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A Final Report of Research on

# STOCHASTIC AND ADAPTIVE SYSTEMS

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Submitted to:

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ABSTRACT

This final report describes the research carried out by members of the Decision and Control Sciences Group at the Laboratory for Information and Decision Systems, M.I.T. during the time period February 1, 1977 to January 31, 1982 with support extended by the Air Force Office of Scientific Research under Grant AF-AFOSR 77-3281B.

The principal investigators were Professor Michael Athans and Professor Sanjoy Mitter. The contract monitors were Major C.L. Nefzger, Dr. J. Bram and Dr. B. Epstein of the AFOSR Directorate of Mathematical and Information Sciences.

Research was carried out on the following main topics:

1. Realization Theory for Stochastic Systems
2. Linear and Non-linear Filtering
3. Stochastic Control
4. Adaptive Control

Technical details of the research may be found in the reports, theses, and papers cited in the references. A list of publications supported wholly or partially by this grant is included at the end of this report.

## 0. Introduction.

In this report we describe the work on stochastic and adaptive systems which has been done during the period February 1, 1977 to January 31, 1982 supported by the grant. In the initial stages of this grant, Professors T.L. Johnson and A.S. Willsky also participated in this research program. Professor Johnson's work was concerned with control of stochastic systems with finite-state compensators. This work was subsequently continued by Professor T.L. Johnson under AFOSR Grant AFOSR/F49620-80-C-0002 and we refer the reader to the final report of this last-named grant for details of this work. Professor Willsky's work on random fields was initially partially supported by this grant. We do not give details of this work in this report but refer the reader to the publications cited at the end of this report.

The main work performed under this grant may be categorized as follows:

1) Stochastic Realization Theory
2) Linear and Nonlinear Filtering Theory
3) Stochastic Control
4) Adaptive Control.

This research was carried out by Professors M. Athans and S.K. Mitter and Dr. L. Valavani assisted by several graduate students. Professors M.H.A. Davis of Imperial College, London and Professor John Baras of University of Maryland also participated in this research as visiting scientists with support extended by this grant.

1. Contributions Made in this Research Program.

We feel that the major contributions of this work have been the following:

1) Proof of the Innovations conjecture of Kailath (originally made in 1967), a major open problem in non-linear filtering since 1967.

2) Development of a new theory of non-linear filtering based on the Zakai equation leading to new insights into existence and non-existence of finite-dimensional filters and approximation methods.

3) The demonstration of the close connection between mathematical problems of non-linear filtering and quantum physics.

4) Multiple Integral Expansions for non-linear filtering and equations for best quadratic filters.

5) Relationship between non-linear filtering and stochastic control — again, a major open problem since the mid-sixties.

6) Initiation of a theory of non-linear stochastic realization.

7) Insights into Robust Stochastic Control.

8) Pinpointing the deficiencies of existing adaptive control algorithms and a new approach to the transient analysis of adaptive control schemes.

## 2. Framework and Motivation of this Research.

Our research has been concerned with fundamental aspects of controlling linear and non-linear stochastic systems in the presence of measurement and parameter uncertainties. In case the uncertainties reside in the state description of the physical system and measurements, then we refer to the control problem as a stochastic control problem. If, in addition, there are parameter uncertainties, then the problem is referred to as an adaptive control problem. This is because, in addition to state estimation, some form of parameter identification scheme will be needed and almost always the control and estimation-identification functions will interact in a non-trivial nonlinear and time-varying way.

A subproblem of the stochastic and adaptive control problem is the state estimation problem. Suppose for a moment we make the assumption that there are no parameter uncertainties present in the dynamical description of the state of the system. Then all the probabilistic information that one can extract about the "state" of the system on the basis of noisy measurements of the state is contained in the conditional probability density of the state given the observations. Indeed, this is the probabilistic state of the joint physical-measurement system. The recursive computation of the probabilistic state is the state estimation problem. If this could be done, then one could look for the best controller as a function of this probabilistic state, best being judged in terms of a suitable performance criterion.

We believe that we have only scratched the surface in understanding some of these fundamental issues in modelling, estimation, and control of linear systems with multiplicative uncertain parameters and, more generally, non-linear systems. Our proposed research deals with those issues that we feel are <u>both</u> fundamental and relevent from an engineering point of view.

We would like to mention that many of the theoretical questions we are posing have been motivated by problems which are of importance to the U.S. Air Force. Modelling, estimation, and identification is clearly fundamental for application of control theory, especially the theory developed in the last fifteen years or so. Kalman filtering has found widespread application in the guidance and control of aerospace vehicles, laser tracking and pointing systems and missile designs. However, very few applications of non-linear filtering are known. The extended Kalman filter which is often used in practice is basically not understood from a scientific point of view. The success of passive tracking algorithms requires advances in filtering.

The control of future aircraft, possibly statistically unstable, is a problem of continuing importance from the point of view of designing adaptive stability augmentation systems that operate reliably over a wide operating envelope. Similarly, the control of advanced jet engines, whose dynamical characteristics change rapidly with operating conditions, poses a challenging problem if one wishes to design a control system which accommodates commanded thrust level changes rapidly, while maintaining fan and compressor stability margins, and minimizing excess temperature durations. Furthermore, adaptive control appears particularly attractive to handle less predictable things such as changes in atmospheric gust (particularly wind shears on final approach) and changes to aircraft weight and mass center (e.g., due to unusual loading distribution or dropping of external stores).

The use of optical sensors and laser pointing and tracking systems leads to new problems in both stochastic estimation and control. For example, in a laser tracking system the "glint" phenomenon corresponds to noise in the measurement equation that is multiplicative rather than additive in nature. Any stochastic interception problem involving a maneuvering target and with "angle only" sensors corresponds to a very complex nonlinear estimation and control problem.

# 3. Description of Research.

## 3.1 NONLINEAR FILTERING AND STOCHASTIC CONTROL

### 3.1.1 Nonlinear Filtering

Significant progress had made in the area of non-linear filtering by Professor Mitter and his graduate students, Daniel Ocone and Larry Vallot. This work has been reported in various papers and two theses (References [1] to [10]) all supported by this grant. The progress that has been made is on several fronts:

(i) The innovations problem of non-linear filtering in the form conjectured by Kailath has been finally settled [1].

(ii) A new attack on the non-linear filtering problem based on the analysis of the Zakai equation, proposed independently by Brockett and Mitter has had considerable success. In fact, a major part of the summer school sponsored by NATO in Les Arcs, France held in July 1980 was devoted to ideas proposed in [2].

(iii) A rigorous formulation of the variational principle underlying non-linear filtering has been presented in [3]. This gives new insight into questions of approximation for non-linear filters.

(iv) It has been shown that there are, indeed, deep connections between non-linear filtering and recent ideas of quantum physics from a stochastic viewpoint ([2], [3]).

(v) In [8] it has been shown how these recent developments can be used to obtain sub-optimal filters (with guaranteed performance improvement) for bilinear stochastic systems.

Previous work on non-linear filtering has concentrated on the equation for the conditional density of the state given the observations over a time interval. This equation, the so-called Kushner-Stratanovich equation, is a non-linear stochastic partial differential equation with an integral term. In contrast, the recent work of Mitter has concentrated on the equation for the unnormalized conditional density, the so-called Duncan-Mortensen-Zakai equation. This is a bilinear stochastic partial differential equation which has as its input the observation process and, hence, is considerably simpler to analyze than the Kushner-Stratanovich equation. This equation acts as a universal recursive filter, in the sense that any conditional statistic can be computed by an integration with respect to this unnormalized density and an appropriate normalization.

A general program for analyzing this equation and its invariance properties was proposed in [2]. In this program a certain Lie algebra of operators with two generators $L_0^* - \frac{1}{2} L_1^2$ and $L_1$ where $L_0^*$ is the formal adjoint of the generator of a diffusion process and $L_1$ is a multiplication operator has an important role to play. If this Lie algebra is a finite dimensional than it is an indication that the Zakai equation has a finite-dimensional statistic. In fact, the n-dimensional Kalman filtering problem gives rise to a (2n+2)-dimensional Lie algebra which is an n-dimensional generalization of the Oscillator algebra of some fame in quantum physics. Indeed, the Kalman filter occupies the same role as the Harmonic oscillator or the free field in quantum physics ([2], [3]). A by-product of this approach is that the Zakai equation for the Kalman filter has a finite-dimensional statistic even if the initial density is a non-gaussian. Soon after these ideas became known, Benes exhibited finite-dimensional filters for a class of filtering problems with non-linear drift and it was shown by Mitter [2] that the filtering problems of Benes were "gauge equivalent" to the Kalman filtering. In the doctoral dissertation of D. Ocone [7] it is proved that for scalar diffusion processes defined over the whole real line, the Kalman filtering problem and the gauge equivalent Benes filtering problems were the only problems admitting finite-dimensional filters.

In [2], it was shown that the Lie algebra of operators corresponding to a large class of filtering problems was infinite-dimensional and simple. In particular, it was shown that the Lie algebra of operators corresponding to the cubic sensor problem (extensively investigated by R.S. Bucy) was the Weyl algebra (the Lie algebra of partial differential operators of all order) which is known to be infinite-dimensional and simple. This result was obtained independently by M. Hazewinkel and S. Marcus. It was conjectured in [2] that no finite-dimensionally computable statistic existed for this problem. This has now been proved by H. Sussman.

In proving the above result, one constructs a proof of the Ansatz of R.W. Brockett, namely, that if there exists a finite-dimensional filter than there must exist a certain Lie Algebra homomorphism between the Lie Algebra of operators corresponding to the filtering problem and the Lie Algebra of certain vector fields. For the cubic sensor problem this was done by M. Hazewinkel and S. Marcus. To prove the theorem of the non-existence of finite-dimensionally computable statistic one needs a rigorous proof of the existence and uniqueness of solutions of the Zakai equation. In joint work of

Mitter with J.S. Baras and D. Ocone (preliminary report in [9]) this has been accomplished when the observation map is a polynomial.

In D. Ocone's doctoral thesis [7], the problem of expanding the unnormalized conditional density in a Ito-Wiener Series has been solved. Ocone also obtains a multiplication formula for multiplying two multiple Ito integrals and subsequently expanding it in an Ito-Wiener Series. He has also obtained integral equations for the kernels corresponding to best polynomial filters and shown existence and uniqueness of solutions.

In L. Vallot's S.M. thesis [8], a theoretical and experimental investigation of filtering for bilinear systems has been undertaken. As examples, he has considered the phase-lock loop problem and also the cubic sensor problem. He has shown how tensor and Fock space ideas expressed in [2] can be usefully utilized to obtained sub-optimal filters which guarantee performance improvement (in the mean square sense).

In summary, a framework has been established for the theoretical and experimental investigation of a broad class of non-linear filtering problems. It is believed that these ideas can be used to design practical non-linear filters and evaluate their performance.

### 3.1.2 On the Relationship Between Non-Linear Filtering and Stochastic Control

It has long been felt that there is a variational principle underlying non-linear filtering. Indeed, in the early sixties, a formulation of the non-linear filtering problem as a non-linear least-squares problem was proposed first by Bryson and Frazier and, subsequently, by Mortensen. These formulations were, however, deterministic formulations and had the inherent mathematical difficulty of working with white noise, the formal differential of the Wiener process (which is almost surely, not differentiable). It was also known that these formulations gave the correct answer for the Kalman Filtering problem.

In [11] Professor Mitter (joint work with Professor Fleming of Brown University) has shown that associated with the Zakai equation of non-linear filtering is a Bellman-Hamilton-Jacobi equation which corresponds to a perfectly observable stochastic control problem. The solution of one equation leads to a solution of the other. In particular, the Kalman filtering problem corresponds to a perfectly observable linear quadratic gaussian stochastic control problem. Indeed, this explains in the clearest possible way the duality principle originally proposed by Kalman. The difficulty of the non-differentiability of the Wiener paths is avoided by working with the robust form of the non-linear filtering equations as developed by Clark and Davis (M.H. Davis of Imperial College was a visitor at M.I.T. in 1979 and was partially supported by this grant).

These ideas have important consequences. It opens the way for developing algorithms for non-linear filtering using stochastic control ideas. It, for the first time, explains how the extended Kalman filtering algorithm really works. Finally, it has important implications in the development of a theory of stochastic dissipativeness.

### 3.1.3 Concluding Remarks

The developments in non-linear filtering reported here have many consequences. For example, parameter identification problems can be considered to be special cases of non-linear filtering problems and the ideas discussed here will be important in obtaining a better understanding of identification. It is also felt that the relationship between non-linear filtering and stochastic control exhibited here will play a role in obtaining a better understanding of stochastic adaptive control.

## 3.2 STOCHASTIC REALIZATION THEORY

The problem of stochastic realization theory could be thought of as the construction of state-space representations for stochastic processes. The problem is essentially solved in the case where the observed stochastic process is a stationary Gaussian process [12]. In joint work with A. Lindquist and G. Picci [12], we have initiated work where the stationary process has finite-energy and has an innovation representation in the form of a Wiener-Ito series. We have also investigated the special case where the observed process is the output of a bilinear stochastic differential equation. We feel that this work has important ramifications for non-linear filtering theory.

## 3.3 ADAPTIVE CONTROL

For the past several years an intensive study of characteristics of existing direct adaptive control algorithms has been conducted by Drs. Athans, and Valavani with the assistance of Drs. Stein and Sastry and several students. The initial emphasis was to understand the transient behavior of existing direct adaptive control algorithms and their robustness to unmodelled dynamics and observation noise.

The first phase of this research was devoted to digital simulation studies and a brief paper [13] described the simulation results. A major part of the research that was carried out during this phase was reported in the annual report prepared for the AFOSR for the previous year.

From the simulation results, it became self-evident that no consistent pattern with respect to the adaptation process could be predicted. Nonetheless, the simulation results confirmed our suspicions that the class of adaptive algorithms considered were characterized by:

(a) high-frequency control signals characteristic of a high-bandwidth system

(b) the extreme sensitivity of the algorithm to unmodelled high-frequency dynamics which can result in unstable closed loop behavior

(c) lack of robustness to observation noise.

Motivated by the simulation results in [13] a decision was made to initiate an analytical investigation into the nature and properties of several available direct control algorithms. The focus of the analytical effort was to understand:

(a) the dependence of the closed loop adaptive system bandwidth upon the amplitude and frequency content of the reference input signal

(b) the robustness of the adaptive control system to unmodelled high frequency dynamics

(c) the impact of sensor noise.

To gain a basic understanding it was assumed that the controlled plant was a simple first-order system; the rationale was that if undesirable performance and robustness characteristics were encountered for first-order systems, one could certainly conjecture that the same problems would arise in more interesting high order systems.

### 3.3.1 An Analytical Methodology in the Study of Convergence Patterns of Adaptive Algorithms.

A recent paper [14] summarizes our progress to date. We have been successful in devising an analytical technique, based upon linearization, which we call the final approach analysis, that can be used to analyze the dynamic properties of several available direct adaptive control algorithms, both in the continuous-time case and the discrete-time case. In particular, this method can be used to predict the behavior of the adaptive systems with respect to parameter convergence, sensitivity to unmodelled dynamics, and impact of observation noise.

As explained in more detail in [14] the final approach analysis method is valid during the final stage of adaptation in which the output error is small. During this phase one can linearize the general nonlinear time-varying differential (or difference,) equations linking the dynamics of the output error to those of the parameter adjustment algorithm. One then obtains a set of linear differential or difference equations which are either time-varying or time-invariant depending upon the nature of the reference (command) inputs and outputs. It then becomes possible to analyze the behavior of the linearized dynamics using available results in linear systems theory. When the resultant dynamics are time-invariant, even simple root-locus type of plots can be used to predict the asymptotic perforamance of the adaptive system with respect to oscillatory behavior and possible instability in the presence of unmodelled dynamics.

The technical results in [14]demonstrate the value of the final approach analysis; it has been used to analyze the behavior of the adaptive systems when the algorithms of Narendra and Valavani [15] Feuer and Morse [16] Narendra, Lin, Valavani [17] Morse [18] Narendra, Lin [19] Landau and Silveira [20] [21] and Goodwin, Ramadge and Caines [22] were employed. For the base-line first-order example considered, all algorithms considered were found (in different degrees) to suffer from the viewpoint of yielding high-bandwidth closed-loop systems which can excite high-frequency dynamics.

It is important to place the method of final approach analysis into perspective regarding both completed results as well as relevant current research directions which include:

(1) High-order Systems - It is possible to extend the final approach analysis method to high order systems, using either output feedback or state feedback, and examine analytically the coupled dynamic characteristics of the output error and parameter adjustment logic, through the study of the resultant linear time-varying differential or difference equations. This has been already done (but not formally documented, as yet) for several of the algorithms already considered in [14]

(2) Effects of Input and Output - The linear time-varying dynamical equations that arise from the final approach analysis have a very special structure. By this, we mean that the analysis leads to vector differential equations of the form $\dot{\underline{Z}}(t) = \underline{F}(t)\ \underline{Z}(t)$; however, the matrix $\underline{F}(t)$ has a very special form that allows one to study its characteristic polynomial in a relatively straight-forward manner. In particular, (as demonstrated in the specific examples analyzed in [14],) the command reference input r(t) and output y(t) appear non-linearly in certain elements of the matrix $\underline{F}(t)$; study of the eigenvalues of $\underline{F}(t)$ reveals that both the magnitude and the frequency of the input and output signals impact the bandwidth of the adaptive closed-loop system and hence its robustness to high-frequency dynamics.

(3) Constructive Use of the Final Approach Analysis - The linear dynamic equations that arise through the use of the final approach analysis in conjunction with any adaptive algorithm can be used in a constructive way to modify the algorithm so as to alleviate to the extent possible its practical shortcomings. As a specific example, the results in Section 4.3 of [14] demonstrate how to adjust the control gains as a nonlinear function of the input and output so as to improve the performance of the Narendra, Lin, Valavani [17] and Morse [18] algorithms in the presence of unmodelled dynamics. Similarly, the results in Section 5.2 of [14] demonstrate how to properly select the gains in the Landau-Silveira [20], [21] algorithms.

The main motivation for the modifications introduced in the above can essentially be attributed to the need to limit the bandwidth of the adaptive system so as not to excite the high frequency unmodelled dynamics which invariably drive it to instability. In the specific cases cited above, this was achieved by appropriately controlling the resulting final approach root locus pattern, either by keeping the root locus parameter (gain) within bounds, as a function of the reference input and process output, or by introducing an additional zero at an appropriate location, so as to reduce the order of the root-locus pattern, which is already enhanced by the presence of the unmodelled poles. Unfortunately, these improvements cannot be generalized to deal with the truly transient phase without seriously affecting the stability arguments, or to handle any number of unmodelled poles beyond a certain frequency. Clearly, a more substantial remedy is necessary. We defer a discussion on this to the subsequent subsection. Also, a discussion of the effects of observation noise and disturbances on the performance of adaptive algorithms is included in a subsection by itself.

Justifiably, since the final approach analysis is based upon dynamic linearization under the assumption that the output error is small, it cannot predict the dynamic behavior of the adaptive system during its transient (start-up) phase. The simulation results of [13] suggest that even more complex dynamic effects are present. Thus, the final approach analysis must be viewed as a necessary, but by no means sufficient, step in the analysis and design of adaptive algorithms.

In spite of its limitations, we believe that the final approach analysis is a useful tool, since it can predict undesirable characteristics of wide classes of adaptive algorithms in the final phase of the adaption process; these characteristics are apt to persist (or get even worse) in the transient start-up phase. Moreover, the final approach analysis can suggest ways of modulating the control gains, in a nonlinear manner, to improve performance in the absence of high frequency modelling errors. At this stage of understanding the resultant transient start-up characteristics can be evaluated only by simulation; current research is focusing on analytical results for this aspect of the adaptive problem.

### 3.3.2 Model Assumptions and Fundamental Questions

From the foregoing discussion, the inevitable conclusion is that a good (practical) adaptive control loop must adjust its bandwidth (cross-over frequency) in such a manner so that it does not excite unmodelled high frequency dynamics. To put it another way, the adaptive loop must remain stable in the presence of *unstructured* modelling uncertainty which always exists and cannot be adequately modelled in any physical system. On the other hand, the adaptive control system must also be able to provide performance improvement in the case of plant *structured* uncertainty, typically exhibited when the parameters in the differential equations that are used to model the plant in the low frequency region vary within a bounded set. The adaptive system must exhibit good command - following and disturbance - rejection properties in the low frequency region where the structured model uncertainty predominates.

Our research experience to date points to the existence of a fundamental conflict in many adaptive control schemes. To compensate for structured uncertainty and performance the adaptive scheme may wish to increase the cross-over frequency. On the other hand, the presence of unstructured uncertainty places an upper bound on the cross-over frequency in order to maintain stability. Good practical adaptive algorithms must be "smart" enough to recognize this fundamental conflict, and adjust their cross-over frequency.

The mathematical assumptions that have led to all available adaptive control algorithms have taken into account the existence of structured uncertainty but they have neglected completely the issue of unstructured uncertainty. To avoid such an undesirable behavior as discussed in the preceding, the mathematical assumptions must include information on the unstructured uncertainty. The question is: what level of detail can one provide to the mathematics about the unstructured uncertainty.

As a natural consequence of the above considerations, we have adopted currently a fundamentally different structure of the model for the plant to be controlled, described below:

#### 3.3.3 Frequency Domain Description

In the single-input-single-output (SISO) case, we assume that the unknown plant has the transfer function, from the scalar control u(t) to the scalar output y(t), given by:

$$g(s) = g_o(s,\underline{\theta})\ [1 + \ell(s)]$$

The parameter vector $\underline{\Theta}$ in the transfer function $g(s,\underline{\Theta})$ gives rise to the structured modelling uncertainty, and we assume that it is a good model for the plant at low frequencies. The parameter $\underline{\Theta}$ is assumed to be in a closed bounded set; the relative degree of $g_o(s,\underline{\Theta})$ will not change as the parameter vector $\underline{\Theta}$ changes*. In fact, a nominal value of $\underline{\Theta}$ may even be known which can be used in the adaptive control algorithm before any real time measurements (initialization problem).

The transfer function $\ell(s)$ gives rise to the unstructured modelling uncertainty (high frequency bending modes, small time-delays, non-minimum phase zeros at high frequencies, etc.). $\ell(s)$ represents a multiplicative perturbation to $g_o(s,\underline{\Theta})$ under the present formulation.

An analogous formulation with parallel definitions as in the SISO can be applied to the MIMO problem as follows:

(a) For unstructured modelling uncertainty reflected at the plant input

$$\underline{G}(s) = \underline{G}_o(s,\underline{\Theta})[\underline{I} + \underline{L}(s)]$$

(b) For unstructured modelling uncertainty reflected at the plant output

$$\underline{G}(s) = [\underline{I} + \underline{L}(s)]\underline{G}_o(s,\underline{\Theta}).$$

*In aerospace applications, the parameter $\underline{\Theta}$ models the effect of changing dynamic pressure and geometry upon the aerodynamic coefficients in the equations of motion (typically rigid translational and rotational dynamics).

Our approach takes the point of view that we know very little about the unmodelled high frequency dynamics. At sufficiently high frequencies, $\ell(s)$, and the elements of $\underline{L}(s)$, will be characterized by $\pm 180°$ phase uncertainty. We also do not know very much about the order of such high-frequency dynamics.

We claim that the only reasonable information that a control engineer has about high frequency errors is as follows:

(a) They are negligible at low frequencies

(b) There is a frequency, $\omega_u$, in which the magnitude of the unstructured modelling uncertainty becomes significant

(c) They are dominant at high frequencies.

The way we model the existence of $\ell(s)$ or $\underline{L}(s)$ in the adaptive control problem is to assume that there exists a scalar function of frequency, $m(\omega)$ such that

$$|\ell(j\omega)| < m(\omega), \quad \forall \omega \text{ for the SISO case,}$$

and $\sigma_{max}(\underline{L}(j\omega)) < m(\omega)$, $\forall\, \omega$ for the MIMO case,

where $\sigma_{max}$ denotes the maximum singular value of $\underline{L}(j\omega)$.

The function $m(\omega)$ is <u>small</u> at low frequencies and is a monotonically increasing function of $\omega$. The frequency $\omega_u$ at which $m(\omega_u) = 1$, will be referred to as the uncertainty cross-over frequency.

The existence of the unstructured uncertainty places an upper bound on the bandwidth of the closed-loop system, whether the closed-loop system is adaptive or not. If one assumes that the "best" non-adaptive controller has a maximum gain crossover frequency for all $\underline{\theta}$

in the allowable parameter set of $\omega_c^*$, then for guaranteed stability in the presence of unstructured uncertainty we must have $\omega_c^* < \omega_u$. On the other hand, use of an adaptive controller is justified if the performance of the "best" nonadaptive controller is not good enough for a specific problem. In many applications, whenever the parameter vector $\underline{\theta}$ is not at its worst value, an adaptive algorithm, after the transient adaptation phase, should converge to a compensator with crossover frequency $\omega_{c\ell}$ that satisfies

$$\omega_c^* < \omega_{c\ell} < \omega u$$

for performance and robustness. Unfortunately, our investigations have shown that none of the currently existing algorithms satisfy the above criterion.

The problem as we see it, arises with the nature of the information available to the explicit or implicit adaptive controller which relies for its parameter adjustments, on some form of output signal(s) correlation. This, incidentally, is true of <u>every</u> adaptive algorithm proposed in the literature (dual control, self-tuning regulators, model-reference adaptive control). When the requirement $\omega_{c\ell} < \omega_u$ is "temporarily" violated (due to the initial "hunting" behavior of adaptive algorithms), the closed loop system becomes temporarily unstable, its high frequency dynamics in $\ell(s)$ (or $\underline{L}(s)$) get excited, and hence the measured outputs contain signals (the models of $\ell(s)$ or $\underline{L}(s)$) for which by <u>assumption</u> we know nothing about, in view of the magnitude and phase uncertainty for $\omega > \omega_u$. The output correlation algorithm, consequently, must be <u>able to quickly separate the relevant information in the output signals from those that the algorithm knows nothing about</u>, and at the same time carry out rapidly the necessary (explicit or implicit) change in the controller parameters in the correct direction! The results in [14] demonstrate that many of the existing adaptive control algorithms cannot handle this phenomenon, and the closed loop system breaks into total instability.

The focal point of our current research is how to characterize the impact of the unstructured uncertainty upon the output measurements and to consequently reflect this in the parameter adjustment mechanism. Both time-domain as well as frequency domain methods are being employed in these investigations, the baseline for which is being provided by the final approach analysis, and our results will be reported at a future date, upon completion.

#### 3.3.4 Disturbances and Sensor Measurement Noise

Although the MRAC algorithms are explicitly designed for good command-following (the theory neglected to date disturbance rejection) the specific mechanism employed for achieving the necessary high loop gain,for this purpose, is unsatisfactory from a practical point of view. All MRAC type algorithms that we are aware of do not follow the time-honored rule of augmenting the plant dynamics with integrators and other disturbance rejecting transfer functions which naturally make the loop transfer function have a large magnitude in the command-following/disturbance-rejection frequency range. We have found that standard MRAC algorithms increase the forward loop amplification at all frequencies, thus forcing the closed-loop system to have a wide bandwidth, which further exasperates the problem of exciting the high-frequency dynamics. Our current work here primarily relies on extending the robustified LQG approach to suitably modify existing MRAC and self-tuning regulator algorithms so that they can handle both known and unknown dynamics in the definition of the structured uncertainty.

Sensor measurement noise, although it initially enters additively, it readily becomes multiplicative by entering the closed loop system through the parameter adjustment law.

The final approach analysis can also be employed in this case to analyze the effects that the addition of the observation noise has on the adaptive algorithm. Section 6 of [14] illustrates how the methodology can be used to analyze a particular algorithm, and how to modify the algorithm so that parameter errors are not increased thus leading to a wide bandwidth system.

More specifically, the analysis in [14] shows that the noise n(t) enters the (linearized) system both linearly as well as in a quadratic form $n^2(t)$. The effect of the term $n^2(t)$ is always to increase the parameter errors which correspond to the difference of the actual closed-loop pole(s) from the model reference pole(s). Hence, the bandwidth of the closed-loop system increases and, although it remains stable for exact modelling [13] it is clear that if the presence of sensor noise causes the adaptive loop bandwidth to increase, eventually the high frequency dynamics will get excited and instability will result.

We emphasize here that the increase in bandwidth rather than stability (under assumptions of proper modelling) is the crucial issue. For the discrete-time systems, in particular, mean-square boundedness proofs for the output errors have been obtained [23] under the assumptions of proper modelling and positive realness of the noise transfer function. Earlier, Ljung [24], obtained local convergence results for stochastic approximation type adaptive algorithms also, under the assumptions of stationarity of the closed-loop signals and positive reality of the error transfer function. Section 7 of [14] yet contains what we consider to be the first proof in the literature to yield mean-square boundedness of the parameter errors as well.

The problem, therefore, with sensor noise is again to characterize its effect on the output(s) and to appropriately compensate the adaptive laws for this. Some discussion for final approach improvement is contained in [14], although the problem in its generality has to be treated using the same methodology discussed in the preceding subsection, in the context of unmodelled dynamics.

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