

# AGKY Hadronization Model Tuning in GENIE v3

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(Dated: June 11, 2021)

The GENIE neutrino Monte Carlo describes neutrino-induced hadronization with an effective model, known as AGKY, which is interfaced with PYTHIA at high invariant mass. Only the low-mass AGKY model parameters were extracted from hadronic shower data from the FNAL 15 ft and BEBC experiments.

In this paper, the first hadronization tune on averaged charged multiplicity data from deuterium and hydrogen bubble chamber experiments is presented, with a complete estimation of parameter uncertainties. A partial tune on deuterium data only highlights the tensions between hydrogen and deuterium datasets.

## I. INTRODUCTION

The next generation of neutrino oscillation experiments will rely on the precise understanding of neutrino interactions at the percent level. Experiments such as T2K [1], NOvA [2], MINERvA [3] and MicroBooNE [4] study neutrino interactions over a broad energy range. In the few GeV region,  $0\pi$  and  $1\pi$  contributions dominate the event rate. Hence, most of the effort has been focused on the theoretical understanding of these interactions [5–9] as well as the precise measurement of quasielastic [10–16] and pion production cross sections [10, 17–23]. Pions, before FSI, can be produced by either neutrino resonance interactions [24] or hadronization processes. Hadronization models provide information about the multiplicities and kinematics of the hadrons before final state interactions (FSI) given the neutrino-nucleon interaction and the event kinematics. The knowledge of the exact mixture of hadrons in showers affects the efficiency to distinguish between NC/CC events, the event topological characterization [11, 20], impacts the estimation

of backgrounds [25] and calorimetric energy reconstruction. FSI interaction modeling and detector efficiency corrections are also crucial to avoid confusion in measurements of neutrino-induced hadron production. Unfortunately, due to the lack of unified models for exclusive hadronic multiparticle production over the energy range of interest for neutrino experiments, one must resort to stitching together different modeling ingredients. The GENIE neutrino Monte Carlo event generator [26] uses the Andreopoulos-Gallagher-Kehayias-Yang model (AGKY) hadronization model [27] whose validity extends down to the inelastic threshold. At low hadronic invariant mass  $W$  the model is based on the Koba-Nielsen-Olesen scaling law (KNO), while at high- $W$  it is based on PYTHIA [28].

Future experiments will operate at high energies, where potential biases originating from hadronization mismodeling become important. For instance, DUNE [29], PINGU [30] and ORCA [31] will focus on the 2 to 20 GeV energy range where deep inelastic (DIS) events are dominant. The neutrino energy dependence of the main inelastic components of the expected event rate for charged-current (CC)  $\nu_\mu$ -<sup>40</sup>Ar scattering is shown in Fig. 1. Some relevant neutrino fluxes of interest are shown in Fig. 1(top). It is seen that the contribution to the event rate from

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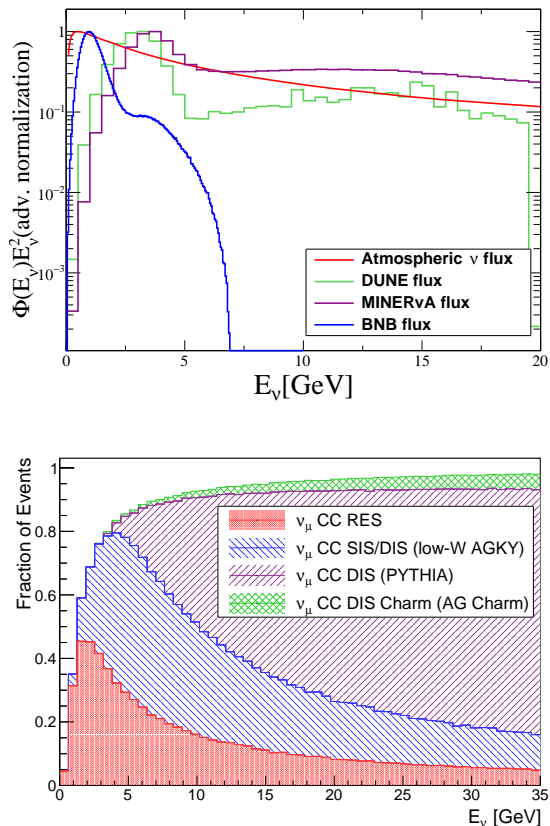


FIG. (1) Breakdown of CC events as a function of the neutrino energy from  $\nu_\mu$  scattering on  $^{40}\text{Ar}$  (bottom plot). The plot was obtained with GENIE v3.00.06 using tune G18\_02a\_02\_11a. The main components are: resonance (RES), shallow and deep inelastic scattering (SIS/DIS) and deep inelastic charm production (DIS Charm). DIS contributions are split according to the hadronization model used: low- $W$  AGKY and PYTHIA. On the top plot, normalized neutrino fluxes are shown for the atmospheric neutrino flux at Kamioka [33], DUNE [34], MINERvA [35] and BNB [36] flux predictions.

GeV neutrinos is mainly driven by CC RES events and SIS/DIS events from the low- $W$  AGKY model, whereas PYTHIA events dominate at high neutrino energies.

The description of the AGKY hadronization model implementation in GENIE is described in Sec. II. There is a separate hadronization model to simulate DIS charm production, the Aivazis, Olness and Tung model [32]. Hadronic remnants produced in the interaction are hadronized with PYTHIA.

The AGKY model parameters controlling hadronization at low invariant masses were extracted from some of the FNAL 15 ft bubble

chamber and the Big European Bubble Chamber (BEBC) analyses [37, 38]. PYTHIA has never been tuned to low-energy neutrino-induced hadronization data. In 2010, GENIE revisited the AGKY parameter values and modified a number of PYTHIA parameters using information from the NUX PYTHIA tune [39], as discussed in Sec. II. Thus, we refer to this parameter set as the *2010* GENIE AGKY tune or *2010* GENIE tune. Despite the modifications, several discrepancies between the model and neutrino-induced hadron shower data remained [40, 41].

This paper summarises the results of the first tune of the AGKY hadronization model against averaged charged multiplicity data on hydrogen and deuterium targets from bubble chamber experiments. The work was performed within the GENIE v3.00.06 global analysis framework [24]. The base configuration used for all the plots presented here is the G18\_02a\_02\_11a. The AGKY model specifics relevant for this work are described in Sec. II, followed in Sec. III by an explanation of the analysis procedure applied to the hydrogen and deuterium datasets. Section V discusses the free parameters in the model, and Sec. VI presents the construction of the likelihood function used for fitting. The AGKY best-fit results are summarised in Sec. VII.

## II. THE AGKY MODEL

The AGKY [27] model is the main hadronization model used in GENIE. As a function of hadronic invariant mass  $W$ , three different regimes are defined: an empirical model anchored to bubble chamber data at low- $W$  ( $W < W_{\min}^{\text{tr}}$ ), a pure PYTHIA region for high- $W$  ( $W > W_{\max}^{\text{tr}}$ ) and a transition region that connects them. In the transition region, the probability to produce a PYTHIA event increases linearly with  $W$ , from zero for  $W_{\min}^{\text{tr}}$  to 1 at  $W_{\max}^{\text{tr}}$ . The values of the transition region limits are  $W_{\min}^{\text{tr}} = 2.3 \text{ GeV}/c^2$  and  $W_{\max}^{\text{tr}} = 3.0 \text{ GeV}/c^2$ . The empirical low- $W$  model and PYTHIA are valid in different mass ranges and they are combined accordingly.

The low- $W$  AGKY and PYTHIA algorithms are described in the Sec. IIA and Sec. IIB respectively. The contribution of the main inelastic components as a function of  $W$  for events generated with the DUNE flux [34] is shown in Fig. 2. Most of the DIS/SIS events use the low- $W$  AGKY model while the PYTHIA events are coming from the high-energy tail of the beam.

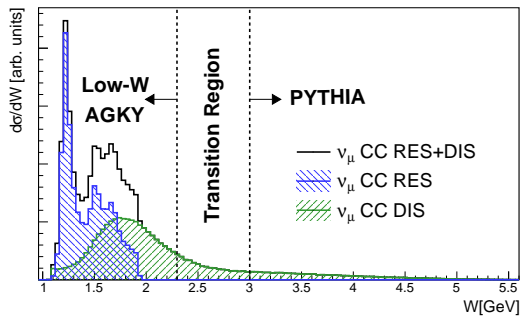


FIG. (2) Flux integrated CC inelastic differential cross section as a function of the hadronic invariant mass for a DUNE  $\nu_\mu$  beam on  $^{40}\text{Ar}$ , obtained with the G18.02a\_02\_11a tune. The distribution is decomposed in RES and DIS contributions. The DIS contribution to the total number of events is 38% and 36% for RES events. The  $\nu_\mu$  flux maximum is between 1 and 5 GeV.

#### A. Effective low- $W$ AGKY hadronization model

At low- $W$ , the showers are made of one baryon and any number of  $\pi$  or  $K$  consistent with momentum, charge, baryon and strange number, isospin and parity conservation laws:

$$\begin{aligned} \nu_\mu + p &\rightarrow \mu^- + X^{++} \\ \nu_\mu + n &\rightarrow \mu^- + X^+ \\ \bar{\nu}_\mu + p &\rightarrow \mu^+ + X^0 \\ \bar{\nu}_\mu + n &\rightarrow \mu^+ + X^- \end{aligned}$$

For instance, when approaching the pion production threshold, the  $\nu_\mu p$  interaction would produce a shower made of a proton and a  $\pi^+$ . In general, the hadron multiplicity at the lowest possible  $W$  is 2 as the hadronic final state can only be made of a pion and a nucleon.

As  $W$  increases, more possibilities are available. The model draws random integer numbers from the simulated hadronic multiplicity distribution to generate the number of particles in the shower, then the particles are labeled so that baryon number, charge, and strangeness are conserved. The particle content of a shower is selected so that the total mass is not exceeding  $W$ . The four-momenta of the hadronic shower particles are generated by a weighted phase space decay of a particle of mass  $W$  to the selected hadronic-multiparticle state. There are many ingredients in the simulation of the hadronic probability distribution: average hadronic multiplicity data, the KNO scaling law, particle content rules, phase space weighting and others, as discussed in detail in

TABLE (I) 2010 GENIE tune low- $W$  AGKY parameters.

Parameter	$\nu_\mu p$	$\nu_\mu n$	$\bar{\nu}_\mu p$	$\bar{\nu}_\mu n$
$\alpha_{\text{ch}}$	0.40	-0.20	0.02	0.80
$\beta_{\text{ch}}$	1.42	1.42	1.28	0.95
$c$	7.93	5.22	5.22	7.93

Ref. [27]. In this paper we focus on the description of the hadronic multiplicity. The hadronic multiplicity probability distribution depends on two ingredients: the measured average as a function of  $W$ , and an empirical parameterization of multiplicity dispersion. Both parameterizations must be extracted from data.

Empirical observation suggests that the average charged multiplicity is linear with  $\ln W^2$ :

$$\langle n_{\text{ch}} \rangle(W) = \alpha_{\text{ch}} + \beta_{\text{ch}} \ln \left( \frac{W^2}{\text{GeV}^2/c^4} \right) \quad (1)$$

The coefficients  $\alpha_{\text{ch}}$  and  $\beta_{\text{ch}}$  depend on the initial state and their values can be extracted from neutrino-induced hadronization data, see Sec. III. This behaviour has also been proved to be true for heavier nuclear targets [42, 43]. From fits to  $\pi^0$  production data, it is known that  $\langle n_{\text{ch}} \rangle \sim 0.5 \langle n_{\pi^0} \rangle$  [44]. Therefore, the total hadronic multiplicity is obtained from the the charged one as

$$\langle n \rangle(W) \equiv 1.5 \langle n_{\text{ch}} \rangle(W) \quad (2)$$

Given the average  $\langle n \rangle$ , the hadronic multiplicity distribution,  $n$ , can be obtained from the KNO scaling law, which relates the dispersion of hadron multiplicities with a universal scaling function [45],

$$\langle n \rangle P(n) = f \left( \frac{n}{\langle n \rangle} \right) \quad (3)$$

The scaling function  $f(n/\langle n \rangle)$  is parametrized with the Levy function  $L(n/\langle n \rangle; c)$

$$L(n/\langle n \rangle; c) = \frac{2e^{-c} c^{c \frac{n}{\langle n \rangle} + 1}}{\Gamma \left( c \frac{n}{\langle n \rangle} + 1 \right)} \quad (4)$$

where  $\Gamma$  is the gamma function and  $c$  is the free parameter that has to be extracted from data and depends on the interaction isospin. By construction, the dispersion of the hadronic multiplicity distribution is independent from the average, see Fig. 3. The 2010 GENIE AGKY values of  $\alpha_{\text{ch}}$ ,  $\beta_{\text{ch}}$  and  $c$  are specified in Tab. I.

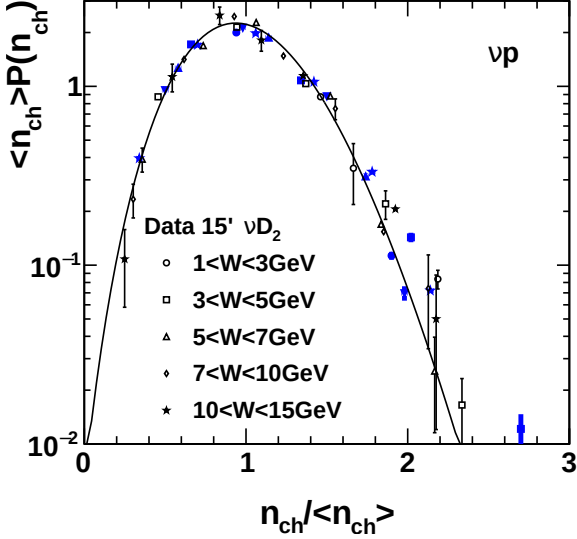
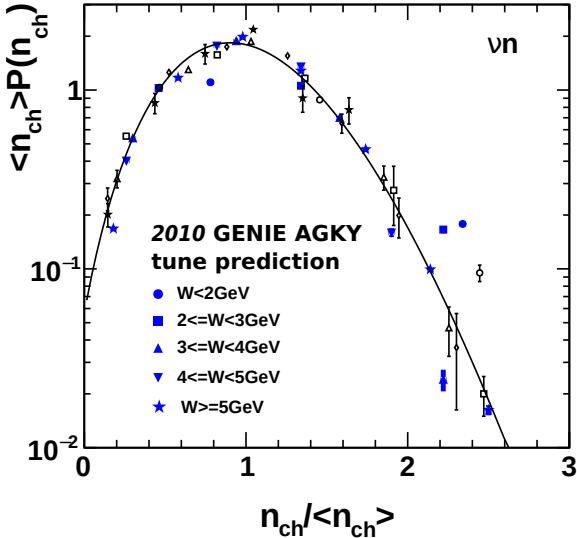
(a) KNO distribution for  $\nu p$  interactions.(b) KNO distribution for  $\nu n$  interactions.

FIG. (3) KNO scaling distributions for neutrino interactions on deuterium [27]. The solid line is the best fit result of the Levy function to FNAL 15 ft bubble chamber data [37]. Blue dots show the 2010 GENIE AGKY prediction for a given  $W$  range specified in the legend.

## B. PYTHIA in GENIE

The PYTHIA algorithm is well known for its wide use in high-energy collider experiments to simulate the evolution from a few-body hard process to a multi-hadronic final state [28, 46]. The PYTHIA hadronization model is based on the Lund string fragmentation framework which describes the hadronization process as break-ups in a string

throughout production of new  $q\bar{q}$  pairs [47]. Each string represents a color flux which is subject to a linear confined potential. In the Lund model, the  $q\bar{q}$  pairs break by tunneling, which, together with causality, defines the Lund symmetric fragmentation function,

$$f(z) \propto \frac{(1-z)^a}{z} \exp\left(\frac{-bm_{\perp}^2}{z}\right) \quad (5)$$

with the transverse mass of the hadron defined as  $m_{\perp}^2 \equiv m^2 + p_{\perp}^2/c$  and  $z$  being quantities that characterise the hadronic shower [48]. The transverse momentum is defined as  $p_{\perp}^2 = p_x^2 + p_y^2$ .  $z$  describes the fraction of available light cone momentum  $E + p_z$  transferred to the hadrons produced with energy  $E$ , and it is defined as  $z = E/\nu$ . The parameters  $a$  and  $b$ , known as Lund  $a$  and Lund  $b$ , are free parameters of the model that are responsible to distribute the longitudinal energy of the hadronic system after the interaction and should be tuned to reproduce experimental data [48]. In terms on the effect on  $\langle n_{\text{ch}} \rangle$ , as Lund  $a$  increases, the multiplicity increases as well, while the opposite is happening for Lund  $b$ .

In GENIE, PYTHIA is used to simulate the hadronization at high energy invariant masses. Specifically, GENIE v3.00.06 uses PYTHIA 6. Future GENIE releases will slowly transition to PYTHIA 8. In particular, in v3.00.06, PYTHIA 8 is partially integrated in GENIE and it is fully integrated in the AGKY model. After the partial integration of PYTHIA 8, simulation outputs remained unchanged. Hence, the tune presented in this paper is also valid for PYTHIA 8. Moreover, different GENIE Comprehensive Model Configurations (CMC)'s have no impact on the hadronization predictions.

The default PYTHIA parameters shown in Tab. II come from fits to high energy  $e^+ - e^-$  experiments [49–60] ( $\sqrt{s} \sim 35$  GeV). PYTHIA's description to data at low energy, such as modern neutrino oscillation experiments (1 – 10 GeV) or even lower energy  $e^{\pm} - p$  experiments such as the HERMES experiment (at 27 GeV) [61], is not accurate, see Sec. IV. The first attempt to improve this disagreement was in 2010, where some of the PYTHIA parameters were modified according to a NUX PYTHIA tune [39]. The parameters modified by the NUX PYTHIA tune are:

- $P_{s\bar{s}}$  controls the  $s\bar{s}$  production suppression
- $\langle p_{\perp}^2 \rangle$  determines the average hadron transverse momentum squared
- $E_{\text{CutOff}}$  is the energy cut off for the fragmentation process

These parameters are related to important hadron shower characteristics. The assumption of tunneling break-ups implies the suppression of heavy-quark

TABLE (II) Summary of different PYTHIA parameterizations. The parameter configuration for PYTHIA, NUX, HERMES and 2010 GENIE tunes are specified. The details on the HERMES tune are given in Sec. IV B.

Parameter	Name in PYTHIA	PYTHIA default	NUX tune	HERMES tune	2010 GENIE tune
$P_{s\bar{s}}$	PARJ(2)	0.30	0.21	0.25	0.30
$\langle p_{\perp}^2 \rangle$ [GeV <sup>2</sup> /c <sup>2</sup> ]	PARJ(21)	0.36	0.44	0.42	0.44
$E_{\text{CutOff}}$ [GeV]	PARJ(33)	0.80	0.20	0.47	0.20
Lund $a$	PARJ(41)	0.30	0.30	0.68	0.30
Lund $b$ [c <sup>4</sup> /GeV <sup>2</sup> ]	PARJ(42)	0.58	0.58	0.35	0.58

production, limiting its production in soft fragmentation processes. The suppression factor for heavy quarks is  $u\bar{u}:d\bar{d}:s\bar{s}:c\bar{c} \sim 1:1:0.3:10^{-11}$  [48]. This is supported by  $\eta$  production data, Fig. 5. Previous tunes are in agreement with this fact, see Tab. II. Each quark anti-quark pair receive opposite  $p_{\perp}$  kicks at each string breaking point according to a Gaussian distribution. The  $\langle p_{\perp}^2 \rangle$  parameter controls the variance of the Gaussian distribution used at the breaking point. There is different datasets available to constrain this parameter [27], see for instance Fig. 4. Finally,  $E_{\text{CutOff}}$  determines the minimum energy at which the fragmentation of the parton system can occur, set to 0.8 GeV in PYTHIA. 2010 GENIE uses the best-fit-value from the NUX PYTHIA tune, where  $E_{\text{CutOff}} = 0.20$  GeV.

In GENIE v3 and previous releases, there is only one parameter set configuration for the low- $W$  AGKY model (Tab. I) and PYTHIA (Tab. II) that is common for all CMC's.

### III. NEUTRINO-INDUCED HADRONIZATION DATA REVIEW

The characterization of the AGKY parameters relies on neutrino-induced hadronization data from BEBC and FNAL 15 ft experiments. Experiments published a variety of observables related to hadronization. This work is based mainly on charged multiplicity data as a function of the hadronic invariant mass and hence it is what this review is focusing on. The analyses procedure for both experiments is similar and it depends on the target type that can be hydrogen or deuterium. The different analysis requirements need to be implemented in the GENIE hadronization analysis for a meaningful data/MC comparison, see Sec. VI. In this section, the analyses of interest for this work are discussed in detail.

#### A. Hydrogen data

The bubble chamber at Fermilab (FNAL 15 ft) and BEBC at CERN follow similar analyses procedures. The data considered in this work are those listed in Tab. III.

Both experiments look for  $\nu_{\mu}$  and  $\bar{\nu}_{\mu}$  CC interactions on hydrogen to study the averaged charged multiplicity of the final state. The main requirement to select CC events is to detect a muon track. Muons are detected with a External Muon Identifier (EMI), and a minimum muon momentum,  $p_{\mu}$ , is usually required to guarantee good muon identification (ID). This is a consequence of the muon ID efficiency dependence on the muon momentum energy. For instance, in BEBC experiment, the muon ID efficiency varies from 40% to 100% in the muon momentum range of  $3 \text{ GeV}/c \leq p_{\mu} \leq 10 \text{ GeV}/c$ , with an average efficiency of 95%. The FNAL 15 ft experiment also uses a kinematic technique to identify negative muons in neutrino interactions [63]. Under this  $\mu^{-}$ -ID method, only events in which the  $\mu^{-}$  candidate has transverse momentum,  $p_{\perp}^{\mu}$ , of at least 1 GeV/c are accepted.

Selected events, which satisfy the conditions specified above, are analyzed to reconstruct the event topology and kinematics. In particular, BEBC uses the HYDRA program [64–66] and FNAL 15 ft a modified version of the TVGP program [67]. Only a small fraction of the charged final state hadrons is identified by using energy loss, range in hydrogen, break point probability and kinematic fits [64]. If left unidentified, the remaining charged hadrons are assumed to be pions: this assumption can cause migration of particles from the backward to the forward going hemisphere. For instance, the BEBC experiment is able to identify about 30% of the protons using the HYDRA algorithm, while the rest are classified as pions [65].

For  $\nu_{\mu}$  CC interactions, because of charge conservation, the experiments scan for events with three or more charged particles in the final state.

The topology of neutrino and antineutrino events

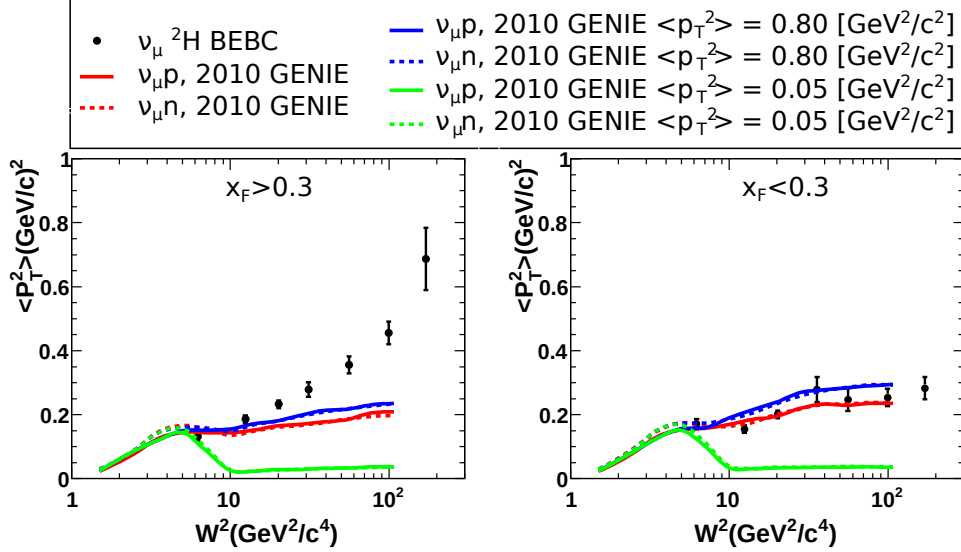


FIG. (4) Effect of the  $\langle p_T^2 \rangle$  parameter on the  $\langle p_T^2 \rangle$  distributions as a function of  $W^2$  for  $\nu_\mu$  data on  $^2\text{H}$  from the BEBC experiment under different Feynman- $x$  ( $x_F = p_L/p_{L,\text{max}}$ ) conditions [27]:  $x_F > 0.3$  (left) and  $x_F < 0.3$  (right). The 2010 GENIE parameter value is  $\langle p_T^2 \rangle = 0.44 \text{ GeV}^2/c^2$ . The validation range used for this plot is specified in the legend.

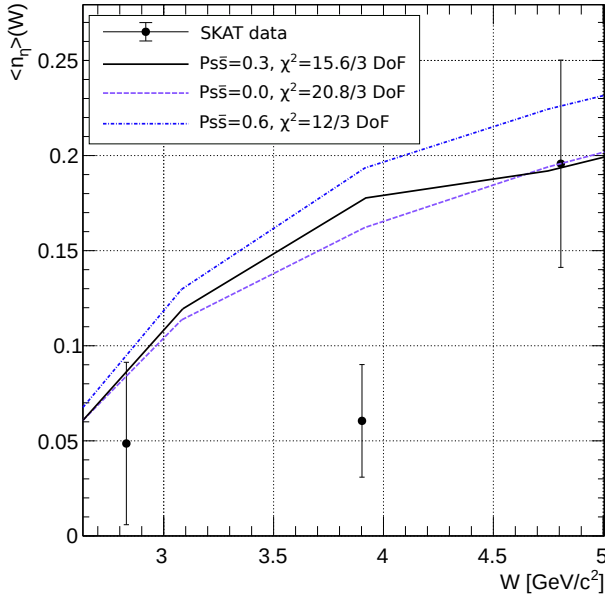


FIG. (5) Parameter impact on the averaged  $\eta$  production data from SKAT [62].

is expected to be different. In anti-neutrino events, interactions with only one charged track can occur ( $n_{\text{ch}} = 0$ ). Such events are not negligible at low  $E_\nu$  and low- $W$ . However, these are removed due to low scanning efficiency and poor anti-neutrino energy reconstruction. Both BEBC and FNAL 15 ft correct for the effect of removing one-prong contributions

in anti-neutrino samples using Monte Carlo (MC) calculations [64–66, 68]. One-prong MC events are weighted so that the fraction of one-prong events agree with the experimental estimate. The scanning efficiencies for three prong events are higher than 90%, improving as the number of charged secondaries increases ( $\geq 95\%$ ).

In hydrogen and deuterium bubble chambers, the identification of neutral particles, such as  $\pi^0$ , is difficult due to the low  $Z$  of the medium. As a consequence, the transverse momentum balance method is used to estimate the neutrino energy by assuming undetected neutral particles in the event [69],

$$E_\nu^{\text{reco}} = p_L^\mu c + p_L^{\text{ch}} c \left( 1 + \frac{|\mathbf{p}_\perp^\mu + \mathbf{p}_\perp^{\text{ch}}|}{\sum_{i=1}^{n_{\text{ch}}} |\mathbf{p}_{\perp i}|} \right) \quad (6)$$

The subscript  $L$  and  $\perp$  refer to longitudinal and transverse components of the momenta relative to the neutrino direction, whereas the  $\text{ch}$  and  $\mu$  labels denote the charged-hadron system and the muon respectively. The index  $i$  runs over the charged hadrons in the hadronic system. By using this method, there is a non-negligible bias for the neutrino energy reconstruction. For instance, the BEBC experiment estimated the reconstructed neutrino energy to differ from the true energy by  $\sim 10 - 15\%$  [37, 64]. Both bubble chambers corrected for this effect, see Sec. III C. In some analyses, cuts on the reconstructed neutrino energy,  $E_\nu^{\text{reco}}$  are applied [37, 64, 70].

Backgrounds from NC events, quasi-elastic (QEL) CC events or neutral particle induced events are removed from the final sample using kinematic cuts that depend on each analysis. NC events can mimic CC events as a consequence of muon-hadron miss-ID. On the one hand, for the FNAL 15 ft experiment, the muon-hadron miss-ID increases at high Bjorken inelasticity values ( $y_B$ ) and a cut on  $y_B$  is required to guarantee a good efficiency in selecting CC events [68]. In  $\bar{\nu}_\mu$  events, backgrounds from low-energy neutrons as well as events caused by incoming hadron tracks that rescatter within the chamber are controlled by requiring the total momentum in the forward hemisphere,  $p_{FW}^{\text{tot}}$ , to be greater than 2 GeV/c [67, 68]. Moreover, FNAL 15 ft removes backgrounds from  $K_L^0$  mesons by requiring the minimum total momentum from charged particles,  $p_{\text{ch}}$ , to be higher than 5 GeV/c [68]. On the other hand, the BEBC experiment applies kinematic cuts on either  $W$  or/and  $Q^2$  to remove quasielastic events [64–66]. All cuts applied to the different analyses are shown in Tab. III.

## B. Deuterium data

The analyses algorithm followed by the FNAL 15 ft and BEBC bubble chamber experiments operating with deuterium aims to discriminate between interactions on proton and neutron. The data on deuterium considered in this work are those listed in Tab. IV.

Before classifying the event as a neutrino interaction on either proton or neutron, the analyses procedure is equivalent to the one described in Sec. III. Each event has to contain a muon, identified with the EMI, that satisfies the cuts summarised in Tab. IV. The information about the event topology and kinematics is obtained using the TVGP-SQUAW or HYDRA algorithms for FNAL 15 ft [37] and BEBC respectively [38, 71, 72]. Particles are classified as pions if the algorithms fail to identify them as any other particle. The neutrino energy is reconstructed using the transverse momentum balance method. Similar kinematic cuts to those specified for the hydrogen analyses are applied.

The main difference between both analyses is the particle identification of struck nucleons in the event. A neutrino event is classified as a neutrino interaction on proton if the event topology has an odd number of prongs. The event is classified as an interaction on neutron if the event has an even number of prongs with no visible spectator or an odd number of prongs that include a visible proton. See a graphical interpretation in Fig. 6. The anti-neutrino case is similar except that the minimum prong multiplicity

on proton is 1, instead of 3. Because of the selection criteria explained in Sec. III, interactions with  $n_{\text{ch}} = 0$  are not considered, effectively making the selection criteria for anti-neutrinos the same as for neutrinos.

In the analyses, a prong is classified as a proton if it corresponds to a particle moving backwards relatively to the beam direction ( $\cos\theta_p < 0$ ) or a forward-going particle with low momentum. The maximum momentum cut is dataset dependent, see Tab. IV. If these conditions are not satisfied, the proton is not reconstructed and for the purpose of the analyses it is considered invisible. In the FNAL 15 ft analyses, for a proton to be detected as a prong, its momentum has to be  $p_p > 200$  MeV/c.

The deuterium target can induce rescattering of the hit nucleon with the spectator: this can increase the number of hadrons in the final state [63]. An odd number of prongs can occur in any possible neutrino interaction because of rescattering, independently of the hit nucleons, so the  $\nu_\mu p$  sample will contain  $\nu_\mu n$  events. In contrast, the  $\nu_\mu n$  sample can only contain  $\nu_\mu p$  events because of detector inefficiencies. Rescattering events have an impact on the event kinematics, that be quantified defining an energy balance as,

$$\varepsilon \equiv \sum_i (E_i - p_{L_i}c) - Mc^2 \quad (7)$$

where  $E_i$  and  $p_{L_i}$  are the  $i$ -th charged particle energy and longitudinal momentum component relative to the neutrino direction respectively while  $M$  is the mass of the target nucleon assumed in the selection sample. Eq. 7 assumes that the nucleon is at rest and that the neutrino direction is known. In an ideal detector where all final state particles are identified,  $\varepsilon = 0$  [73]. In a bubble chamber experiment, where only the charged particles are detected,  $\varepsilon < 0$ . Possible particle miss-ID further reduces the  $\varepsilon$  value, as particles are assigned to be pions as a default, unless identified otherwise. On the other hand, rescattering events have a  $\varepsilon > 0$  with a maximum value of  $M_{2H}c^2 - M_n c^2$ , see Fig. 7. The BEBC experiment eliminates rescattering events from the sample by imposing a cut on the energy balance [38, 71, 72]. An event is rejected due to rescattering if the following conditions are satisfied:

- If  $\varepsilon > 0.1$  GeV
- If  $\varepsilon > -0.1$  GeV and the transverse missing momenta squared differs from zero,  $(p_{\perp}^{\text{miss}})^2 > 0.075$  (GeV/c)<sup>2</sup>.

The FNAL 15 ft experiment did not correct for rescattering events. In some of the analysis, additional cuts are considered for the deuterium analyses to remove backgrounds. For instance, the FNAL

TABLE (III) Compilation of historical data from the BEBC and FNAL 15 ft bubble chamber experiments on averaged charged hadron multiplicity in muon (anti)neutrino on hydrogen interactions. Information about the number of points in each dataset,  $N_p$ , the  $W^2$  range covered and the cuts applied in each analyses is provided. Unless specified, the systematic errors were not included in the data error bands and have been added in quadrature by the amount specified in this table, see details in Sec. III C. The sixth column specifies whether the dataset is included, discarded or partially included in the fit, see Sec. IV C. The data points removed in this case are specified in the Sec. VII.

Experiment	$N_p$	$W^2$ [GeV $^2/c^4$ ]	Cuts	Syst.	In Fit	Ref.
$\nu_\mu + p \rightarrow \mu^- + X^{++}$						
FNAL 15 ft (1976)	25	[1.5, 150]	$E_\nu^{\text{reco}} \geq 15$ GeV $p_L^{\text{visible}} \geq 10$ GeV/c $p^\mu \geq 5$ GeV/c $p_T^\mu \geq 1$ GeV/c	Included	$W^2 < 20$ GeV $^2/c^4$	[70]
BEBC (1983)	11	[9, 121]	$p^\mu \geq 3$ GeV/c $E^{\text{visible}} \geq 5$ GeV $W^2 \geq 9$ GeV $^2/c^4$	3 – 5%	×	[64]
BEBC (1990)	6	[6, 150]	$Q^2 \geq 1$ (GeV/c) $^2$ $p^\mu \geq 3$ GeV/c $W^2 \geq 4$ GeV $^2/c^4$	Statistical	$W^2 < 9$ GeV $^2/c^4$	[65]
BEBC (1992)	5	[12, 144]	$p^\mu \geq 3$ GeV/c	Included	✓	[66]
$\bar{\nu}_\mu + p \rightarrow \mu^+ + X^0$						
FNAL 15 ft (1981)	10	[16, 100]	$p_{\text{ch}} \geq 5$ GeV/c $p_{\text{FW}}^{\text{tot}} \geq 2$ GeV/c $y_B \geq 0.1$ $y_B \leq 0.8$ $E_{\bar{\nu}}^{\text{reco}} \geq 5$ GeV	Statistical	$W^2 < 30$ GeV $^2/c^4$	[68]
BEBC (1983)	10	[9, 121]	$p^\mu \geq 3$ GeV/c $E^{\text{visible}} \geq 5$ GeV $W^2 \geq 9$ GeV $^2/c^4$	3 – 5%	×	[64]
BEBC (1990)	6	[6, 144]	$Q^2 \geq 0.1$ GeV $^2$ $p^\mu \geq 3$ GeV/c $W^2 \geq 4$ GeV $^2/c^4$	Statistical	$W^2 < 10$ GeV $^2/c^4$	[65]
BEBC (1992)	5	[12, 144]	$p^\mu \geq 3$ GeV/c	Included	$W^2 < 60$ GeV $^2/c^4$	[66]

15 ft bubble chamber reduces the background from neutral hadron-induced events by applying a cut on the total charged-particle longitudinal momentum,  $p_{\text{ch}}^L$ , in the final state system [37].

### C. Sources of systematic uncertainties in the FNAL 15 ft and BEBC experiments

MC studies were performed by the FNAL 15 ft and BEBC bubble chamber experiments to correct for possible sources of errors. In particular, the different analyses correct for the following effects:

- EMI geometrical inefficiency [38, 65, 66, 72].

- Efficiency losses due to possible hadron miss-ID and migration of particles from the forward to backward hemispheres [64–66, 72].
- $W^2$  smearing due to the uncertainty in the neutrino energy reconstruction [37, 38, 64–66, 68, 70–72].
- Neutrino energy uncertainty associated the transverse balance method [37, 38, 64–66, 68, 70–72]
- Neutral particle decays ( $\gamma$ ,  $K^0$  and  $\Delta$ ) into charged particles that can lead to a higher charged multiplicity if the decay vertex is close to the primary one [37, 38, 64–66, 68, 70].



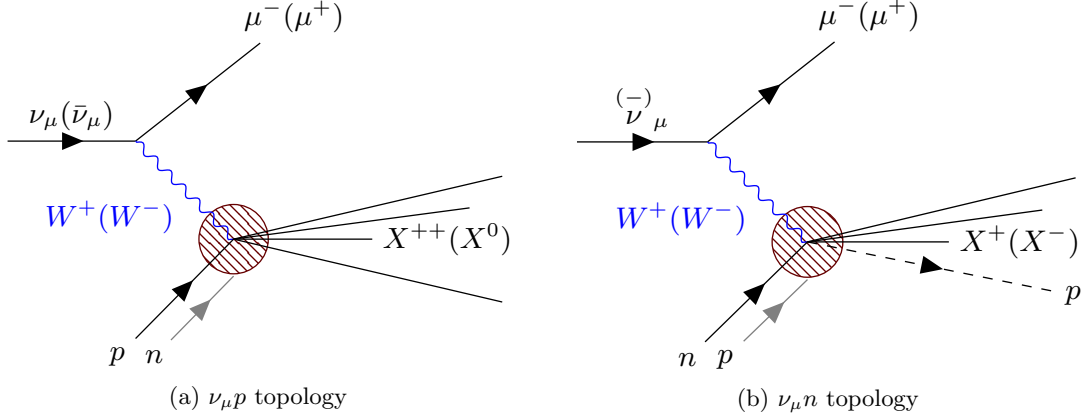
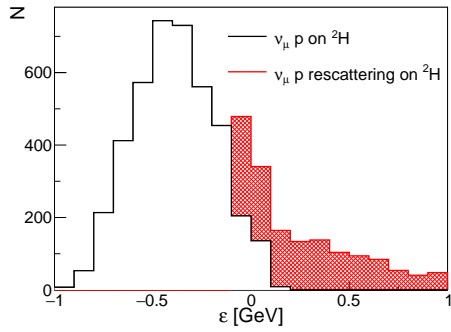
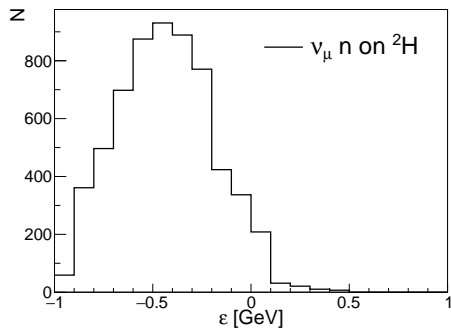


FIG. (6) Bubble chamber analyses of  $\nu_\mu$  and  $\bar{\nu}_\mu$  on  ${}^2\text{H}$  data schematic procedure. The topology definition is based mainly on the number or prongs in each event. Possible visible proton spectators that satisfy the momentum requirements specified in Tab. IV are represented with the dashed lines.  $\bar{\nu}_\mu$   ${}^2\text{H}$  one prong events are not considered.



(a)  $\nu_\mu p$  events under the odd prong topology assumption.



(b)  $\nu_\mu n$  events under the even prong topology assumption. Neutron events with a spectator proton are not included.

FIG. (7) Energy balance distribution for  $\nu_\mu$  events on neutron and proton candidates digitalised from the BEBC analyses paper [72]. Events that do not satisfy the  $\epsilon_{\text{reco}}$  correspond to rescattering events and are highlighted in red. No rescattering contribution is observed in the  $\nu_\mu n$  sample.

- One prong event corrections [64–66, 68, 72]. This kind of events occur for low- $W$   $\bar{\nu}p$  interactions in which only the  $\mu^+$  is observed.
- Efficiency to detect CC events [37, 68]
- Corrections due to the Fermi motion in Deuterium [72].
- Possible measurement errors [37, 64–66, 72].

The information about the BEBC systematic errors was obtained by using two MC programs: the LUND MC and a Longitudinal Phase Space model (LPS) [74]. Both MC were tuned to describe the BEBC experiment. From the MC generations, two samples are created: the initial,  $d_{\text{initial}}^{\text{MC}}$ , and modified,  $d_{\text{modified}}^{\text{MC}}$ , samples. The initial sample contains the truth information of the event. The modified sample includes modifications to mimic the analyses procedure. The ratio between the samples provides with a correction factor that it is applied to the data.

BEBC systematic errors are obtained from the difference between both MC calculations. The FNAL 15 ft corrected for some of the effects, but no clear information about the methodology followed to estimate the systematics is provided. Some of these experiments provide error bars which already include an estimation of systematics, however, this is not the case for most of the data. In particular, there is three different ways in which the BEBC and FNAL 15 ft experiments quote the systematic errors: (1) the systematic errors are already included in the total error, (2) the systematic uncertainty was quoted as a percentage with respect to the central value or (3) the systematic error is considered to be approximately of the same size of the statistical error.

TABLE (IV) Compilation of historical data from the BEBC and FNAL 15 ft bubble chamber experiments on averaged charged hadron multiplicity in muon (anti)neutrino on deuterium interactions. Information about the number of points in each dataset,  $N_p$ , the  $W^2$  range covered and the cuts applied in each analyses is provided. Unless specified, the systematic errors were not included in the data error bands and have been added in quadrature by the amount specified in this table, see details in Sec. III C. The sixth column specifies whether the dataset is included, discarded or partially included in the fit, see Sec. IV C.

Experiment	$N_p$	$W^2$ [GeV $^2/c^4$ ]	Cuts	Syst.	In Fit	Ref.
$\nu_\mu + p \rightarrow \mu^- X^{++}$						
FNAL 15 ft (1983)	14	[1, 225]	$p_\mu \geq 5$ GeV/c $p_\mu^\perp \geq 1$ GeV/c $p_{ch}^L \geq 5$ GeV/c $p_p \leq 340$ MeV/c $p_p \geq 200$ MeV/c $W \geq 1.5$ GeV/c $^2$ $E_\nu^{reco} \geq 10$ GeV	10%	$W^2 > 4$ GeV $^2/c^4$ †	[37]
BEBC (1989)	6	[4, 196]	$\epsilon^{cut}$ $p^\mu \geq 4$ GeV/c $p_p \leq 300$ MeV/c	<i>Not Included</i>	×	[71]
$\nu_\mu + n \rightarrow \mu^- X^+$						
FNAL 15 ft (1983)	14	[1, 225]	$p_\mu^\perp \geq 1$ GeV/c $p_{ch}^L \geq 5$ GeV/c $E_\nu^{reco} \geq 10$ GeV $p_p \leq 340$ MeV/c $p_p \geq 200$ MeV/c	10%	✓	[37]
BEBC (1984)	8	[6, 112]	$\epsilon^{cut}$ $p_\mu \geq 4$ GeV/c $Q^2 \geq 1$ (GeV/c) $^2$ $W^2 \geq 5$ GeV $^2/c^4$ $p_p \leq 300$ MeV/c	<i>Statistical</i>	✓	[72]
BEBC (1989)	6	[4, 196]	$\epsilon^{cut}$ $p_\mu \geq 4$ GeV/c $p_p \leq 300$ MeV/c $W \geq 5$ GeV/c $^2$	<i>Included</i>	×	[71]
$\bar{\nu}_\mu + p \rightarrow \mu^+ X^0$						
BEBC (1982)	8	[5, 75]	$p_\mu \geq 4$ GeV/c $p_p \leq 300$ MeV/c	<i>Statistical</i>	✓	[38]
BEBC (1989)	6	[4, 196]	$\epsilon^{cut}$ $p^\mu \geq 4$ GeV/c $p_p \leq 300$ MeV/c	<i>Not Included</i>	×	[71]
$\bar{\nu}_\mu + n \rightarrow \mu^+ X^-$						
BEBC (1982)	8	[1.5, 56]	$p_\mu \geq 4$ $p_p \leq 300$ MeV/c	<i>Statistical</i>	✓	[38]
BEBC (1989)	6	[4, 196]	$\epsilon^{cut}$ $p_\mu \geq 4$ GeV/c $p_p \leq 300$ MeV/c	<i>Not Included</i>	×	[71]

For the cases (2) and (3), the systematic errors are added in quadrature to the statistical ones in this analysis. In Tab. III and Tab. IV, the information on the systematic error is provided. We label the different categories as (1) *Included*, (2) with the percentage, or (3) *Statistical*, respectively. Particularly

for the datasets from Ref. [71], information on systematic errors is not provided in the data release. No correlation matrices are provided by any of these experiments.

#### IV. REVIEW OF PREVIOUS TUNES TO HADRONIZATION DATA

While summarising the experimental fits to averaged charged multiplicity data, this section also explains the origin of the *2010* GENIE tune parameters. This is necessary to define proper selection criteria for a dataset to be included in a global fit.

##### A. Fits to bubble chamber data

Both BEBC and FNAL 15 ft experiments provided estimations of the  $\alpha_{ch}$  and  $\beta_{ch}$  parameters for every released dataset. The individual fits were performed by fitting Eq. 1 in each channel. Fit results are summarised in Tab. V.

There are 6 channels in total:  $\nu_\mu$  or  $\bar{\nu}_\mu$  on proton or neutron, while the information on interactions on proton can be from data with hydrogen or deuterium targets. Information about neutrino interaction on neutron can only be extracted from deuterium samples. The BEBC and/or FNAL 15 ft experiments performed individual fits to each of the available channels.

From the best-fit values extracted for each dataset, we observe clear discrepancies for the  $\alpha_{ch}$  and  $\beta_{ch}$  values between data releases and between the BEBC and FNAL 15 ft data (e.g. for  $\nu_\mu p$  interactions on hydrogen). Discrepancies between hydrogen and deuterium samples are also present. This target-related discrepancy can also be observed in fits to OPERA and CHORUS data [42, 75]. These discrepancies could have different origins: the  $W^2$  range, the beam energy or the kinematic cuts applied in the analyses.

The *2010* GENIE AGKY parameter values presented in Tab. I correspond to the analyses on deuterium targets highlighted in Tab. VII. Notice that the parameters used in the *2010* GENIE prediction come from fits to Eq. 1 over the whole  $W^2$  range. This procedure is not adequate as the  $\alpha_{ch}$  and  $\beta_{ch}$  should be extracted from a fit to data over the low- $W$  validity range given that the AGKY model differs from the simplified linear behaviour.

The description of the shower particle content is linked to several observables whose correlation is still unknown. For instance, the averaged charged multiplicity and dispersion observables can be correlated. The full list of available hadronization data is shown in Ref. [27]. Ideally, the AGKY tune should improve the agreement with all hadronization related observables. The extraction of the averaged charged multiplicity parameters, such as  $\alpha_{ch}$  and  $\beta_{ch}$ , strongly rely on the precise understanding of the datasets described in Sec. III and Sec. IIIB. However, the

analysis of historical averaged charged multiplicity datasets already show clear disagreements between each of the different data releases, as summarised in Tab. VII. For these reasons, on this work, we focus on the description and tune of averaged charged multiplicity data on hydrogen and deuterium samples.

##### B. The HERMES tune

The PYTHIA parameters are extracted from high energy  $e^-e^+$  experiments, see Sec. IIB. So far, the PYTHIA contribution in GENIE has not been tuned using data from neutrino experiments. As a consequence, PYTHIA underestimates the averaged charged multiplicity, see Fig. 8.

The *2010* GENIE tune, summarised in Sec. IIB, aimed to improve the agreement with different hadronization observables by incorporating the results from a NUX PYTHIA tune [39]. However, this was not sufficient to improve the agreement of PYTHIA with average charged multiplicity data neutrino data from bubble chambers experiments. Moreover, the tune lacked of information about the uncertainties of the fit parameters.

Information on PYTHIA parameters at lower energy was provided by the HERMES experiment, which tuned PYTHIA using  $e^\pm p$  data at 27 GeV [61]. It has been proved that the HERMES tune improves the agreement with neutrino data [40, 47], as seen in Fig.8. The main differences between the HERMES tune and the GENIE *2010* re-tune are the modification of the Lund  $a$  and Lund  $b$  parameters, suggesting higher (lower) values of Lund  $a$  (Lund  $b$ ).

The PYTHIA parameters with most impact on the average charged multiplicity for the *2010* GENIE AGKY and HERMES tunes are summarised in Tab. II.

##### C. Requirements for including a dataset in the AGKY multiplicity tune

Only the averaged charged multiplicity data on hydrogen and deuterium are taken into account in this AGKY fit. If possible, only the latest analysis of each experiment is included. Previous analyses are considered only if:

1. Its reanalyses did not cover all the original  $W^2$  range,
2. The prediction interpolation by Professor fails to describe the GENIE prediction (see Sec. VI),

TABLE (V) Compilation of best fit values for the intercept  $\alpha_{\text{ch}}$  and slope  $\beta_{\text{ch}}$  obtained from individual fits to Eq.(1) against mean charged hadron multiplicity data as a function of  $W^2$ . The parameters for charged-current  $\nu_\mu$  and  $\bar{\nu}_\mu$  scattering data on hydrogen, deuterium,  $^{207}\text{Pb}$  and the Fuji ET-B7 emulsion are shown in the table. 2010 GENIE parameters are extracted from the analyses highlighted in bold.

Experiment	$[W^2 \text{ GeV}^2/c^4]$	Target	$\alpha_{\text{ch}}$	$\beta_{\text{ch}}$	Ref.
$\nu_\mu + p \rightarrow \mu^- X^{++}$					
FNAL 15 ft (1976)	[1.5, 150]	H	$1.09 \pm 0.38$	$1.09 \pm 0.03$	[70]
BEBC (1983)	[12, 112]	H	$-0.05 \pm 0.11$	$1.43 \pm 0.04$	[64]
<b>FNAL 15 ft (1983)</b>	[1.5, 160]	$^2\text{H}$	$0.05 \pm 0.07$	$1.42 \pm 0.03$	[37]
BEBC (1990)	[6, 150]	H	$0.911 \pm 0.224$	$1.131 \pm 0.086$	[65]
BEBC (1992)	[12, 144]	H	$0.40 \pm 0.13$	$1.25 \pm 0.04$	[66]
$\nu_\mu + n \rightarrow \mu^- X^+$					
BEBC (1984)	[6, 112]	$^2\text{H}$	$1.75 \pm 0.12$	$1.31 \pm 0.04$	[72]
<b>FNAL 15 ft (1983)</b>	[1.5, 160]	$^2\text{H}$	$-0.20 \pm 0.07$	$1.42 \pm 0.03$	[37]
$\bar{\nu}_\mu + p \rightarrow \mu^+ X^0$					
FNAL 15 ft (1982)	[1.7, 74]	H	$-0.44 \pm 0.13$	$1.48 \pm 0.06$	[68]
<b>BEBC (1982)</b>	[5, 75]	$^2\text{H}$	$0.02 \pm 0.20$	$1.28 \pm 0.08$	[38]
BEBC (1983)	[12, 96]	H	$-0.56 \pm 0.25$	$1.42 \pm 0.08$	[64]
BEBC (1990)	[6, 144]	H	$0.222 \pm 0.0362$	$1.117 \pm 0.100$	[65]
BEBC (1992)	[12, 144]	H	$-0.44 \pm 0.20$	$1.30 \pm 0.06$	[66]
$\bar{\nu}_\mu + n \rightarrow \mu^+ X^-$					
<b>BEBC (1982)</b>	[1.5, 56]	$^2\text{H}$	$0.80 \pm 0.09$	$0.95 \pm 0.04$	[38]
$\nu_\mu + A$					
OPERA (2018)	[1.6, 54.6]	Pb	$-0.19 \pm 0.18$	$0.76 \pm 0.07$	[42]
CHORUS (2007)	[1, 148]	Fuji ET-B7	$1.07 \pm 0.05$	$1.32 \pm 0.11$	[75]

- The data release lack of sufficient information about systematic errors

In Tab. III-IV, the information about which datasets are included in the tune is specified. In the first case, previous analyses are used to complement the covered  $W^2$  range as those points were not documented in the revisited analyses. If the datasets are included partially only, the approximate  $W^2$  range used is provided. An example is the BEBC  $\nu_\mu$  on H data, see Fig. 13 (b) and (c). In this case, the data point at  $W^2 < 10 \text{ GeV}^2/c^4$  from the earlier release is included in the fit, while the others are not. This approach has already been implemented in other studies [41]. The exact  $W^2$  range after requirement (2) is given in Figs. 13-16.

Global fits can be used to expose datasets that pull the results in different directions. This is the case of the most recent  $\bar{\nu}_\mu$  measurement by BEBC experiment [71], which did not provide information on sys-

tematic errors and, consequently, the total error on this data tends to be much smaller than the rest, see Sec. VII. Such small errors give a strong preference to this dataset and, as a consequence, this measurement is in tension with other data, including older  $\bar{\nu}_\mu$  BEBC measurements [38] which information on the systematic uncertainty was provided, see Tab. IV. Given that the data release did not provide enough information on the systematic errors and the dataset is in clear disagreement with the other analysis, this datasets are not considered in the tune and they are shown for comparison only.

## V. PARAMETERISATION OF MODEL UNCERTAINTIES

This section discusses the impact on AGKY parameters on the predictions. Particularly, we are in-

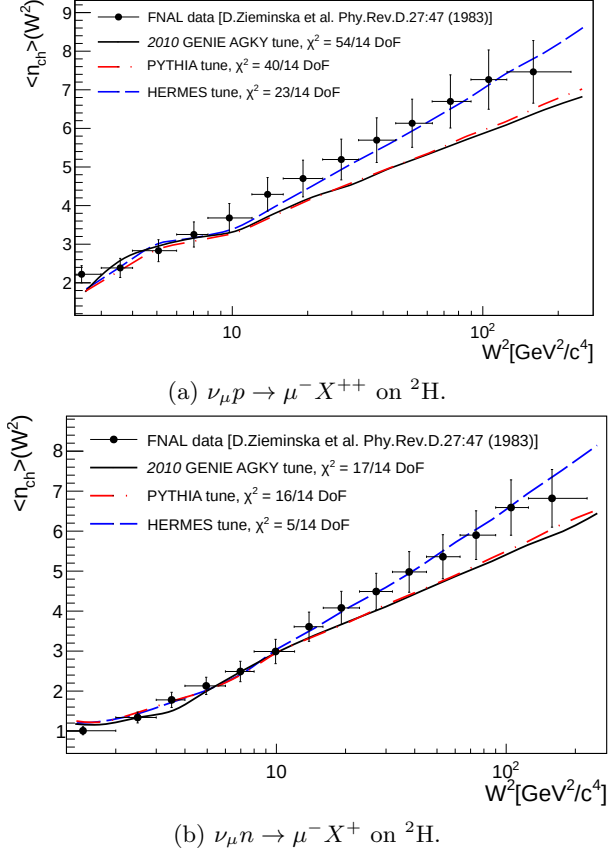


FIG. (8) Comparison of FNAL average charged multiplicity deuterium data against GENIE predictions obtained with the parameterisations specified in Tab. II

terested on the subset of parameters controlling the averaged charged hadron multiplicity. The list contains the parameters  $\alpha_{\text{ch}}$  and  $\beta_{\text{ch}}$  defined in Eq. 1 and the five PYTHIA parameters discussed before in Sec. II B. The parameter ranges that defines the parameter space are defined in Tab. VII. The ranges for  $\alpha_{\text{ch}}$  and  $\beta_{\text{ch}}$  parameters are defined in a such a way that they cover the values reported by experimental fits, see Tab. V. The same approach is followed to define the PYTHIA parameters range from the HERMES tune, see Tab. II.

The impact of each parameter range on the predictions of averaged charge multiplicity for  $\nu_{\mu}$  CC interactions on proton is shown in Fig. 9. As expected both  $\alpha_{\text{ch}}$  and  $\beta_{\text{ch}}$  act on low- $W$  and their effect vanishes gradually over the transition region. In the PYTHIA region, the largest contribution comes Lund  $a$  and Lund  $b$  parameters. In the transition region, the prediction will be determined by both sets of parameters: as a consequence we anticipate a correlation between PYTHIA and the low- $W$  AGKY

TABLE (VI) Complete list of models used for the G18.02a.02.11a/b CMC in GENIE v3.

Simulation domain	Model
Nuclear model	Fermi Gas [76]
QEL	Llewellyn Smith [78]
QEL Charm	Kovalenko [79]
QEL $\Delta S = 1$	Pais [80]
RES	Rein-Sehgal [81]
SIS/DIS	Bodek-Yang [77]
DIS $\Delta S = 1$	Aivazis-Olness-Tung [82]
Coherent $\pi$ production	Rein-Sehgal [81]
Hadronization	AGKY [27]
FSI	INTRANUKE hA [83]

parameters after the fit.

## VI. CONSTRUCTION OF THE GENIE PREDICTIONS AND EVALUATION OF THE LIKELIHOOD

In order to build the hadronization prediction for the data described in Sec. III,  $\nu_{\mu}$  and  $\bar{\nu}_{\mu}$  charged current (CC) events on H and  ${}^2\text{H}$  are simulated. Events are generated using a " $1/E$ "-like flux, with a 0.1 – 200 GeV energy range. This is sufficient as the observables are given in terms of  $W$ , hence, the neutrino flux is factorized out.

The predictions are generated with the G18.02a.02.11a tune of GENIE version 3.0.6 [24]. This CMC was previously tuned to improve the agreement with pion production data on free nucleon. The complete model list for this CMC is summarised in Tab. VI. As introduced in Sec. II, hadronization is modelled with the AGKY model [27]. Interactions with nuclei are calculated within the relativistic Fermi Gas framework, using the Bodek-Ritchie model [76], and hadronic re-interactions are simulated using INTRANUKE hA. The main contributions to the averaged charged multiplicity predictions come from CC DIS and non-resonance SIS [77]. As the DIS and models are common for all GENIE v3 tunes, the choice of the base configuration does not affect the hadronization predictions. An updated version of the G18.02a.02.11a tune, named G18.02a.02.11b, has been recently released in Ref. [24]. In terms of the hadronization predictions, these CMC's are interchangeable and the results of this work are valid with in the updated version.

In order to compute the prediction associated to the  $i$ -th dataset from Tab. III and Tab. IV, we select events simulated with the neutrino flux and target

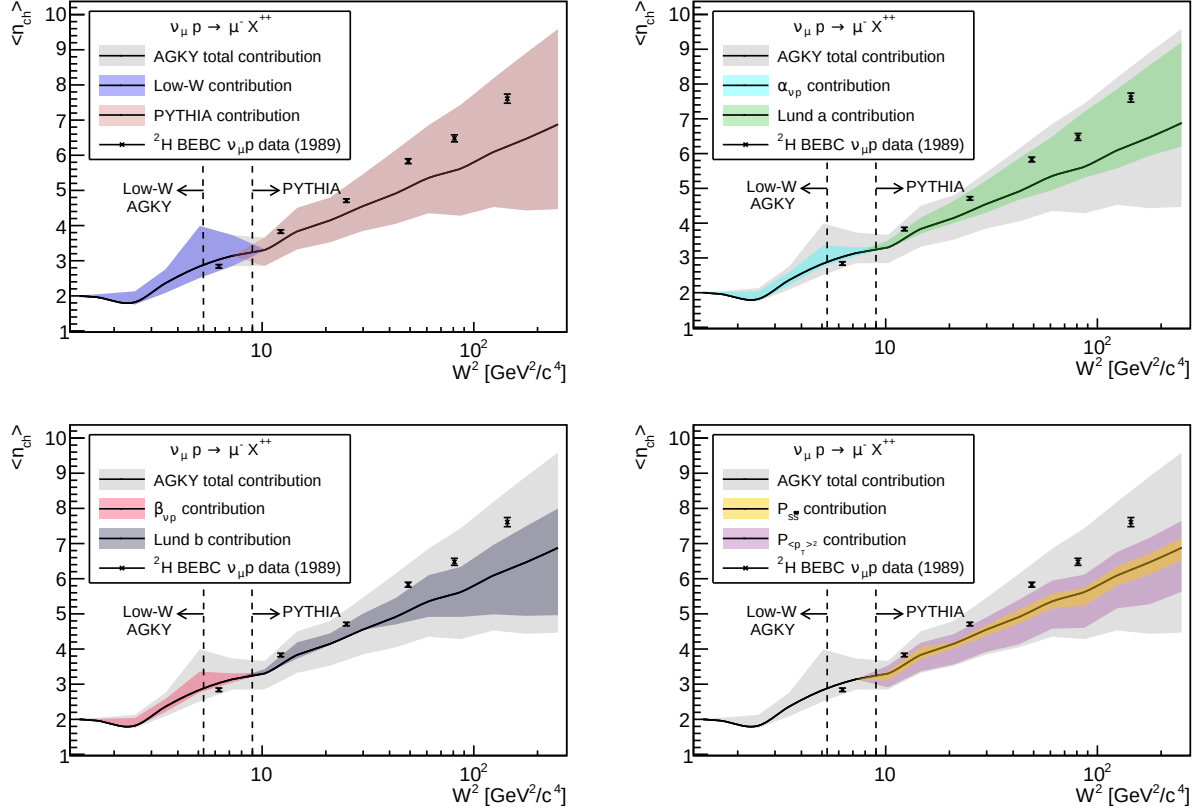


FIG. (9) Impact of fit parameters on the prediction of the averaged charged multiplicity, as a function of  $W$ , for  $\nu_{\mu} p \rightarrow \mu^- X^{++}$  interaction on a deuterium target. Each parameter has been varied within the range of study specified in Tab. VII. The top left plot shows the total contributions from low- $W$  and PYTHIA parameters. All the other plots specify the contribution from specific parameters compared to the total which is always rendered with the grey area. Dashed lines correspond to  $W_{\min}^{\text{tr}}$  and  $W_{\max}^{\text{tr}}$ , defining the transition region. BEBC data [71] are shown for reference.

of the corresponding experiment and processed using the same experimental cuts. For each selected event we reconstruct  $E_{\nu}$  and  $W$  following the recipes described in Sec. III. The events are divided in bins according to the reconstructed  $W$  and for each bin we evaluate the average charged multiplicity  $\langle n_{ch} \rangle_i(W)$ . This operation is repeated for a number of points in the parameter space  $\theta$  defined in Tab. VII. Each experiment has a different binning system and therefore we identify the  $W$  bins using two indices: one for the dataset and for the bin index inside the dataset. Labeling with  $\theta$  the vector of coordinates of point in the parameters space we can define our predictions associated to the  $i$ -th dataset and a given  $j$ -th  $W$  bin as  $\langle n_{ch} \rangle_i(W_{ij}|\theta)$ . The statistical error due to the MC sample size is also evaluated and this is referred to as  $\sigma_{ij}(\theta)$ .

We use Professor [84] to generate a parameterisation denoted as  $\tilde{n}_{ij}(\theta)$  and  $\tilde{\sigma}_{ij}(\theta)$  interpolating the values of  $\langle n_{ch} \rangle_i(W_{ij}|\theta)$  and  $\sigma_{ij}(\theta)$  as a function

of  $\theta$ . The parameterisation is a generic polynomial of order  $M$  in the  $P$ -dimensional space [84], whose analytical form is

$$\begin{aligned} \tilde{n}_{ij}(\theta) = & \alpha_0^{ijk} + \sum_{n=1}^P \beta_n^{ijk} \theta_n + \sum_{n \leq m} \gamma_{nm}^{ijk} \theta_n \theta_m \\ & + \dots + \sum_{n_1 \leq \dots \leq n_M} \xi_{n_1 \dots n_M}^{ijk} \prod_{\ell=1}^M \theta_{n_\ell} \end{aligned} \quad (8)$$

where  $\theta_n$  is the coordinate of the  $n$ -th parameter and  $M$  the polynomial order, set to 4<sup>th</sup> order in this work. The coefficients  $\alpha_0^{ijk}$ ,  $\beta_n^{ijk}$ ,  $\gamma_{(nm)}^{ijk}$ ,  $\dots$ ,  $\xi_{(n_1 \dots n_M)}^{ijk}$  are determined by Professor fitting the parameterisation against the computed  $\langle n_{ch} \rangle_i(W_{ij}|\theta)$  obtained by computing  $O(10^4)$  points uniformly spread within parameter space defined in Tab. VII. Non-physical regions in the sampled parameter space are avoided applying a veto function. In particular, every combination of  $\theta$  has

to verify that  $\langle n_{\text{ch}} \rangle \geq 0$  at the pion production threshold. The parameterisation  $\tilde{n}_{ij}(\boldsymbol{\theta})$  is used instead of the exact predictions in order to estimate the best-fit parameters by minimising the  $\chi^2$ . The main advantage of this method is the reduction of the brute-force scans computational complexity while allowing for massive parallelisation.

As mentioned in Sec. IV C, since the Professor interpolation is just an approximation, it can fail to describe the actual prediction. When this happens, we remove from the analysis data points whose Professor interpolation, of the predicted mean value or predicted error, disagree too much with the GENIE prediction corresponding to that data point. The relative difference between the interpolation and the GENIE prediction is known as residual. For each data point, we calculate the bin central value and bin error residuals for all the points in our parameter space. The corresponding distribution associated to the bin central value and bin error residual for the last bin of the BEBC [66] and the two last bins of the FNAL 15 ft [37] datasets are shown in Fig. 10 (b-c) and (e-f) respectively. The distributions of the residual for central values and errors are monitored and whenever the means or the variances of a bin are too far from the average values among all bins, the corresponding data point is removed. An example of the cutoff condition on the error residual distribution for all data points is shown in Fig. 11. The cut off condition requires that any data points with a mean central value or error that exceeds the average values among all bins by  $0.25\sigma_{\mu_\mu}$  or  $0.25\sigma_{\mu_\sigma}$  respectively is removed from the analysis. Two examples are given in Fig. 10: a dataset in which the interpolation is accurate for all the  $W^2$  range and a dataset in which the interpolation fails for some of the dataset points. This criteria allows us to ensure that the Professor parameterisation does not fail for the data considered in the tune. A total of  $\sim 18\%$  of the data points have been removed due to this requirement. In this work, it has been observed that the residual variance increases with  $W^2$ , with few exceptions. See Figs. 13–16 for the complete list of removed datapoints.

The variance of the residual distribution for a given data point can be improved by increasing the order of the polynomial used for the Professor interpolation. In this case, a polynomial of order four is used. However, specifically in this particular tune where 13 parameters are tuned, an increase of the order requires the generation of a much higher number of MC samples that can be very computationally demanding.

Our parameters of interest affect other hadronization observables and not only the averaged multiplicity. This is taken into account by using Gaussian

priors. The  $s\bar{s}$  suppression factor not only impacts the averaged multiplicity data but also the  $\eta$  multiplicity production, see Fig.5. A prior of  $0.30 \pm 0.05$  is considered in the tune in order to preserve a good agreement with the SKAT data [62]. Variations of  $E_{\text{CutOff}}$  affect the shape of  $F(x_F)$  invariant distribution defined as:

$$F(x_F) = \frac{1}{N_{\text{ev}}} \cdot \frac{1}{\pi} \cdot \frac{E}{p^L \max_c} \cdot \frac{dN}{dx_F} \quad (9)$$

where  $x_F$  is the Feynman variable,  $N_{\text{ev}}$  the total number of events, and  $E$  and  $p^L \max_c$  are the energy and maximum longitudinal momentum of the final state hadron in the hadronic center of mass. The  $F(x_F)$  invariant distribution describes the fragmentation process for the forward and backward hemispheres and it allows to study the symmetry between this two fragmentation regions. In Fig. 12, the  $F(x_F)$  invariant distribution for  $\bar{\nu}_\mu$  data on  ${}^2\text{H}$  [72] is compared against the GENIE predictions obtained by varying the  $E_{\text{CutOff}}$  within a  $[0, 2]$  GeV range. The main conclusion is that small values of this parameter preserve the agreement with data. In order to avoid an increase of  $F(x_F)$  at  $|x_F| \sim 1$ , we apply a prior on  $E_{\text{CutOff}}$  of  $0.25 \pm 0.05$  GeV. Another parameter that has a strong impact on other observables is  $\langle p_\perp^2 \rangle$ . As demonstrated in Fig. 4, low values of  $\langle p_\perp^2 \rangle$  are not in agreement with data for  $\langle p_T^2 \rangle$  distributions. Thus, we also apply a prior on the parameter to guarantee the agreement with this data of  $0.44 \pm 0.05$  (GeV/c) $^2$ . No priors are applied to the remaining parameters.

Using the parameterisation and the corresponding set of points belonging to the  $i$ -th dataset,  $\mathcal{D}_{ij} \pm \delta\mathcal{D}_{ij}$ , we seek to estimate the best-fit parameters  $\hat{\boldsymbol{\theta}}$  by minimising the quantity

$$\chi^2(\boldsymbol{\theta}) = \sum_{i,j} w_{ij} \frac{(\tilde{n}_{ij}(\boldsymbol{\theta}) - \mathcal{D}_{ij})^2}{\tilde{\sigma}_{ij}^2(\boldsymbol{\theta}) + \delta\mathcal{D}_{ij}^2} + \sum_l \frac{(\theta_l - \mu_l)^2}{\sigma_l^2} \quad (10)$$

The first term allows the minimization between data and prediction while applying weights,  $w_{ij}$ , that allows to consider only the desired data points in the fit. The second term adds uncorrelated Gaussian priors for parameter; the vectors of central values and variances are denoted  $\mu_l$  and  $\sigma_l$  respectively.

## VII. AGKY TUNE RESULTS

Starting from  $\nu_\mu$  and  $\bar{\nu}_\mu$  hadronization data, two tunes were considered: a global tune (2021 GENIE global) and a tune using only deuterium data (2021  ${}^2\text{H}$ ). The reason for a deuterium only fit is because other studies showed tensions between data on H and

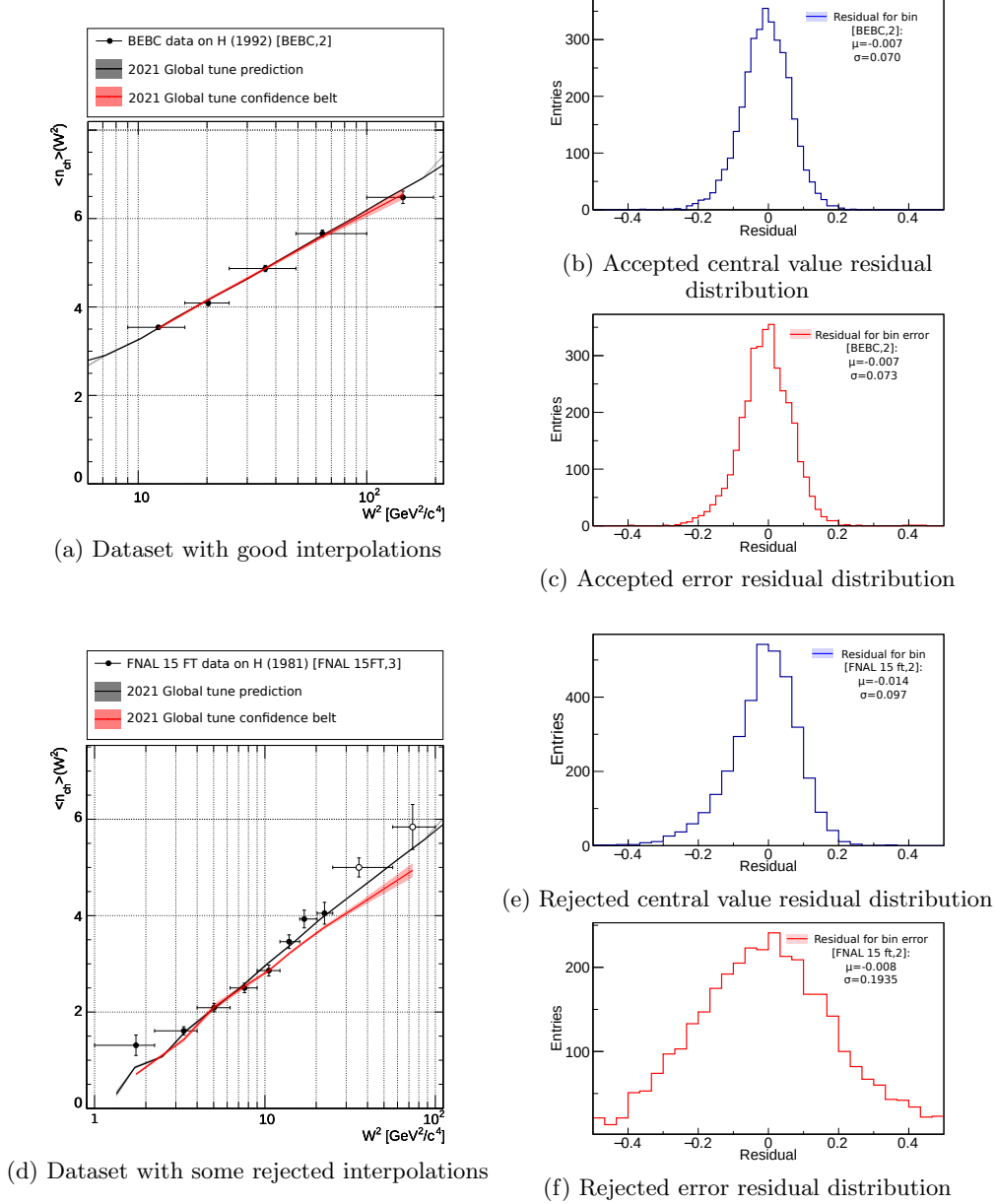


FIG. (10) Comparisons of  $\nu_\mu$  data on H against predictions obtained from the Professor parametrisation (red confidence belt) at the best-fit value for the AGKY global tune and the actual GENIE prediction (black line). For the Professor parameterisation, the uncertainties of the tuned parameters are propagated to the prediction considering the full covariance matrix. For three selected bins (the bin with highest  $W$  for [BEBC,2] (b-c) and the two higher  $W$  bins for the [FNAL 15FT,3] plot (e-f) on the left column), the central values and error residual distributions are shown, blue and red respectively: accepted parameterisations at the top, rejected parameterisation at the bottom. It can be seen that the residual distribution of the rejected bins is wider than its accepted counterpart. In this particular case, the two data points with higher  $W$  are neglected as the parameterisation of the bin value and error do not satisfy the required criteria.

$^2\text{H}$  targets on bubble chamber experiments [41]. The goal of the global tune is to improve the agreement with hydrogen and deuterium targets, regardless of these tensions, while the deuterium only tune was

performed to quantify the tensions within the same framework. Because of our data selection criteria, the hydrogen only data were not considered sufficient to allow a reliable hydrogen only tune. The



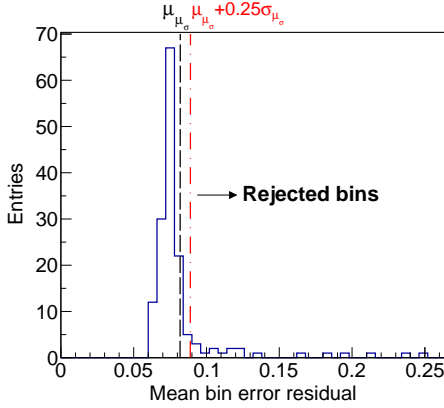


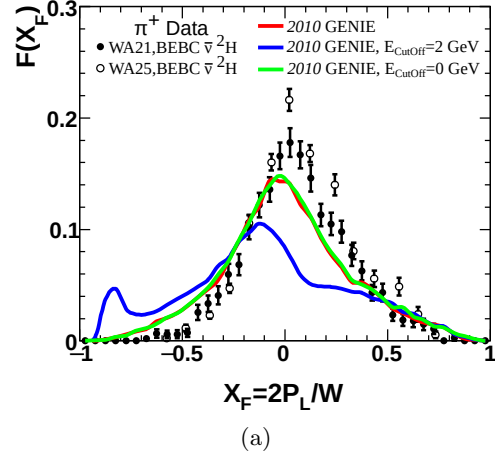
FIG. (11) Distribution of mean bin error residual for all data points. The distribution mean value ( $\mu_{\mu_\sigma}$ ) is shown with a dashed black line. Data points with a mean value higher than 0.25 of the mean bin error variance ( $\sigma_{\mu_\sigma}$ ) are rejected. This cutoff value is shown with the dashed red line.

analyses procedure outlined in the previous sections was applied to both tunes. The likelihood function was minimized against averaged charged multiplicity data, see Sec. III. The best-fit parameter set for tunes and the  $\chi^2$  values obtained using the Professor parameterisations and Eq. 10 are summarised in Tab. VII.

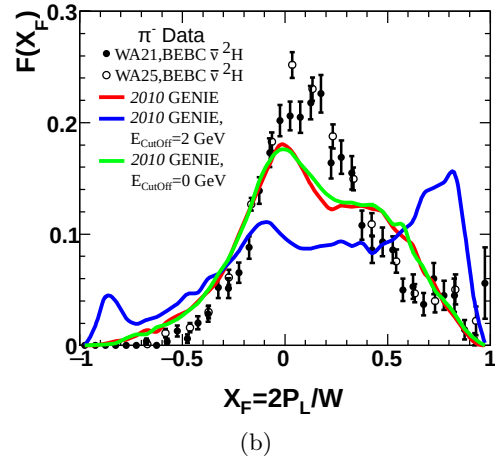
GENIE predictions for all the averaged charged multiplicity data available are shown in Figs. 13–16 before and after the AGKY tunes. The results show the prediction for the *2021* GENIE global tune in red and *2021* GENIE  $^2\text{H}$  tune in green. To distinguish data points used in the analyses from those that were not, the used one points have completely black markers, the others are empty circles. Vertical error bars include statistical and systematic uncertainties following our data review. Horizontal bars correspond to the bin width used in the data release, and are only shown if those are available in the original paper.

In terms of prediction differences, the *2021* GENIE global tune tends to underpredict deuterium data whereas the *2021* GENIE  $^2\text{H}$  tune overpredicts the hydrogen data. This especially true for the PYTHIA region, at high- $W$ . This is translated in the parameters with an increase (decrease) of Lund  $a$  (Lund  $b$ ) for the deuterium tune with respect to the global tune.

The summary of the  $\chi^2$  values per dataset as well as the total contributions are shown in Tab. VIII. Three different  $\chi^2$  values are presented:  $\chi_{\text{def}}^2$ ,  $\chi_{\text{AGKY}}^2$  and  $\chi_{^2\text{H}}^2$  using, respectively, the *2010* GENIE, *2021* GENIE global and *2021* GENIE  $^2\text{H}$  tune



(a)



(b)

FIG. (12) Effect of  $E_{\text{CutOff}}$  on the  $x_F$  invariant distributions for  $\pi^+$  (a) and  $\pi^-$  (b) in  $\bar{\nu}_\mu$  data on  $^2\text{H}$  from the BEBC experiment [72]. The *2010* GENIE tune value for the energy cut off is  $E_{\text{CutOff}} = 0.2$  GeV.

parameters. The  $\chi^2$  values per dataset are computed by comparing the GENIE predictions against all the data points in each dataset, regardless of the fact that the point was used in the fit or not. Differences between the  $\chi^2$  obtained with Eq. 10 and the one calculated using the GENIE predictions are expected as Eq. 10 considers the datapoints included in the tune only and the Professor parameterisation  $\tilde{n}_{ij}$  is not exact, as seen in Sec. VI.

It is important to stress that the total  $\chi^2$  from Tab. VIII are not providing any information related to goodness of fit, but simply a general agreement with respect to available datasets. A sense of the goodness of fit can be obtained looking at the total  $\chi^2$  using only dataset that were used in the fits, see Tab. IX.

The parameters covariance matrices for the tunes are obtained by inverting the Hessian of the log-

TABLE (VII) Parameters of interest for the study of averaged charged multiplicity data within the AGKY model. The range of study and priors used in the tune are specified in the table. See Sec. VII A and Sec. VII B for the details on the error estimation. Posterior distributions are not always symmetric: in that case the interval is reported accordingly. The total  $\chi^2$  obtained from each fit is obtained from the minimization of Eq. 10.

Parameter	GENIE parameter name	2010 GENIE	Allowed range	2021 Global Fit	2021 $^2\text{H}$ Fit
Low-W empirical model					
$\alpha_{\nu p}$	KNO-Alpha- $\nu p$	0.40	$[-1.0, 2.0]$	$1.1 \pm 0.3$	$1.2 \pm 0.4$
$\alpha_{\nu n}$	KNO-Alpha- $\nu n$	-0.20	$[-1.0, 2.0]$	$1.75^{+0.14}_{-0.11}$	$-0.58 \pm 0.07$
$\alpha_{\bar{\nu} p}$	KNO-Alpha- $\nu bp$	0.02	$[-1.0, 2.0]$	$1.32^{+0.16}_{-0.14}$	$1.9 \pm 0.08$
$\alpha_{\bar{\nu} n}$	KNO-Alpha- $\nu bn$	0.80	$[-1.0, 2.0]$	$1.11 \pm 0.09$	$1.07 \pm 0.3$
$\beta_{\nu p}$	KNO-Beta- $\nu p$	1.42	$[0.0, 2.5]$	$0.79 \pm 0.15$	$0.9 \pm 0.3$
$\beta_{\nu n}$	KNO-Beta- $\nu n$	1.42	$[0.0, 2.5]$	$0.5 \pm 0.1$	$1.9 \pm 0.3$
$\beta_{\bar{\nu} p}$	KNO-Beta- $\nu bp$	1.28	$[0.0, 2.5]$	$0.8 \pm 0.1$	$0.3 \pm 0.1$
$\beta_{\bar{\nu} n}$	KNO-Beta- $\nu bn$	0.95	$[0.0, 2.5]$	$0.88^{+0.09}_{-0.08}$	$0.9 \pm 0.2$
PYTHIA					
$P_{s\bar{s}}$	PYTHIA-SSBarSuppression	0.30	$[0.0, 1.0]$	$0.27 \pm 0.04$	$0.29 \pm 0.05$
$\langle p_{\perp}^2 \rangle$ [ $\text{GeV}^2/c^2$ ]	PYTHIA-GaussianPt2	0.44	$[0.1, 0.7]$	$0.43 \pm 0.05$	$0.43 \pm 0.04$
$E_{\text{CutOff}}$ [GeV]	PYTHIA-RemainingEnergyCutoff	0.20	$[0.0, 1.0]$	$0.30 \pm 0.04$	$0.24 \pm 0.05$
Lund $a$	PYTHIA-Lunda	0.30	$[0.0, 2.0]$	$1.53 \pm 0.13$	$1.85 \pm 0.15$
Lund $b$ [ $\text{c}^4/\text{GeV}^2$ ]	PYTHIA-Lundb	0.58	$[0.0, 1.5]$	$1.16 \pm 0.09$	$1.0 \pm 0.2$
$\chi^2 =$				87.9/62 DoF	29.5/32 DoF

likelihood function at the best fit point, see Tab. XI and Tab. X. As expected, the low- $W$  AGKY and PYTHIA parameters are now correlated in both tunes because of the interplay of the models in the transition region, with a number of parameters showing a correlation above 50%. See a graphical representation of the correlation matrix in Fig. 17.

The results from the 2021 GENIE AGKY tunes will be available in GENIE v3.2.0. Users can run the 2021 GENIE tunes global and  $^2\text{H}$  tunes out of the box using the G18\_02a\_03\_330 and G18\_02a\_03\_320 comprehensive configurations respectively.

#### A. The 2021 GENIE AGKY global tune

After the AGKY global tune, the GENIE predictions show a better agreement to the data. In particular, for the datasets included in the 2021 GENIE global tune, the  $\chi^2$  associated to the prediction is  $\chi^2_{2010} = 486/109$  DoF. After the tune, the  $\chi^2_{2021(\text{global})}$  is 242/109 DoF. This is clearly an improvement although the agreement is not completely satisfactory since the p-value is  $4 \cdot 10^{-12}$ . The im-

provement in the data description is general and both deuterium and hydrogen samples have a better agreement. Moreover, after the tune both samples have similar goodness of fit hence in general the level of agreement is the same. This can be noted from the  $\chi^2$  contributions from Tab. IX.

The agreement with the datasets not included in the tune has also improved, as shown in Tab. VIII. The total  $\chi^2$  computed using all available data is reduced significantly for both H and  $^2\text{H}$  datasets. Particularly, the global tune shows a better agreement against all hydrogen data. As expected from Sec. IV C, the datasets with highest contribution to the total  $\chi^2$  after the global tune are [BEBC,3] and [BEBC,5].

The main effect of the tune is observed in the PYTHIA region, at  $W > 3 \text{ GeV}/c^2$ , where the prediction of  $\langle n_{\text{ch}} \rangle$  increased. This is a direct consequence of the increase on Lund  $a$  and Lund  $b$ . This behaviour is consistent with the HERMES tune, summarised in Sec. II B.

For each parameter, the corresponding uncertainty is obtained with the profiling method under the condition  $\Delta\chi^2_{\text{profile}}(\theta_i) < 1$ . The profiles are calculated by fixing the value of the parameter under

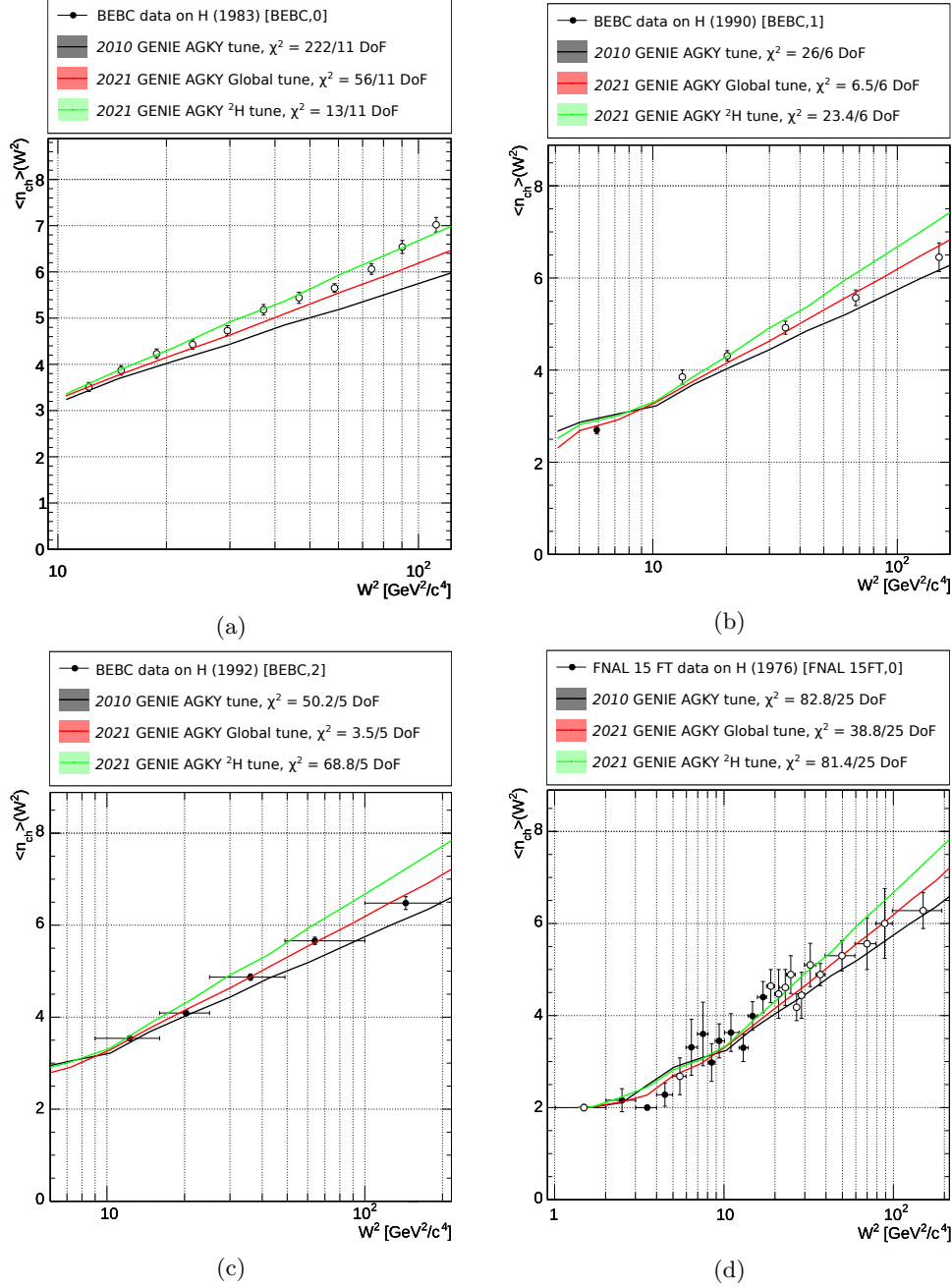


FIG. (13) Comparison of  $\langle n_{\text{ch}} \rangle$  against neutrino-induced hadronization data on  $\nu_\mu + p$  interactions on H from BEBC [64–66] and FNAL [70] bubble chamber experiments filled with H. The  $^2\text{H}$  tune prediction is shown for comparison only. The predictions are computed using the parameters specified in Tab.VII. The  $\chi^2$  values are calculated against all the data from each experiment. See definition of *Tags* in Tab. XII.

study  $\theta_i$  to a desired value and minimizing the quantity  $\Delta\chi^2(\boldsymbol{\theta}) = \chi^2(\boldsymbol{\theta}) - \chi_{\text{min}}^2$  with respect to all others parameters that were allowed to float in the fit. The constant  $\chi_{\text{min}}^2$  corresponds to the global minimum value of  $\chi^2(\boldsymbol{\theta})$ . Some parameters have a good Gaussian behaviour and a symmetric profile. For some others this is not true and this gives rise to asym-

metric uncertainties for the parameters. Example of a symmetric parameter profile compared to the non-Gaussian ones is shown in Fig. 18. The contours for some pairs of the AGKY parameters are shown in Fig. 19.

The fit covariance matrix can be propagated back to the GENIE predictions giving a posterior confi-

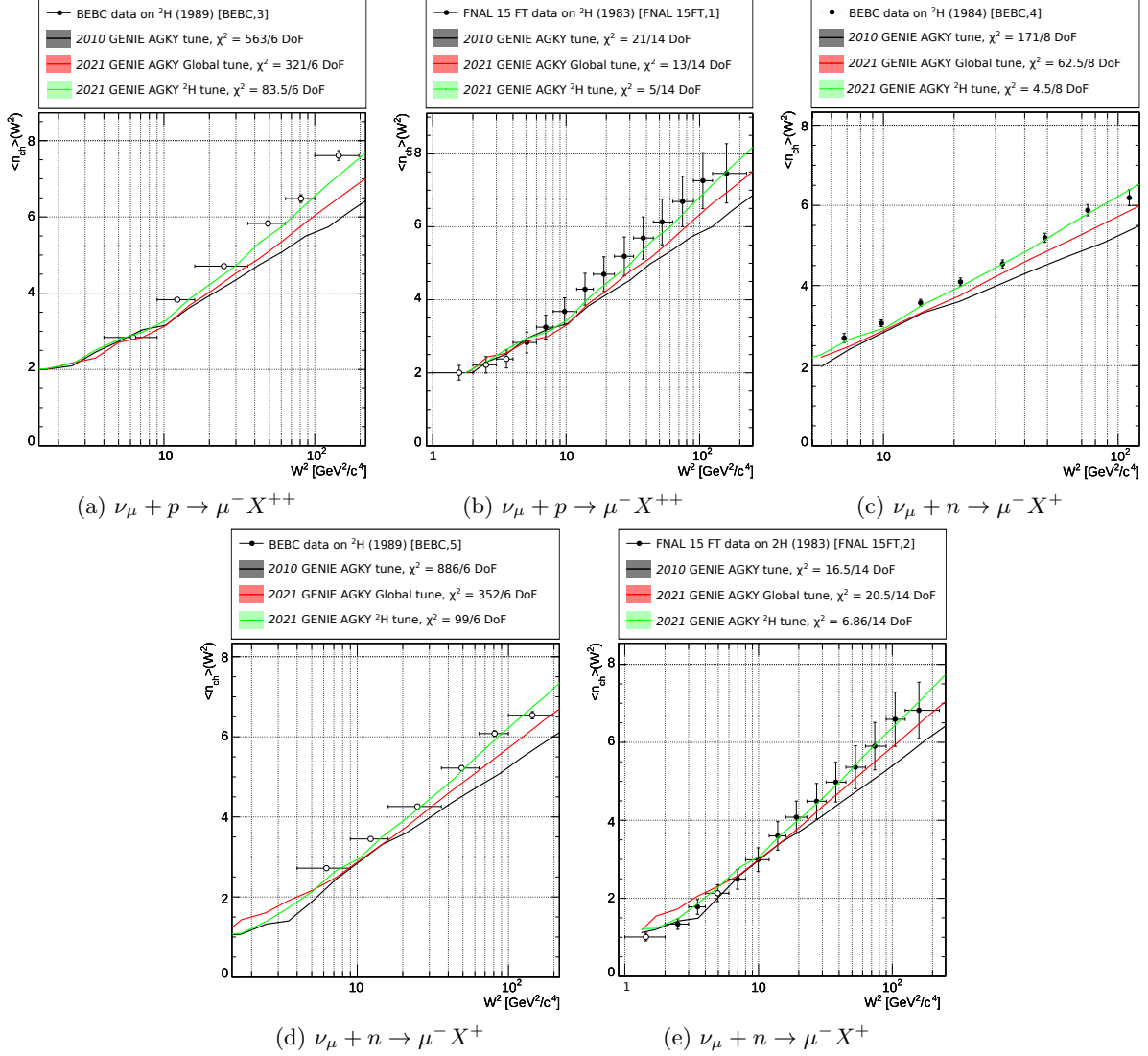


FIG. (14) Comparison of  $\langle n_{\text{ch}} \rangle$  against neutrino-induced hadronization data on  $\nu_\mu$  interactions on p and n from the BEBC bubble chamber experiment filled with  $^2\text{H}$  [38, 71]. The predictions are computed using the parameters specified in Tab.VII. The  $\chi^2$  values are calculated against all the data from each experiment. See definition of *Tags* in Tab. XII.

dence belt for the prediction associated to the tune. As an example, a comparison of the global tune prediction and the associated posterior confidence belt is shown in Fig. 10.

### B. The 2021 GENIE AGKY $^2\text{H}$ tune

For the datasets included in the deuterium only tune, the  $\chi^2$  associated to the 2010 GENIE AGKY prediction is  $\chi^2_{2010} = 230/52$  DoF. After the tune, the total  $\chi^2_{2021(^2\text{H})}$  is 37/52DoF that corresponds to a p-value of 0.94. Being the deuterium only goodness

of fit so much better than the global tune is a further confirmation of the high tension between H and  $^2\text{H}$  datasets.

Surprisingly, the deuterium only tune shows a better agreement than the global tune when all neutrino-induced hadronization data are considered, see Tab. VIII. Yet, this does not imply that the deuterium only fit is a better tune, it simply reinforces that the discarded dataset are not compatible with the data used in the fit.

The tension between hydrogen and deuterium data were already observed by other studies where a modified KNO-based model was tuned to averaged

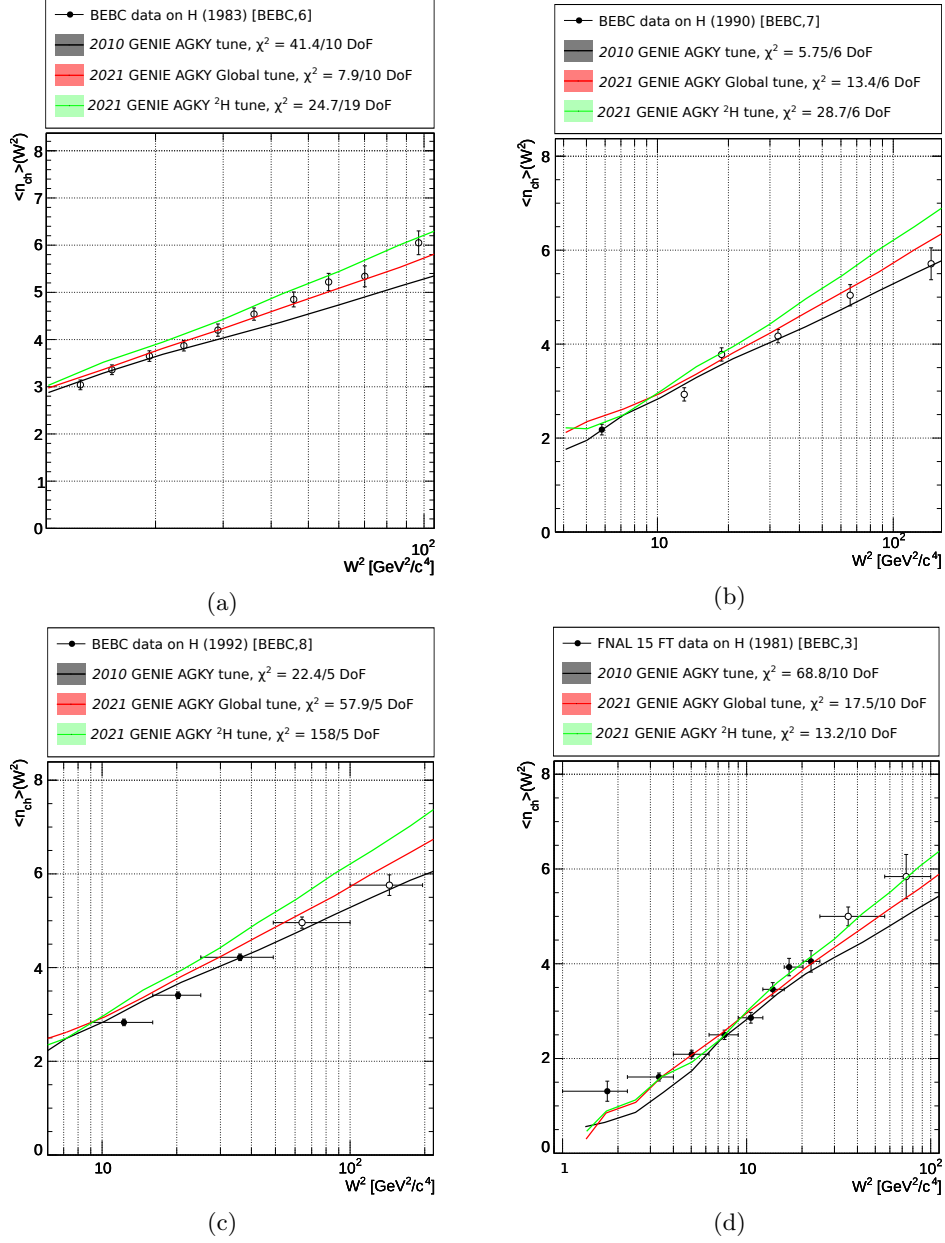


FIG. (15) Comparison of  $\langle n_{\text{ch}} \rangle$  against neutrino-induced hadronization data on  $\bar{\nu}_\mu + p$  interactions on H from the BEBC [64–66] and FNAL [68] bubble chamber experiment filled with H. The predictions are computed using the parameters specified in Tab.VII. The  $\chi^2$  values are calculated against all the data from each experiment. See definition of  $T_{\text{ags}}$  in Tab. XII.

charged multiplicity data from bubble chamber experiments [41]. They suggest that the origin of tensions between H and  $^2\text{H}$  could be due to rescattering effects on deuterium. As explained in Sec. III, the bubble chamber experiments claim that rescattering effects have a smaller effect on neutron samples as a consequence of the classification into  $\nu_\mu$  on  $p$  or  $\nu_\mu$  on  $n$  events. This is a consequence of the neutron re-interaction with the proton from the

deuterium, which is then kicked out and, therefore, miss-identified as a  $\nu_\mu p$  event. If the disagreements were only due to rescattering, the global tune would have a better agreement than the deuterium only tune on  $\nu_\mu$  and  $\bar{\nu}_\mu$  on neutron data. However, a better agreement of the global tune on neutron samples is not observed.

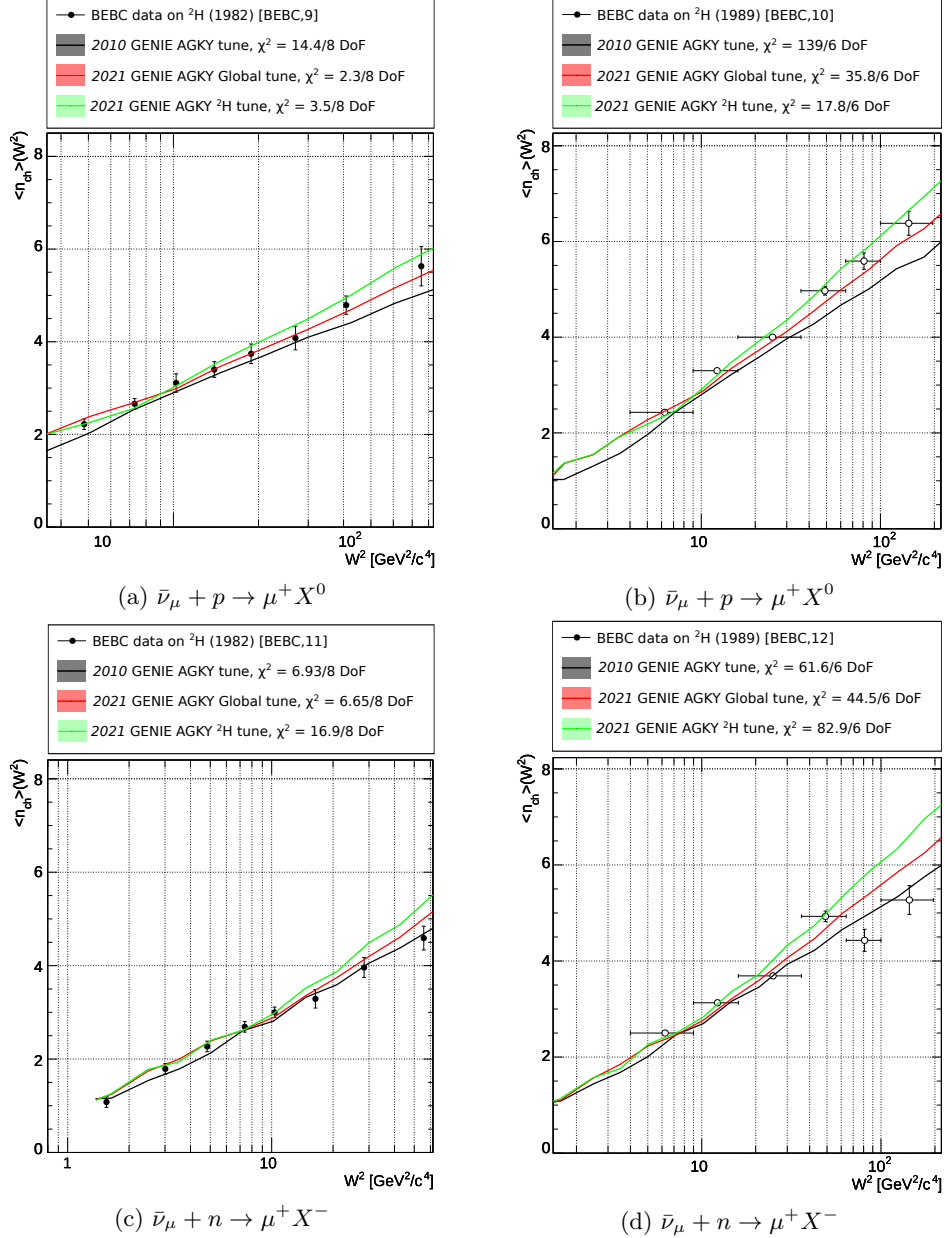


FIG. (16) Comparison of  $\langle n_{\text{ch}} \rangle$  against neutrino-induced hadronization data on  $\bar{\nu}_\mu$  interactions on p and n from the BEBC bubble chamber experiment filled with  ${}^2\text{H}$  [38, 71]. The predictions are computed using the parameters specified in Tab.VII. The  $\chi^2$  values are calculated against all the data from each experiment. See definition of  $T_{\text{ags}}$  in Tab. XII.

### C. AGKY Global and deuterium only tunes impact on other neutrino-induced hadronization observables

The analyses procedure discussed in this paper focuses on the description of the charged averaged multiplicity. However, as discussed in Sec. IV A, different observables are linked with the shower particle content description. In this section, the effect of the

global tune on different hadronization observables is discussed. A wider comparison against all available hadronization observables for the G00\_00a.00\_00a AGKY predictions is reported in [27]. Some information provided by these observables were included in the tune using priors, see Sec. VI. The agreement of the 2021 GENIE AGKY global tune with these observables was not compromised.

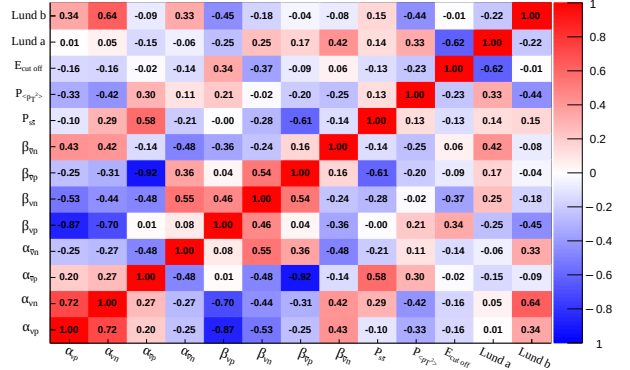
There are, however, other observables that show

TABLE (VIII) Summary of  $\chi^2$  values for the datasets shown in Figs. 14, 16, 13, and 15. The table shows the  $\chi^2$  per dataset and interaction channel as well as the total and per channel  $\chi^2$ . The  $\chi^2$  values are calculated using the GENIE predictions for each tune: *2010* GENIE,  $\chi_{2010}^2$ , *2021* GENIE,  $\chi_{2021(\text{global})}^2$ , and *2021* GENIE,  $\chi_{2021(2\text{H})}^2$ .

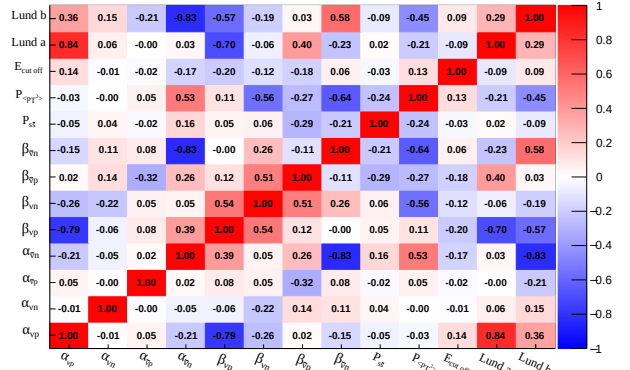
Experiment	$\chi_{2010}^2$	$\chi_{2021(\text{global})}^2$	$\chi_{2021(2\text{H})}^2$	DoF	In tune
$\nu_\mu + p \rightarrow \mu^- X^{++}$					
Data on hydrogen					
FNAL 15 ft,0	83	39	81	25	Partially
BEBC,0	222	56	13	11	×
BEBC,1	26	7	23	6	Partially
BEBC,2	50.2	3.5	68.8	5	✓
Data on deuterium					
FNAL 15 ft,1	21	13	5	14	Partially
BEBC,3	563	321	84	6	×
<b>Total for <math>\nu_\mu p</math></b>	<b>965</b>	<b>447</b>	<b>275</b>	<b>67</b>	
$\nu_\mu + n \rightarrow \mu^- X^{+}$					
FNAL 15 ft,2	17	21	7	14	Partially
BEBC,4	171	6	5	8	✓
BEBC,5	886	352	99	6	×
<b>Total for <math>\nu_\mu n</math></b>	<b>1,074</b>	<b>435</b>	<b>111</b>	<b>28</b>	
$\bar{\nu}_\mu + p \rightarrow \mu^+ X^0$					
Data on hydrogen					
FNAL 15 ft,3	69	18	13	10	Partially
BEBC,6	41	8	25	10	×
BEBC,7	5.8	13.4	28.7	6	Partially
BEBC,8	22.4	57.9	158.0	5	✓
Data on deuterium					
BEBC,9	14	2	4	8	✓
BEBC,10	139	36	18	6	×
<b>Total for <math>\bar{\nu}_\mu p</math></b>	<b>292</b>	<b>135</b>	<b>246</b>	<b>45</b>	
$\bar{\nu}_\mu + n \rightarrow \mu^+ X^-$					
BEBC,11	6.9	6.7	16.9	8	✓
BEBC,12	61.6	44.5	82.9	6	×
<b>Total for <math>\bar{\nu}_\mu n</math></b>	<b>69</b>	<b>51</b>	<b>100</b>	<b>14</b>	
$\chi^2$ Summary					
<b>All data</b>	<b>2,398</b>	<b>1,068</b>	<b>731</b>	<b>154</b>	
<b>All <math>^2\text{H}</math> data</b>	<b>1,879</b>	<b>858</b>	<b>320</b>	<b>76</b>	
<b>All H data</b>	<b>519</b>	<b>202</b>	<b>411</b>	<b>78</b>	

TABLE (IX) Total  $\chi^2$  from Tab. VIII using only datasets used in each fit: *2010* GENIE,  $\chi_{2010}^2$ , *2021* GENIE,  $\chi_{2021(\text{global})}^2$ , and *2021* GENIE,  $\chi_{2021(2\text{H})}^2$ .

Datasets	$\chi_{2010}^2$	$\chi_{2021(\text{global})}^2$	$\chi_{2021(2\text{H})}^2$	DoF
All Data in tune	486	242	410	109
$^2\text{H}$ Data in tune	230	105	37	52
H Data in tune	256	138	374	57



(a) Global tune correlation matrix.



(b) Deuterium only tune correlation matrix.

FIG. (17) Parameter correlation matrix for the *2021* GENIE AGKY tunes against averaged charged multiplicity data.

tensions with the averaged charged multiplicity data. The neutral pion averaged multiplicity is related with the charged hadron multiplicity via Eq. 2: an increase on the charged averaged multiplicity is equivalent to a higher neutral pion averaged multiplicity. This result is incompatible with the data, as demonstrated in Fig. 20. Another example is the dispersion observable, defined as  $D = \sqrt{\langle n^2 \rangle - \langle n \rangle^2}$ . The comparison of data on the ratio of  $D/\langle n_{\text{ch}} \rangle$  vs. the different tunes is shown in Fig. 21. In this case, the disagreement also increases with  $W$ .

The tension between charged averaged multiplic-

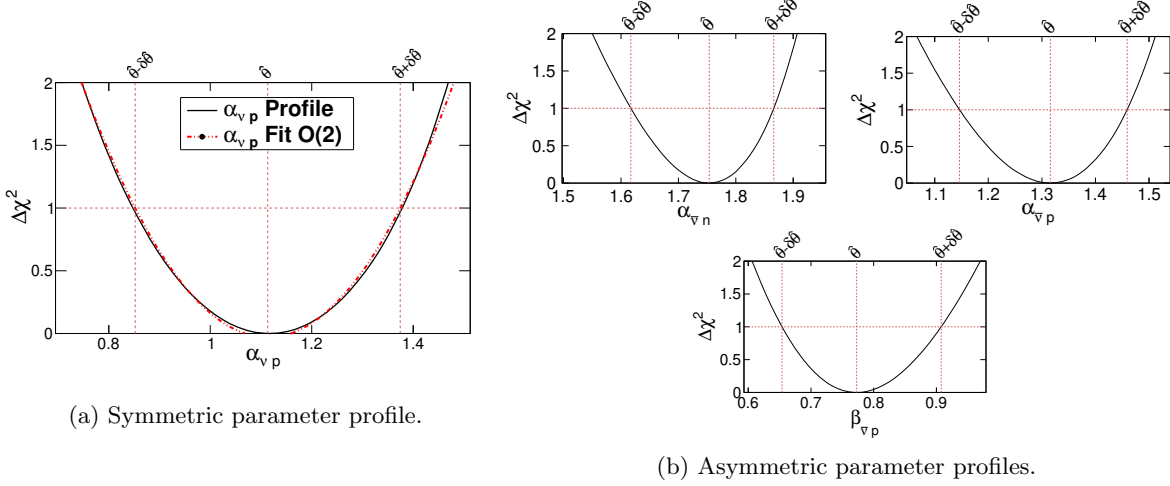


FIG. (18) Joint function obtained fixing the two parameters under study and minimizing  $\Delta\chi_{\text{profile}}^2(\theta)$  respect the other parameters in the *2021* GENIE global tune. The dashed lines represent the parameter range that satisfies the condition  $\Delta\chi_{\text{profile}}^2(\theta_i) < 1$ . This is also denoted as  $\hat{\theta} \pm \delta\hat{\theta}$ .

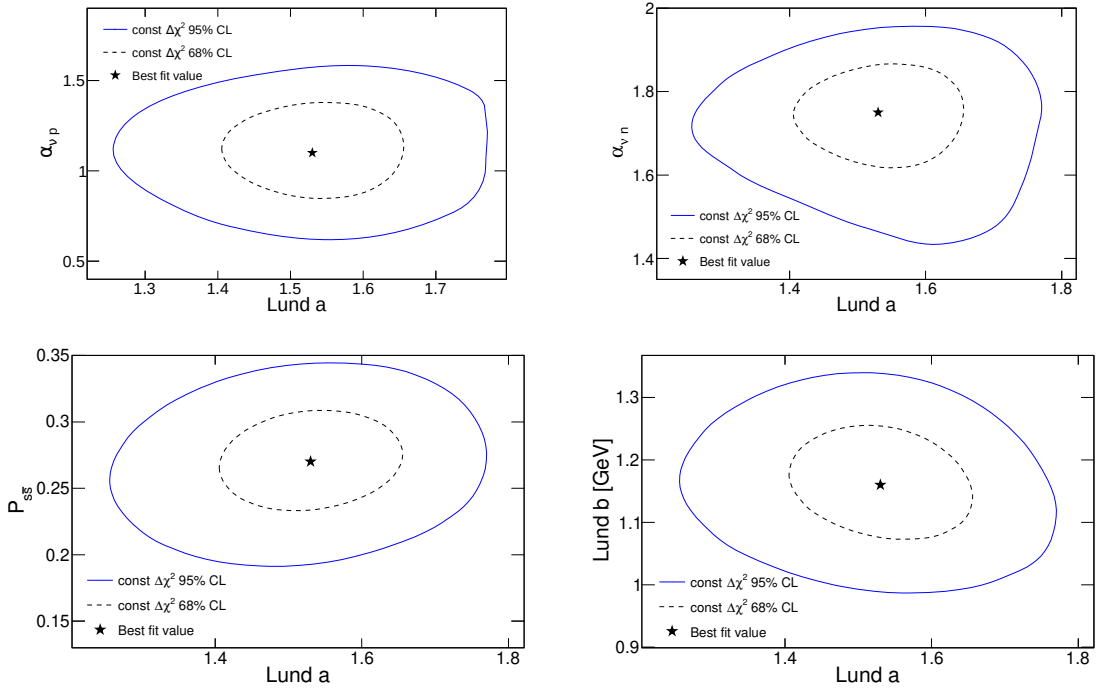


FIG. (19) Joint  $\Delta\chi_{\text{contour}}^2(\theta_i, \theta_j)$  function obtained fixing the two parameters under study and minimizing  $\Delta\chi_{\text{contour}}^2(\theta)$  respect the other parameters in the *2021* GENIE global tune. The 95% and 68% contour lines are shown as well as the best fit values for the global tune.

ity with  $\langle n_{\pi^0} \rangle$  and dispersion data was already observed when using the HERMES parameterisation described in Sec. IV B. The origin of these tensions is beyond the scope of this paper as we aim for a better description of the charged averaged multiplicity data only. The further understanding of the connection

between the different observables would require to repeat the analyses procedure of this paper including other hadronization related observables. Yet, it is important to understand how the *2021* GENIE AGKY global tune impacts other non-hadronization related observables.



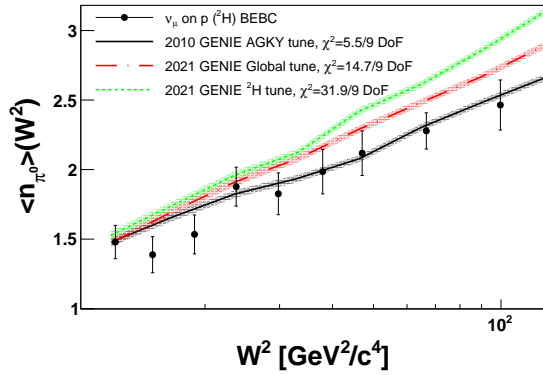


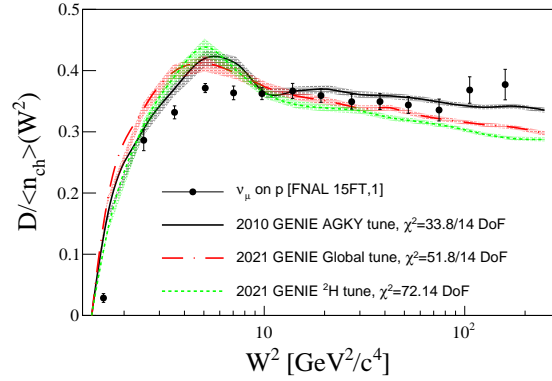
FIG. (20) Comparison of the predicted  $\langle n_{\pi^0} \rangle$  against neutrino-induced hadronization data on  $\nu_\mu$  interactions on p from the BEBC bubble chamber experiment filled with  $^2\text{H}$  [38, 71]. The predictions shown correspond to the 2010 GENIE AGKY (black), the 2021 GENIE AGKY global (red) and the 2021 GENIE AGKY  $^2\text{H}$  (green) tunes.

#### D. 2021 GENIE AGKY global tune impact at the SIS region

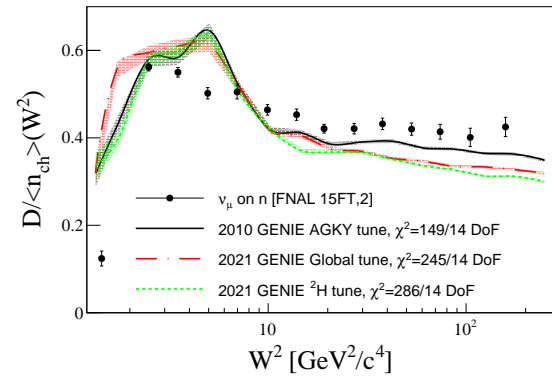
Other observables can be affected by this tune. The main impact is on the description of the shallow inelastic scattering (SIS) region in GENIE, since it is linked with final state multiplicities [24]. In GENIE, the SIS is modeled applying scaling factors to the DIS cross section and these factors depend on the multiplicity of the process. Hence, varying the final state multiplicity probabilities (Eq. 3) changes the scaling applied to the DIS cross section, affecting the DIS contribution to the SIS. The  $P_n^{\text{had}}$  probability distributions for the 2010 GENIE AGKY tune and for the AGKY global tune are shown in Fig. 22.

The impact of the AGKY tune on CC inclusive cross sections is summarised in Fig. 23. When applying the AGKY global tune to the SIS region, an increase of CC inclusive cross section is observed, for both  $\nu_\mu$  and  $\bar{\nu}_\mu$ . The exclusive cross sections for different pion multiplicities show that the AGKY tune enhances the  $2\pi$  production whilst the  $1\pi$  production remains similar, see Fig. 24. As a consequence, the agreement with inclusive and  $\nu_\mu$  CC  $\pi^+\pi^-$  data is lost.

Both the bare nucleon tune [24] and the 2021 GENIE global tune show a preference to increase the two pion production, suggesting that a joint tune could preserve the agreement with inclusive and exclusive data at low- $W$ . This was neglected in previous analyses to minimise the complexity but this analysis is clearly suggesting otherwise. The high- $W$  AGKY parameters don't need anymore refinements.



(a) Comparison against  $\nu_\mu$  on p data.



(b) Comparison against  $\nu_\mu$  on n data.

FIG. (21) Comparison of the predicted  $D/\langle n_{\text{ch}} \rangle$  against neutrino-induced hadronization data on  $\nu_\mu$  interactions on p (a) and n (b) from the FNAL 15 ft bubble chamber experiment filled with  $^2\text{H}$  [37].

The predictions shown correspond to the 2010 GENIE AGKY (black), the 2021 GENIE AGKY global (red) and the 2021 GENIE AGKY  $^2\text{H}$  (green) tunes.

On the contrary, the low- $W$  parameters requires a joint tune in order to have a satisfactory result that can be used to extract data driven parameter uncertainties.

## VIII. CONCLUSIONS

In this paper, we present the first GENIE tune of the AGKY model [27, 83] which was possible thanks to the Professor framework [84]. The analyses goal was to improve the GENIE agreement with neutrino charged averaged multiplicity data and to produce the first data driven constraints on the hadronization parameters. Specifically, we constraint parameters of both low- $W$  empirical model and PYTHIA. The

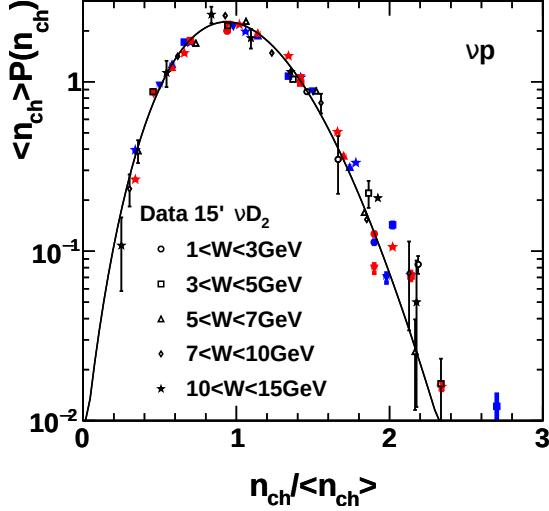
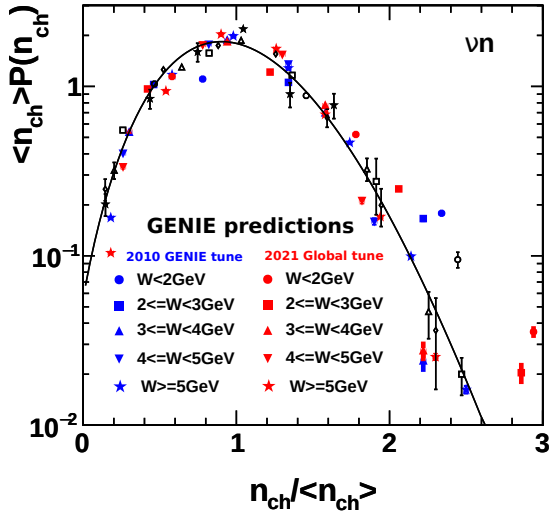
(a)  $\nu_\mu p$  KNO scaling distribution(b)  $\nu_\mu n$  KNO scaling distribution

FIG. (22) Comparison of the KNO scaling distributions for neutrino interactions on deuterium against the predictions for 2010 GENIE tune (blue) and the 2021 GENIE global tune (red). The solid line is the best fit result of the Levi function to FNAL 15 ft bubble chamber data [37]. The  $W$  range used for each data and predicted point is specified in the legend of Fig. 22 (a) and (b).

data were from the BEBC and FNAL 15 ft bubble chamber experiments filled with hydrogen and deuterium.

Tensions between hydrogen and deuterium data were observed and two separate tunes were performed: a global tune and a deuterium only. In particular, the global tune AGKY prediction underpredicts the deuterium data at the PYTHIA region whereas the deuterium only tune overpredicts the

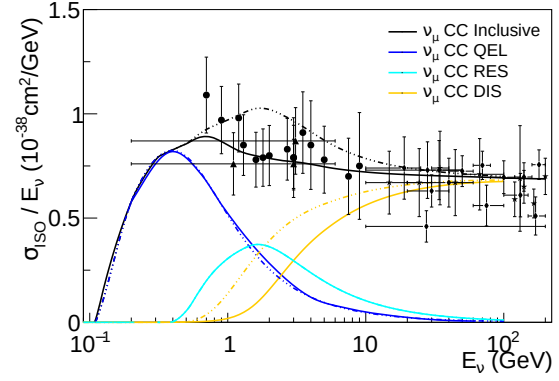
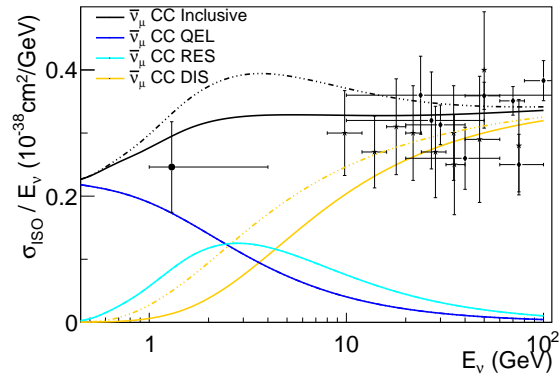
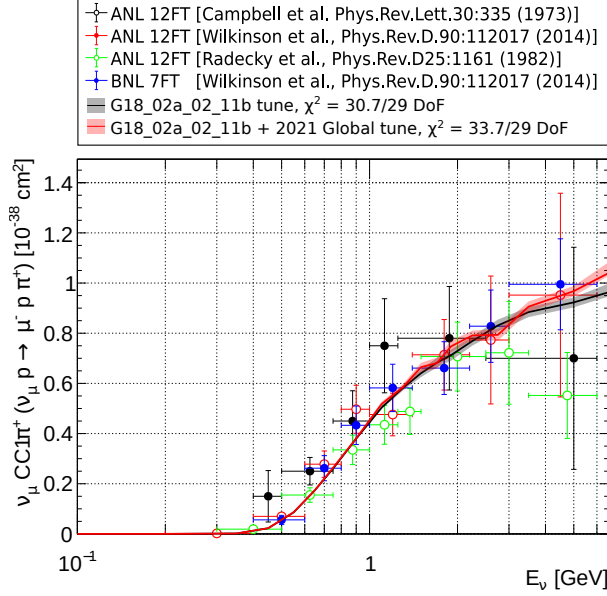
(a)  $\nu_\mu$  CC inclusive cross section(b)  $\bar{\nu}_\mu$  CC inclusive cross section

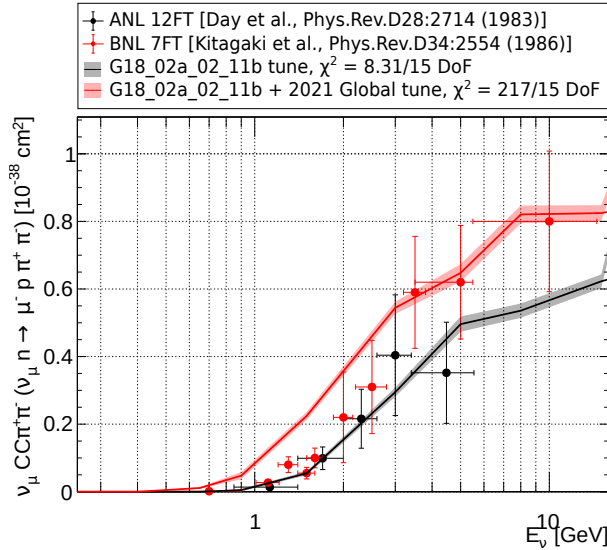
FIG. (23) Comparison of the  $\nu_\mu$  and  $\bar{\nu}_\mu$  CC inclusive cross section on free nucleon for the 2010 GENIE AGKY tune [24] (continuous lines) and the 2021 GENIE global tune (dashed lines) against hydrogen and deuterium data from ANL 12FT ( $\Delta$ ), BNL 7FT ( $\bullet$ ), BEBC ( $\diamond$ ) and FNAL ( $\star$ ). The breakdown of the CC QEL, CC RES and CC DIS contributions is shown for before and after the 2021 GENIE AGKY global tune.

hydrogen data. Further investigations on hadronization samples are needed in order to clarify the origin of this discrepancy. A possible solution can come from more recent neutrino experiments that released data on neutrino-induced hadronization. This is the case of NOMAD [85, 86] for  $\nu_\mu$  on mainly carbon target, CHORUS for  $\nu_\mu$  and  $\bar{\nu}_\mu$  on Fuji ET-7B emulsion [43, 87], OPERA for  $\nu_\mu$  on lead [42, 88] and MicroBooNE for  $\nu_\mu$  on argon [89]. But of course these samples include nuclear effects and therefore are not in the scope of this work.

Despite the tensions, the global tune shows a better agreement with the charged averaged multiplicity data and provides the first data driven analyses of this kind using neutrino interactions. This



(a) One pion prediction



(b) Two pion production

FIG. (24) Comparison of the  $\nu_\mu$  CC exclusive cross section data on free nucleon for the *2010* GENIE AGKY tune [24] (black) and the *2021* GENIE global tune (red) against ANL 12FT and BNL 12FT data.

statistical analyses can be a useful input for proper systematic studies of modern neutrino experiments. The main effect of the tune is the increase of the averaged charged multiplicity for  $W^2 > 10 \text{ GeV}^2/c^4$ , modelled with PYTHIA. The low- $W$  region is also affected but constraints due to energy, momentum, charge, baryon number and strangeness conservation laws reduce the available phase space and the effect

of the tuning procedure.

The effect of the *2021* GENIE AGKY global tune at the shallow inelastic scattering region is an increase on the two pion production cross section, which affects the current agreement with CC inclusive data [24]. Therefore, we conclude that this tune is more appropriate at higher energies where the contribution of the SIS region is not relevant. The information on the systematic uncertainties coming from the low- $W$  AGKY parameters is still valuable for neutrino experiments interested in the  $W < 2 \text{ GeV}/c^2$  region. A joint tune of the shallow inelastic scattering region and hadronization datasets would address this disagreement.

## IX. ACKNOWLEDGEMENTS

We would like to thank Andy Buckley (University of Glasgow, UK) and Holger Schultz (Institute of Particle Physics Phenomenology, University of Durham, UK) for their support interfacing the Professor tool with the software products that underpin the GENIE global analyses. We would like to thank the CC-IN2P3 Computing Center, as well as the Particle Physics Department at Rutherford Appleton Laboratory for providing computing resources and for their support. This work, as well as the ongoing development of several other GENIE physics tunes was enabled through a PhD studentship funded by STFC through LIV.DAT, the Liverpool Big Data Science Centre for Doctoral Training (project reference: 2021488). The initial conceptual and prototyping work for the development of the GENIE / Professor interfaces, as well as for the development of the GENIE global analyses framework that, currently, underpins several analyses, was supported in part through an Associateship Award by the Institute of Particle Physics Phenomenology, University of Durham.

This document was prepared by the GENIE collaboration using the resources of the Fermi National Accelerator Laboratory (Fermilab), a U.S. Department of Energy, Office of Science, HEP User Facility. Fermilab is managed by Fermi Research Alliance, LLC (FRA), acting under Contract No. DE-AC02-07CH11359.

## LIST OF ACRONYMS

- KNO** Koba-Nielsen-Olesen scaling law
- AGKY** Andreopoulos-Gallagher-Kehayias-Yang model
- BEBC** Big European Bubble Chamber

TABLE (X) Parameter covariance matrix extracted from the 2021 GENIE AGKY global tune.

	$\alpha_{\nu p}$	$\alpha_{\nu n}$	$\alpha_{\bar{\nu} p}$	$\alpha_{\bar{\nu} n}$	$\beta_{\nu p}$	$\beta_{\nu n}$	$\beta_{\bar{\nu} p}$	$\beta_{\bar{\nu} n}$	$P_{s\bar{s}}$	$\langle p_{\perp}^2 \rangle$	$E_{\text{CutOff}}$	Lund <i>a</i>	Lund <i>b</i>
$\alpha_{\nu p}$	1.8E-1	-2.2E-4	1.5E-3	-2.8E-2	-1.1E-1	-1.9E-2	9.2E-4	-1.5E-2	-1.0E-3	-5.8E-4	2.7E-3	5.6E-2	3.4E-2
$\alpha_{\nu n}$	-2.2E-4	5.4E-3	-3.0E-5	-1.3E-3	-1.4E-3	-2.9E-3	1.1E-3	1.8E-3	1.4E-4	0.0	-3.0E-5	7.2E-4	2.4E-3
$\alpha_{\bar{\nu} p}$	1.5E-3	-3.0E-5	6.2E-3	5.7E-4	2.2E-3	7.7E-4	-2.9E-3	1.4E-3	-8.0E-5	1.8E-4	-6.0E-5	-3.0E-5	-3.7E-3
$\alpha_{\bar{\nu} n}$	-2.8E-2	-1.3E-3	5.7E-4	1.0E-1	4.3E-2	2.9E-3	9.5E-3	-6.1E-2	2.4E-3	7.1E-3	-2.6E-3	1.6E-3	-5.9E-2
$\beta_{\nu p}$	-1.1E-1	-1.4E-3	2.2E-3	4.3E-2	1.2E-1	3.3E-2	4.7E-3	-6.0E-5	7.7E-4	1.6E-3	-3.3E-3	-3.8E-2	-4.4E-2
$\beta_{\nu n}$	-1.9E-2	-2.9E-3	7.7E-4	2.9E-3	3.3E-2	3.2E-2	1.1E-2	1.1E-2	4.9E-4	-4.2E-3	-1.1E-3	-1.8E-3	-7.5E-3
$\beta_{\bar{\nu} p}$	9.2E-4	1.1E-3	-2.9E-3	9.5E-3	4.7E-3	1.1E-2	1.3E-2	-3.0E-3	-1.6E-3	-1.3E-3	-9.9E-4	7.2E-3	7.7E-4
$\beta_{\bar{\nu} n}$	-1.5E-2	1.8E-3	1.4E-3	-6.1E-2	-6.0E-5	1.1E-2	-3.0E-3	5.2E-2	-2.4E-3	-6.1E-3	6.3E-4	-8.5E-3	2.9E-2
$P_{s\bar{s}}$	-1.0E-3	1.4E-4	-8.0E-5	2.4E-3	7.7E-4	4.9E-4	-1.6E-3	-2.4E-3	2.3E-3	-4.8E-4	-7.0E-5	1.2E-4	-9.6E-4
$\langle p_{\perp}^2 \rangle$	-5.8E-4	0.0	1.8E-4	7.1E-3	1.6E-3	-4.2E-3	-1.3E-3	-6.1E-3	-4.8E-4	1.8E-3	2.7E-4	-1.4E-3	-4.2E-3
$E_{\text{CutOff}}$	2.7E-3	-3.0E-5	-6.0E-5	-2.6E-3	-3.3E-3	-1.1E-3	-9.9E-4	6.3E-4	-7.0E-5	2.7E-4	2.3E-3	-7.0E-4	9.2E-4
Lund <i>a</i>	5.6E-2	7.2E-4	-3.0E-5	1.6E-3	-3.8E-2	-1.8E-3	7.2E-3	-8.5E-3	1.2E-4	-1.4E-3	-7.0E-4	2.5E-2	1.0E-2
Lund <i>b</i>	3.4E-2	2.4E-3	-3.7E-3	-5.9E-2	-4.4E-2	-7.5E-3	7.7E-4	2.9E-2	-9.6E-4	-4.2E-3	9.2E-4	1.0E-2	5.0E-2

TABLE (XI) Parameter covariance matrix extracted from the 2021 GENIE AGKY  $^2\text{H}$  tune.

	$\alpha_{\nu p}$	$\alpha_{\nu n}$	$\alpha_{\bar{\nu} p}$	$\alpha_{\bar{\nu} n}$	$\beta_{\nu p}$	$\beta_{\nu n}$	$\beta_{\bar{\nu} p}$	$\beta_{\bar{\nu} n}$	$P_{s\bar{s}}$	$\langle p_{\perp}^2 \rangle$	$E_{\text{CutOff}}$	Lund <i>a</i>	Lund <i>b</i>
$\alpha_{\nu p}$	7.7E-2	2.5E-2	8.6E-3	-6.3E-3	-3.6E-2	-1.6E-2	-8.7E-3	1.0E-2	-1.1E-3	-2.9E-3	-1.7E-3	3.8E-4	8.8E-3
$\alpha_{\nu n}$	2.5E-2	1.5E-2	5.2E-3	-3.1E-3	-1.3E-2	-5.8E-3	-4.8E-3	4.4E-3	1.4E-3	-1.6E-3	-7.8E-4	7.9E-4	7.2E-3
$\alpha_{\bar{\nu} p}$	8.6E-3	5.2E-3	2.4E-2	-6.9E-3	3.0E-4	-8.1E-3	-1.8E-2	-1.9E-3	3.4E-3	1.5E-3	-1.2E-4	-2.9E-3	-1.2E-3
$\alpha_{\bar{\nu} n}$	-6.3E-3	-3.1E-3	-6.9E-3	8.5E-3	1.1E-3	5.5E-3	4.1E-3	-3.8E-3	-7.1E-4	3.3E-4	-5.0E-4	-7.1E-4	2.8E-3
$\beta_{\nu p}$	-3.6E-2	-1.3E-2	3.0E-4	1.1E-3	2.2E-2	7.4E-3	7.2E-4	-4.5E-3	-2.0E-5	1.0E-3	1.9E-3	-4.6E-3	-6.2E-3
$\beta_{\nu n}$	-1.6E-2	-5.8E-3	-8.1E-3	5.5E-3	7.4E-3	1.2E-2	7.2E-3	-2.1E-3	-1.1E-3	-6.0E-5	-1.5E-3	3.3E-3	-1.8E-3
$\beta_{\bar{\nu} p}$	-8.7E-3	-4.8E-3	-1.8E-2	4.1E-3	7.2E-4	7.2E-3	1.6E-2	1.7E-3	-2.8E-3	-8.0E-4	-4.2E-4	2.6E-3	-4.9E-4
$\beta_{\bar{\nu} n}$	1.0E-2	4.4E-3	-1.9E-3	-3.8E-3	-4.5E-3	-2.1E-3	1.7E-3	7.1E-3	-4.3E-4	-6.6E-4	1.8E-4	4.4E-3	-6.1E-4
$P_{s\bar{s}}$	-1.1E-3	1.4E-3	3.4E-3	-7.1E-4	-2.0E-5	-1.1E-3	-2.8E-3	-4.3E-4	1.4E-3	1.6E-4	-1.9E-4	6.3E-4	5.3E-4
$\langle p_{\perp}^2 \rangle$	-2.9E-3	-1.6E-3	1.5E-3	3.3E-4	1.0E-3	-6.0E-5	-8.0E-4	-6.6E-4	1.6E-4	9.9E-4	-2.8E-4	1.3E-3	-1.3E-3
$E_{\text{CutOff}}$	-1.7E-3	-7.8E-4	-1.2E-4	-5.0E-4	1.9E-3	-1.5E-3	-4.2E-4	1.8E-4	-1.9E-4	-2.8E-4	1.5E-3	-2.9E-3	-2.0E-5
Lund <i>a</i>	3.8E-4	7.9E-4	-2.9E-3	-7.1E-4	-4.6E-3	3.3E-3	2.6E-3	4.4E-3	6.3E-4	1.3E-3	-2.9E-3	1.5E-2	-2.5E-3
Lund <i>b</i>	8.8E-3	7.2E-3	-1.2E-3	2.8E-3	-6.2E-3	-1.8E-3	-4.9E-4	-6.1E-4	5.3E-4	-1.3E-3	-2.0E-5	-2.5E-3	8.5E-3

**CMC** Comprehensive Model Configurations

**MC** Monte Carlo

**EMI** External Muon Identifier

**LPS** Longitudinal Phase Space model

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TABLE (XII) Summary of data used for comparisons in Figs. 7, 15, 14 and 16. This table links the experiment and the tag used for the legend in each plot to the corresponding reference.

Experiment	Target	Tag	Ref.
$\nu_\mu + p \rightarrow \mu^- X^{++}$			
FNAL 15 ft (1976)	H	FNAL 15 ft,0	[70]
BEBC (1983)	H	BEBC,0	[64]
BEBC (1990)	H	BEBC,1	[65]
BEBC (1992)	H	BEBC,2	[66]
FNAL 15 ft (1983)	$^2\text{H}$	FNAL 15 ft,1	[37]
BEBC (1989)	$^2\text{H}$	BEBC,3	[71]
$\nu_\mu + n \rightarrow \mu^- X^+$			
FNAL 15 ft (1983)	$^2\text{H}$	FNAL 15 ft,2	[37]
BEBC (1984)	$^2\text{H}$	BEBC,4	[72]
BEBC (1989)	$^2\text{H}$	BEBC,5	[71]
$\bar{\nu}_\mu + p \rightarrow \mu^+ X^0$			
FNAL 15 ft (1981)	H	FNAL 15 ft,3	[68]
BEBC (1983)	H	BEBC,6	[64]
BEBC (1990)	H	BEBC,7	[65]
BEBC (1992)	H	BEBC,8	[66]
BEBC (1982)	$^2\text{H}$	BEBC,9	[38]
BEBC (1989)	$^2\text{H}$	BEBC,10	[71]
$\bar{\nu}_\mu + n \rightarrow \mu^+ X^-$			
BEBC (1982)	$^2\text{H}$	BEBC,11	[38]
BEBC (1989)	$^2\text{H}$	BEBC,12	[71]

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