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Research Article

Study of Local Mineralized Intensity Using Rescaled Range Analysis and Lacunarity Analysis

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Abstract

In this paper, the gold grade series along drifts have been analyzed using two methods, rescaled range analysis and lacunarity analysis which are commonly used in nonlinear systems analysis. The aim of this study is to better understand the ore-forming processes and identify the local mineralized intensity and interactions that influence the spatial structure of gold element grade distribution, in the Dayingezhuang fault-controlled, disseminated-veinlet gold deposit in the Jiaodong gold province, eastern China. The result shows that the efficiency of two methods, in distinguishing between weakly mineralized, moderately mineralized and intensely mineralized of ore-forming area. It is obvious that the two parameters of both Hurst and lacunarity index in the weakly mineralized drifts are distinguished from those in the mineralized drifts, and the lower the index is, the more homogeneously distributed of the elements and the mineral intensity is relatively smaller. The methods used in this paper provide a relatively comprehensive description for local mineral intensity, offering an evidence for the identification of mineralization intensity and providing a guidance for further determination to the extent of deposit concentration and delineation of target mineralization zone.

Keywords: rescaled range analysis, lacunarity analysis, nonlinear, mineralized intensity, gold grade

1. Introduction

Mineral intensity is a basic concept for the appraisal of enrichment strength of ore-forming elements, which can give information for both the ore-forming process and the exploitation. Isotopes and fluid components data acquired by geochemical testing contain abundant information about metallogeny, material sources, etc., but their indications for mineralized intensities are still poor, and geology observations are also difficult to identify the mineralized intensity accurately. The serie of metallogenic elements grade is still a key clue for mineralized intensity, and geo-mathematics theory becomes an effective tool to quantitatively identify the mineralized intensity[1,2,3]. However, metallogenic element distribution shows a highly irregular structure, and to exhibit scale-dependent changes in structure, one needs nonconventional statistical methods. For a better comprehension of the element grade of volatility function properties, it is necessary to ascertain the element contents changes by investigating the structure of latency at the microscopic level.

In recent studies, it has been shown that a lot of time series in nature are characterized by self-similarity and scale in variance [4,5,6,7,8,9]. These methods have become widespread and valuable tools in studying geological data. However, each method has its characteristic parameters. Some parameters have the same effect on the feature description of series and some have not, such as the scaling index H, by rescaled range (R/S) analysis, which is known as long-term memory or high persistence that implies a strong correlation between the successive data points, and lacunarity index($\Lambda(r)$), by lacunarity analysis, which is a scale-dependent measure of spatial complexity or texture of a dataset.

Since the gold deposits usually contain irregular ore bodies due to the complicated distribution of grade, the nonlinear characteristics of the grade distribution on different locations with various mineralization ranks are still ambiguous. In this paper, the Dayingezhuang structure controlled alteration-rock type gold ore deposit in Jiaodong gold province, China is selected for a case study. The gold elements grade along drifts will be analyzed, the aim of this study is to describe the local mineral intensity, using rescaled range analysis and lacunarity analysis. Meantime, the relationship between the characteristic parameters of the two methods will be discussed.

2. Analysis Techniques

2.1 Rescaled Range Analysis

If the series $x(t)$ is a self-affine fractal, then $x(bt)$ is statistically equivalent to $b^H x(t)$, where H is the Hurst exponent. The Hurst exponent is frequently calculated for the data sets obtained experimentally to characterize noisy data series. It is also used in characterizing stochastic processes. For example, Brownian noise is a self-affine

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fractal with a Hurst exponent H = 0.5; white noise may be considered as having H = 0; and an exactly self-similar process would be characterized by H=1. An H less than 0.5 indicates presence of negative correlations while an H greater than 0.5 indicates presence of positive correlations in the time series. The Hurst exponent is commonly used as a measure of the geometric (fractal) scaling in the data series. In many cases, when dealing with geological and geophysical data, the Hurst exponent is calculated from a series that consists of a short discrete set of values. One of the commonly used methods for calculating the Hurst exponent is rescaled range analysis, by denoted R/S analysis[5,6].

Rescaled range analysis was created by Hurst while studying the statistical properties of the Nile River overflows[9]. This technique was inspired by Einstein's work on pure random walks, where the absolute value of the particles displacement scaled as the square root of time. The calculation starts with the whole observed data set $\{\xi\}_{i=1}^{N}$, that covers the range n and calculates its mean over the available data in the range n.

$$(E\xi)_n = \frac{1}{n}\sum_{i=1}^{n}\xi_i \tag{1}$$

Sum the differences from the mean to get the cumulative total at each data point, $X(i,n)$, from the beginning of the range up to any point of the range：

$$X(i,n) = \sum_{t=1}^{i}[\xi_t - (E\xi)_n] = \sum_{t=1}^{i}\xi_t - i(E\xi)_n, 1 \le i \le n \tag{2}$$

Find the $\max X(i,n)$ representing the maximum of $X(i,n)$, $\min X(i,n)$ representing the minimum of $X(i,n)$ for $1 \le i \le n$, and calculate the range $R(n)$:

$$R(n) = \max X(i,n) - \min X(i,n) \tag{3}$$

Calculate the standard deviation $S(n)$ of the values over the range n:

$$S(n) = \{\frac{1}{n}\sum_{i=1}^{n}[\xi_i - (E\xi)_n]^2\}^{1/2} \tag{4}$$

Then, we can calculate

$$R(n)/S(n) \propto n^H \tag{5}$$

For the first step, n covers a fraction of the dataset, typically is equal to N/2, determined R/S for each segment of a data set, then take the mean value of R/S, $E(R/S)_n$. Using successively shorter n at each step, we can divide the data set into more non-overlapping segments and find the mean R/S of these segments.

$$\ln E(R/S)_n = \ln C + H\ln n \tag{6}$$

In plotting ln ($E(R/S)_n$) against ln n, we expect to get a line whose slope determines the Hurst exponent.

2.2 Lacunarity Analysis

Allain and Cloitre presented an algorithm to calculate lacunarity index by utilizing a moving window[10]. This algorithm is briefly summarized below. Suppose a ordered set of one-dimensional $A \subset R$ is considered, and the total length of the set is N_t and individual segments have unit length (r=1). Consider a moving window of length size r (r=1, 2 ..., N_t/2), which is translated in unit increment. Let us define $n(M, r)$ to be the number of gliding boxes with size r and mass (measure) $M(r)$, and M_i is created, where i is a counter-variable used to enumerate the specific mass M. The probability function $P(M, r)$ is obtained by dividing $n(M, r)$ by the total number of boxes $N(r) = (N_t - r+1)$.

$$P(M,r) = \frac{n(M,r)}{N(r)} \tag{7}$$

The statistical moments $Z^{(q)}(r)$ of this distribution now can be determined as[11-12]:

$$Z_P^{(q)}(r) = \sum_i M_i^q(r)P(M_i,r) = \frac{1}{N(r)}\sum_i M_i^q n(M_i,r) \tag{8}$$

The definition of the lacunarity function $\Lambda(r)$ uses only the first and the second moments of M, that is

$$\Lambda(r) = \frac{Z_P^{(2)}(r)}{[Z_P^{(1)}(r)]^2} = 1 + \frac{\sigma^2(r)}{[Z_P^{(1)}(r)]^2} \tag{9}$$

Where $\sigma^2(r)$ is the sample variance and $Z^{(1)}(r)$ is the mean of the probability function $P(M, r)$. It can be seen easily that lacunarity defined in (2) has the following properties: (ⅰ) $\Lambda(r) > 1$ because the variance $\sigma^2(r) > 0$; and (□) $\Lambda(r) = 1$ if and only if $\sigma^2(r) = 0$, which implies that M is constant over all gliding boxes; therefore M has translational invariance and the lacunarity reaches its minimum value.

However, as Cheng[12]points out, lacunarity analysis may be skewed by edge effects when quantifying a finite pattern. The problem occurs because as the gliding-box of size r is moved about the pattern, the values around the edges are under-sampled by the gliding-box, due to the fact that the gliding-box cannot overlap beyond the edge of the pattern. The effect is particularly strong when the size r of the gliding-box is large relative to the extent of the pattern[8,13].

A new algorithm considering edge effects is put forward where the lacunarity algorithm was altered so that once the gliding-boxes reached the edge of the ordered set, and it could go beyond it by wrapping around to the opposite side. For example, assume that the pattern size is N and the gliding-box is composed of r cells. After the gliding-box is moved across the tail and the edge is reached, the gliding-box is positioned such that the r_1 grid cell of the gliding-box is superimposed over the last of cells on the right side of the pattern, and the r_2 grid cells of the gliding-box is over the last column of cells on the left side of the pattern, r_1+ r_2= r (see Fig.1).

For self-similar set, the lacunarity follows a power law of the form

$$\Lambda(r) = \alpha r^{-\beta} \tag{10}$$

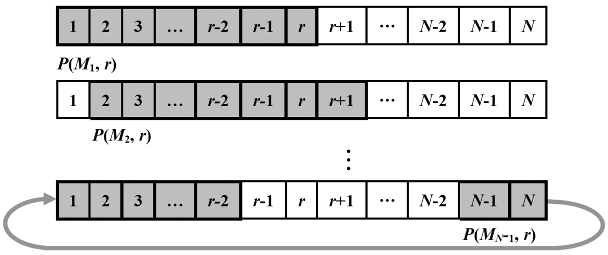

Fig. 1. Illustration of the slid-window method used to determine the lacunarity

Logarithmic equation (9)

$$\ln \Lambda(r) = \ln \alpha - \beta \ln r \tag{11}$$

Where α is a pre-factor and the lacunarity scaling index β satisfies the equality

$$\beta = D - E \tag{12}$$

Where D and E are the fractal and Euclidean dimensions, respectively.

3. Research Material

3.1 Geological Settings

The Dayingezhuang ore deposit is located in the middle segment of the Zhaoping fault zone in Jiaodong gold province, China. The Jiaodong gold province is famous for its gold production, and structure controlled alteration rock gold deposits formed in the Mesozoic dominate the province [14-16]. The reserves of the Dayingezhuang are more than 100 t, with an estimated annual production greater than 2.6 t, and the pyrite-sericite-quartz altered rock (pay rock) is distributed through the whole mineralized zone. And it is with 3000m in length, 30m~140m in width; moreover, ore bodies of altered rocks extend like sloping wave in a strike and in trend. The explored ore-bodies with irregular shape, embedded primarily in the fractural-altered rocks under the main fractural plane of the Zhaoyuan-Pingdu fault, distributed within the exploration lines 66 and 84 between the levels -26m and -492m. The main explored levels are -140m, -175m, -210m, -290m[17,18,19].

The original gold grade data are obtained from the continuous samples with 1m in length along different drifts. The non-mineralized drift is named in this paper when the most contents are lower than the cut-off grade and the oredoby can not be demarcated clearly; and the mineralized drift is defined when part of the grades are greater than the cut-off and there exists orebody.

3.2 Data preparation

This paper focuses on the analysis of the mineral intensity on -210m level in the No.II-1 orebody and the surrounding alteration zone for drift CM1 to drift CM7. Along different drifts at several levels below the fault plane, channel samples of 1 min length were collected continuously across the intense altered-mineralization zone. These samples were assayed. The results are used for reserve calculation as well as for this study. The gold orebody described in this study is delimited based on a cutoff grade of 2 g/t.

The mineralization degree in these drifts can be categorized into various ranks according to the development of the orebodies as follows[20,21]: (I) weakly mineralized areas, where the contents are rarely greater than 2 g/t, and the orebodies are barely developed; (II) moderately mineralized areas with discontinuous orebodies; and (III) intensely mineralized areas, where more than half of the contents are greater than the cut-off and the orebodies are huge or continuous in the sample range (Table 1).

Table 1. Description of mineralization rank

Mineralization rank	Name	Gold contents distribution	Orebody distribution
I	Weakly mineralized area	Rarely greater than 2 g/t	The orebody is very thin
II	Moderately mineralized area	A few grades are greater than the cut-off	The orebody is discontinuous
III	Intensely mineralized area	Half of the contents are greater than the cut-off	The orebody is huge or continuous

4. Results and Discussion

4.1 Summary Statistics and Normal Test

To describe the main features of a collection of data in the difference drifts, a statistical treatment of the data was performed using SPSS. We implement a Jarque-Bera (JB) test for normality with a relative large sample here. The statistical quantity for the JB test is

$$JB = n\left[\frac{S^2}{6} + \frac{(K-3)^2}{24}\right] \tag{13}$$

Where S is skewness, K is kurtosis, n is the volume of a sample, and JB is a statistical quantity for JB test. The results of the JB test are in Table 2. The table shows that the time series of element contents is not normally distributed as depicted by the Jarque-Bera statistic. The kurtosis values lower than 3 are an indication of the presence of platykurtosis in the probability distribution, the skewness values greater than 0 are an indication of the positive skewness.

Table 2 Descriptive statistics of gold contents and normal test

Drift	Mean(g/t)	Standard deviation	Kurtosis	Skewness	Jarque-Bera statistic	Probability*
CM1	0.4932	0.4273	1.8716	3.2753	60.8163	0.0000
CM2	0.9783	1.7273	4.5462	22.9222	1925.6524	0.0000
CM3	0.6244	1.4426	6.9465	55.0082	12070.9573	0.0000
CM4	0.6801	0.6724	1.3410	0.8740	23.2089	0.0000
CM5	2.2920	2.8862	2.4683	8.9881	788.6710	0.0000
CM6	0.4073	0.3693	3.2562	13.6116	1109.9714	0.0000
CM7	2.6428	4.5495	5.2739	31.6053	2174.0490	0.0000

Note: *Statistical significance at the 5% level.

For the JB test, the null hypothesis is no rejection to a normal distribution. But the results of the JB test show that, at the 5% significant level, the values of probability are almost to 0.0000 (but not exactly 0). So we can reject the null hypothesis of following a normal distribution to total samples.

4.2 Evaluation of Hurst and launarity index

For carding out the Hurst exponent calculation, Let the sample number be N, choose step $N/2 > n > 5$ to divide the dataset into $[N/n]$ groups, calculate the R/S in each group and get the mean value $(R/S)_n$. Plot $(R/S)_n$ and n in the ln-ln coordinates, the slope of the fitting line is the Hurst exponent. Fig.1 shows the calculation diagram of the Hurst exponent by R/S analysis method.

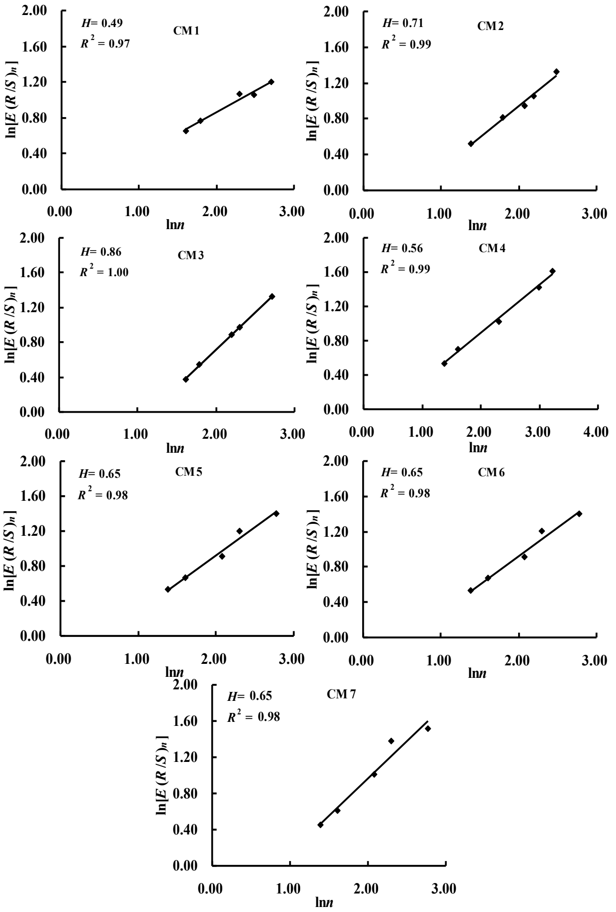

Fig.2. Calculation diagram of the Hurst exponent by R/S analysis method from CM1 to CM7

In Fig.2, it shows the calculation diagram of the Hurst exponent by R/S analysis method. The fitting goodness is greater than 0.93, in plotting ln $(E(R/S)_n)$ against ln n, which indicates that the gold element grade series are characterized by the scale-invariant. Hurst is 0.49 closing to 0.5 in CM1, and is to obey the Brownian motion in weakly mineralized area. The Hurst is greater than 0.56 in moderately mineralized, as CM2, CM4, CM5 and CM6, and the Hurst is greater than 0.80 in intensely mineralized areas, as CM3 and CM7, which indicate that the sequence of the stochastic process is of scale invariant and long-range correlations, and the continued strength of the sequence is consistent with the mineralization.

Lacunarity was calculated for each test drift using the gliding-box algorithm outlined above, from Formula (7) to (12), with a range of moving-window sizes varying from r =1m up to r =20m. The algorithm of lacunarity was plotted against the gliding box size, as illustrated in Fig.3.

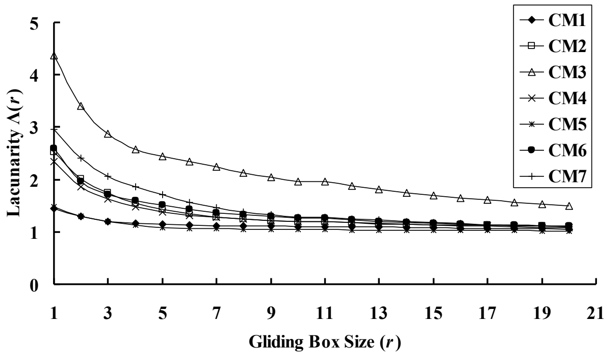

Fig. 3. Lacunarity plotted against box size for different drifts form CM1 to CM7

The lacunarity results for boundary algorithm for seven drifts in Table 4. In Fig. 3, the lacunarity decreases with the increase of box size r, and the lacunarity scaling exponent $\Lambda(r)$ follows a power law with high fit goodness on the exploration lines (see Table 4), mostly above 0.88. That is, the larger β values indicate fast decay of lacunarity as the scale is increased. In contrast, lacunarity is not scale-dependent as $\beta \to 0$. The lacunarity β ranges from 0.09 to 0.35, and the lacunarity index size can be divided into different rank. We can see that α<1.36 and β<0.1 are relatively lower in weakly mineralized, whereas the 1.36<=α<2.37 and 0.2<β<0.3 are relatively larger in moderately mineralized area, and α>2.9 and β > 0.3 are intensely mineralized area.

Table 4 Gliding box algorithm lacunarity estimations in Dayingezhuang gold ore deposit, China

Drift	Type	R/S		Lacunarity		
		H	R^2	α	β	R^2
CM1	I	0.49	0.97	1.36	0.09	0.93
CM2	II	0.71	0.99	2.32	0.27	0.96
CM3	III	0.86	1.00	4.32	0.35	1.00
CM4	II	0.56	0.99	2.15	0.24	0.97
CM5	II	0.65	0.98	1.36	0.11	0.88
CM6	II	0.69	0.98	2.37	0.26	0.98
CM7	III	0.81	0.96	2.94	0.34	0.99

Whereas the average, Hurst exponent and α for the three types are respectively H= 0.49, α = 1.36, H= 0.651, α =2.05, and H= 0.834, α = 3.63, in weakly mineralized, moderately mineralized and intensely mineralized of ore-forming area. The Hurst and lacunarity index of the mineralized intensity basically show positive relation, the correlation coefficient reaches to 0.85.

The lacunarity is more sensitive to the special structure of the spatial gold grade distribution, especially when r is small. The lacunarities are mostly great in the mineralized area, showing the gold grade distribution along the non-mineralized drifts is more homogeneous in general. The gold grade of the drift in non-mineralize areas is more homogeneously distributed, and the mineral intensity is relatively smaller.

5. Conclusions

The geochemical element grade is inhomogeneity, and the local enriching and depleting are to indicate the intense and bare mineralization respectively. By selecting seven drifts in Dayingezhuang gold ore deposit in the eastern Shandong Province, and using rescaled range analysis and lacunarity analysis, the distributions of the ore-forming elements have been analyzed to better understand the element transport mechanism, in which the combinations of the two parameters were applied for identification of local mineral intensity. The result shows the efficiency of two methods, in distinguishing between weakly mineralized, moderately mineralized and intensely mineralized of ore-forming area. It is obvious that the two parameter of both Hurst and lacunarity index in the weakly mineralized drifts are distinguished from those in the mineralized drifts, the lower the index is, the more homogeneously distributed of the elements and the mineral intensity is relatively smaller. The analysis for the drifts in this paper is also suitable for the other exploration engineering, e.g. drills, trenches. Two methods used in this paper provide more comprehensive description and comparison for local mineral intensity, and offering an evidence for the identification of mineralization intensity and providing a guidance for further determination to the extent of deposit concentration and delineation of target mineralization zone.

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