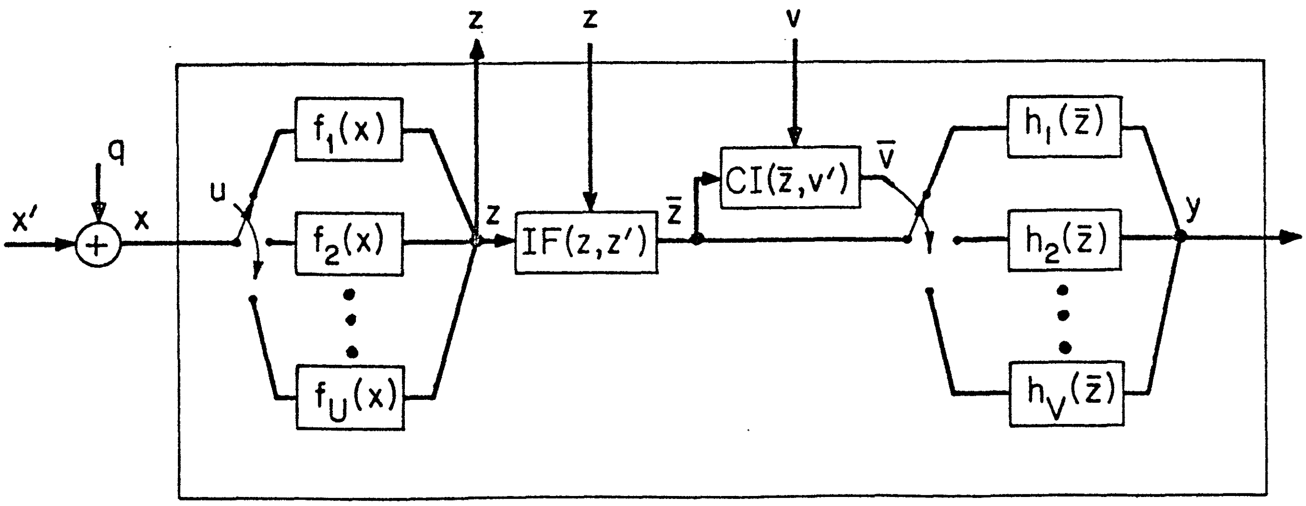


Figure 3. Detailed Model of the Interacting Decisionmaker

A more detailed description of the decisionmaker model without buffer or memory is shown in Figure 3. This figure shows the internal structure of the four processing stages: SA, IF, CI, and RS. The situation assessment stage consists of a set of U algorithms (deterministic or not) that are capable of producing some situation assessment z. The choice of algorithms is achieved through specification of the internal variable u in accordance with the situation assessment strategy p(u) or p(u|x), if a decision aid (e.g., a preprocessor) is present. A second internal decision is the selection of the algorithm in the RS stage according to the response selection strategy $p(\bar{v}|\bar{z},v')$. The two strategies, when taken together, constitute the internal decision strategy of the decisionmaker.

The analytical framework presented in Section 2, when applied to the single interacting decisionmaker with deterministic algorithms in the SA and RS stages, yields the four aggregate quantities that characterize the information processing and decisionmaking activity within the DM [2,11]:

Throughput:

$$G_t = T(x,z',v':z,y) \tag{4.1}$$

Blockage:

$$G_b = H(x,z',v') - G_t \tag{4.2}$$

Internally generated information:

$$G_n = H(u) - H_{\bar{z}}(v) \tag{4.3}$$

Coordination:

$$G_c = \sum_{i=1}^{U} p_i g_c^i (p(x)) + \alpha_i H(p_i) + H(z) + g_c^{IF}(p(z,z')) + g_c^{CI}(p(\bar{z},v'))$$

$$+ \sum_{j=1}^{V} p_j g_c^j (p(\bar{z}|\bar{v} = j)) + \alpha_j H(p_j) + H(y)$$

$$+ H(z) + H(\bar{z}) + H(\bar{z},\bar{v}) + T_z(x':z') + T_{\bar{z}}(x',z':v') \qquad (4.4)$$

The expression for G_n shows that it depends on the two internal strategies $p(u)$ and $p(v|\bar{z})$ even though a command input may exist. This implies that the command input v' modifies the DM's internal decision after $p(v|\bar{z})$ has been determined.

In the expressions defining the system coordination, p_i is the probability that algorithm f_i has been selected for processing the input x and p_j is the probability that algorithm h_j has been selected, i.e., $u = i$ and $\bar{v} = j$. The quantities g_c represent the internal coordinations of the corresponding algorithms and depend on the distribution of their respective inputs; the quantities α_i, α_j are the number of internal variables of the algorithms f_i and h_j, respectively. Finally, the quantity H is the entropy of a binary random variable:

$$H(p) = - p\log_2 p - (1 - p)\log_2(1-p) \qquad (4.5)$$

Equations (4.1) to (4.4) determine the total activity G of the decisionmaker according to the partition law of information (2.6). The activity G can be evaluated alternatively as the sum of the marginal uncertainties of each system variable. For any given internal decision strategy, G and its component parts can be computed.

Since the quantity G may be interpreted as the total information processing activity of the system, it can serve as a measure of the workload of the organization member in carrying out his decisionmaking task.

The qualitative notion that the rationality of a human decisionmaker is not perfect, but is bounded [12], has been modeled as a constraint on the total activity G:

$$G = G_t + G_b + G_n + G_c \leq F \tau_o \tag{4.6}$$

where τ_o is the symbol interarrival time and F is the maximum rate of information processing that characterizes a decisionmaker. This constraint implies that the decisionmaker must process his input at a rate that is least equal to the rate with which they arrive. For a detailed discussion of this particular model of bounded rationality, see Boettcher and Levis [2].

Weakening the assumption that the algorithms are deterministic changes the numerical values of G_n and of the coordination term G_c [4]. If memory is present in the model, then additional terms appear in the expressions for the coordination rate and for the internally generated information rate [7,8].

5. ORGANIZATIONAL FORM

In order to define an organizational structure, the interactions between the human decisionmakers that constitute the organization must be specified. The interactions between DMs and the environment have already been described in Section 3. The internal interactions between DMs consist of receiving inputs from other DM's, sharing situation assessments, receiving command inputs, and producing outputs that are either inputs or commands to other DM's. The detailed specification of the interactions requires the determination of what information is to be passed among individual organization members and the precise sequence of processing events, i.e., the standard operating procedure or communication and execution protocol of the organization.

Information structures that can be modeled within this analytical framework are those that represent synchronized, acyclical information flows. Since inputs are assumed to arrive at a fixed average rate, the organization is constrained to produce outputs at the same average rate. The overall response is made up, in general, of the responses of several members; therefore, each member is assumed to complete the processing corresponding to a particular input at the same average rate.

Within this overal rate synchronization, however, processing of a specific input symbol or vector takes place in an asynchronous manner. If the requisite inputs for a particular stage of processing are present, then processing can begin without regard to any other stage, which implies that concurrent processing is present. For example, as soon as the organization input arrives and is partitioned through π, processing of x begins to obtain z. The IF stage must wait, however, until both the z and z' values are present. Each stage of processing is thus event-driven; a well-defined sequence of events is therefore an essential element of the model specification.

Acyclical information structures are those whose directed graphs representing the flows of information do not contain any cycles or loops. This restriction is made to avoid deadlock and circulation of messages within the organization. Deadlock occurs when one DM is waiting for a message from another in order to proceed with his task, while the second one is in turn waiting for an input from the first.

The system theoretic representation of the organizational form is useful for showing the various processing stages or subsystems. For example, in Figure 4, a two person organization is shown in block diagram form in which the second member sends information to the first (z^{21}), who in turn can issue commands to the second DM.

Evaluation of the various information theoretic quantities, including total activity, can be accomplished readily, using the decomposition property of the information theoretic framework [5]. However, the internal

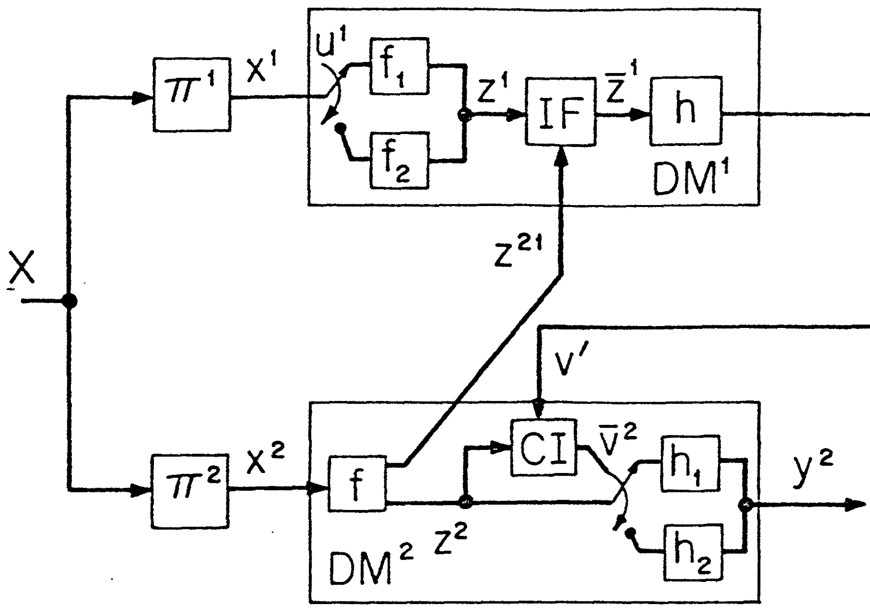


Figure 4. Block diagram representation of two person organization

information structure of the organization is often ambiguous when represented in block diagram terms. For example, the requirement that both z^1 and z^{21} be present before IF^1 processing can begin is not apparent from Figure 4. An alternate representation is needed which shows explicitly the information structure without compromising the usefulness of the information theoretic decomposition property.

The data-flow schema [1,6] has been developed as a model of information flow for systems with asynchronous, concurrent processing activities. Three basic elements are used in their structure: places, transitions, and directed arcs which connect the two. Places and transitions represent conditions and events, respectively. No event occurs unless the requisite conditions are met, but the occurrence of an event gives rise to new conditions. Tokens are used to mark which conditions are in effect; when all input places to (conditions for) a transition contain a token (are satisfied), then the event can occur, which in turn results in the generation of tokens for output places. Since tokens are carriers of data, each transition is a processor which generates a result from the input data and deposits it on an output token which then moves according to the schema's structure along a directed arc to the next stage of processing.

To represent the information theoretic decisionmaking model using a data-flow formalism, a simple translation in structure is made: distinct inputs and outputs of each subsystem are assigned places and the processing within a subsystem is represented by a transition. Associated with each transition is the set of internal variables of the subsystem, exclusive of the input variables, which are accounted for separately by the input places. By assuming a probability distribution on the organization's inputs, distributions are also included on the places in the structure. Therefore, distributions are also present on subsystem variables, and all information theoretic quantities are well-defined and can be computed as before.

The organization structure shown in Figure 4 can be represented in data-flow terms, as shown in Figure 5. In addition to places, transitions, and directed arcs, the structure contains two new elements, the switches u^1 and $\bar{v}^2$. These are logical elements which direct the flow of tokens. The switch u^1 takes values independently, while the value of $\bar{v}^2$ is determined as a result of the processing by algorithm B^2 contained in CI^2. Since the structure shown in Figure 5 is equivalent to the system theoretic structure in Figure 4, the internal variable definition and all information theoretic quantities remain unchanged. However, the information structure of the organization is made explicit in Figure 5. Once an input X is partitioned, the processing by each DM in his respective SA stage (algorithms f) begins concurrently and asynchronously. The information fusion processing (algorithm A^1) must wait until both z^1 and z^{21} have arrived at the input places of IF^1. Similarly, DM^2 must wait until DM^1 issues a command input v^{12} before the process of command interpretation can begin. This sequence of processing is evident from the representation. Note that because of the assumed synchronization with respect to organization inputs, there can be at most one data token in any single place. The structure is obviously acyclical and deadlock in the organization is prevented.

While the data-flow framework provides an equivalent representation for the class of synchronous, acyclical information structure , it is also able to model more general structures, many of which are of interest in the context of organizations. For example, the framework can easily model the

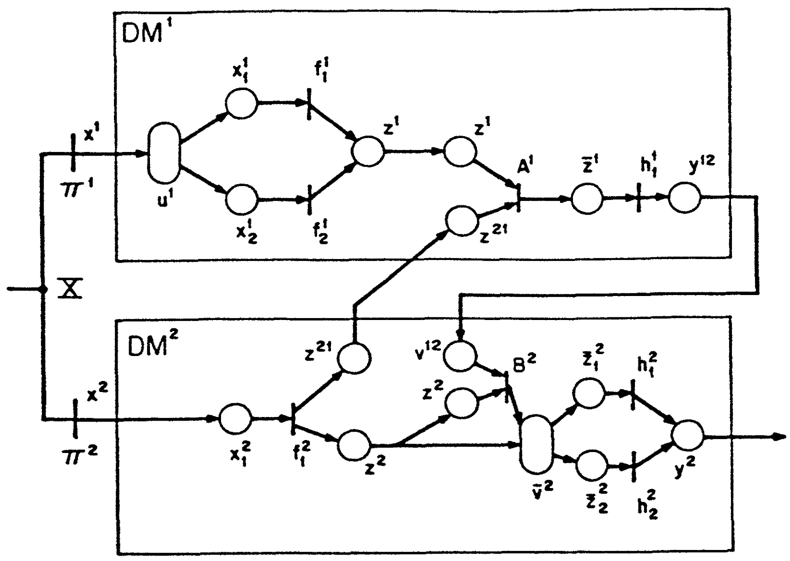


Figure 5. Data-flow representation of organization structure.

cyclic structures which arise when a two-way exchange of information is present in an organization. Such protocols are, of course, common. In addition, fully asynchronous structures can be represented within the framework. Since in a large organization members do not operate at the same rate (same tempo), asynchronous processing is of much interest. The study of these structures and their implications in terms of the n-dimensional information theoretic framework are subjects of current research.

A second advantage of the data flow framework is that it provides a natural way for describing in a precise manner the interactions between the DM's and the data bases and decision aids present in the organization.

The presence of data bases, an integral part of a C^3 system, requires the introduction of two additional modeling elements. The first is the query-response process. The second is the modeling of the data storage devices themselves. Consider, for example, the situation assessment subsystem shown in Figure 6. An accordance with the internal strategy u, an algorithm is chosen to process the input x. However, this algorithm may require parameters (e.g., terrain information, meteorological data) or past situation assessments in order to do the processing. The data base is accessed and queried for this information through the signal D_I. The data

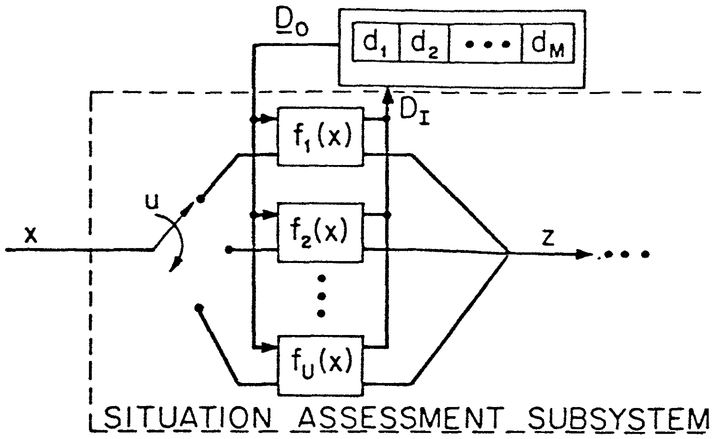


Figure 6. Model of SA subsystem with data base access

from the data base are provided to the SA subsystem of the DM through Do. The same link, DI, can be used to update the information in the data base. Clearly, the block diagram representation is ambiguous; the data flow formalism allows for the precise modeling of the fact that data is requested only when certain conditions are met.

Consider next the effect of a data base containing data that do not change during the execution of a task, i.e., the data are fixed. At first glance, it might seem that the addition of the data base with fixed values would have no effect on the total information theoretic rate of activity of the system, i.e.,

$$\bar{H}(d_i) = 0 \qquad i = 1,2,\ldots,M \tag{5.1}$$

However, the problemn is more complex. For example, if each algorithm f_i accesses β_i parameter values from the data base (in contrast to having these values fixed within the algorithm itself) then the rates of throughput, blockage, and noise of the combined system will not be affected, but the coordination term will have additional activity rate:

$$\Delta\bar{G}_c = \sum_{i=1}^{U} \beta_i \, H[p(u=i)] \tag{5.2}$$

Since a data base increases the overall activity of the system without creating any change in its input-output characteristic, one would question its presence. There are several advantages: (a) reduction in the information that needs to exist within the algorithms or within the decisionmaker model, (b) increased flexibility in the use of algorithms and hence possible reduction in the number of algorithms, and (c) access to common data by several organization members. Even though there is increased coordination activity due to the interaction between the DM and the data base, the total activity of the DM may be reduced — the task may be redesigned to fall within the bounded rationality constraints.

Similar arguments apply to the modeling and analysis of decision aids. Preliminary results indicate that an inappropriately designed decision aid may not reduce a decisionmaker's information processing load, but may actually increase it [4].

In this section, an approach to modeling the organizational form — the specification of the protocols for interaction between DM's — and the supporting command, control, and communication system has been presented. It is based on an integration of the data flow formalism with the information theoretic framework used in the quantitative modeling of the decisionmaking process.

6. ANALYSIS OF ORGANIZATIONS

As stated in Section 3, it is assumed that the designer knows a priori the set of desired responses Y to the input set X. Then the performance of the organization in accomplishing its tasks can be evaluated using the approach shown in Figure 7.

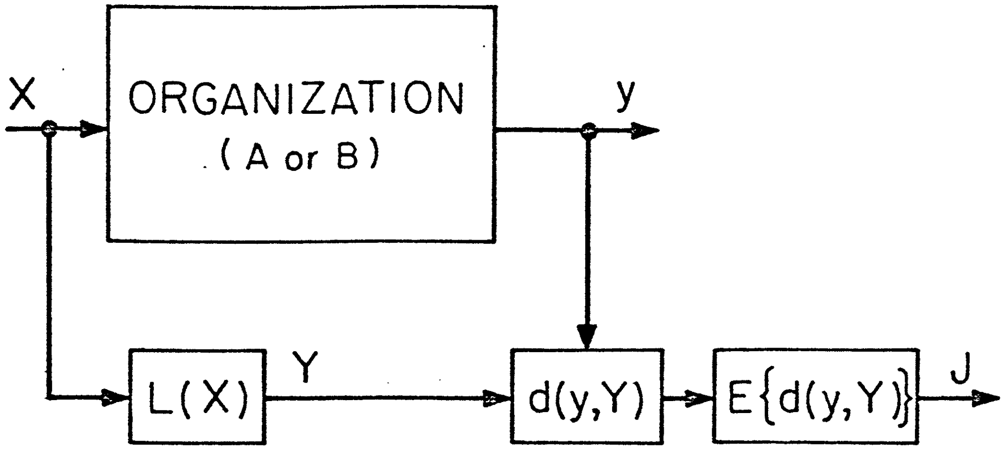


Figure 7. Performance evaluation of an organization

The organization's actual response y can be compared to the desired set Y_d and a cost assigned using a cost function d(y,Y). The expected value of the cost, obtained by averaging over all possible inputs, can serve as a performance index, J, for the organization. For example, if the function d(y,Y) takes the value of zero when the actual response is one of the desired ones and unity otherwise, then

$$J = E\ \{d(y,Y)\} = p(y \neq Y_d) \tag{6.1}$$

In this case, J represents the probability of the organization making the wrong decision, i.e., the probability of error. Once the organizational form is specified, the total processing activity G and the value of organizational performance J can be expressed as functions of the internal decision strategies selected by each decisionmaker. Let an internal strategy for a given decisionmaker be defined as pure, if both the situation assessment strategy p(u) and the response selection strategy $p(v|\bar{z})$ are pure, i.e., an algorithm f_i is selected with probability one and an algorithm h_j is selected also with probability one when the situation assessed as being $\hat{z}$:

$$D_k = \{p(u=i) = 1 \quad ; \quad p(v=j\,|\,\bar{z}=\hat{z}) = 1\} \tag{6.2}$$

for some i, some j, and for each $\hat{z}$ element of the alphabet $\bar{Z}$. There are n possible pure internal strategies,

$$n = U \cdot V^M \qquad (6.3)$$

where U is the number of f algorithms in the SA stage, V the number of h algorithm in the RS stage and M the dimension of the set $\bar{Z}$. All other internal strategies are mixed [14] and are obtained as convex combinations of pure strategies:

$$D(p_k) = \sum_{k=1}^{n} p_k D_k \qquad (6.4)$$

where the weighting coefficients are probabilities.

Corresponding to each $D(p_k)$ is a point in the simplex

$$\sum_{k=1}^{n} p_k = 1, \quad p_k \geq 0 \quad \forall k \qquad (6.5)$$

The possible strategies for an individual DM are elements of a closed convex hyperpolyhedron of dimension n-1 whose vertices are the unit vectors corresponding to pure strategies.

Because of the possible interactions among organization members, the value of G depends not only on $D(p_k)$ but also on the internal decisions of the other decisionmakers. A pure organizational strategy is defined as a M-tuplet of pure strategies, one from each DM:

$$\Delta_{1,2,\ldots,M} = \{D_{k_1}, D_{k_2}, \ldots, D_{k_M}\} \qquad (6.6)$$

Independent internal decision strategies for each DM, whether pure or mixed, induce a behavioral strategy [14] for the organization, which can be expressed as

$$\Delta = \sum_{1,2,\ldots,M} (\Delta_{1,2,\ldots,M} \prod_{i=1}^{M} p_{k_i}) \qquad (6.7)$$

where p_k is the probability of using pure strategy, D_k . Because each DM is assumed ^{i} to select his strategy independently of other ^{i} DM's, the strategy space of the organization, S^O, is determined as the direct sum of the individual DM strategy spaces:

$$S^O = S^1 \oplus S^2 \oplus \ldots \oplus S^M \tag{6.8}$$

where S^i denotes the individual DM strategy space. The dimension of S^O is given by

$$s = \dim S^O = \sum_{i=1}^{M} (n_i - 1)$$

Thus, the organizational strategies are elements of an s-dimensional closed convex hyperpolyhedron.

As Δ ranges over S^O, the corresponding values of the performance index J and the activity or workload of each individual organization member can be computed . In this manner, the set S^O is mapped into a locus on the M+1 dimensional performance-workload space, namely the space $(J, G^1, G^2, \ldots, G^M)$. Note that only the internal processing activity of the decisionmakers is presented in the locus and not the total activity of the system which includes the activity of the decision aids, data bases, and other components of the supporting C^3 system. Consequently, the bounded rationality constraints become hyperplanes in the performance-workload space. Since the bounded rationality constraint for all DM's depends on τ, the admissible internal decision strategies of each DM will also depend on the tempo of operations. The unconstrained case can be thought of as the limiting case when $\tau \to \infty$.

The methodology for the analysis of organizational structures allows for the formulation and solution of two problems: (a) the determination of the organizational strategies that minimize J and (b) the determination of the set of strategies for which $J \leq \bar{J}$. The first problem is one of optimization while the latter is formulated so as to obtain satisficing strategies with respect to a performance threshold $\bar{J}$. The satisficing condition also defines a plane in the performance-workload space that is normal to the J axis and intersects it at $\bar{J}$. All points on the locus on or below this plane which also satisfy the bounded rationality constraint for each decisionmaker in the organization define the set of satisficing decision strategies. Analytical properties of this locus as well as a computational approach to its efficient construction have been discussed in [2,3,11].

A qualitative evaluation of an organizational structure can be made by comparing the performance-workload locus to the space defined by the satisficing and bounded rationality constraints. In the same manner, alternative organizational strutures can be compared by considering their respective loci.

Since individual decisionmakers select their own decision strategies independently of all other organization members, a particular organizational form can yield a broad range of performance as illustrated by the locus in the performance-workload space. The designer must assess, therefore, the likelihood that strategies which lead to satisficing performance will be selected. A possible measure of this mutual consistency between individually selected strategies can be obtained by comparing the locus of the satisficing strategies to the locus of the organization's strategy space S^o. Let R^i be the subspaces of organization strategies which are feasible with respect to the bounded rationality constraint of each DM, i.e.,

$$R^i = \{\Delta \mid G^i(\Delta) \leq F^i \tau\} \tag{6.9}$$

and let R^J contain the strategies that satisfy the performance threshold $\bar{J}$:

$$R^J = \{\Delta \mid J(\Delta) \leq \bar{J}\} \tag{6.10}$$

The subspace of satisficing strategies R^o is given by :

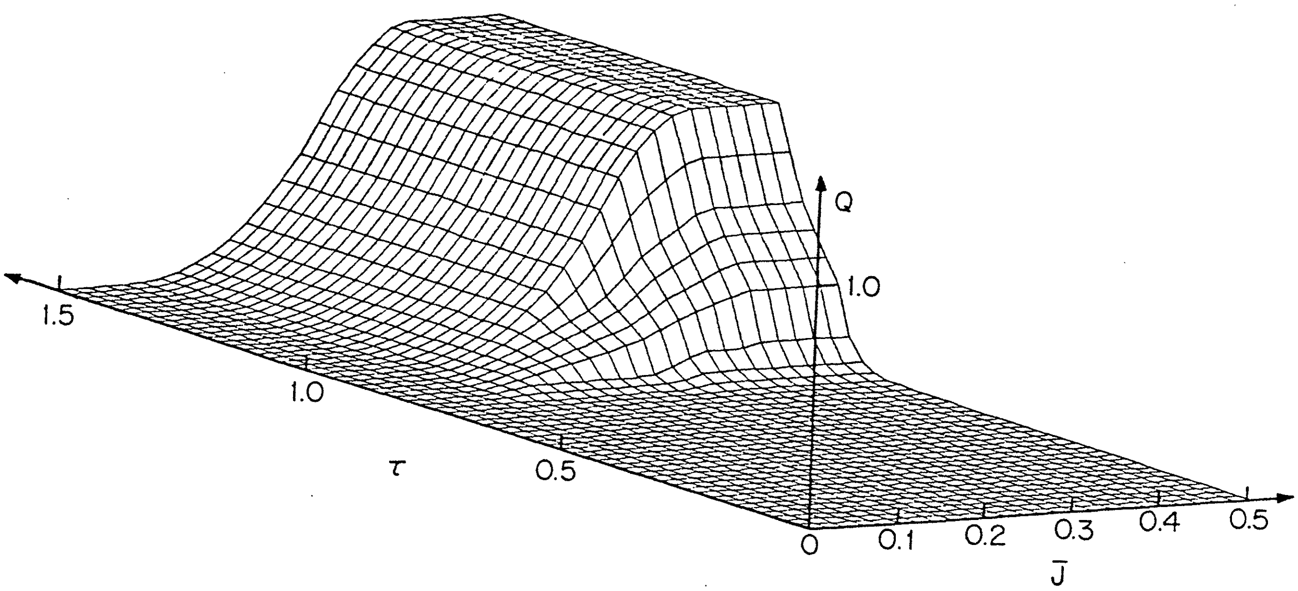
$$R^{o} = R^{1} \cap R^{2} \cap \ldots \cap R^{M} \cap R^{J} \quad (6.11)$$

The volume of R^{o}, denoted by $V(R^{o})$ is compared with that of S^{o}, $V(S^{o})$, to determine the measure of mutual consistency, Q, i.e.,

$$Q = V(R^{o})/V(S^{o}) \quad (6.12)$$

The ratio Q is a monotonic function of $\bar{J}$ and τ with minimum zero and maximum one. A null value for Q implies that no combination of strategies of the individual decisionmakers will satisfy the design specifications, while unity implies that all organizational strategies are feasible, i.e., satisfy the bounded rationality constraints and the performance specifications.

Since Q can be expressed as a function of $\bar{J}$ and τ only, it can be plotted in the three-dimensional space $(Q, \bar{J}, \tau)$. A typical plot from a three DM example [3] is shown in Figure 8.



8. Mutual consistency measure Q versus $\bar{J}$ and τ.

7. CONCLUSIONS

An analytical approach to modeling organizational structures for teams of decisionmakers supported by command, control, and communication (C^3) systems has been described. The integration of n-dimensional information theory with the data flow schema provides tools for describing the activities and interactions within each decisionmaker model, among decisionmakers, and between a decisionmaker and the supporting C^3 system. While only synchronous processing with acyclical information structures has been considered in detail, the approach shows promise for the modeling and analysis of asynchronous information processing and decisionmaking. Furthermore, the introduction of memory in the decisionmaker model, and data bases in the organizational structure has broadened the class of organizations and tasks that can be analyzed using this approach.

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