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AEROSPACE GUIDANCE COMPUTER ANALYSIS AND SYNTHESIS

by

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FOREWORD

The research reported in this document was made possible through the support extended the Massachusetts Institute of Technology, Electronic Systems Laboratory, Department of Electrical Engineering, Cambridge, Massachusetts, by the Air Force Avionics Laboratory, Wright-Patterson Air Force Base, Ohio. This work was initiated June 15, 1963 under Contract AF-33(657)-11311 (M.I.T., DSR Project No. 79891). The M.I.T. Electronic Systems Laboratory designation of this report is ESL-FR-341. The Air Force Project Monitor is Mr. Dewey Brewer (AVNE). The M.I.T. Project Engineer is Mr. Robert W. Roig. Mr. John O. Silvey and Mr. John E. Ward gathered and edited the material for this report.

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Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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ABSTRACT

This report summarizes the results of studies made under the contract of ways in which the state-of-the-art of navigation and control of aerospace vehicles could be advanced by the use of digital computers and techniques. The specific problems studied were:

1. The navigation errors that will occur in a strapped-down navigation system because of quantization and sampling of gyro and accelerometer data, and because of the noncommutability of the measured angles.

2. Characteristics of an accelerometer that uses digital computation to provide the accelerometer decision element and lead compensation of the accelerometer feedback loop.

3. Systems whereby the accuracy of a position-augmented inertial navigation can be optimized in the presence of gyro drift and noise-corrupted position data.

4. Study of steering of space vehicle during rendezvous with constant, or almost-constant, thrust.

5. The ways in which data from high-resolution side-looking radars can be processed for real-time, on-board presentation of the radar map and the amount and type of equipment required for such processing.

Results of the studies are briefly reported, and references are given to reports containing detailed analyses and results.

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I. INTRODUCTION

A. PURPOSE AND SCOPE OF REPORT

This is the final summary report for Contract AF-33(657)-11311, M.I.T. Project 79891, the purpose of which was to study the application of digital computers and techniques to advance the state-of-the-art in the navigation and guidance of aerospace vehicles and in the real-time processing of data from high-resolution synthetic-aperture radars. The various studies completed during the contract period from June 15, 1963 to August 15, 1967 have all been documented in detail in separate technical reports, and the purpose of this report is to present summaries and conclusions.

Of the five principle areas of investigation, four were previously discussed in an Interim Summary Report ESL-IR-266, April 1966. These summaries, modified and updated where necessary, are included, together with a summary of the more recent investigations. Thus this report completely supplants ESL-IR-266.

B. PROBLEMS STUDIED

The problems which have been studied in detail on this project are as follows:

1. Strapped-down Navigation Systems

In strapped-down navigation systems, errors result from quantization, roundoff, and the algorithm used in the digital coordinate conversion. The purpose of the study was to determine the relations required among sampling rate, data quantization level, computation algorithm, and computation rate to obtain a prescribed navigation accuracy. This work is covered in Report ESL-R-244, "A Study of Coordinate-Conversion Errors in Strapped-Down Navigation," by Frank B. Hills. A summary is given in Section II.

2. Novel Accelerometer

The purpose of this study was to determine if an accelerometer which uses a digital computer as the decision element and lead computer of the accelerometer feedback loop has any advantage over the pulse-rebalance accelerometers now being used. This study is reported in ESL-R-263, "A New Technique for Obtaining Digital Data from Specific Force Sensors (Novel Accelerometer System)," by Ashok Malhotra. A summary is given in Section III.

3. Augmented Inertial Navigation Systems

The performance of a position-augmented, inertial navigation system deteriorates if the inertial system has random drift and if the position data is corrupted by noise. The purpose of this study was to determine how the navigation error of such a system can be minimized, and what computation procedure would result in the minimum demand on the computer. Details of this study are reported in Report ESL-R-264, "Augmented Inertial Navigation Systems," by Donald R. Knudson. A summary is given in Section IV.

4. Rendezvous Guidance

The steering equations that are required to accomplish a space vehicle rendezvous maneuver within a predetermined time by use of essentially constant vehicle thrust have been determined. Three somewhat simplified cases were studied in detail: 3D/constant-gravity/constant mass, 3D/constant gravity/variable-mass, and 2D (orbital plane)/ constant-mass/variable gravity. This study is detailed in Report ESL-R-265, "A Rendezvous Guidance Scheme," by Albert Vezza. A summary is given in Section V.

5. Digital Processing of High-resolution Radar Data

A study was made to determine how digital computing techniques can be effectively applied to the processing of data from high-resolution radars, with the objective of immediate, on-board presentation. Particular goals were to determine how such processing can be accomplished, to determine the amount and kind of equipment needed in a practical processor, and to determine what device developments are needed for practical processors. Details of this study, including the results of simulations, are presented in report No. ESL-R-319, "Digital Processing of High-Resolution Radar Data (U)," by Robert W. Roig and John O. Silvey, CONFIDENTIAL. A summary is given in Section VI.

II. STRAPPED-DOWN NAVIGATION SYSTEMS

A. INTRODUCTION

The purpose of this study has been to determine computation errors peculiar to navigation systems that have all motion-sensing devices (accelerometers and gyros) attached to the vehicle frame. Computations peculiar to this type of navigation system arise from the necessity of performing a coordinate transformation on the quantized information from the accelerometers. The transformation, from vehicle body coordinates into the coordinates in which navigation computations are made, is computed from the quantized angular information from the gyros. It was assumed that coordinate transformation was performed by direction cosines.

The studies conducted pertain to the following subjects:

1. The choice of algorithm for computing direction cosines.

2. Errors produced by "rotational non-commutability." These errors are produced by the combined effects of sampled and quantized body rotation angles and the non-commutative properties of these rotations.

3. Errors produced by "acceleration non-commutability." These errors are produced by the combined effects of sampled and quantized acceleration, and the non-commutative properties of rotations and accelerations.

It was concluded from these studies that it is impossible to reduce significantly the navigation errors produced by non-commutability through the choice of an algorithm. For the particular examples chosen in the analysis of non-commutability errors it was found that the navigation errors can be significant. It was also concluded that only by simulation can the magnitude of combined errors produced by roundoff and non-commutability be determined.

B. EFFECT OF ALGORITHM USED TO COMPUTE DIRECTION COSINES

The direction cosines calculated by a strapped-down navigation computer are determined from gyro information that comes to the computer as rotation increments. The sampled and quantized nature of this data results in a loss of information of the behavior of angular velocity and of position. An infinite number of angular velocity histories would yield the same three histories of angular increments from the three body-axis gyros. Consequently, after a sequence of body attitude changes, there is an uncertainty in computed attitude because of uncertainties inherent in data supplied to the computer. This is illustrated in Fig. 1 for the two-axis case. The shaded area corresponds to the uncertainty region.

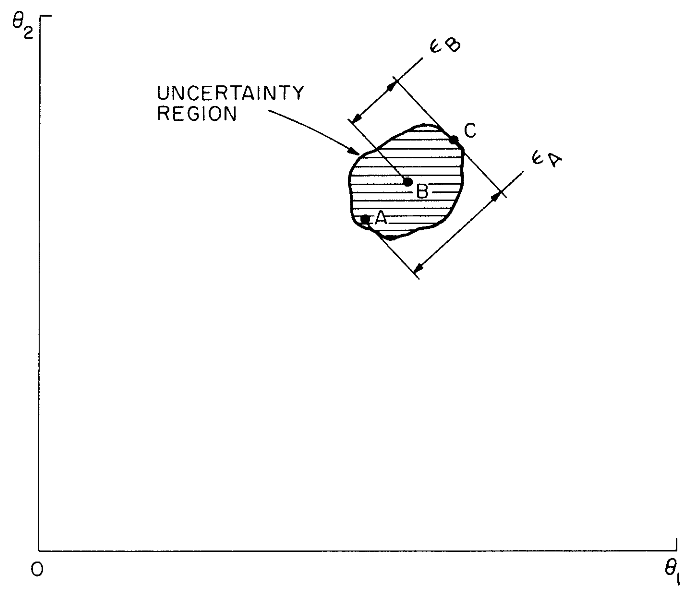


Fig. 1 Uncertainty Region

Figure 1 can be used to demonstrate the effect of the choice of algorithm. The best algorithm possible would be one which would produce the computed attitude B. The maximum error would then be that indicated as ϵ_B, if the actual attitude is that indicated by C. The worst possible algorithm that yields a solution within the uncertainty region would be that indicated by point A, on the edge of the uncertainty region, where the error is $2\epsilon_A$ if actual attitude is at C. Hence, the best possible algorithm can reduce the possible error by a factor of two at most.

The direction cosines, which must be computed to transform coordinates, are solutions to the differential equations

$$[C'] = [C][\Omega] \tag{2.1}$$

where $[C]$ is the matrix of direction cosines, the prime indicates time derivative, and

$$[\Omega] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

ω_i = angular velocity about axis i

A matrix equation that could be used in numerical calculation of $[C]$ at time t_n from its value at t_{n-1} is obtained from Eq. 2.1 by means of a Taylor's series expansion

$$[C(t_n)] = [C(t_{n-1})] \left\{ [I] + [\Theta] + \frac{[\Theta^2]}{2} + \frac{[\Theta^3]}{3!} + \Big([\Omega'(t_{n-1})][\Omega(t_{n-1})] - [\Omega(t_{n-1})][\Omega'(t_{n-1})] \Big) \frac{\nabla t_n^3}{12} - - - \right\}$$

$$\text{where } [\Theta] = \begin{bmatrix} 0 & -\theta_3 & \theta_2 \\ \theta_3 & 0 & -\theta_1 \\ -\theta_2 & \theta_1 & \theta \end{bmatrix} \qquad (2.2)$$

and

$$\theta_i = \int_{t_{n-1}}^{t_n} \omega_i \, dt$$

To evaluate the direction cosines with great accuracy requires not only angle increments but angular velocity, acceleration, and higher time derivative increments as well. Normal instruments do not measure any angular derivatives.

It can be shown that a solution for the direction cosines that lies within the uncertainty region produced by input sampling and quantization is obtained from Eq. 2.2 truncated to exclude all derivatives of angle. It is assumed that angular velocity component about each axis is proportional to the total angular velocity vector, although the velocity may vary with time. That is

$$\overline{\omega}_i(t) = \begin{bmatrix} p_{1,n} \\ p_{2,n} \\ p_{3,n} \end{bmatrix} \omega(t) \quad \text{for} \quad t_{n-1} < t < t_n$$

and the p's are constants such that

$$\sqrt{p_{1,n}^{2} + p_{2,n}^{2} + p_{3,n}^{2}} = 1$$

The resulting matrix expression for the direction cosines is

$$[C(t_n)] = [C(t_{n-1})] \left\{ [I] + [\Theta] + \frac{[\Theta^2]}{2!} + \frac{[\Theta^3]}{3!} + --- \right\}$$

Or

$$[C(t_n)] = [C(t_{n-1})] \left\{ [I] + [P] \sin \theta_n + [P^2] \ (1 - \cos \theta_n) \right\}$$

Where $\theta_n = \sqrt{\theta_{1,n}^{2} + \theta_{2,n}^{2} + \theta_{3,n}^{2}}$

$$[P] = \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix}$$

C. ANALYTICAL STUDIES OF ROTATIONAL NON-COMMUTABILITY ERROR

The procedure for this analysis was to compute the exact solution for a specific example, and to compute a second solution by updating previous data, as would be done by a digital computer. The difference between the solutions represents the error. This method is practical only for the simplest problems because only in these problems can an analytical solution be found.

The specific problem chosen was that of rotation such that the angular velocity about one axis is K (an integer) times the velocity about a second axis, with zero rotation about the third axis. Since angular velocities are proportional, rotations, if they are held to very small angles, are also proportional. The navigational position error was found in terms of the parameter K, and the gyro angular quantization level. It's magnitude was found to be approximately 0.34 mile per hour if a velocity change of 1000 ft/sec along a body axis occurs and the angular quantization level is 10^{-3} radian. Even larger errors can occur if rotation is such that a component occurs about all three axes.

D. ACCELERATION NON-COMMUTABILITY ERROR

This error results from the assumption that velocity increments occur instantaneously at constant body angles. The resulting computed velocity in space is not in the proper direction. Assume the body in Fig. 2 is moving in a

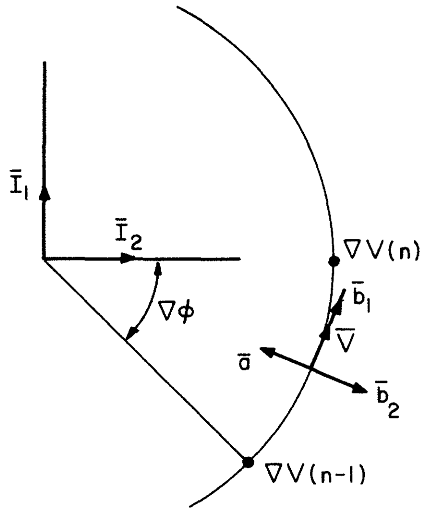


Fig. 2 Non-Commutability of Acceleration and Rotation

circle and rotating so that its velocity v is always along body axis $\bar{b}_1$. The positions at which velocity samples n-1 and n are taken are shown in Fig. 2. The velocity component differences are due to acceleration $\bar{a}$. Assume that inertial velocity is computed from body coordinates by $\nabla\bar{V}_n = [C_n] \nabla\bar{v}_n$, where $\nabla\bar{V}_n$ is in inertial coordinates and ∇v_n is in body coordinates. If this is applied directly to the case shown in Fig. 2 at t_n, only velocity change along axis I_2 would be computed because body acceleration, a, was acting along that space coordinate at $t = t_n$. However, there were actually velocity changes along both the I_1 and I_2 axes between t_{n-1} and t_n.

The errors produced in a few specific cases have been computed by solving the problem twice, once by obtaining an exact solution and second by use of an algorithm; the difference between solutions is the error.

The error produced by acceleration noncommutability is reduced to an insignificant quantity for the examples considered if the direction cosines are computed for the average values of the angles during the intersample interval, provided all accelerometers are oriented so that they continually measure a significant acceleration. However, programming the computer to use average interval values is not simple.

III. NOVEL ACCELEROMETER

A. SUMMARY OF STUDY

The analysis and simulation of a novel accelerometer system was conducted under this contract. Although this accelerometer system uses a pendulous sensor, just as a conventional system does, it produces an output pulse each time acceleration changes by one acceleration quantum rather than each time the accelerometer receives a quantum change in velocity. It utilizes a digital computer to obtain the phase lead necessary to stabilize the accelerometer closed loop and as the accelerometer decision element.

The advantages of the novel accelerometer are:

1. It can be made less subject to calibration errors than either a continuous or a pulse-rebalance accelerometer by using the associated digital computer to correct for these errors.

2. Velocity and position errors produced by acceleration transients are less than for the pulse-rebalance accelerometer studied.

3. The accurate pulse generator of the pulse-rebalance accelerometer is not required.

The disadvantages are:

1. Phase lead compensation must be computed on a digital computer.

2. Sufficiently large acceleration steps can throw the system into oscillation, although this limiting acceleration can be made so high that it need never be reached in a practical application.

The conclusion reached on the basis of these analytic and simulation studies is that the novel accelerometer appears to be a practical device that probably deserves to be tried out in practice.

B. DESCRIPTION OF SYSTEM

A block diagram of the novel accelerometer system is shown in Fig. 3. A pendulous mass in the sensor is assumed damped, but with no spring restraint. The sensor signal is sampled at uniformly-spaced instants of time, and these samples converted to digital numbers. Phase compensation required to maintain stability of the closed loop is then

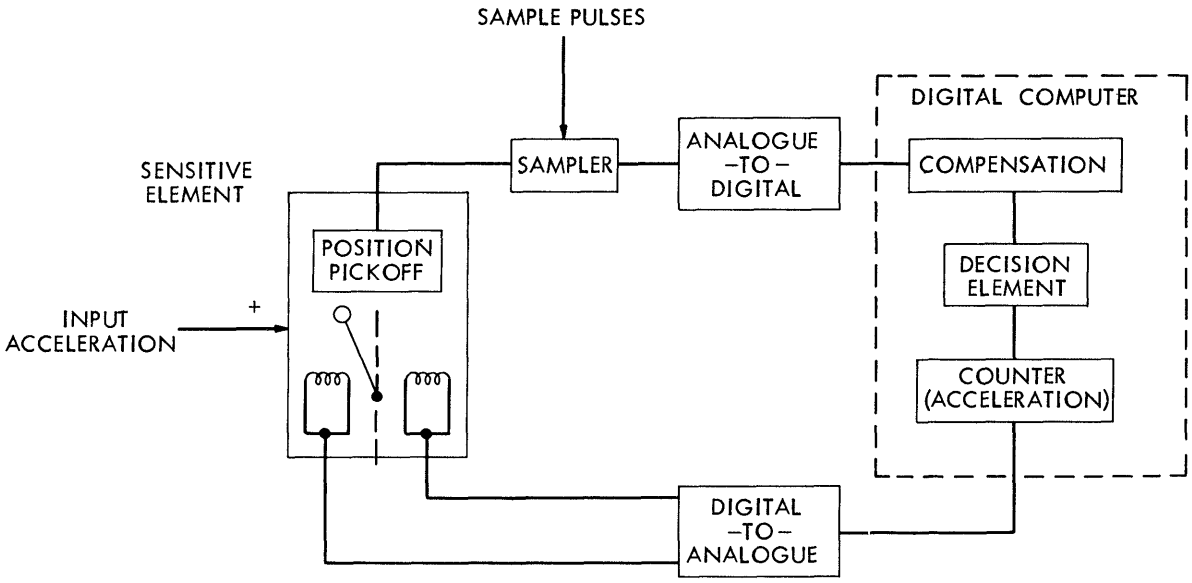


Fig. 3 Block Diagram of Novel Accelerometer

added in a digital computer. The decision element, which follows phase compensation, operates according to the following rules:

1. The direction of decision element output is the same as the direction of the sum of the last m inputs (m is constant once the rule is finalized).

2. If the direction of the present decision is not the same as the direction of the last decision, the magnitude of the output is one.

3. If the directions of the present decision and of the last decision are the same, but not the same as the next-to-last decision, output magnitude is one.

4. If the directions of present decision, last decision, and next-to-last decisions are all the same, output magnitude of present output is equal to twice the magnitude of the next-to-last output, unless this exceeds a limit of 2^{ℓ}, in which case the output is 2^{ℓ}.

Each output of the decision element is added to the contents of a counter. A digital-to-analog converter produces a current in the torque generator proportional to the contents of the register. This current flows in a direction to reduce the pendulum deflection to zero, and thus closes the accelerometer feedback loop.

C. ANALYSIS AND SIMULATION OF SIMPLE SYSTEM

Analyses of the novel accelerometer were made for the special case where the magnitude of the decision element output was limited to unity ($\ell = 0$) by describing function techniques and by assuming a quasi-linear model for the accelerometer system. The describing function analysis was used to determine the lead required to obtain the limit cycle having minimum amplitude and maximum frequency, (the best possible performance of the system).

The quasi-linear analysis was based on the system of Fig. 4 and was used to obtain information about the steady-state errors resulting

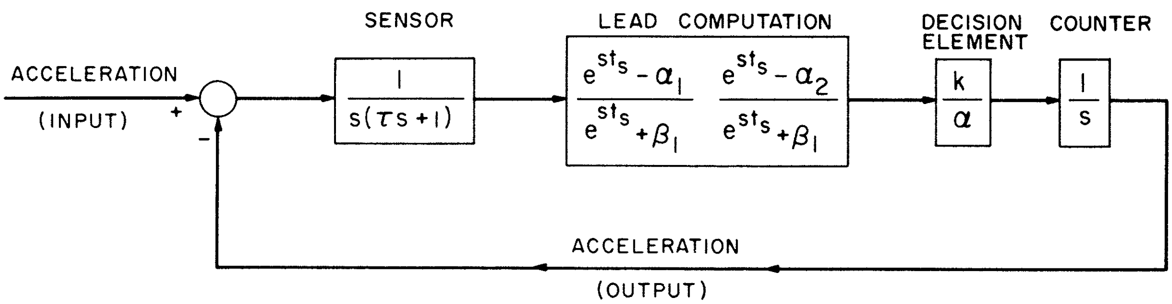
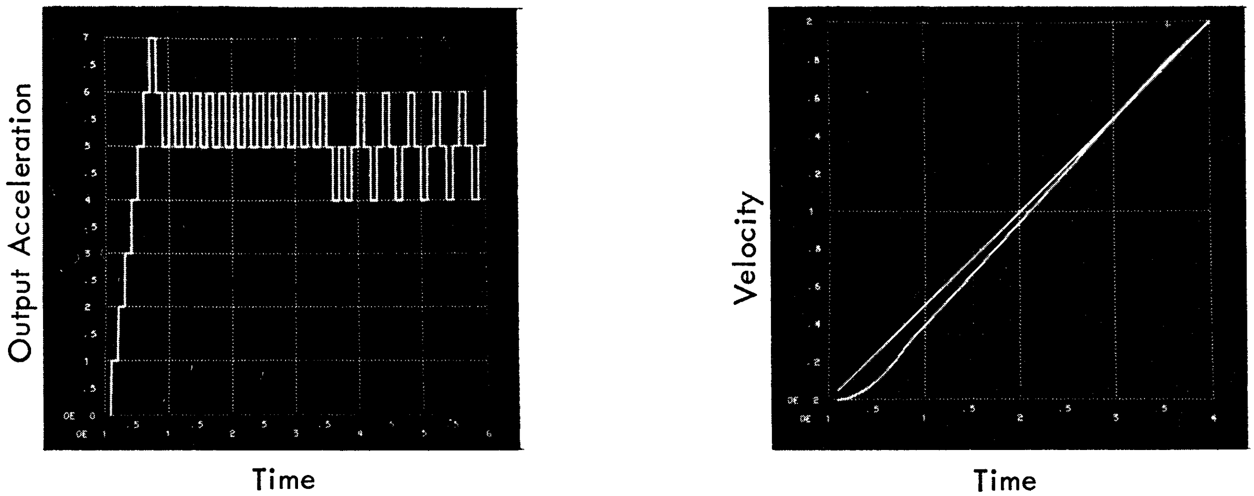
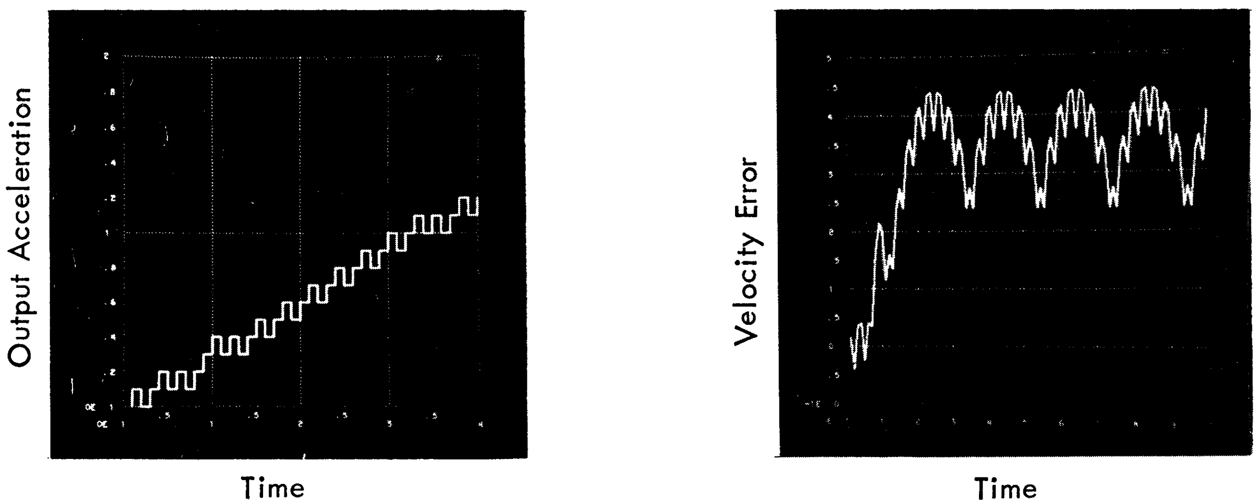


Fig. 4 Quasi-Linear System Block Diagram

from step functions and ramps of acceleration. This analysis shows that it is possible to obtain a system which has only a constant position error in response to a step of acceleration and a ramp of position error and a constant velocity error in response to an input ramp of acceleration. Here velocity error is defined as the difference between input velocity and the first integral of output acceleration, and position error is defined as the difference between input position and the second integral of output acceleration. These effects are illustrated by the simulation results shown in Fig. 5. Figure 5a shows that velocity error resulting from an acceleration step ultimately becomes zero, and Fig. 5b shows that velocity error resulting from an acceleration ramp attains a constant value.



a) Response to Acceleration Step of 5 Units



b) System Response to a Ramp of Slope 0.3 $\dot{A}_{max}$

Fig. 5 Accelerometer Response with Unity Output from Decision Element

D. RESULTS OF SIMULATION WITH GENERALIZED DECISION RULE

The speed of response of the accelerometer system can be improved significantly by permitting the output of the decision element to exceed unity. However, increasing the maximum output of the decision element effectively introduces a low-pass filter network into the system, and thus introduces phase lag which must be offset by increased lead. Furthermore, there seems to be a value for the maximum output which is optimum.

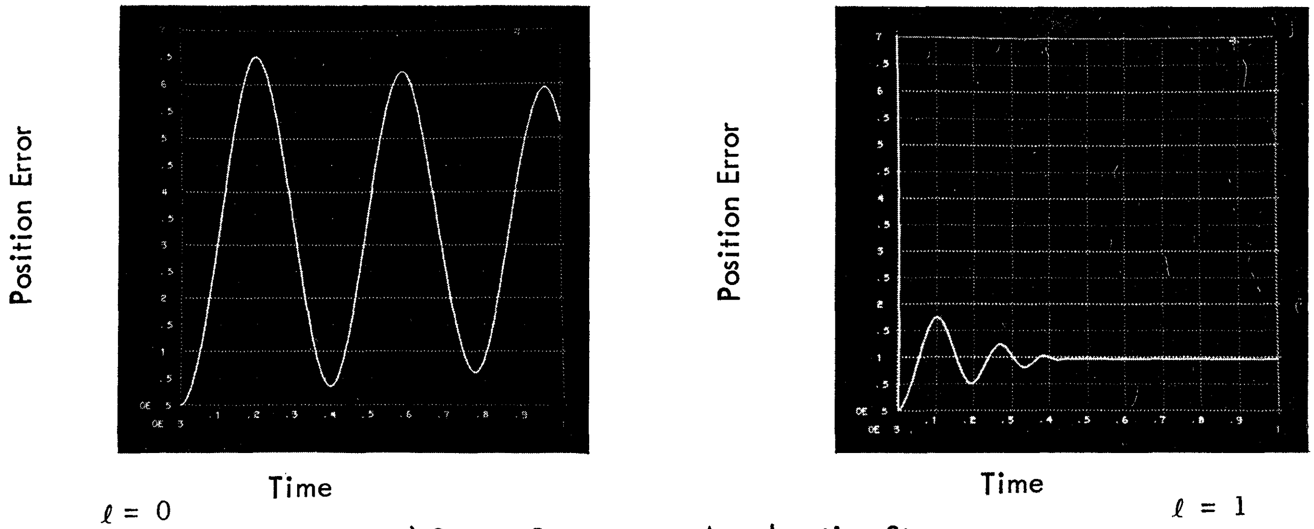
The beneficial effects of changing the decision rule are shown in Fig. 6. In Fig. 6a the position error resulting from an acceleration step is shown for a maximum decision element output of one ($\ell = 0$) and a maximum decision element output of four ($\ell = 2$). Obviously the higher decision element output reduces the magnitude and duration of the transient. (It should be noted that the acceleration step is much greater in Fig. 6a than in Fig. 5a.) Acceleration errors resulting from a step of acceleration are shown in Fig. 6b for the same decision element output maxima. It is easily seen why the transient disappears more rapidly for the higher output maximum. Position errors resulting from acceleration ramps are shown in Fig. 6c. It is seen from this figure that for the ramp slope chosen, there is little reduction of the error by the higher decision element output. At lower slopes of the acceleration ramp, the position error of the system with the lower decision-element maximum output is lower than the error of the system with $\ell = 2$.

It appears that a value of ℓ of two or three produces the best compromise between rapid reduction of position errors produced by step functions of accelerations and the slope of the position error ramp resulting from a ramp of acceleration.

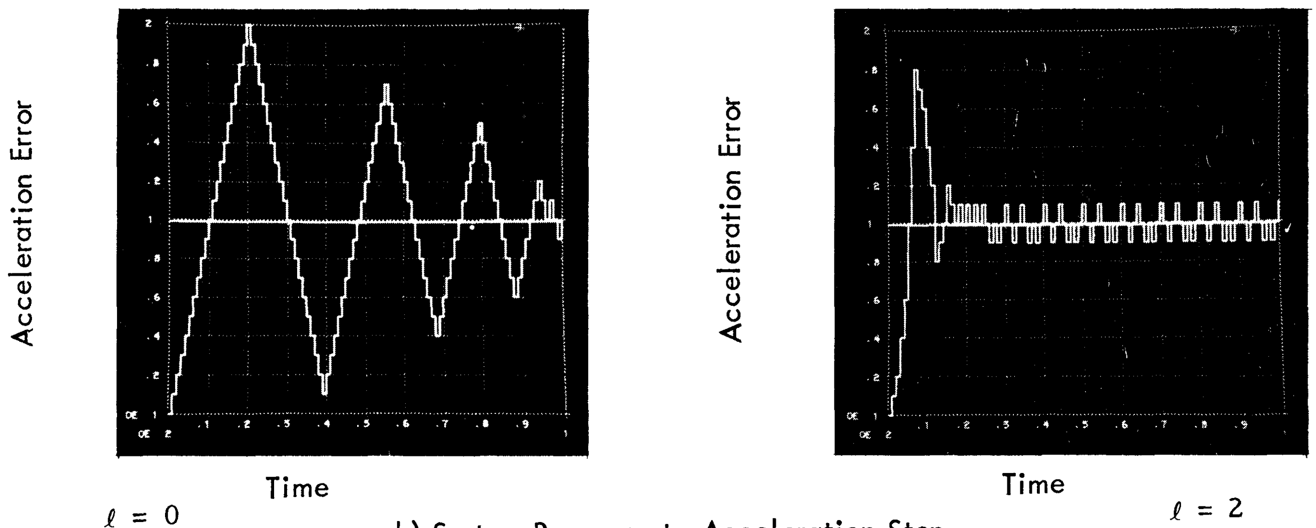
E. COMPARISON OF NOVEL AND PULSE-REBALANCE ACCELEROMETERS

To obtain a meaningful comparison of the performance of the novel accelerometer with the performance of a comparable, better-known device, analysis and simulation study were conducted on the pulse-rebalance accelerometer.

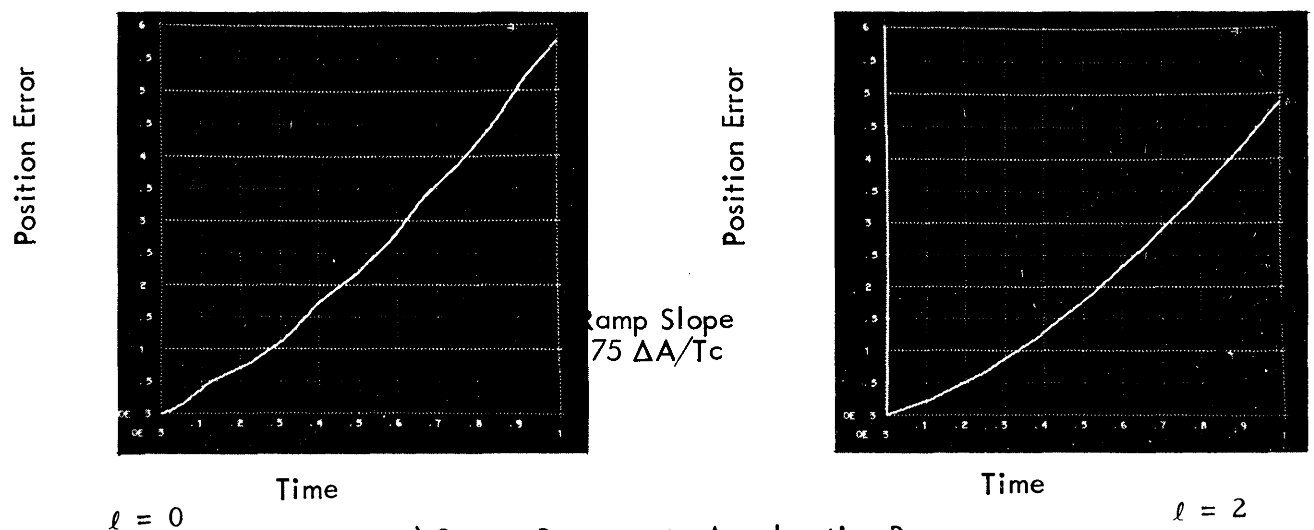
The pulse-rebalance accelerometer produces pulses which correspond to velocity increments. Hence, the "roundoff" error will be $\pm\Delta V/2$ where ΔV is the velocity equivalent of the rebalance pulse.



a) System Response to Acceleration Step



b) System Response to Acceleration Step



c) System Response to Acceleration Ramp

Fig. 6 Effect of Decision Element Output on Accelerometer Response

The novel accelerometer, on the other hand, contains a digital register, the content of which is instantaneous acceleration. It will produce, therefore, an acceleration "roundoff" error of $\pm\Delta A/2$, where ΔA is the acceleration equivalent of the least significant bit of the register. It can be shown that both types of accelerometers can have a velocity standoff error and a position rate error due to these roundoffs. The ratio of the velocity standoff errors is

$$\frac{E_{rebalance}}{E_{novel}} = \frac{\Delta V}{\Delta A T_c}$$

where ΔV and ΔA are defined above and T_c is the time between decisions of the novel-accelerometer. Typical figures indicate that the velocity standoff of the novel accelerometer will be one hundred times smaller than that of the pulse-rebalance accelerometer.

Another factor in the comparison is that the pulse-rebalance accelerometer closed loop contains one less integration than is contained in the novel accelerometer closed loop. Hence, the pulse-rebalance accelerometer will produce a constant velocity error and ramp position error in response to an input acceleration step, and constant acceleration, velocity ramp, and time-squared position errors in response to an acceleration ramp.

IV. AUGMENTED INERTIAL NAVIGATION SYSTEMS

A. INTRODUCTION

The study of augmented inertial navigation systems was instigated by a suggestion from the Tactical Air Reconnaissance Center at Shaw Air Force Base that an inertial system augmented with a means to introduce corrections obtained from position fixes would best fulfill the navigation and guidance requirements of tactical aircraft.

The objective of the study wasto investigate, by means of a simplified model, feasible methods of combining position information with an inertial system in such a way that the variance of the position error contained in the resulting system output is minimized when the navigation system inputs, as well as position information, are corrupted with noise. Methods that accomplish this objective were determined. The quantity of calculations that result and the accuracy required make necessary a degree of flexibility and computational sophistication that is available only in a digital computer.

B. REASONS FOR AUGMENTING INERTIAL NAVIGATION SYSTEMS

The "Schuler tuning" used in inertial navigation systems yields either a completely undamped system or a system affected by vehicle accelerations, unless velocity or position data from an external source is provided to provide system damping. The position errors of undamped, Schuler-tuned systems increase with time, and can become quite significant for aircraft missions that last for several hours. Augmented systems should be designed to exploit the short-term accuracy of inertial systems and to limit the time-increasing errors to values comparable to the errors of the external navigation data supplied.

So-called optimum systems are designed to minimize a statistical measure of the navigation error. The presence of a digital computer in the navigation loop permits considerable flexibility and sophistication in the processing of the sensor outputs.

Two different digital techniques have been studied for application to an aircraft inertial navigation system augmented by an external system which provides position measurements at discrete intervals of time. The computations for each system and an estimate of the computer load have been determined for each system.

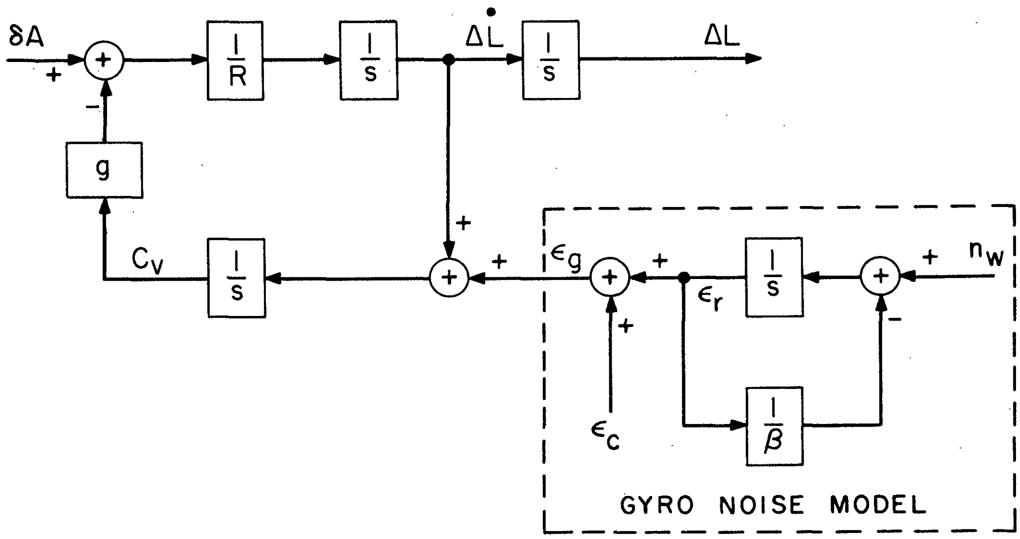
C. OPTIMUM AUGMENTED SYSTEMS

If the components of the augmented inertial navigation system are subject to error sources which are random variables, an optimum system can be defined as one in which some weighting function of the navigation parameters is adjusted to minimize the output error in a statistical sense. This, in general, may be difficult to accomplish. However, if the random variables are assumed to be Gaussian, a linear estimate of the navigation parameters can be computed with a minimum mean-square error in that estimate. Further, in the Gaussian case, the system which minimizes the mean-square error will also minimize any monotonically increasing function of the estimation error.

The inertial navigation system must handle all trajectories of the vehicle and therefore its inputs, in general, will be time-varying and nonlinear. If only errors around these trajectories are considered, the problem can be reduced to that of analyzing statistical variation in a linear system. A linear error model for a single-channel, Schuler-tuned, inertial navigation system is shown in Fig. 7.

In order to simplify the analysis, only one coordinate is considered and cross-coupling effects of errors in other coordinates are neglected. Also, only one error source (gyro drift rate) is included. Extending the results from this simplified case to a more realistic case should be relatively straightforward.

The noise model used to represent gyro drift rate is a commonly-used one which consists of a constant (random from gyro to gyro) plus white noise filtered through a first-order time lag. The presence of noise, in both the inertial system and the augmenting data, leads to a stochastic approach to the analysis and design of these systems. If the statistical characteristics of the noise is known a priori, the augmenting data can be used to form an optimum estimate of the inertial system error. Any optimum technique, if it is truly optimum, will provide the same accuracy. However, the available techniques do vary significantly in terms of their suitability for digital computation. This study has investigated the application of different techniques of optimum linear estimation to an aircraft inertial navigation system augmented by position measurements at discrete time intervals.



ΔL	=	Latitude error of inertial system
$1/\beta$	=	Correlation time of random gyro drift rate
n_w	=	White noise
R	=	Radius from center of earth
g	=	Magnitude of gravity
δ_A	=	Accelerometer error
ϵ_g	=	Gyro drift rate
ϵ_c	=	Constant component of gyro drift rate
ϵ_r	=	Random component of gyro drift rate
C_v	=	Error angle to local vertical

Fig. 7 Linear Model of Inertial Navigation System

The two optimizing techniques that have been studied use noise-corrupted, external position measurements and the outputs of the inertial system to form an optimum estimate of the navigation errors. The two techniques are called the <u>growing-memory digital filter</u> and the Kalman filter. They are basically quite different in that the first processes the position measurements as a "batch" and the second processes each measurement individually.

The system incorporating the growing-memory digital filter calculates the estimated position error which results from using weighting factors that are adjusted for each position measurement to reduce the mean square expected error to zero. Expected error is determined from assumed a priori statistical characteristics of gyro drift and noise that perturbs position measurements.

The major disadvantage of the system is that the computation involves the inversion of a matrix, the size of which is directly proportional to number of position measurements that have been made. However, since approximations to matrix inversion can be made, a compromise between accuracy and computation complexity is possible.

The Kalman filter is a method of optimum linear estimation which incorporates the use of the system state vector and the transition matrix. In this case, weighting factors are adjusted such that the estimate of the state vector produces a minimum expected mean-square error. The calculations for optimizing the system at the time of any measurement involve only the results of the next previous calculations and position difference at the time for which the calculations are made. This recursive nature of the computation means that no large memory will be required.

An analysis of the Kalman Filter system shows that the true accuracy of the calculations depends upon the accuracy of the model chosen to represent the system, including the noise. A digital computer would be required for the determination of these errors. Although no similar study was made of the growing-memory system, it seems obvious that its true accuracy would also be dependent upon the accuracy of the assumed noise model.

A comparison of the computations involved in the optimization process shows clearly that fewer computations will be required with the Kalman filter than would be required with the growing-memory filter.

In order to apply the results of this study to any specific navigation system, a more complete model of the navigation system is required than was used in this study, where only a single coordinate and a single error source (gyro drift rate) have been considered in the inertial system. Other error sources, cross coupling between channels, etc., must be considered for a complete representation of the system. These factors depend on the specific system and are influenced by such design considerations as gimbal configuration, types of accelerometer, gyro spin axis orientation, etc. However, it is believed that the extension of the study to include these factors for a specific system is relatively straightforward.

V. RENDEZVOUS GUIDANCE

A. PURPOSE AND RESULTS OF STUDY

The particular rendezvous guidance problem studied under this contract involved that portion of a rendezvous maneuver which occurs after the two vehicles are at approximately the same altitude and while they are still too far apart to permit the chase vehicle to track the vehicle with which it is to rendezvous. We have attempted to determine the simplest maneuver the chase vehicle can execute with constant thrust to accomplish the rendezvous in a particular time. A rendezvous is said to have occurred when the chase vehicle theoretically has the same location and velocity as the vehicle with which it is to rendezvous.

Three simplified cases were studied, each of which would be applicable to some practical cases. The three cases are:

1. Three-dimensional problem with uniform gravitational field, and constant chase vehicle mass (constant acceleration).

2. Three-dimensional problem with uniform gravitational field, and constant negative time rate of change of mass of chase vehicle.

3. Two-dimensional problem with a central force field lying in the plane considered, and no rate of change of chase vehicle mass.

These studies show that for the constant-gravity-constant-mass case the thrust can remain truly constant, and the maximum required maneuver would consist of four changes of course; each course would be held for one-quarter of the time of the rendezvous maneuver. Under the assumptions of this study there is a minimum thrust that produces minimum fuel consumption.

The constant-gravity-variable-mass case is complicated. It can be solved for a program of four equal time intervals in which constant thrust is applied in the proper direction to obtain a rendezvous with minimum fuel. It has not yet been demonstrated that a solution exists for higher thrusts.

The variable-gravity-constant-mass case can be reduced to a simpler case by use of variable thrust to counteract gravity changes. This assumption makes a solution possible.

B. CONSTANT-GRAVITY-CONSTANT-MASS CASE

The basic assumptions of this case, as well as of the other cases considered, were:

1. Rendezvous is to occur at a prescribed time after the rendezvous maneuver has started.

2. Initial position and velocity of the chase vehicle are known.

3. Position and velocity at the instant of rendezvous of the vehicle with which rendezvous is to be made are either known or can be computed before the rendezvous maneuver.

The equations, upon which the solution of this problem is based, are:

$$\left.\begin{aligned} \underline{V}_D - \underline{V}_o - \underline{g}T &= \int_{t_o}^{t_f} \underline{a}_T(t)\,dt = \underline{\Delta V} \\ \underline{R}_D - V_o T - \underline{g}\frac{T^2}{2} &= \int_{t_o}^{t_f}\int_{t_o}^{t} \underline{a}_T(p)\,dp\,dt = \underline{\Delta R} \end{aligned}\right\} \quad (5.1)$$

where the underscore indicates a vector and

$\underline{V}_d$ = desired velocity (velocity of both vehicles at rendezvous)

$\underline{R}_d$ = desired position (position of both vehicles at rendezvous)

$\underline{V}_o$ = initial velocity of chase vehicle

$\underline{R}_o$ = initial position of chase vehicle

T = time required for rendezvous

$\underline{a}_T(t)$ = acceleration of chase vehicle produced by thrust. For this case, $|\underline{a}_T(t)| = a_T$ = constant.

t = time (variable)

The two requirements for an analytic solution are that it be possible to integrate the acceleration, a(t), twice, and that there be enough degrees of freedom in $\underline{a}(t)$ to satisfy Eqs. 5.1. Since we have restricted ourselves to constant thrust, we can write thrust acceleration in cartesian coordinates thus:

$$\underline{a}_T(t) = a_T[\underline{i} \cos \psi(t) + \underline{j} \sin \psi(t)]$$

where ψ is the conventional angle of the vector and $\underline{i}$ and $\underline{j}$ are conventional unit vectors. If the time to rendezvous is divided into four equal time intervals, and ψ is held at a fixed, constant value during each of these intervals, enough degrees of freedom are available to obtain a solution to Eqs. 5.1 and we obtain

$$\underline{\Delta V} = a_T \frac{T}{4}\left[\underline{i} \sum_{n=1}^{4} \cos \psi_n + \underline{j} \sum_{n=1}^{4} \sin \psi_n\right]$$

$$\underline{\Delta R} = \frac{a_T}{2}\left(\frac{T}{4}\right)^2\left[\underline{i} \sum_{n=1}^{4} (9-2n) \cos \psi_A + \underline{j} \sum_{n=1}^{4} (9-2n) \sin \psi\right]$$

The method of Lagrange multipliers can be used to minimize the acceleration while maintaining the constraints of Eqs. 5.1. Thus, the minimum thrust with which the desired maneuver can be made in the permitted time can be determined. It is reasonably obvious that a maneuver can be executed in the allotted time by a vehicle having greater thrust than the minimum if the auxiliary maneuver is made perpendicular to the plane containing the change in position and the change of velocity. If the components of acceleration in the desired direction are kept at their values for the minimum acceleration and the components of acceleration normal to the plane containing position and velocity change vectors are controlled so that the resulting velocity or position change relative to this plane is zero, the desired final position and velocity will result.

If thrust is above minimum, and if acceleration is confined to the plane containing position- and velocity-change vectors a transcendental equation which is not readily solved results. It is believed, however, that there is a solution to this problem, although its determination would require the use of a digital computer.

C. CONSTANT-GRAVITY-VARIABLE-MASS CASE

The basic equations to be solved are Eqs. 5.1, but in this case acceleration is of the form:

$$\underline{a}_T(t) = a_T(t) \left[\underline{i} \cos \psi(t) + \underline{j} \sin \psi(t) \right]$$

where $a_T(t) = \frac{V_e \beta}{m_o - \beta t}$

V_e = gas exhaust velocity

β = rate of mass loss

m_o = initial chase vehicle mass

If a sequence of four thrust angles is assumed, equations for $\underline{\Delta V}$ and ΔR can again be obtained. For the case of minimum thrust, the solution for the sequence of angles can be determined through the use of Lagrange multipliers. For the constant mass case, the resulting set of linear equations are theoretically easily solved. When thrust is greater than the minimum required to accomplish the rendezvous within the prespecified time, the problem becomes more complex, and the simple expedient of applying thrust normal to the plane containing the velocity and position increments cannot be used. In fact, the existence of a solution when thrust is above optimum has not been proved.

D. CONSTANT-MASS-VARIABLE-GRAVITY CASE

The complexity of the constant-mass-variable-gravity case is reduced immediately by the following assumptions:

1. Gravity is known as a function of position for all possible chase vehicle positions.
2. The chase vehicle carries navigation equipment which continually computes position.
3. Magnitude and direction of chase vehicle thrust are continually changed to compensate for gravity variations encountered by change in chase vehicle position.

4. The rendezvous maneuver is conducted in an orbital plane; no attempt is made to change directions.

This reduces the problem to one involving constant mass and gravity with constant magnitude, but not direction. The solution to the resulting differential equations are quite comparable to the solutions for the constant-mass-constant-gravity case. Again, by using four thrust angles, an optimum thrust to accomplish the rendezvous can be shown to exist. Furthermore, it can be shown that if excessive thrust exists, fuel can be expended by velocity changes which total to zero in a plane perpendicular to the orbital plane, to produce almost zero position change perpendicular to the orbital plane for most practical cases.

VI. DIGITAL PROCESSING OF HIGH-RESOLUTION RADAR DATA

A. INTRODUCTION

It is theoretically possible to process the returns of high-resolution, side-looking radars by means of digital computers to obtain immediate, on-board presentations of their outputs. Furthermore, it is theoretically possible, by means of a digital computer, to process these returns properly regardless of perturbations of the radar from the desired flight path, provided that information from an adequate navigation system is available. This section summarizes the results of a study made to determine how such processing can be accomplished, to obtain some idea of the amount and kind of equipment required in a practical processor, and to determine what devices (if any) will need to be developed in order to obtain a practical digital processor.

In the study, a mathematical analysis was made of the operations to be performed by the processor, and certain results (not reported here) were obtained by computer simulation. Three different processor configurations were postulated and studied in detail. On the basis of these studies, it is concluded that:

1. A minimum amount of processing equipment will be required if the processing is done at video frequency, rather than at intermediate (in superhetrodyne radars) or radio frequencies.

2. The amount of processing equipment is minimized if the radar is designed for digital processing. For example, a drastic reduction results if the radar emits pulses at equal distances along its trajectory, rather than at equal time intervals.

3. Processing equipment is significantly reduced by accomplishing motion compensation by analog means prior to presenting the radar data to the digital processor, although resolution may suffer somewhat.

4. Digital radar processors which compute the signal from each resolution cell as the radar moves a distance equal to the synthetic aperture length require the same amount of computing equipment as that required by processors of equal resolution which store returns until

all data for a resolution cell has been received and then perform all the computation for the cell during the next interpulse period. The amount of memory, however, depends upon other details, and no general statement as to which system requires less equipment can be made.

5. The two most-needed devices for a practical digital processor of high-resolution radar data are small, inexpensive sample-and-hold devices and analog-to-digital converters.

B. MATHEMATICAL ANALYSIS

A general mathematical analysis of processors was performed in hopes that it might reveal means whereby processing could be accomplished more efficiently. Unfortunately, nothing radically new which is useful on this problem was discovered. The analysis is included in the report, however, because so far as is known, an analysis of this type has not been published previously. A general outline of the analysis is given below.

The mathematical expression for the output of a high-resolution radar produced by a point scatterer is

$$\underline{e}_{rm}(t) = \underline{\psi}_m \frac{g_m^2(t)}{R_m^2(t)} \underline{e}_c(t-\tau_m) + \underline{n}(t)$$

where

$e_{rm}(t)$ = receiver output voltage resulting from m^{th} scatterer

$\underline{\psi}_m$ = complex reflection coefficient of the m^{th} scatterer

g_m = ratio of field strength at scatterer position to that which would be expected if antenna were isotropic.

R_m = distance between radar and scatterer

$e_c(t)$ = transmitted voltage

τ_m = round trip time delay for electromagnetic radiation

$n(t)$ = receiver noise

t = time

and the underscore indicates a complex quantity.

We now define the function

$$\underline{I}(\vec{u}_m) = \int_{\infty}^{\infty} \underline{e}_r(t)\, \underline{f}^*(t, \vec{u}_m)\, dt$$

where $\underline{e}_r(t)$ = the received signal

$\underline{f}(t, \vec{u}_m)$ = complex weighting function

$\vec{u}_m$ = coordinates of the m^{th} target

and the asterisk indicates the complex conjugate. It can be shown that if we choose $f(t, \vec{u}_m)$ so that

$$\underline{f}(t, \vec{u}_m) = \frac{\underline{e}_r(t) - \underline{n}(t)}{\underline{\psi}(t)}$$

the signal-to-noise ratio of the function $\underline{I}(\vec{u}_m)$ will be maximized. In addition, the probability that the function $\underline{I}(\vec{u}_m)$ will be maximum when a scatterer is at the coordinates $\vec{u}_m$ is maximized.

Both the return, $\underline{e}_r(t)$ and the weighting function, $\underline{f}(t, \vec{u}_m)$ are analytic signals. As a consequence, it can be demonstrated that no more than two integrations need be performed to evaluate the function $\underline{I}(\vec{u}_m)$, and that under certain conditions only one integral need be evaluated.

From the standpoint of a practical digital processor, evaluation of the function $\underline{I}(\vec{u}_m)$ can be considered as simply the cross-correlation of the radar return with the return expected from targets at all expected coordinates $\vec{u}_m$. Two integrals must be evaluated for each target. Quite obviously from the definition of $\underline{I}(\vec{u}_m)$ it is possible to evaluate the cross-correlation function of the radio-frequency return and the weighting function representing the expected return, using time as the variable of integration. It can also be shown that an equally accurate cross-correlation function can be obtained by using either an intermediate-frequency signal (for a superhetrodyne receiver) or simply the video signals.

Although scatterers can be located as a function of time by the use of the above expression, it is far easier to construct a map if the signals and the correlating function are expressed as function of two variables: distance traveled along the desired flight path, and range from the radar to the scatterer. When variables are changed to accomplish this, it can be shown that one possible form of $\underline{I}(\vec{u}_m)$ is

$$\underline{I}(\vec{u}_m) = \int_{\infty}^{\infty} e_r(t) f^*(t, \vec{u}_m)$$

$$= \frac{c'}{2} \sum_k e^{-j\omega_i T_k} \int_0^{\frac{c'}{2}(T_{k-1} - T_k)} e^{-j\frac{2\omega_i}{c'}R}$$

$$w(R_k, {}^kS_1) e_{ra}(R, {}^kS_1) e^{-j\arg[e_k(R, {}^nS_1)]} dR$$

where

c	=	speed of light
c'	=	effective speed of light $= c + \frac{dR}{dt}$
$\underline{e}_k(R_k, {}^nS_1)$	=	actual radar return of the k^{th} pulse expressed as a function of range and along-track target distance.
$e_{ra}(R_k, {}^kS_1)$	=	expected radar return of the k^{th} pulse expressed as a function of range and along-track distance.
k	=	pulse index
R_k	=	range variable during reception of returns from the k^{th} pulse
${}^kS_1$	$\equiv$	along-track advance during reception of returns from the k^{th} pulse (during interval $T_k \leq t \leq T_{k+1}$)
${}^nS_1$	=	approximate distance along reference trajectory by which target is designated
T_k	=	time of emission of k^{th} pulse
$w(R_k, {}^kS_1)$	=	amplitude function similar to, but not necessarily equal to $e_{ra}(R, {}^kS_1)$ expressed as a function of range and along-track distance

arg [ ] = angle of the complex quantity in the brackets

ω_i = intermediate frequency of radar receiver

This is one form of the basic equation describing the functions of the radar processor. The factors $e^{-j\omega_i T_k}$ and $e^{-j \frac{2\omega_i}{c'} R}$ are considered to be introduced by the processor, and are used so that only the video signal is ultimately processed.

It was found in simulation studies that the amplitude of the function with which the actual return is correlated is not required to conform exactly with the expected return. Thus, the processing can be simplified by relatively coarse quantization of this quantity. Also, the phase of this function, $\arg\left[e_k(R, {}^kS_1)\right]$ can deviate somewhat from the desired value, but must have no more than a few degrees error for most applications.

The mathematical expression described by the processor relation will in general be non-analytic. It can be shown that for such a processor, two integrals must be evaluated to obtain the desired evaluation of $\underline{I}(\vec{u}_m)$ to obtain a point on the map.

C. DIGITAL PROCESSOR CONFIGURATIONS

This discussion of digital processing assumes that the following processes have been accomplished prior to transmitting the radar return to the processor:

1. radar pulses are emitted at predetermined points along the reference trajectory,
2. the radar return has been "compressed in range" (that is, the short pulse length required for range resolution has been restored),
3. any necessary motion compensation has been applied, and
4. the resulting signal has been coherently detected so that only the video signal remains in two detectors, the signals from which are in quadrature.

Under these conditions, two possible all-digital processor configurations have been established, which we have termed "delayed" (Fig. 8) and "immediate" (Fig. 9). In addition, a hybrid (partly digital, partly analog) version of the immediate processor has been established (Fig. 10). which appears to be simpler to implement than the all-digital version.

1. Delayed Processor

The system of Fig. 8 is termed a delayed processor because returns are stored until all information pertaining to a particular resolution cell is received, and the processing of this information is accomplished during the next interpulse interval. Starting at the top of the figure, operation of the system is as follows: the signals from the two detectors (termed the direct and quadrature detectors) are passed to separate banks of sample-and-hold devices. The sample gate of each device is opened during a time interval corresponding to the range resolution of the system, and as the sample gate of one device is closed, the gate of the next device is opened.* At the end of pulse reception, therefore, the returns for the range interval of interest will be stored in the sample-and-hold devices. These are converted to digital form by the analog-to-digital converters during the interpulse interval, and stored in the raw data registers. The sample-and-hold units are cleared prior to reception of the next pulse.

Data from each range resolution element contained in the raw data registers is next multiplied by the aperture weighting function. This function, which is a maximum at the antenna beam center and a minimum at the beam edges, modifies the radar returns so that the minor lobes that necessarily occur in all applications of mapping radars are reduced. A different antenna weighting function will theoretically be required for each along-track resolution element to be used at each range resolution interval. Hence, when the data are

*In Fig. 8, the sample-and-hold devices are shown actuated by a ramp. Alternately, a pulse passed through a delay network or line could also accomplish this successive opening and closing of sample gates.

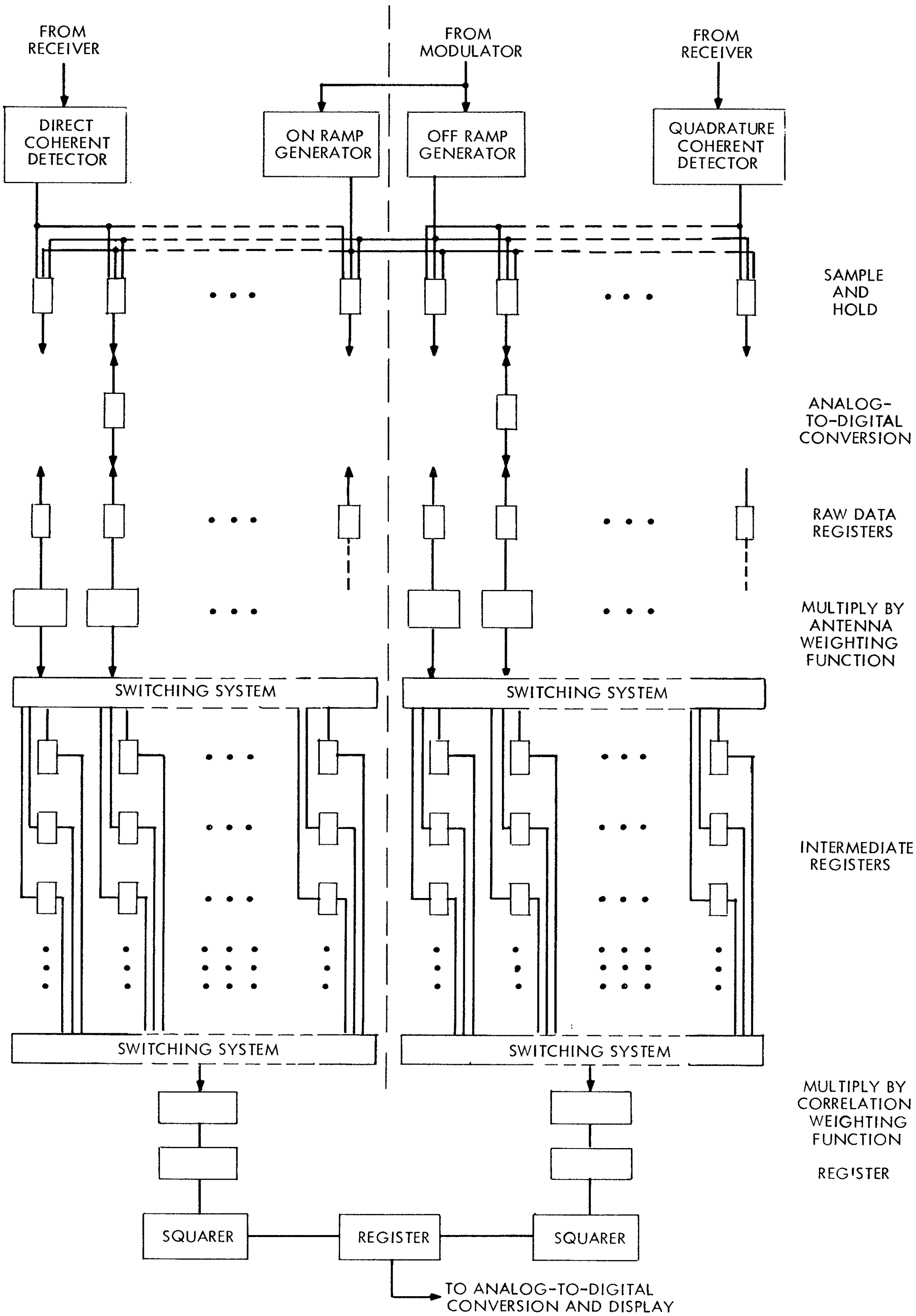


Fig. 8 Delayed Processor Block Diagram

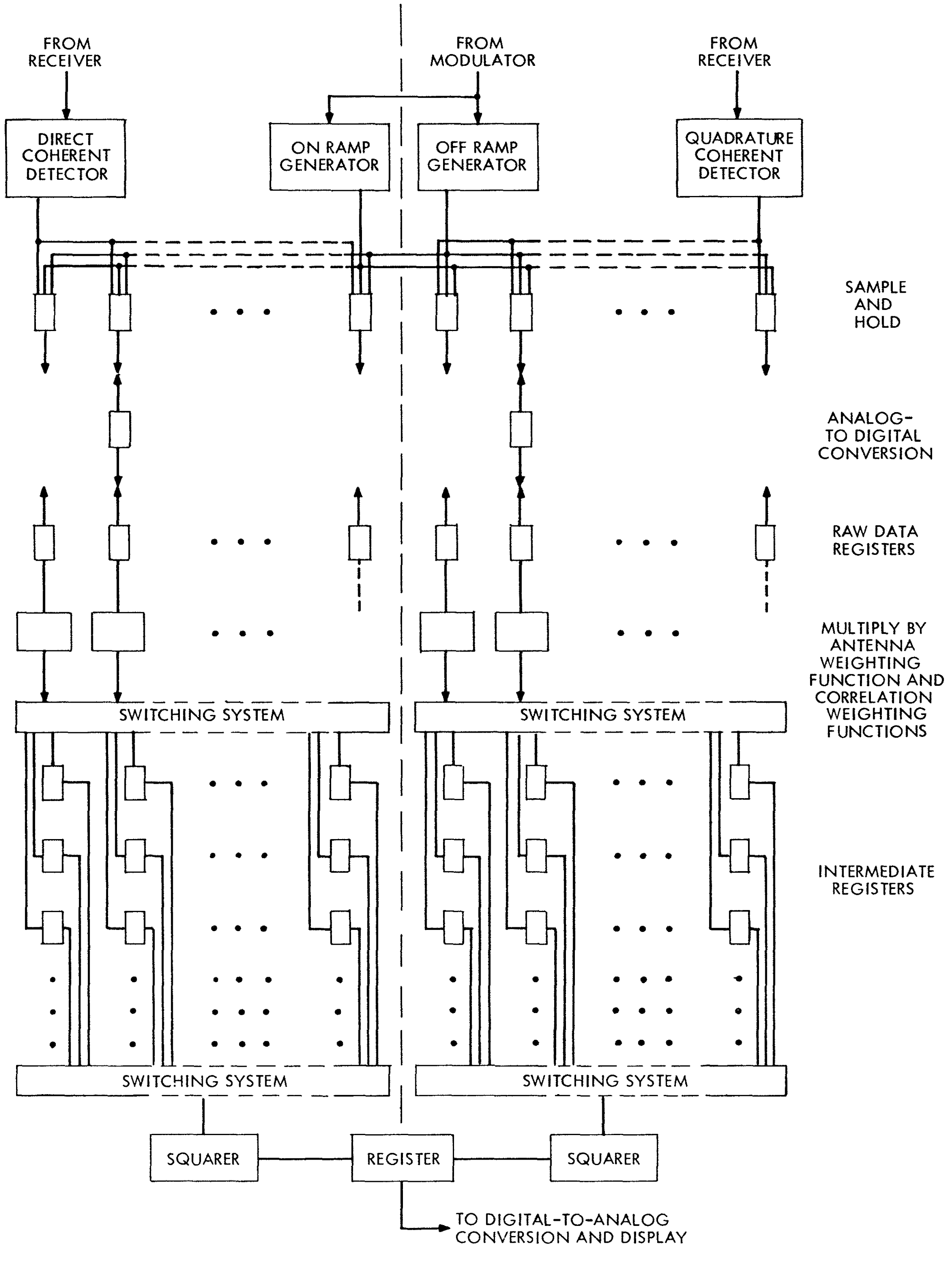


Fig. 9 Immediate Processor Block Diagram

transferred to the intermediate storage registers, there will need to be one register for each along-track resolution element for each range resolution element for each pulse.

After the radar has moved through the distance required for the desired along-track resolution, a correlation function is formed to produce an amplitude value for each range resolution interval. To do this, the data in the intermediate registers are multiplied by the appropriate correlation weighting functions, and summed. The sums for the same resolution element from the direct and quadrature channels are then squared and added, and transmitted for display or storage. At the end of this procedure, there will be one set of intermediate registers for each range resolution distance which contains data for which there is no further use. These registers are cleared and used to record the returns from the next pulse.

Although the processor shown in Fig. 8 indicates that products of scatterer return amplitude and aperture weighting function are stored in the intermediate register, the processor could be made differently. The unmodified scatterer return for each pulse at each range resolution interval could be stored in the intermediate registers, and multiplication by the antenna weighting function and the correlation weighting function could be combined in one operation.

Two changes in the processor would be required if antenna squint angle is changed:

1. the correlation weighting functions, which are assumed stored in memory, must be altered, and

2. the switching system program must be modified so that the cross-correlation is formed from returns from resolution elements which have the range-azimuth angle history expected of a scatterer.

2. Immediate Processor

The immediate processor, shown in Fig. 9 also samples the incoming signals, converts them to digital form, and puts them in raw data storage. However, the data in the raw data storage are multiplied immediately by both the aperture weighting function and the correlation weighting functions, and the product applied to an intermediate register. An individual intermediate register in this processor collects all the information pertaining to a particular resolution cell as the radar moves past it. Hence, the signals from successive pulses are processed as they arrive, and when the radar has moved along the trajectory the distance required for the desired resolution, the processing is complete, and the numbers in the direct and quadrature channel registers for that particular resolution element can be squared and added, and the sum displayed or recorded.

3. Hybrid Immediate Processor

If hybrid computing techniques are employed, rather than a pure digital system, a simpler immediate processor such as that shown in Fig. 10 might be made. In this system, the scatterer return is simply gated on and then off so that its time duration corresponds to the range resolution interval. Analog multiplication by the antenna weighting function and the correlation weighting function is accomplished by means of a bank of attenuators. The attenuator (multiplier) outputs are held during each interpulse interval, converted to digital form, and summed in a bank of registers, which has one register per resolution cell within the area of interest. Squaring and addition of the correlator outputs of the direct and quadrature channels is done as in the systems discussed earlier.

D. DISCUSSION OF COMPUTING REQUIREMENTS

It is impossible to make any general statements concerning the relative complexity of the two types of processors studied, because so much depends on the parameters of a particular application. Obviously, the amount of computing equipment and the memory size required by a processor will depend upon the number of resolution cells within a synthetic aperture length and the number of functions to be performed by the processor. For the particular cases studied it was found that the immediate processor would require less memory than would the delayed processor.

It is clear, however, that either type of processor will be quite complex for practical applications. The assumptions made in the

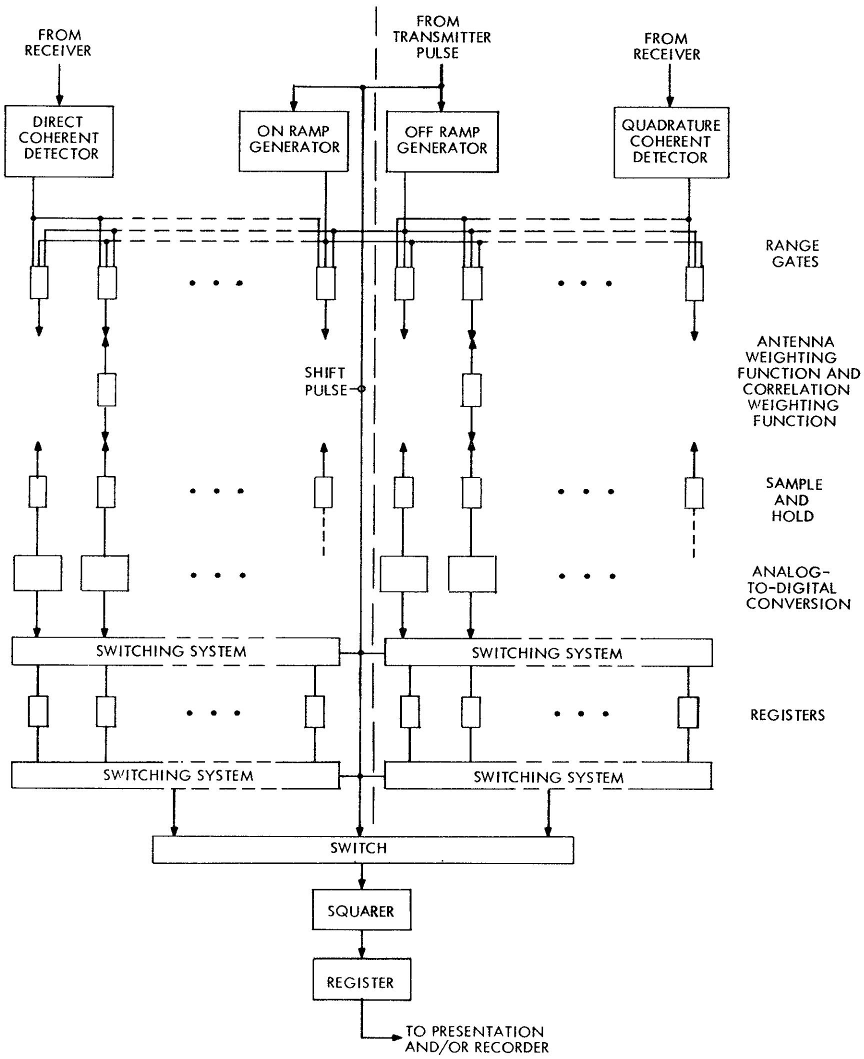


Fig. 10 Hybrid Immediate Processor for Straight and Level Trajectory

beginning of this discussion minimize the functions to be performed by the computer. Even with this minimization, it is not difficult to postulate resolution requirements and a range swath width which makes it necessary to use several parallel arithmetic elements and memory capacities between 10^5 and 10^6 bits.

If processors not subject to these assumptions are proposed, the computation and memory requirements become significantly larger. For example, if pulses are emitted at a constant frequency rather than at constant intervals of distance along the track, the expected return of each pulse for each range resolution cell (correlation weighting function) must be computed prior to the correlation process. Similarly, if motion compensation is not performed prior to analog-to-digital conversion, it must be inserted digitally, either by modifying the return or by computing the correlation weighting function before the correlation can be accomplished. It should be mentioned that the flexibility of the radar-processor combination is increased if the correlation weighting function is computed for actual aircraft path. However, computer and memory requirements imposed by such a system make it prohibitively large at the present time.

The devices which are essential to the success of any numerical processor are: analog-to-digital converters, sample-and-hold devices, and arithmetic elements. Since large numbers of sample-and-hold devices and analog-to-digital converters will be required for any practical digital processing systems for side-looking radars, it is concluded that development of integrated-circuit versions of all of these devices must be carried out before a practical real-time digital processor can be built.

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