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An Interim Report of Research on

STOCHASTIC AND ADAPTIVE SYSTEMS

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by

Professor Michael Athans
Professor Sanjoy K. Mitter
Dr. Lena Valavani

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Dr. Joseph Bram
Directorate of Mathematical and Information Sciences
Air Force Office of Scientific Research (AFSC)
Bolling Air Force Base
Washington, D.C. 20332

Laboratory for Information and Decision Systems
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

SUMMARY

This interim report describes research advances on stochastic and adaptive systems by faculty, research staff, and students of the M.I.T. Laboratory for Information and Decision Systems with support provided by the United States Air Force Office of Scientific Research under Grant AFOSR 77-3281C. The principal co-investigators were Professors Michael Athans and Sanjoy K. Mitter. The Grant Monitor was Dr. Joseph Bram. The time period covered by this report is February 1, 1980 to January 31, 1981.

Substantial progress is reported in the areas of nonlinear filtering theory, stochastic control, and adaptive control systems.

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1. INTRODUCTION

The research we have conducted over the past several years and in particular during the period February 1, 1980 to January 31, 1981 has been concerned with fundamental aspects of controlling linear and non-linear stochastic systems in the presence of measurement and parameter uncertainties. In case the uncertainties reside in the state description of the physical system and measurements then we refer to the control problem as a stochastic control problem. If in addition there are parameter uncertainties, then the problem is referred to as an adaptive control problem. This is because in addition to state estimation some form of parameter identification scheme will be needed and almost always the control and estimation-identification functions will interact in a non-trivial nonlinear and time-varying way.

A subproblem of the stochastic and adaptive control problem is the state estimation problem. Suppose for a moment we make the assumption that there are no parameter uncertainties present in the dynamical description of the state of the system. Then all the probabilistic information that one can extract about the "state" of the system on the basis of noisy measurements of the state is contained in the conditional probability density of the state given the observations. Indeed this is the probabilistic state of the joint physical-measurement system. The recursive computation of the probabilistic state is the state estimation problem. If this could be done, then one could look for the best controller as a function of this probabilistic state, best being judged in terms of a suitable performance criterion.

We believe that we have only scratched the surface in understanding some of these fundamental issues in modelling, estimation, and control of linear systems with multiplicative uncertain parameters and,more,generally non-linear systems. Our proposed research deals with those issues that we feel are both fundamental and relevant from an engineering point of view.

We would like to mention that many of the theoretical questions we are posing have been motivated by problems which are of importance to the U.S. Air Force. Modelling, estimation, and identification is clearly fundamental for application of control theory, especially the theory developed in the last fifteen years or so. Kalman filtering has found widespread application in the guidance and control of aerospace vehicles, laser tracking and pointing systems and missile designs. However, very few applications of non-linear filtering are known. The extended Kalman filter which is often used in practice is basically not understood from a scientific point of view. The success of passive tracking algorithms requires advances in the state of the art.

The control of future aircraft, possibly statistically unstable, is a problem of continuing importance from the point of view of designing adaptive stability augmentation systems that operate reliably over a wide operating envelope. Similarly, the control of advanced jet engines, whose dynamical characteristics change rapidly with operating conditions, poses a challenging problem if one wishes to design a control system which accomodates commanded thrust level changes rapidly, while maintaining fan and compressor stability margins, and minimizing excess temperature durations. Furthermore, adaptive control appears particularly attractive to handle less predictable things such as changes in atmospheric gust (particularly wind shears on final approach) and changes to aircraft weight and mass center (e.g., due to unusual loading distribution or dropping of external stores).

The use of optical sensors and laser pointing and tracking systems leads to new problems in both stochastic estimation and control. For example, in a laser tracking system the "glint" phenomenon corresponds to noise in the measurement equation that is multiplicative rather than additive in nature. Any stochastic interception problem involving a maneuvering target and with "angle only" sensors corresponds to a very complex nonlinear estimation and control problem.

Our research progress during the past year is described under two main headings. Section 2 contains a discussion of progress in nonlinear filtering and stochastic control. Section 3 contains a discussion of progress on adaptive control systems. Section 4 is a list of references, while Section 5 contains a list of publications published or generated during the past year, supported in part or in full by AFOSR Grant 77-3281.

2. NONLINEAR FILTERING AND STOCHASTIC CONTROL

2.1 Nonlinear Filtering

Significant progress has been made in the area of non-linear filtering by Professor Mitter and his graduate students, Daniel Ocone and Larry Vallot. This work has been reported in various papers and two theses (References [1] to [10]) all supported by this grant. The progress that has been made is on several fronts:

(i) The innovations problem of non-linear filtering in the form conjectured by Kailath has been finally settled [1].

(ii) A new attack on the non-linear filtering problem based on the analysis of the Zakai equation, proposed independently by Brockett and Mitter has had considerable success. In fact, a major part of the summer school sponsored by NATO in Les Arcs, France held in July 1980 was devoted to ideas proposed in [2].

(iii) A rigorous formulation of the variational principle underlying non-linear filtering has been presented in [3]. This gives new insight into questions of approximation for non-linear filters.

(iv) It has been shown that there are indeed deep connections between non-linear filtering and recent ideas of quantum physics from a stochastic viewpoint ([2], [3]).

(v) In [8] it has been shown how these recent developments can be used to obtain sub-optimal filters (with guaranteed performance improvement) for bilinear stochastic systems.

Previous work on non-linear filtering has concentrated on the equation for the conditional density of the state given the observations over a time interval. This equation, the so-called Kushner-Stratanovich equation, is a non-linear stochastic partial differential equation with an integral term. In contrast, the recent work of Mitter has concentrated on the equation for the unnormalized conditional density, the so-called Duncan-Mortansen-Zakai equation. This is a bilinear stochastic partial differential equation which has as its input the observation process and, hence, is considerably simpler to analyze than the Kushner-Stratanovich equation. This equation acts as a universal recursive filter, in the sense that any conditional statistic can be computed by an integration with respect to this unnormalized density and an appropriate normalization.

A general program for analyzing this equation and its invariance properties was proposed in [2]. In this program a certain Lie algebra of operators with two generators $L_0^* - \frac{1}{2} L_1^2$ and L_1 where L_0^* is the formal adjoint of the generator of a diffusion process and L_1 is a multiplication operator has an important role to play. If this Lie algebra is finite dimensional then it is an indication that the Zakai equation has a finite-dimensional statistic. In fact, the n-dimensional Kalman filtering problem gives rise to a (2n+2)-dimensional Lie algebra which is an n-dimensional generalization of the Oscillator algebra of some fame in quantum physics. Indeed the Kalman filter occupies the same role as the Harmonic oscillator or the free field in quantum physics ([2], [3]). A by-product of this approach is that the Zakai equation for the Kalman filter has a finite-dimensional statistic even if the initial density is non-guassian. Soon after these ideas became known Benes exhibited finite-dimensional filters for a class of filtering problems with non-linear drift and it was shown by Mitter [2] that the filtering problems of Benes were "gauge equivalent" to the Kalman filter. In the doctoral dissertation of D. Ocone [7] it is proved that for scalar diffusion processes defined over the whole real line, the Kalman filtering problem and the gauge equivalent Benes filtering problems were the only problems admitting finite-dimensional filters.

In [2] it was shown that the Lie algebra of operators corresponding to a large class of filtering problems was infinite-dimensional and simple. In particular, it was shown that the Lie algebra of operators corresponding to the cubic sensor problem (extensively investigated by R.S. Bucy) was the Weyl algebra (the Lie algebra of partial differential operators of all order) which is known to be infinite-dimensional and simple. This result was obtained independently by M. Hazewinkel and S. Marcus. It was conjectured in [2] that no finite-dimensionally computable statistic existed for this problem. This has now been proved by H. Sussman.

In proving the above result, one constructs a proof of the Ansatz of R.W. Brockett, namely, that if there exists a finite-dimensional filter than there must exist a certain Lie Algebra homomorphism between the Lie Algebra of operators corresponding to the filtering problem and the Lie algebra of certain vector fields. For the cubic sensor problem this was done by M. Haxewinkel and S. Marcus. To prove the theorem on the non-existence of finite-dimensionally computable statistic one needs a rigorous proof of the existence and uniqueness of solutions of the Zakai equation. In joint work of Mitter with J.S. Baras and D. Ocone (preliminary report in [9]) this has been accomplished when the observation map is a polynomial.

In D. Ocone's doctoral thesis [7], the problem of expanding the unnormalized conditional density in a Ito-Wiener Series has been solved. Ocone also obtains a multiplication formula for multiplying two multiple Ito integrals and subsequently expanding it in an Ito-Wiener Series. He has also obtained integral equations for the kernels corresponding to best polynomial filters and shown existence and uniqueness of solutions.

In L. Vallot's S.M. thesis [8], a theoretical and experimental investigation of filtering for bilinear systems has been undertaken. As examples he has considered the phase-lock loop problem and also the cubic sensor problem. He has shown how tensor and Fock space ideas expressed in [2] can be usefully utilized to obtained sub-optimal filters which guarantee performance improvement (in the mean square sense).

In summary, a framework has been established for the theoretical and experimental investigation of a broad class of non-linear filtering problems. It is believed that these ideas can be used to design practical non-linear filters and evaluate their performance.

2.2 On the Relationship between Non-Linear Filtering and Stochastic Control

It has long been felt that there is a variational principle underlying non-linear filtering. Indeed in the early sixties, a formulation of the non-linear filtering problem as a non-linear least-squares problem was proposed first by Bryson and Frazier and, subsequently, by Mortensen. These formulations were, however, deterministic formulations and had the inherent mathematical difficulty of working with white noise, the formal differential of the Wiener process (which is almost surely, not differentiable). It was also known that these formulations gave the correct answer for the

Kalman Filtering problem.

In [3], Professor Mitter (joint work with Professor Fleming of Brown University) has shown that associated with the Zakai equation of non-linear filtering is a Bellman-Hamilton-Jacobi equation which corresponds to a perfectly observable stochastic control problem. The solution of one equation leads to a solution of the other. In particular, the Kalman filtering problem corresponds to a perfectly observable linear quadratic gaussian stochastic control problem. Indeed, this explains in the clearest possible way the duality principle originally proposed by Kalman. The difficulty of the non-differentiability of the Wiener paths is avoided by working with the robust form of the non-linear filtering equations as developed by Clark and Davis (M.H.A. Davis of Imperial College was a visitor at M.I.T. in 1979 and was partially supported by this grant).

These ideas have important consequences. It opens the way for developing algorithms for non-linear filtering using stochastic control ideas. It, for the first time, explains how the extended Kalman filtering algorithm really works. Finally, it has important implications in the development of a theory of stochastic dissipativeness.

2.3 Concluding Remarks

The developments in non-linear filtering reported here have many consequences. For example, parameter identification problems can be considered to be special cases of non-linear filtering problems and the ideas discussed here will be important in obtaining a better understanding of identification. It is also felt that the relationship between non-linear filtering and stochastic control exhibited here will play a role in obtaining a better understanding of stochastic adaptive control.

3. ADAPTIVE CONTROL

3.1 Introduction

Significant breakthroughs in our understanding of adaptive control systems have taken place during the past year. The work was carried out by Dr. L. Valavani, Prof. Athans and Mr. C. Rohrs, a doctoral student. Partial results are documented in Refs. [11], [16], and [18].

During the past year theoretical results, backed up by digital computer simulation studies, that relate to the performance of several adaptive control algorithms that are based on model reference techniques were studied. Special emphasis was placed upon the transient performance of these algorithms and the implications upon the bandwidth of the closed-loop adaptive system. The conclusions are as follows:

(1) During the transient phase of the adaptation procedure, the control signal is characterized by excessive high frequency content. Novel analytical studies based upon stability theory can be used to predict this high frequency oscillatory behavior, which depends both upon the amplitude and frequency of the reference input.

(2) During the transient adaptation phase the excessive control loop bandwidth is detrimental to system performance, because it can excite unmodeled high-frequency dynamics and lead to instability.

(3) Similar effects can occur in the presence of stochastic inputs (plant noise and measurement noise).

The analytical techniques that have been developed are constructive, so that modifications to the existing algorithms are suggested to overcome the practical shortcomings of the existing algorithms.

3.2 Background

The development of a systematic design methodology for the synthesis of practical self-adjusting systems which can maintain first stability and second performance improvement in the presence of rapid and large variations in the open-loop dynamics, represents a very important generic goal in control system engineering, in view of its wide applicability to industrial and defense applications. The so-called "adaptive control problem" has received attention by theoreticions and practitioners alike for the past 25 years. About a dozen books and hundreds of articles have been devoted to the subject; different philosophies have been developed (model reference adaptive control, self-tuning regulators, dual control methods, multiple-model adaptive control etc.) and a variety of (mostly academic) examples have been simulated.

In the last few years the theory of adaptive control has matured immensely. Diverse adaptive algorithms have been unified under the same underlying principles [11] and global asymptotic stability proofs have been obtained for a large class of such algorithms [12, 13, 14]. In spite of the intense research activity, however, a significant gap exists between the available methodologies and the potential applications.

As a rule, these algorithms exhibit rather poor behavior in practice. More specifically, although certain versions of the self-tuning regulator have been shown to work on systems with slow dynamics, such as ocean tankers and some industrial processes, in general serious problems can arise when most of the existing algorithms are applied to systems characterized by fast time-constants, oscillatory or unstable dynamics and/or unmodeled high frequency dynamics, and/or signficant stochastic disturbances and noisy sensor measurements. Moreover, all available algorithms can exhibit transient undesirable behavior which makes them dangerous to use in practical applications, e.g. control of aircraft [15]. For the most part, the difficulty in applying theoretical results to real problems, with reasonable margins of confidence, can be attributed to the fact that the hypotheses needed to rigorously prove the theory are too restrictive from a practical point of view. On the other hand, by adopting a more engineering oriented approach to the formulation of adaptive control theories, we have a better chance of understanding the key issues on which to base design improvements. A systematic study and analysis of the convergence properties of already existing adaptive algorithms is the first step in this direction.

3.3 Research Progress

3.3.1 Motivation, Focus, and Summary

The research carried out in the last year was motivated mainly from the considerations above. In the process, it became obvious that the global black box approach should be abandoned and that frequency domain methods should be incoporated with time-domain adjustment mechanisms. Besides, so far, preliminary (limited) experience with most adaptive schemes had shown that they tended to exhibit undesirable high-frequency (behavior characteristics) in the early stages of adaptation. In addition, for practical and reliable design, one should always look for good approximations in the frequency-domain with due consideration for cross-over frequency and high frequency roll-off. However, in the case of the present adaptive schemes, roll-off circuits (at the plant input) cannot be introduced without violating the relative degree assumption which is critical in their stability analysis. Such a modification would change the overall structure of adaptive design so far. On the other hand, future "good" algorithms should also be able to handle plants with oscillatory or unstable poles, lots of zeros, unmodeled dynamics, sensor noise and other disturbances during the entire adaptation process. Therefore, special emphasis was placed upon the transient performance of the adaptive algorithms, during which most of them exhibit rather poor behavior and on the implications upon the bandwidth of the closed-loop adaptive system. Detailed digital computer simulation studies of several adaptive algorithms - primarily based on model reference techniques - were carried out during the past year; the conclusions of this work can be summarized as follows:

(1) During the transient phase of the adaptation procedure, the control signal is characterized by excessive high frequency content. Novel analytical studies based upon stability theory can now be used to predict this high frequency behavior, which depends both upon the amplitude and the frequency of the reference input.

(2) During the transient adaptation phase the excessive control loop bandwidth is detrimental to system performance, because it can excite unmodeled high-frequency dynamics and lead to instability.

(3) Similar effects can occur in the presence of stochastic inputs (plant noise and measurement noise). The analytical techniques that have been developed are constructive, so that modifications to the existing algorithms are suggested to overcome the practical shortcomings of the existing algorithms.

3.3.2 Discussion of Results

The first part of our research is described in more detail in [16]. There, results of digital simulations are reported for cases of accurate modeling, modeling errors which exclude high frequency dynamics and with observation noise present at the output. It was observed that, even when the system was properly modeled, the plant was subjected by the control input to high frequencies of considerable magnitude, during the early stages of adaptation. This generation of high-frequency control inputs is an inherent feature of current adaptive algorithms due to a multiplicative effect both in the parameter adjustment law (i.e., eqns.2c), 2d following) as well as in the control input synthesis (i.e., eqn. 1c), since time-varying signals are used in both the above procedures. Clearly, such high-frequency inputs are undesirable, as they will inevitably excite unmodeled dynamics that can in turn cause instability.

Secondly, it was also shown [16] that, in the presence of an unmodeled high-frequency pole, the adaptive system exhibited wildly oscillatory, and even unstable, closed loop behavior.

Lastly, the presence of even a small amount of observation noise prevented the closed loop system from converging to the desired model by causing it to slowly drift away to an increasingly higher bandwidth system. As a consequence, high frequency unmodeled dynamics will eventually be excited even at an advanced stage of the adaptation process. This increase in the bandwidth of the closed-loop system was also observed by Narendra and Peterson [17] in terms of a scalar example.

For the second part of our work reported in [18], in order to obtain practical insights as well as analytical results, a simple scalar system was first studied in a systematic way. The equations that describe it are given below:

Plant Dynamics: $\dot{y}(t) = -\alpha y(t) + \beta u(t)$ (1.a)

Model Dynamics: $\dot{y}_m(t) = -ay_m(t) + br(t)$ (1.b)

where the plant control is given by $u(t) = \theta_1(t)y(t) + \theta_2(t)r(t)$, (1.c)
with $r(t)$ being the reference input and $\theta_1(t)$, $\theta_2(t)$ adjustable (control) parameters used in the control signal synthesis. The first algorithm for the control parameter adaptation studied was that proposed by Narendra and Valavani [19]; its scalar version consists of the following system of nonlinear differential equations for the output error $e(t)$ and parameter errors $\phi_1(t)$, $\phi_2(t)$ as well as the plant state $y(t)$:

$$\dot{y}(t) = -\alpha y(t) + \beta u(t) \qquad (2.a)$$

$$\dot{e}(t) = -ae(t) + b\phi_1(t)y(t) + b\phi_2(t)r(t) \qquad (2.b)$$

$$\phi_1(t) = -\gamma_{11}y(t)e(t) - \gamma_{12}e(t)r(t) \qquad (2.c)$$

$$\dot{\phi}_2(t) = \gamma_{21}y(t)e(t) - \gamma_{22}e(t)r(t) \qquad (2.d)$$

where $e(t) = y_m(t) - y(t)$ and $\phi_1(t)$, $\phi_2(t)$ are the parameter errors; γ_{ij}, $i,j = 1,2$, are constants and such that $\gamma_{ii} > 0$, $\gamma_{11}\gamma_{22} > \gamma_{12}\gamma_{21}$.

A stability-based time-varying analysis of the properties of the nonlinear time-varying system described by eqns. (2) results in the development of a new analytical method that explains and even predicts the behavior of the adaptive system observed in the simulations. The analytical results thus obtained are valid in the vector case as well; they also provide further insights into the mechanism of adaptation. Due to the inherent nonlinearity of the closed-loop, it was found that the behavior of the system is sensitive to both the magnitude of the reference input and the input frequencies. A large reference input magnitude can create arbitrarily high frequency content in the adjustable (control) parameters $\theta_1(t)$ and $\theta_2(t)$ and, therefore, also in the control signal $u(t)$.

Moreover this analysis demonstrates that, $\phi_1(t)$ and $\phi_2(t)$, do not necessarily converge to zero but, rather, to a linear subspace in the parameter space where the output error is identically zero. This subspace evolves in time, and its evolution is dependent on the time-varying characteristics of the reference input and the plant output. If the reference input is such that the subspace remains fixed, no changes in the parameters occur. If, however, the subspace varies with time, the parameter errors will approach it from a direction orthogonal to it. It is precisely here that the so called "richness" of the adaptive signals could play an important role, insofar as parameter error convergence is concerned. It is becoming increasingly apparent, however, that the closed loop, given the present algorithms, cannot satisfy the "richness" conditions required for asymptotic convergence. This has obvious implications for certain adaptive schemes [20, 21] whose overall global stability hinges heavily on exact parameter convergence. On the other hand, stability results have been obtained recently for the above mentioned class of algorithms, irrespective of "sufficient excitation" and parameter convergence [18]; but they are valid only locally.

When the algorithm described in [19] was studied with an unmodeled high frequency pole, the same analytical results successfully explained the exhibited oscillatory, and eventually unstable, behavior and, moreover, demonstrated the same dependence of such behavior on the reference input.

In the case of observation noise, $n(t)$, our analysis shows that a term containing $n^2(t)$ will in effect keep driving the error system and, therefore, the parameter error can increase within the zero output subspace discussed above. But if the variance of the measurement noise is known, this term can be subtracted out. Unfortunately, the problem still remains of white noise being input into a marginally stable system in the same subspace. However, since the noise also drives (affects) the time-evolution of the subspace, it may very well be the case that the decaying effect of the time evolution of the subspace on the parameter errors is enough to keep their variance bounded.

The analytical framework mentioned in the foregoing was employed to study different adaptive algorithms and consistency of the results was shown. Specifically, our study so far has concentrated on the algorithms obtained using a stability point of view for their derivation, the most important representatives of which are Monopoli's [23] Narendra, Valavani, and Lin's [19, 12], Feuer and Morse's [24], Laudau and coworkers, [25], both for discrete as well as continuous-time systems. In the analysis, the above algorithms all shared the same characteristics, although some exhibited marginally better behavior in some numerical cases. For example, the algorithm described in [12] remained stable, in the presence of one unmodeled pole, with increasing reference input amplitude, although the control signal was heavily oscillatory. However, the presence of two unmodeled high-frequency poles resulted in its instability. Conversely, the observation noise enters here in a much more complicated way than in the rest of the algorithms examined.

3.4 Future Research Plans

A class of adaptive algorithms which are currently being investigated in the framework established in [18], is that containing Åström and Wittenmark's basic self-tuning regulator scheme [26] and those that followed it along the same lines; i.e., some schemes contained in [27], and the algorithms obtained by Goodwin, Ramadge and Caines [13]. These are basically dead-beat algorithms designed for discrete-time systems. Although at first glance they seem to have some advantage over the others with respect to the high frequency content of the control input, we anticipate a much worse response in the presence of unmodeled dynamics. Work is currently in progress to verify this preliminary conjecture.

Based on our theoretical results, we are also developing modifications to the existing schemes that avoid the undesirable behavior described so far. The full paper [18] will describe some of the results currently obtained.

Finally, apart from the modifications and improvements for deterministic adaptive algorithms, the effect of process and/or measurement noise has to be expressly taken into account in all design improvements and the robustness properties of the resulting adaptive schemes have to be analyzed. A lot of future research is required along these lines, before adaptive algorithms can have practical impact, apart from the purely theoretical appeal they have enjoyed so far.

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1. D. Allinger and S.K. Mitter: New Results on the Innovations Problem for Non-Linear Filtering, to appear in Stochastics, 1981.

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