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Implementing Logics in Diagrams

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1 Introduction

Graphical representations, together with the processes which construct them and retrieve information from them, implement logics just as much as linguistic representations do. The distinctive property of graphical representation systems is 'specificity'—they enforce the representation of certain classes of information (e.g. referential and spatial relations). This property curtails their ability to express abstractions, though some but not all abstractions may be reintroduced by interpretative conventions. This curtailment of abstraction gives graphics their dominant cognitive characteristics. Provided that the inferential task at hand does not require abstractions which are inaccessible to graphical systems, the curtailment of abstraction leads to more processible representations. However, if the task at hand does require abstractions that are not available, graphics may substantially hinder processing.

We have been developing a general cognitive theory of graphics from this starting point. Our aim in this paper is to lay out the outlines of our theory so that it can be compared and contrasted with other approaches at this workshop. We will give a brief exposition of the logical treatment and then illustrate the theory in the domain of an Euler's Circles based system for solving syllogisms.

2 PSRS and LARS

Specificity is a property of representational *systems*, not of particular representations. Systems that demand the specification of information are said to be 'specific', and there are degrees of specificity. Such an observation is by no means new; Bishop Berkeley noted that geometry diagrams had this property: a triangle must be irregular, isosceles, or equilateral but the word 'triangle' can denote these three types indifferently.

For current purposes, the best characterisation of specificity is in semantic terms: a representational system (such as one whose elements are diagrams) is *primitively specific* (it is a PSRS) if each of its diagrams has exactly one model. Primitive specificity has a syntactic reflex and it is possible to design a predicate language which is sufficiently syntactically constrained to be a PSRS. Equally, there are diagrammatic schemes which can be considered PSRSs.

However, for any PSRS, there will be a family of *limited abstraction* representational systems (LARSs); semantically, their diagrams have one or more models; that is how they abstract over cases. What makes their abstraction limited is that there is no guarantee that a

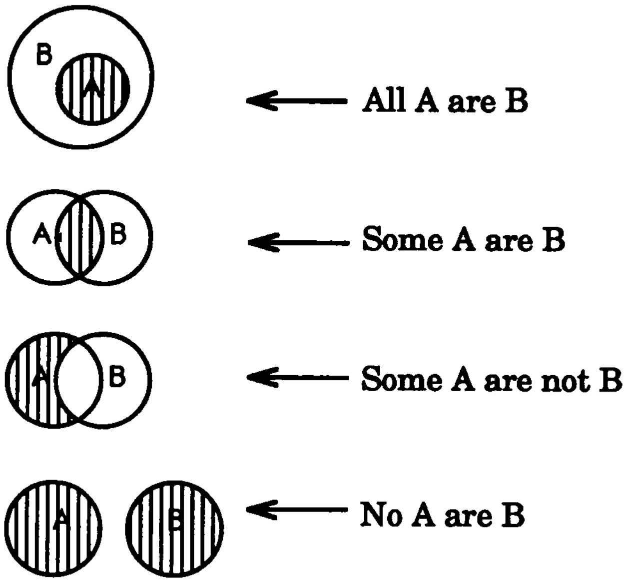

Figure 1: Characteristic diagrams

LARS will have a single representation for an arbitrary set of models. The move from a PSRS to a LARS also has a syntactic reflex; for example, new predicate symbols can be introduced, corresponding to disjunctive predicates.

Actual diagrammatic or graphical systems are almost always LARSs rather than PSRSs. Elsewhere, we have investigated the tradeoffs between expressiveness and tractability involved in the difference. The focus here is on how an actual diagrammatic reasoning system—Euler's Circles—can be used as a system abstractive enough to be expressive, but limited enough to be efficient.

3 Euler's Circles

ECs are an interesting domain for our purposes because they explicitly implement a logic and this requires the expression of a range of abstractions. There has also been considerable argument among psychologists about their relation to the mental representations people use for 'doing syllogisms' without external aids.

Euler's Circles (ECs) as traditionally interpreted by psychologists have been taken to be PSRS (see e.g. Erickson 1974, Guyote & Sternberg 1981, Johnson-Laird 1983); however, it is possible to re-interpret them as LARSs, and this is how they have actually been used in logic teaching.

We begin by defining a graphical algorithm for solving syllogisms and then examine its relation to our general theory. The four syllogistic premisses map many-to-many onto their models. However, each has a unique *maximal* model (see Figure 1). A shading convention can be used to distinguish regions corresponding to types which must be present in any model of the statement. Since each premiss now has a unique diagram, it remains to specify how to 'register' two premiss diagrams and read off conclusions. Premiss diagrams are registered

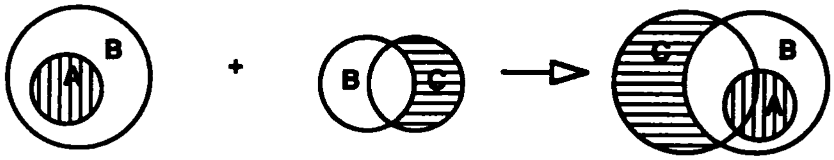

Figure 2: Registration diagram for *All A are B. Some C are not B.*

by making the B circles coincide and arranging the A and C circles so as to represent the *maximum* number of types consistent with the premisses.[1]

There are 21 registration diagrams of which Figure 2 illustrates an example for the syllogism *All A are B. Some C are not B.* If there is a shaded region of a premiss diagram which persists as un-intersected in the registration diagram, then there is a valid conclusion. An existential conclusion can always be drawn by taking the type of individual corresponding to this non-intersected shaded area, deleting the B term and prefacing with an existential quantifier. In Figure 2, on the basis of the region representing individuals which are $\neg A \neg BC$ which persists not intersected from premiss diagram into registration diagram, we may conclude *Some C are not A*. Universal conclusions require the critical non-intersected shaded region to be circular. The term corresponding to the circle is then the subject of a valid universal conclusion.

4 ECs as a LARS

Euler's circles exploit the specificity of graphical representations both in the representation of sets of individuals by static diagrams and in the representation of relations between these models by movements of circles which transform one diagram into another.

The static diagrams exhibit specificity in that every region represents a maximal type of individual because every point in a plane is either inside or outside every circle in that plane. It is a peculiar logical property of syllogisms that they are 'case identifiable'—every syllogism that has a valid conclusion does so in virtue of establishing the existence of a maximal type of individual.

Relations between static diagrams are represented by movements of circles which transform diagram into diagram—a minimal case of 'animation'. Here the specificity of the graphical representations exploits a continuity in the logical space. Movements (including changes of size) of circles lead to the appearance or disappearance of single regions at a time. The movements therefore correspond to sequences of transitions from neighbour to neighbour in a seven-dimensional space, one dimension corresponding to each of the seven maximal types of individual. Movement of circles therefore identifies the types of individual relevant to checking for logical constraint.

[1] We assume as a primitive the ability to detect whether a diagram is consistent with a premiss. This reflects our belief that the problem for a human reasoner is not consistency detection but *premiss combination*.

5 Conclusions

We have argued elsewhere (Stenning & Oberlander (in press), (1992)) that Johnson-Laird's MMs are a notational variant upon the theorem prover and logic implemented by the graphical algorithm sketched here. Our analysis highlights the need for careful examination of the ways that real graphical systems are actually employed in reasoning for their logic to be understood. Careless assumption that they are PRSRs in the case of Mental Models actually lead to the creation of an apparently non-graphical notation equivalent to the real usage of a traditional graphical system. The point that non-graphical and graphical representations may be tacitly equivalent has been made many times (e.g. Palmer 1978).

Although the graphical and non-graphical representations may be equivalent at one level, we would still argue that the graphical notation is more constrained and is a richer guide to correspondences with mental representations. Our approach to graphics through their limiting of the expression of abstraction gives a general theory which explains why these notations have the processing properties they exhibit within general computational concepts of complexity. Our Euler's Circle example illustrates how enforcing specificity leads to processing advantages; why the approach is applicable to categorial syllogisms but not disjunctive ones; how complicating the ontology through interpretation conventions yields a LARS from a PSRS and thus avoids combinatorial explosion.

Our EC example also illustrates the common property of graphical reasoning systems that by defining separate 'construction' and 'reading off' phases, they can appear to make inference 'direct' (see e.g. Lindsay 1988). While we believe that this is an important implementational contrast with more conventional logical procedures, it does not mean that no inference is involved. In graphics, much of the inference is done in constructing the representation, in searching it, and in reading off conclusions. Our approach makes it possible to show exact but complicated correspondences between graphical reasoning and reasoning in a logical calculus.

By formulating an external graphical algorithm, we can raise the question what part of it would require mental implementation and in what sort of memory structure to model human performance in various contexts (see Stenning & Oberlander (1992) for more discussion).

By analysing the logic which graphical systems more generally implement we can relate the facility of processing observed to the constraints on logical expressive and their computational consequences (e.g. Levesque 1988).

By acknowledging that logics require implementation before they can model processes, this approach encourages a more productive relation between logic, computation and psychology.

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