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NETWORK FLOW OPTIMIZATION IN FLEXIBLE MANUFACTURING SYSTEMS

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Abstract

The problem of choosing an optimal mix of operating strategies in a flexible manufacturing system is solved by a network flow optimization approach. Mathematical methods which exploit the structure of the problem to generate manufacturing strategies are outlined. Numerical results show that the method produces results which agree with intuition for a two-workstation system.

1. Introduction

A large proportion of manufacturing activity is at a level which does not justify dedicated automation in the form of assembly lines. In order to increase productivity in this sector of industry, flexible manufacturing systems are being designed and built.

A flexible manufacturing system consists of workstations capable of performing a number of different tasks, interconnected by a transportation system. Workpieces are loaded onto pallets at a loading station, undergo a sequence of operations at the workstations before being finally unloaded at an unloading station. The processes at the workstation are mostly automatic. At certain stations like the loading station for example, there may be some manual operations [Hughes 1977].

Several different kinds of pieces are manufactured simultaneously in the system. Each piece, has a given number of operations necessary for its manufacture. There is a choice in the system as to at which workstation each operation should be performed. Any entering workpiece therefore has the choice of several different routes or manufacturing strategies available. A manufacturing strategy for a piece assigns each operation to a workstation and also specifies the sequence of workstation visits.

In order to gain maximum output and utilization at minimum cost, the overall behavior of the system should be studied. Furthermore mathematical models and algorithms are needed which will enable controllers to make decisions affecting the system with minimum human intervention.

An important problem, which has a fundamental effect on the production rate and the utilization of the system, is the assignment of strategies to the workpieces. Given a flexible manufacturing system with a specified production mix of pieces and given the location at which all the operations can be performed in the system, one wishes to pick the optimal steady-state mix of manufacturing strategies for all the pieces being produced.

Extensive simulation studies of flexible manufacturing systems have been made [Hutchinson, 1977][Horev 1978]. They allow detailed investigation of the effects of parameter variations and strategy assignment on system performance. Solberg [1977] and Ward [1978] model the system as a closed network of queues. Steady state results which are in good agreement with simulation results and observed performance of an actual system are obtained. The use of the closed network of queues model as an analytic method of strategy assignment has been suggested by Secco-Suardo [1978].

In this report, a network flow approach is used. Rather than analyze the movement of individual pieces through the system, the aggregated flow of pieces is analyzed. Network of queues models are used to account for congestion effects at the workstations.

In Section 2, the model is presented and the optimization problem is formulated. Systems where there are non-deterministic arrivals and processing times give rise to non-linear optimization problems. The production rate of the system should be maximized but the build up of queues within the system becomes a constraint. Alternatively, a price can be put on the number of pieces in the system (the in-process inventory) and it can be included in the objective function [Kimemia and Gershwin 1978]. Deterministic systems or systems where the processing and interarrival times have small variances, give rise to linear programs. Assymptotic results for closed queueing models [Baskett et al 1975] and work-rate theorems [Chang and Lavenberg 1972] indicate that the linear programs are valid for maximizing the production rate in systems with general service time distributions.

Mathematical methods which exploit the structure of the problem in order to solve the optimization problems of section 2 are discussed in section 3. Decomposition method [Dantzig and Wolfe 1963] are used to break the linear programs into a set of strategy-generating minimum processing time sub-problems each involving only one piece type. A master problem then finds the optimal combination of strategies for all the pieces.

The lagrange multiplier method of Hestenes [1969] and Powell [1968] converts the non-linear programming problem into a series of optimization problem where a non-linear lagrangian function is minimized subject to linear flow and resource conservation constraints. An extrenal flow method [Cantor and Gerla 1974] [Defenderfer 1977] can

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then be applied to minimize the lagrangian function.

Numerical results for a two workstation system are presented in section 4. The effect of changing system parameters on the optimal strategy assignment, production rate and work-station utilization is investigated.

2. Modelling and Optimization of Flexible Manufacturing Systems

A flexible manufacturing system consists of M workstations connected by a transportation system. There may be P different types of pieces in the system simultaneously. Each piece has S_i strategies available for its manufacture. A strategy is a sequence of operations at assigned workstations which are required to complete a workpiece. Thus strategies are different sequences of the same operations.

The total number of strategies $S=\sum S_i$ may be large if there is a large number of options available in the system. It might not be worthwhile in such a case to identify in advance all possible strategies.

For each piece of type i, the quantity t_{ij}^k is the time to perform operation k at workstation j. The superscript $k=(1,\ldots,K_i)$ represents a particular operation and does not imply that there are strict precedence constraints. For example the 5 different operations required on the piece of Figure 2.1 are indicated. The only precedence constraint in this case is that operation 2 should be done before operation 3.

The definition of the operations is dependent on the capability of the workstations and the tool distribution amongst them. Operation 2, because of close tolerance requirements may need a rough cut and then finishing which might not be done at the same workstation. In this example however, identifying it as one operation specifies that it is done at a single visit to a workstation. The time t_{ij}^k to perform the operation is the total time the piece stays at the workstation including any time for changing tools.

Assuming that the t_{ij}^k matrices are available for every piece, the flow rate of type i pieces to workstation j for operation k is defined as x_{ij}^k. The system controller monitors these variables and can affect them by varying the loading rate and allocating the pieces to the strategies available.

The total arrival rate λ_j at workstation j is

$$\lambda_j = \sum_{i=1}^{P} \sum_{k=1}^{k_i} x_{ij}^k \tag{2.1}$$

The variables x_{ij}^k are related by conservation of flow equations and the production ratio requirement. Conservation of flow requires that the flow rate of type i pieces undergoing any operation k be equal to the production rate of that type of piece. This is expressed as

$$\sum_{j=1}^{M} x_{ij}^k = R_i \quad i=1,\ldots p_n,\ k=1,\ldots,k_i \tag{2.2}$$

where R_i is the production rate of type i pieces. The total production rate is given by

$$R = \sum_{i=1}^{P} R_i \quad \sum_{i=1}^{P} \sum_{j=1}^{M} x_{ij}^1 \tag{2.3}$$

The summation is carried out with k=1 for convenience. The production ratio requirement states that pieces of type i comprise a fraction α_i $(0\le\alpha_i\le1)$ of the total production. This can be expressed as a relationship between R in equation (2.3) and R_i in equation 2.2

$$R_i = \alpha_i R \quad i=1,\ldots,p \tag{2.4}$$

where α_i satisfies

$$\sum \alpha_i = 1 \tag{2.5}$$

An important performance measure is workstation utilization, U_j, defined as the probability that the workstation is occuped. This can be expressed as a function of the flow rates x_{ij}^k and operation times t_{ij}^k ,[Kimemia and Gershwin 1978].

$$U_j = \sum_{i=1}^{P} \sum_{k=1}^{K_i} x_{ij}^k\, t_{ij}^k \tag{2.6}$$

where K_i is the number of operations required to finish a type i piece.

The method of network-of-queues analysis can now be applied so as to express other system performance measures as functions of this x_{ij}^k [Solberg 1977] [Secco-Suardo 1978] [Ward 1978]. Optimization problems can then be formulated so as to pick the assignments x_{ij}^k which maximize the production rate or perhaps some other index of performance.

The application of network-of-queues analysis requires knowledge of the statistical properties of the processing times at the workstations and the arrival process into the network. This is an aspect that requires careful study on actual flexible manufacturing systems.

There are instances where it is either necessary to identify and enumerate strategies in advance or the number of strategies is not large and they can be easily identified. The optimization method in such cases is shown by way of an example in section 4.

The transportation system can be modelled as a network of nodes and arcs. The nodes are merges or diverges of arcs or the actual workstations themselves. It is assumed that the nodes are labelled so that the first M are workstations. This includes the loading and unloading stations which are labelled 1 and M respectively. Operation 1 in this case becomes the loading operation and K_i the unloading operation for a type i piece. This ensures well defined flows for each type of piece from the loading to the unloading station. Define r_{in} as the flow rate of type i pieces on arc n of the network. Then from the definitions

$$\sum_{n\varepsilon A(j)} r_{in} = \sum_k x_{ij}^k \qquad 1\leq j\leq m \tag{2.7}$$

Where A(j) is the set of arcs leading to node j. The conservation of flow at a network node j is expressed as

$$\sum_{n\varepsilon A(j)} r_{in} = \sum_{m\varepsilon D(j)} r_{im} \qquad j>m \tag{2.8}$$

Where D(j) is the set of arcs carrying pieces away from node j.

The in-process inventory I(r,x), which is the average number of pieces in the network, can be expressed as a function of the arc flow rate r_{ij}, and the flow rates through the workstations.

$$I(r,x) = \sum_j q_j(x) + \sum_i \sum_{j>m} t_j r_{ij} \tag{2.9}$$

The average queue length $q_j(\underline{x})$ at workstation j is evaluated by applying queueing network theory. The average number of pieces on the transportation system depends on the time t_j to traverse each arc in the network.

An optimization problem can now be formulated, which maximizes the total production rate subject to a constraint imposed on the average level of in-process inventory, which is required to be less than a certain given value Q. In operating a flexible manufacturing system, the average level of in-process inventory is an important quantity. In general the in-process inventory is a non-linear function of the production rate. Although it is desirable to have the maximum possible production rate, the average level of in-process inventory should not as a consequence become too high. In this formulation this is controlled by setting a level Q and maximizing the production rate subject to the constraint that the average level of in-process inventory does not exceed Q.

NLP 2.1

$$\text{Maximize} \quad \sum_{i=1}^{p} x_{i1}^{1} \tag{2.10}$$

subject to (2.2), (2.4), (2.7), (2.8)

$$\text{and} \quad u_j \leq 1 \qquad j=1,\ldots,m \tag{2.11}$$

$$I(r,x) \leq Q \tag{2.12}$$

$$\sum_i^n r_{ij} \leq d_j \quad r_{ij}, \qquad j>m \tag{2.13}$$

$$r_{ij},\ x_{ij}^k \geq 0 \tag{2.14}$$

The production rate is given by (2.10) and is the total flow rate out of the loading station. The limited capacity of the workstations is expressed by (2.11). The limit on the average level of in-process inventory is given by (2.12). Constraint (2.13) expresses any arc-capacity constraints that might exist in the transportation system.

The optimization problem NLP 2.1 is general and can be adapted to specific situations [Kimemia and Gershwin 1978]. A particular simplification occurs when the processing times and arrival processes into the system are deterministic. If in addition the transportation system has a large enough capacity so that it does not present a constraint on the production rate, the following linear program which maximizes the production rate R results,

LP 2.1

$$\text{Maximize} \quad R \tag{2.15}$$

subject to (2.2), (2.4), (2.11)
and

$$x_{ij}^k \geq 0 \tag{2.16}$$

By applying asymptotic results for closed networks of queues [Gordon and Newell 1968] [Secco-Suardo 1978], results for general queueing networks [Baskett *et al* 1975], the operational results of Denning and Buzen [1977], and the work rate theorems of Chang and Lavenberg [1972], it is found that LP 2.1 also maximizes the production rate for a general class of systems [Kimemia and Gershwin 1978]. The program remains unchanged as the variance of stochastic processes goes to zero. The deterministic case can thus be viewed as a limiting case of a class of stochastic systems.

3. Optimization Techniques for Flexible Manufacturing systems

The general optimization problem can be stated as NLP 3.1

$$\text{minimize} \quad f(x) \tag{3.1}$$

subject to

$$\sum_{i=1}^{p} T_i x_i \leq 1 \tag{3.2}$$

$$c_i x_i = \alpha_i R \tag{3.3}$$

$$A_i x_i = 0 \qquad i=1,\ldots,p \tag{3.4}$$

$$g(x) \leq 0 \tag{3.5}$$

$$x_i \geq 0 \tag{3.6}$$

Where the vector x contains all the flow variables x_{ij}^k and r_{ij} in the problem. This vector is partitioned such that the flow variables concerning only pieces of type i are contained in x_i. The matrix A_i has elements 1, 0 and -1 and defines the flow conservation constraints for type i pieces. The matrix T_i contains the elements t_{ij}^k. The ratio requirement constraints are expressed in (3.3) where c_i is a row vector of 1 and 0 elements such that $c_i x_i$ is the production rate R_i of type i pieces expressed in equation (2.2). The functions f(x) and g(x) in (3.1) and (3.5) may or may not be convex. They express the performance index and the non-linear constraint on the system respectively.

In order to solve this problem efficiently, it is necessary to exploit it's structure. It should be noted that given an optimal solution x_i to the problem, the routing problem and the ordering of operations is not resolved. A solution method therefore should not only give the solution x_i, but it should also provide routing information by generating strategies.
If the set Ω_i is defined as

$$\Omega_i = \{x_i : A_i x_i = 0,\ C_i x_i = \alpha_i,\ x_i \geq 0\}$$

the optimization problem may be stated as
NLP 3.2

$$\underset{x_i \varepsilon \Omega_i}{\text{minimize}}\ f(x) \tag{3.7}$$

subject to $g(x) \leq 0$

The augmented Lagrangian method of Hestenes [1969] and Powell [1968], is used to convert the non linearly constrained optimization problem into a sequence of linearly constrained problems. A number of methods which exploits the convex structure of the set Ω_i to obtain feasible descent directions may then be employed [Assad 1975]. The Cantor-Gerla algorithm [1974] solves the problem over a convex combination of the extreme points of the polygon Ω={set of all feasible flows x}. Let x^j be an extreme points of Ω. Any $x \varepsilon \Omega$ can be expressed as a convex combination of x^j

$$x = \sum_j w_j x^j \tag{3.8}$$

with

$$w_j \geq 0$$

and

$$\sum_j w_j = 1 \tag{3.9}$$

Thus given a set of extreme points x^j, the non linear optimization is carried out over the variables w_j. This reduces the size of the problem considerably.

To generate the extreme points, the flow generating sub-problem is solved. Let $\hat{x}$ be the value of x which minimizes the lagrangian function in the convex hull of x^j $j=1,\ldots,n$. The flow generating linear program is
LP 3.1
minimize

$$\sum_{i=1}^{p} v_i x_i \tag{3.10}$$

subject to (3.2), and $x_i \varepsilon \Omega_i$ $i=1,\ldots,p$ where v_i is. The gradient of the lagrangian function at $x=\hat{x}$

The structure of LP 3.1 allows the Dantzig Wolfe [Dantzig 1963] price directive decomposition principle to be applied. Any $x_i \varepsilon \Omega_i$, can be expressed as a convex combination of the extreme points x_i^j of Ω_i. This gives rise to the following master problem in the variables q_j

LP 3.2

$$\text{minimize} \quad \sum_{i=1}^{p} v_i \sum_j q_j x_i^j \tag{3.11}$$

$$\text{subject to} \quad \sum_{i=1}^{p} T_i \sum_j q_j x_i^j \leq 1 \tag{3.12}$$

$$q_j \geq 0 \tag{3.13}$$

The columns of the master problem are generated as required by solving the following set of linear programs, one for each type of piece
LP 3.4

$$\underset{x_i \varepsilon \Omega_i}{\text{minimize}}\ (v_i - \pi T_i) x_i \tag{3.14}$$

where π is a vector of dual variables associated with the master problem. The column-generating program LP 3.4 has the following interpretation. The vector $(v_i - \pi T_i)$ in (3.17) assigns a cost to each arc in the transportation system and to performing each operation at a permissible workstation. The problem is one of finding a minimum cost path through the system which assigns each operation to a permissible workstation. This is a constrained shortest path problem and can be solved by a labelling type algorithm [Kershenbaum et al. 1976]. The solution x_i^j is thus a strategy. At this stage, the ordering of the operations is determined by the shortest path solution and can be stored.

The extreme point solution x^{n+1} which results is thus a weighted combination of the strategies generated by the solution of LP 3.1. The non linear optimization then finds the optimal mix of the extremal flow-vectors x^j $j=1,\ldots$.

The linear program LP 2.1 has the same structure as LP 3.1 and can be solved using the same decomposition principle. In this case however the cost function in the strategy generating sub-problem can be written for each type of piece as

$$\sum_{j=1}^{M} \sum_{k=1}^{K_i} \pi_j t_{ij}^k x_{ij}^k \tag{3.15}$$

The problem has the same interpretation as LP 3.4, and is easily solved. For each operation k find

$$\pi_s t_{is}^k = \underset{j}{\text{minimum}}\ \pi_j t_{ij}^k \tag{3.16}$$

then set $x_{is}^k = \alpha_i$ and $x_{ij}^k = 0$ for $j \neq s$.

4. Numerical Results for a Two Workstation System

Consider the system depicted in figure 4.1. The workstations and the loading station have exponentially distributed service times with a rate u_i at station i. The service time is independent of the type of piece being processed. Arrivals into the system are assumed to form a poisson process. An arrival in this system is considered to be the command to load a piece issued by the system controller. An assumption made is that there are always "raw" pieces and empty pallets available at the loading station.

There are two types of pieces being manufactured. The first needs one operation which can be done either at workstation 1 or workstation 2. The second type of piece requires two different operations, one at workstation 1 and one at workstation 2. The two operations can be done in any order. The four possible strategies two for each kind of piece are summarized in figures 4.2 and 4.3. The circled number is the workstation, below it is the operation time. A strategy is shown as a path between the loading station L and the unloading station U. The variables y_ℓ, $\ell=1,\dots,4$ represents the flow of strategy ℓ pieces into the network.

The ratio requirement is that two type 2 pieces are produced for every type 1 piece. This is expressed as

$$(y_1+y_2) - 0.5(y_3+y_4)=0 \qquad (4.1)$$

The total production rate is,

$$R = y_1 + y_2 + y_3 + y_4 \qquad (4.2)$$

The pieces travel at constant speed on the transportation system. The average travel time on each arc is taken to be τ (independent of the arc). The average number of pieces on the transportation system can be written down as

$$\tau_1 y_1 + \tau_2 y_2 + \tau_3 y_3 + \tau_4 y_4 \qquad (4.3)$$

τ_ℓ is the average time a strategy ℓ piece spends on the network. The τ_ℓ are derived by noting from figure 4.1 how many arcs each piece traverses in going from the loading to the unloading station, this is summarized in figure 4.4. This assumes that no piece is ever rejected from a workstation. This assumption is consistent with the assumption that the workstations have infinite buffer capacity.

The system is modelled as an open network of queues. The results of Jackson [1963] that each station behaves as an independent m/m/1 queue in the steady state can be applied. The average queue length q_j at workstation j is given by [Kleinrock 1975]

$$q_j = \frac{\sum_{\ell\epsilon M(j)} y_\ell}{u_j - \sum_{\ell\epsilon M(j)} y_\ell} \qquad j=0,1,2 \qquad (4.4)$$

where M(j) is the set of strategies that use station j. Combining (4.3) and (4.4) gives the average level of inprocess inventory

$$I_y(y) = \sum_{j=0}^{2} q_j + \sum_{\ell=1}^{4} \tau_\ell y_\ell \qquad (4.5)$$

The optimization problem which is equivalent to NLP 2.1 is

$$\text{NLP} \quad \text{Maximize } R \qquad (4.6)$$

subject to (4.1) and

$$u_j - \sum_{\ell\epsilon M(j)} y_\ell \leq 1 \qquad j=0,1,2 \qquad (4.7)$$

$$I_y(y) \leq Q \qquad (4.8)$$

$$y_\ell \geq 0 \qquad (4.9)$$

The value of Q is the desired average level of in-process inventory.

The parameter values used are shown in table 4.1. In the first set of results, for fixed system parameters, the value Q is varied from a value of 2 upwards.

Type 2 pieces always follow strategy 3. That is they go to workstation 1 first and then to workstation 2. This is because using strategy 4 increases the in-process inventory without a corresponding increase in production. The proportion of type 1 pieces that go to workstation no.1 (referred to as the optimal split) is shown in figure 4.5 as a function of Q. When the inprocess inventory is low the optimal split is high since 1 is the fastest station. As the number of pieces in the network increases, more are diverted to workstation 2. Secco-Suardo [1978] found a similar result for a system modelled as a closed network. In his case the optimal split depends on the number of pallets available. As can be expected, the production rate increases with Q figure (4.6). A saturation effect is in evidence. The maximum possible production rate when the restriction on the average level of in-process inventory is lifted (i.e. as $Q\to\infty$) is 6.6 pieces per hour. Both stations are then fully utilized.

The effect of increasing Q on the utilization at the workstations is shown in figure 4.7. Thus as the average level of inprocess inventory is increased, the optimal split changes in a way that keeps the workstations balanced in the sence that their levels of utilization are approximately equal despite the difference in their service rates.

In the second set of results, the average inprocess inventory is required to be 10. The speed of workstation no. 1 is varied from 2 to 10 2 pieces per hour. The optimal split for type 1 pieces as a function of the speed of workstation no.1 is shown in figure 4.8.

If the difference in the speeds of the two workstations is great, all of the type 1 pieces go to the faster station. This would indicate that in such a situation it is not worthwhile making the slower station flexible. In the range where the speed of workstation no.1 is between about $\pm$ 40% of the speed of station no. 2, the optimal split changes from zero at the lower speed to unity at the higher speed. The effect on the utilization is shown in figure 4.9. The change in the optimal split keeps the utilizations of the two workstations close to each other. For this system at least the optimization produces a balanced load on the two workstations. The production rate increases with the speed of workstation no.1, (figure 4.10). In the speed range where the optimal split is changing, the increase is linear. Outside that range, the bottle neck effect of the slower station is evident.

5. Conclusion

A network flow optimization approach to the problem of choosing the best mix of operating strategies in a flexible manufacturing system has been presented. An operating strategy is defined as a sequence of operations required to manufacture a workpiece. All possible routes through the system do not have to be identified in advance. The procedure of section 3 generates the paths for each type of piece as part of the solution. The solution gives the optimal proportion of each type of piece to be manufactured under each of the available strategies. Only a subset of the strategies need be considered in order to find the optimal solution.

The method of section 3 solves the problem by a decomposition technique which leads to a set of minimum cost sub-problems each involving only one type of piece. The sub-problem solutions are coordinated by linear and non-linear programs with fewer variables than the original problem.

Numerical results as shown in figures 4.5-4.10 for a two-workstation system are intuitively pleasing. For this case, choosing the routing so that the utilizations of the two workstations are close is optimal when the difference in the speeds of the two workstations is not great. However, when the speed difference at the two workstations is large, the optimal strategy assignment does not produce equal loads at the two stations. The optimal routing as expressed by the optimal split for type 1 pieces is found to be sensitive to the relative speeds of the two workstations.

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Workstation	speed in pieces/hr.	Average Operation time (minutes)
loading	30	2
1	5	12
2	6	10

Average travel time on each arc 1.2 minutes

Table 4.1: System Parameters.

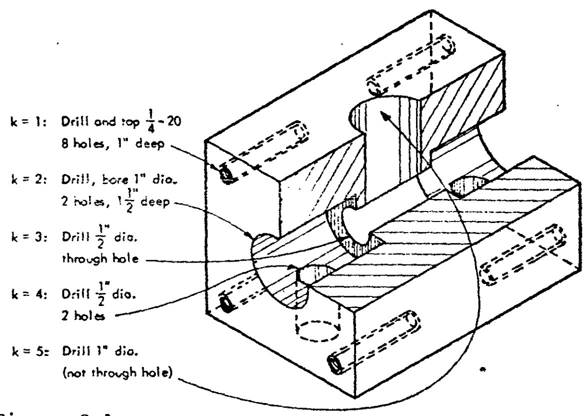


Figure 2.1

strategy	1	2	3	4
τ_L	4τ	4τ	3τ	9τ

Figure 4.4: Average time τ_L on the transportation network for each strategy.

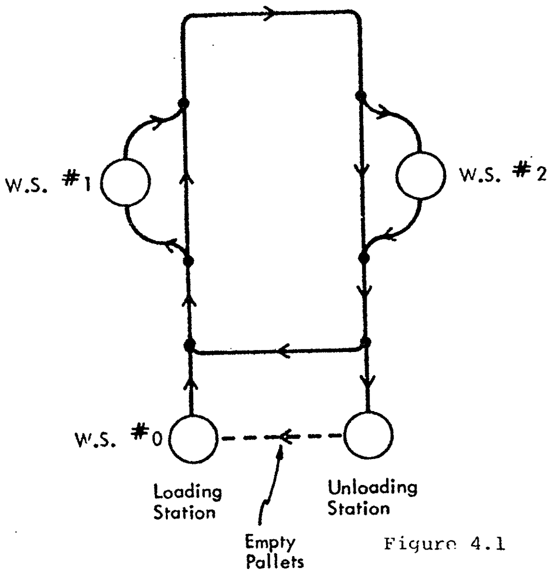
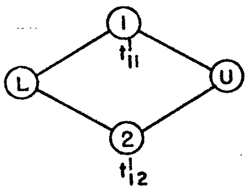
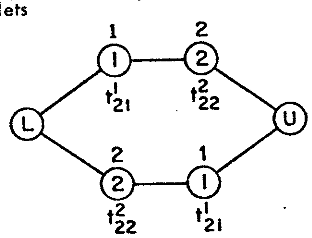


Figure 4.1



strategies 1 and 2

Figure 4.2



strategies 3 and 4

Figure 4.3

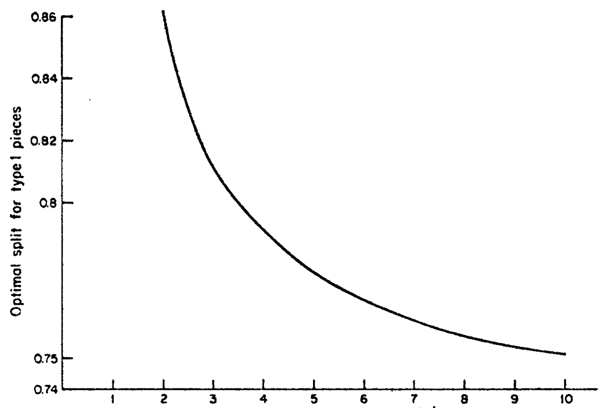


Figure 4.5

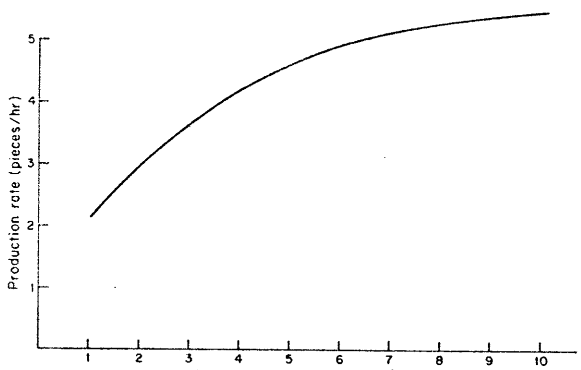


Figure 4.6

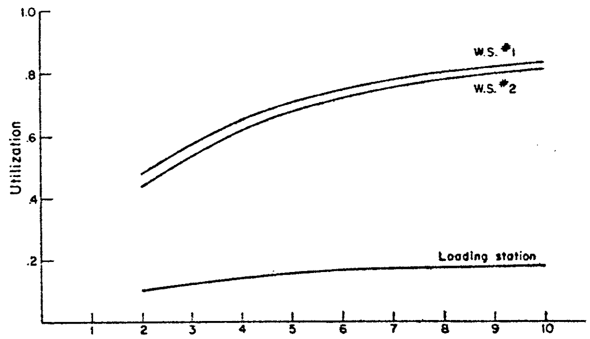


Figure 4.7

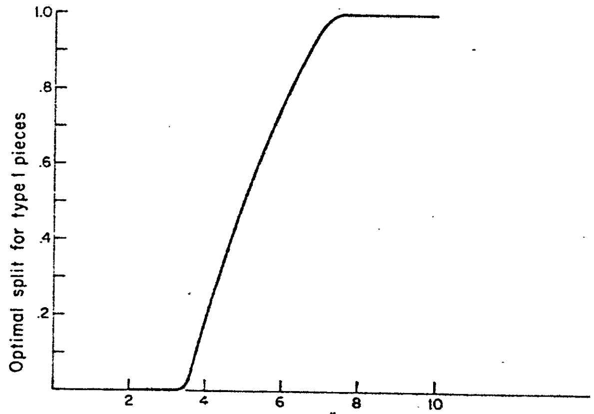


Figure 4.8

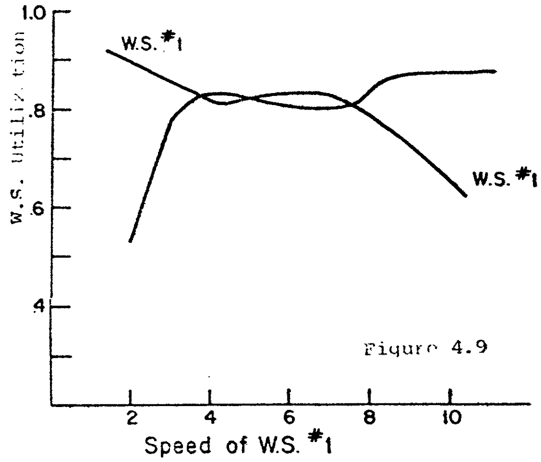


Figure 4.9

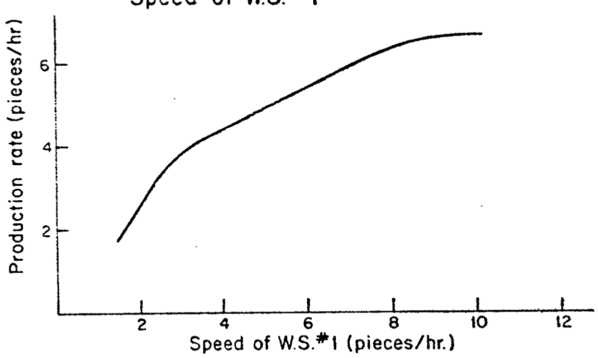


Figure 4.10