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To the highest bidder: The market for talent in sports leagues*

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Abstract

We present a realistic and novel micro-structure for the market for athletes in league sports. In our trading mechanism the clubs bid for individual players, internalizing the effect that a player not hired might play for the competition. For inelastic talent supply, our (wage-minimizing) equilibrium supports the Coasian results of Rottenberg (1956) and Fort and Quirk (1995): talent allocation is independent of initial "ownership" and revenue sharing arrangements. When talent supply is elastic, revenue sharing decreases the aggregate amount of talent hired. This negative effect on the talent level may be efficiency enhancing when the competition for talent results in excess talent being hired. For the first time in the literature, we carry out our entire analysis using a newly formulated, unified club objective, incorporating both pecuniary and non-pecuniary benefits.

*We are grateful for comments from Philipp Kircher, Paul Madden and Fred Palomino.

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1 Introduction

Despite the increasing economic significance of the sports industry,[1] the literature on the economics of sport – kicked off by Rottenberg (1956) – continues to be largely absent from core journals of economics. The biased view of many editors and referees that sport is "just a game", can be only partly blamed for this. More importantly, while there is a generally accepted overall view of the peculiarities of the labor market in this industry – nicely crystallized by Rosen and Sanderson (2001) –, less headway has been made in formal theoretical analysis that not only supports the empirical observations, but can provide generalizable insights for the wider discipline. In this paper we take a step in that direction, putting forward a set of original modelling choices, which together form a basic microstructure of the labor market in sports. This structure is amenable to be built upon with the introduction of further institutional details. At the same time, there is general insight that can be learnt from its generic version, which is transferable to other industries that exhibit oligopolistic competition in both input and output markets.[2]

Two competing views as to clubs' objectives have dominated the sports economics arena since the early days. On the one hand, and more popular among American scholars, it is postulated that clubs maximize profits, just as most firms do. On the other hand, starting with Neale (1964), it has been argued that clubs owned by a "benefactor" – or by a large number of "members" who do not receive dividends, as is often the case with European clubs – do not maximize profits. The usual way of modelling these "utility maximizing" clubs (c.f. Sloane, 1971) is that they hire all the talent they can afford. As the first approach leads to a first-order condition, while the second one is determined by a budget constraint, they often

[1] For example, according to Sport England: "In 2010, sport and sport-related activity generated Gross Value Added (GVA) of £20.3 billion ($30 billion) – 1.9% of the England total. This placed sport within the top 15 industry sectors in England and larger than sale and repair of motor vehicles, insurance, telecoms services, legal services and accounting." And this is before taking into account the savings in health care costs, estimated at $16 billion.

[2] As an example, our model of the labor market could be easily adapted to the market for CEOs, and contribute to an explanation of why they are "overpaid".

lead to drastically different conclusions, even in terms of comparative statics. Our first contribution is to unify the two types of objective function.

Our fundamental observation is that both benefactors and supporter-owned clubs have alternative uses for money that also provide them with "utility". A magnate might wish to buy a yacht, a members' club might want to subsidize its other activities/teams. This fact implies that rather than an inflexible (budget) constraint, what the club has to factor into its decision is the shadow utility – or opportunity cost – of money spent on the team. This construct allows the club to trade off utility against money, typically leading to an interior solution. As a result, we can postulate a general objective function, incorporating both profit and/or non-profit criteria, and still use the first-order approach.

Next, we turn to modelling the player market. What complicates matters is that in this decentralized market, the clubs cannot unilaterally decide the amount of talent they hire: that is jointly determined by the actions of all clubs. It is their strategic interaction that leads to the final talent allocation.[3] Note that an essential characteristic of a sports league is that the clubs are interacting in two different markets. Not only are they competing for the players, but they are also engaged in a tournament – and in fact, in joint production – on the field/pitch/court. As a result, the willingness to pay of a club for an additional player depends on where this player would go if she were not hired by the club. In a two-team league, one could say that if the supply of talent is low/inelastic then the player will go with the rival, while otherwise it will be unemployed (c.f. Madden, 2011, for a continuous version of this scheme). However, this approach clearly breaks down if there are more than two clubs in the league. Moreover, the fundamental issue is that clubs would like to – and, in practice, do – *affect* whether or not they are in direct competition with another club for a player.

We thus posit a market, where clubs can make personalized offers to players.

[3]Alternatively, a club *could* unilaterally decide how much to spend on players (c.f. Madden, 2015b). However, it is unclear how such a game could be implemented in the absence of an "invisible hand".

In actual fact, this may involve complicated forms of multilateral negotiations, but – for simplicity – we model it as a collection of simultaneous sealed-bid auctions, where the clubs bid competitively for individual players.[4] We show that this – highly decentralized – market does not lead to much additional analytical complexity, as the bidding equilibrium always leads to a single wage paid/offered to all hired units of talent. The nature of equilibria is determined by the clubs' demand functions. All the conceptual complications stem from the endogenous nature of these: how much a club values an extra unit of talent depends not just on its own talent level but on the *distribution* of talent across clubs at which we wish to evaluate this marginal effect.

When calculating a club's willingness to pay for another player, *given the other clubs' bids*, we have to consider two scenarios: the new player may be attracted away from a rival club – in the case of "contested" players, who receive an acceptable offer from at least two clubs – or, on the contrary, she may be hired from the pool of non-employed talent – in the case of "uncontested" players, where the other clubs do not compete for them. As it turns out, in most cases,[5] all the hired players are contested: on the one hand, presented with a choice, a club would rather weaken a rival, while on the other hand, even if a club would wish to do so, there is no way to retain exclusive dealing with a player. Consequently, demand and wages incorporate the external effect that hiring a player will not simply add her productivity to the team but it will also subtract her productivity from a competitor.

Finally, we show that revenue sharing simply dampens the incentives to hire talent, with no effect on its allocation. This does not leave us without a motive

[4]In Burguet and Sákovics (2016), we use the same bidding model to analyze the competition for inputs among oligopolists. In that paper we concentrate on the wage enhancing effects of competitive foreclosure, the possibility of unemployment when labor supply is inelastic, the relevance of (non)anonymity, and we also incorporate multiple industries.

De Fraja and Sákovics (2001) also consider the possibility of competitive bidding in a decentralized market, but they have a random matching environment, rather than targeted offers.

Palomino and Sákovics (2004) consider both targeted bidding and externalities, but they have a single player in each team.

[5]It is not always the case as competitive balance considerations may overcome the usual effect.

for intervention though: since the clubs coordinate on bidding for the same players, they may compete too fiercely, even from a social welfare point of view. As revenue sharing softens the competition, it may be welfare improving.

2 Club objectives

Let us flesh out our argument for a unified club objective by formalizing the two traditional views on it, together with the one we wish to put forward.[6] The ingredients are: the amount of talent hired by the club, t; the distribution of talent in the league, $\boldsymbol{t}$; an exogenous budget (which may include future income), B; a revenue function, $R(\boldsymbol{t})$; a cost function, $C(t)$; a utility function measuring the non-pecuniary benefits derived from the hired talent, $U(\boldsymbol{t})$; and a (strictly increasing) indirect utility function measuring the utility derived from the next-best use of money, $V(\$)$. We assume additive separability of the two utility functions, and normalize $R(0,.) = C(0) = U(0,.) = 0$.

The traditional formulations are straightforward:

Profit maximization: $\max_t [R(\boldsymbol{t}) - C(t)]$ – equivalently, $\max_t [B + R(\boldsymbol{t}) - C(t)]$; F.O.C. : $\frac{\partial R(\boldsymbol{t})}{\partial t} = C'(t)$.

Utility maximization: $\max_t U(\boldsymbol{t})$ s.t. $B + R(\boldsymbol{t}) - C(t) \geq 0$. For any increasing $U(.)$, the solution requires a binding budget constraint, $B + R(\boldsymbol{t}) - C(t) = 0$.

We propose a unified formulation, where a club's objective is the sum of its non-pecuniary benefit from hiring talent, $U(\boldsymbol{t})$, and of the additional benefit that it achieves by spending its net money holding, $B + R(\boldsymbol{t}) - C(t)$, elsewhere. That is,

Our unified approach: $\max_t [U(\boldsymbol{t}) + V(B + R(\boldsymbol{t}) - C(t))]$; F.O.C. :

$$\frac{\partial U(\boldsymbol{t})}{\partial t} = V'(B + R(\boldsymbol{t}) - C(t))(C'(t) - \frac{\partial R(\boldsymbol{t})}{\partial t}). \tag{1}$$

Looking at (1), note that V' measures the marginal utility of an extra unit of

[6] For simplicity, we assume that all the expected revenue can be invested in talent: there are no credit constraints. This assumption might be relevant for our negative result on revenue sharing.

money, and $C' - \frac{\partial R(\boldsymbol{t})}{\partial t}$ is the money an extra unit of talent costs the club. At the optimal choice, the product of these two values must, therefore, equal the marginal utility from an additional unit of talent.

It is immediate that, since $V(.)$ is increasing, when non-pecuniary effects are not present – $U(.) \equiv 0$ – the new formulation leads to the same solution as profit maximization. To recover "utility maximization", we would need to make tortuous assumptions on V to recreate the notion of a binding (in both directions) budget constraint (e.g. that V is zero for non-negative values but it is minus infinity for negative ones).

Condition (1) can be rewritten as

$$\frac{\partial R(\boldsymbol{t})}{\partial t} + \frac{\frac{\partial U(\boldsymbol{t})}{\partial t}}{V'(B + R(\boldsymbol{t}) - C(t))} = C'(t). \tag{2}$$

When the club maximizes profits, the optimal (interior) solution equates the marginal cost of one more unit of talent to the marginal revenue it brings to the club. In general, in our unified approach, the optimal solution equates the marginal cost of one more unit of talent to the marginal increase in the clubs' objectives. The left hand side of (2) can be viewed as a modified "revenue" function, one that includes not only the direct revenue effect of one more unit of talent, but also the (non-pecuniary) effect on the utility of members/owners, measured in money terms, where the exchange rate between money and utility is given by $V'(B + R(\boldsymbol{t}) - C(t))$.

If neither risk aversion nor wealth effects are significant, the slope of the indirect utility function, $V'(.)$, may be approximated with a constant in the relevant range.[7] We will maintain this assumption throughout the rest of the paper. It will allow us to work with a crisp model of wage determination and talent allocation that focuses on the interaction between clubs without having to disentangle these interactions from less informative income effects. We will then come back to these effects when we discuss revenue sharing in Section 5.

Despite allowing for the objectives of the club to include both utility and profit

[7] See Friedman and Sákovics (2015) for a detailed motivation and analysis of a similar model in a consumer choice context.

considerations, we will continue to refer to the primitive – with respect to its own talent – of the left hand side of (2), $Z(\boldsymbol{t}) := R(\boldsymbol{t}) + U(\boldsymbol{t})/V'$, as the "revenues."[8] Note that, given our formulation of $Z(\boldsymbol{t})$, not only is there no budget constraint (V' is capturing borrowing costs instead) but there is also no individual rationality constraint: $Z(\boldsymbol{t}) - C(t) = 0$ has no special economic meaning. The outside opportunities are embodied in the indirect utility function.

3 A simple model of the player market

On the supply side, we assume that there is a continuum of talent of measure T, available for hire at (or above) a reservation wage of r per unit.[9] In order to avoid technical difficulties arising from indivisibilities, we do not explicitly model athletes incorporating different measures of talent. Instead, we treat each infinitesimal unit of talent as a separate entity – that is, a "player" in our market game. The wage of an athlete embodying several units of talent can be calculated as the sum (integral) of the wages of each unit. As it turns out, each unit will be paid the same wage in equilibrium, so aggregation is easy.

The demand comes from two competing clubs,[10] whose payoffs depend on the distribution of acquired talent. In particular, if t_i units of talent are hired by Club i, it earns "revenue" $Z^i(t_i, t_{3-i})$, for $i = 1, 2$.[11] Then, the payoff functions are $Z^i(t_i, t_{3-i}) - C^i$, where C^i is Club i's wagebill (for simplicity, its only cost). We do not write $C^i(t_i)$ – not even $C^i(t_i, t_{3-i})$ – for the wagebill, to emphasize that the cost of hiring t_i units of talent is endogenous, even conditional on t_{3-i}, as it depends on

[8] Madden (2015a) proposes a similar functional form but without the microfoundations we provide.

[9] For simplicity, we assume that all players have the same reservation wage. It is straightforward to generalize to an increasing supply function. In fact, with reservation wage dispersion the incentives to bid for the same – the low r – players is even greater, see below.

[10] For clarity's sake we describe our framework for a two-team league. The generalization to more teams is conceptually straightforward.

[11] For ease of exposition, we assume that the revenue functions are twice differentiable in both arguments.

the clubs' bidding behavior in the player market. To retain generality and focus, we treat the relationship between talent distribution and revenues as a black box, and will simply make a few restrictions on the functional form of Z^i.

Before putting forward our assumptions on how the payoffs vary as a function of the final talent allocation, we describe how talent gets hired.

Each Club i, simultaneously, sets a – deterministic and Lebesgue measurable – wage schedule, $W_i(\tau)$, $\tau \in [0, T]$, specifying an individual wage offer to each[12] player. Players then accept the highest bid above their reserve wage that they have received – if any. Note that this is conceptually equivalent to each (infinitesimal) player holding a first-price auction with a reserve price of r. Importantly, the clubs are committed to honor all the offers they have made (if accepted).

When, in a hypothetical equilibrium, all the talent is contested, holding Club j's bids fixed, Club i's revenues can be written as a function of a single variable, its own talent level, t_i: $Z^i(t_i, T - t_i)$. This follows from the fact that any player Club i does not hire will play for Club j. Club i's willingness to pay for a marginal unit of talent is equal to its marginal revenue (c.f. (2)), given by $\frac{dZ^i(t_i,T-t_i)}{dt_i} = \frac{\partial Z^i(t_i,T-t_i)}{\partial t_i} - \frac{\partial Z^i(t_i,T-t_i)}{\partial t_{3-i}} = Z_1^i(t_i, T - t_i) - Z_2^i(t_i, T - t_i)$. We assume that this residual demand function is downward sloping (c.f. Remark 1, below).

When $Z_2^i < 0$, the residual demand exceeds Z_1^i, as the club internalizes the effect poaching a player from the opponent has on its revenues. Note that Z_2^i could be positive only if the competitive balance is so much tilted in favor of Club i that it would prefer its rival's talent level to rise. Since the competitive balance cannot possibly be tilted in favor of both clubs, we can safely assume:

Assumption 1 *If $Z_2^i(t_i, t_{3-i}) > 0$, then $Z_2^{3-i}(t_{3-i}, t_i) < 0$, $i = 1, 2$.*

When Club i considers bidding for an uncontested player, its marginal willingness to pay for an additional unit of talent, *given that t_{3-i} units of talent would currently*

[12] We require that each player receive an offer for mathematical simplicity. If a club wishes not to make an offer to some players, we model it as it offering them a wage below r.

be hired by the rival, is again equal to its marginal revenue, which then is simply $\frac{dZ^i(t_i,t_{3-i})}{dt_i} = Z_1^i(t_i, t_{3-i})$. We assume that this residual demand function is also downward sloping for all t_{3-i} and $t_i < T - t_{3-i}$.

Remark 1 *The fact that marginal revenue is strictly decreasing in t_i represents that the incentive to attract a player away from the rival is decreasing in the amount of talent the club has already hired. This assumption is standardly made in the literature for the entire support of t_i, just as we have done. There are two points worth discussion. First, our revenue function is a more complex object than usual, consisting of both a monetary (R) and a non-monetary (U/V') part. For their sum to be concave, a sufficient condition is that both of these functions are concave. Second, in practice it is likely that for small t_i marginal revenue is actually increasing (an issue first pointed out in Madden, 2010): when t_i is small (and all talent is employed) competitive balance is low, so the pie to divide is small, so – as the effect of an extra unit of talent on the winning probability is also small – the combined effect is small and therefore smaller than when competitive balance is high. A non-monotonic marginal revenue function will lead to a residual demand function with jumps. This is because for any given price the demand is always on the marginal revenue function but when there are multiple talent distributions leading to the same MR, there is one of them selected (the one maximizing $Z^i(t_i, T - t_i) - pt_i$ or $Z^i(t_i, t_{3-i}) - pt_i$). Thus, in the likely case that the MR curve is single-peaked, we would have a "minimum viable scale" (MVS), given by the talent level maximizing MR.*[13] *For no talent price will (residual) demand be positive but less than MVS_i.*

4 Characterization

We now turn to the analysis of the model. Our first result – proved in the Appendix – shows that despite the flexibility available to wage discriminate, in equilibrium

[13]Note that in our model it is the marginal, not the average revenue function, that determines the MVS. Apart from the conceptual difference, this also implies a lower MVS as the MR curve must already be decreasing at the peak of the AR curve.

not only does each club pay the same wage to all of its players,[14] but the wages paid by the two clubs equalize as well.

Proposition 1 *In equilibrium, (almost) all talent hired is paid the same wage per unit.*

An intuitive way of seeing this remarkable result is to note that each club must be willing to make the same offer – its marginal willingness to pay for attracting a player away from its competitor – to each player it is competitively bidding for, as the maximization problem, given the expected outcome in the other auctions, is exactly the same. Next, note that, in equilibrium, any uncontested player who is hired must be paid r. Therefore, there is necessarily competition for each player paid above r, taking us to a common wage across all.

4.1 Equilibrium

Because of the personalized nature of offers, the equilibria of our game involve a great deal of coordination. Unsurprisingly, this may lead to multiplicity. Therefore, to ensure that our comparative statics exercises are meaningful, we need to select a unique equilibrium. To this end, in our main proposition we characterize "the" market solution, assuming that whenever multiple equilibria exist, the clubs coordinate on the one with the lowest wage. Besides being a "focal extreme" of the set of equilbria that is always well defined, this selection has an additional, convenient feature: it is the only equilibrium that does not involve involuntarily unemployed[15] players. As we will see later, in any other equilibrium, some players who would strictly prefer to be employed at the "market" wage are not hired.

In order to ensure that the equilibrium selection is consistent across different revenue functions, we need to make a further restriction. The following assumption

[14] Recall, that the correct interpretation is that the wage per efficiency unit of talent is equalized.

[15] Recall, that not being hired by either club need not mean that the player is literally unemployed. For example, a basketball player not hired by the NBA might play in Europe (or a lower league, say, ABA 2000). Nonetheless, for ease of exposition we will label them as unemployed.

is sufficient.

Assumption 2 $Z^i_{12}(t_i, t_{3-i}) - Z^i_{22}(t_i, t_{3-i}) < 0$, *for all* (t_i, t_{3-i}) *and* $i = 1, 2$.

In words, we require that the marginal valuation for hiring a contested player be decreasing in the rival's talent level. As we will see, this guarantees that a decrease in the aggregate talent level leads to higher market wages.

Let $(t^*, T - t^*)$ be the "market clearing" talent distribution when all players are contested, defined as the talent distribution that equates inverse contested demands: the – by the assumed monotonicity – unique solution to $Z^1_1(t, T-t) - Z^1_2(t, T-t) = Z^2_1(T-t, t) - Z^2_2(T-t, t)$. Denote the corresponding wage by

$$w^* = Z^1_1(t^*, T - t^*) - Z^1_2(t^*, T - t^*). \tag{3}$$

Proposition 2 *Unless the reservation wage is prohibitively high, there exists an equilibrium with wage* $\max\{w^*, r\}$. *When* $w^* > r$, *this equilibrium leads to full employment and* $t_1 = t^*$, $t_2 = T - t^*$. *When* $w^* \leq r$, *the equilibrium talent allocation* (t_1, t_2) *is such that*

$$\max\left\{Z^i_1(t_i, t_{3-i}) - Z^i_2(t_i, t_{3-i}), Z^i_1(t_i, t_{3-i})\right\} = r, \; i = 1, 2, \tag{4}$$

whenever r *is sufficiently low so that a solution to (4) exists. Moreover, when Assumption 2 is satisfied, there exists no equlibrium with a lower wage.*

The outcome displayed by Proposition 2 (when $w^* > r$) is similar to the one often put forward in the literature ever since El-Hodiri and Quirk (1971). Our innovations are still fourfold: i) we extend the result to "utility maximization"; ii) we explicitly incorporate the external effects; iii) we extend to the important[16] case of $w^* \leq r$; and iv) we derive the outcome as the equilibrium of a strategic market game.

[16] Note that when talent supply is an increasing function with large/infinite support, the equilibrium level of aggregate talent is not a corner solution and wage is equal to the marginal reservation wage, making this the relevant scenario.

When $w^* > r$, in our equilibrium, both clubs offer the equilibrium wage – equalling their marginal revenue – to (almost) all players and the players accept each club's offer with the probability corresponding to the equilibrium proportion of talent hired by that club, t_i/T.[17] Since $w^* > r$ implies that the aggregate demand for contested players exceeds supply, there is no opportunity to hire any uncontested player and the equilibrium is characterized via the willingness to pay for contested players.[18] As all the players that are hired are contested, each club knows that if it lets a player go, this player will end up playing for the other team. It is *as if* there was a technological constraint requiring that trades can only happen between clubs. Thus, we have proved that Rottenberg/Coase were right:

Corollary 1 *When $w^* > r$,[19] the final allocation of talent will be the same as if clubs started with arbitrarily sharing the "ownership" of players, but they were allowed to frictionlessly trade among themselves.*

To see that there is no other equilibrium with a lower wage, note that the only other possibility would require that less talent is hired. However, by Assumption 2, the marginal revenue is increasing in the level of unemployment, leading to higher wages.

When $w^* \leq r$, two differences arise. First, it may no longer be possible to hire all the talent in equilibrium. Second, one of the clubs may prefer – and manage – to hire uncontested players. When only the first effect is active (both clubs have a higher

[17] Alternatively, different players could accept with different probabilities, as long as the aggregate probabilities of acceptance are t_i/T.

[18] Recall that this willingness to pay includes the value of attracting a player away from the rival and therefore the equilibrium wage is higher than the one normally interpreted from the literature, where the revenue functions have a single parameter (own talent level).

[19] Note that $w^* > r$, is only a sufficient condition. The only scenario where the equivalence breaks down is where there are uncontested players hired in equilibrium. For that to happen, we must have that the marginal benefit of hiring an unemployed worker is higher than attracting one away from the rival. That is, the concerns about aggregate revenue must outweigh the concerns about performance on the pitch. For example, when the revenues are shared in a non-performance-related manner. See Section 5.

contested than uncontested demand at the equilibrium values), the equilibrium is as before, except that not all players will receive an offer: $r = Z_1^1(t_1, t_2) - Z_2^1(t_1, t_2) = Z_1^2(t_2, t_1) - Z_2^2(t_2, t_1)$ and the players who receive offers accept Club i's one with probability $\frac{t_i}{t_1+t_2}$. When uncontested demand plays a role, the wage must equal r. We may have one club hiring uncontested players only, but also making offers to other players, who nonetheless choose to work for the rival club, which only hires contested players.[20]

As we have already mentioned, the outcome discussed in Proposition 2 need not be the unique equilibrium outcome.[21] First, under certain conditions, there also exist equilibria with "involuntary unemployment" where wages exceed r but some players are not hired (despite homogeneity of talent). These equilibria result from a coordination game – the clubs coordinate on who to make an offer to – and as such there are a continuum of them, whenever they exist. Second, when $w^* \leq r$, there may be more than one equilibrium, always with wage r, as the system of two equations for t_1 and t_2 in the second part of the proposition may have more than one solution.

As we have also anticipated, the equilibrium selection in Proposition 2 makes it possible to discuss an important comparative statics exercise: revenue sharing. We turn to that next.

5 Revenue sharing

One of the most debated questions with regard to the player market (c.f. Fort and Quirk, 1995) is whether teams with high revenues should be forced to share them with poorer teams – presumably – in order to increase the overall quality of

[20] Market sharing – i.e., each club targeting a different set ot players – cannot be equilibrium since that would require that R_2 be positive for both clubs, which contradicts Assumption 1.

[21] Though it is easy to find sufficient conditions for uniqueness. All that is needed is that the uncontested demand of one of the clubs (at wage r) exceed the amount of (contested) talent it is supposed to hire in any hypothetical equilibrium with unemployment. Thus, if $t^{**}(E)$ solves $Z_1^1(t, E-t) = Z_1^2(E-t, t)$, and we let $w^{**} = \min_E Z_1^1(t^{**}, T - t^{**})$, then $w^{**} > r$ is sufficient.

the league (taking into account competitive balance considerations). The resolution of the problem of optimal revenue sharing could also help in determining whether imposing the collective sale of TV rights – a procedure which makes redistribution much more practical – is a good idea.[22] We cannot provide a full answer in this paper, but we wish to highlight a few implications of our approach.

Let us denote the net revenues accrued to Club i after revenue sharing by S^i, and consider a simple revenue sharing scheme, where a proportion $1-\beta$ of each club's (monetary) revenues is transferred to the rival. That is, taking into account non-pecuniary – and therefore non-transferable – benefits:

$$\begin{aligned} S^1(t_1,t_2;\beta) &= \frac{U^1(t_1,t_2)}{V_1'} + \beta R^1(t_1,t_2) + (1-\beta)R^2(t_2,t_1), \\ S^2(t_2,t_1;\beta) &= \frac{U^2(t_2,t_1)}{V_2'} + \beta R^2(t_2,t_1) + (1-\beta)R^1(t_1,t_2). \end{aligned} \tag{5}$$

Note that $\beta = 1$ corresponds to no revenue sharing, while at the other extreme, $\beta = 1/2$ captures full sharing of the (expropriable) revenues. We may define the analogue of w^* when these are the new "revenue" functions, as

$$\begin{aligned} w^*(\beta) &= S_1^1(t_1^*, T-t_1^*;\beta) - S_2^1(t_1^*, T-t_1^*;\beta) = \\ & S_1^2(T-t_1^*, t_1^*;\beta) - S_2^2(T-t_1^*, t_1^*;\beta). \end{aligned} \tag{6}$$

The following irrelevance result generalizes Fort and Quirk (1995):

Proposition 3 *As long as $w^*(\beta) > r$ and everyone is hired, revenue sharing has no effect on the talent distribution, while it decreases the market wage:* $w^*(\beta) = w^*(1) - (1-\beta)\left(\frac{dR^1(t_1^*(\beta), T-t_1^*(\beta))}{dt_1} - \frac{dR^2(T-t_1^*(\beta), t_1^*(\beta))}{dt_1}\right)$.

When all talent is hired, the only effect of revenue sharing is to redistribute revenue from players to clubs. That revenue sharing does not affect the allocation of talent follows from the combined effect of two things. One is that it is the marginal revenues being equal that defines equilibrium. The other is the fact that in equilibrium all players are contested, implying that the marginal increase of one

[22] See Falconieri et al. (2004).

club's talent level leads to the same marginal decrease of that of the other club ($\frac{\partial t_i}{\partial t_{3-i}} = -1$). Together, these imply that the change in the marginal revenue of Club i as a result of receiving a transfer – e.g. $(1-\beta)R^{3-i}$ as in revenue sharing as defined above – is the marginal effect of increasing its talent level on the revenue transferred from the other team, which is exactly the same as -1 times the effect of increasing Club $3-i$'s talent level would have been on the same revenue:

$$\begin{aligned} \frac{dR^{3-i}}{dt_i} &= \frac{\partial R^{3-i}}{\partial t_i} + \frac{\partial R^{3-i}}{\partial t_{3-i}} \cdot \frac{\partial t_{3-i}}{\partial t_i} = \\ \frac{\partial R^{3-i}}{\partial t_i} - \frac{\partial R^{3-i}}{\partial t_{3-i}} &= -\left(\frac{\partial R^{3-i}}{\partial t_i} \cdot \frac{\partial t_i}{\partial t_{3-i}} + \frac{\partial R^{3-i}}{\partial t_{3-i}} \right) = -\frac{dR^{3-i}}{dt_{3-i}}. \end{aligned} \tag{7}$$

Thus, the transferred revenue has exactly the same (negative) effect on the marginal revenues of both the giving and the receiving team. Therefore, if the marginal revenues were equal to start with for a given talent distribution (in the equilibrium before the transfer), they will continue to be so following redistribution (so the same talent distribution still leads to equilibrium after the transfer).[23]

As the clubs' incentives to win and thus their willingness to pay for talent are unambiguously reduced by revenue-sharing, wages are lower the fuller the revenue sharing arrangement is.

When not all talent is hired, revenue sharing may also affect the aggregate amount of talent hired by the league. Since clubs are less willing to pay for talent – for a *given* total amount of talent in the league –, this should be expected to lead to lower demand for talent, and so less talent hired by the league. Needless to say, when the amount of talent that one club hires changes, this also changes the other club's willingness to pay for talent. Therefore, the talent-reducing effect does not dominate for all revenue functions.

[23] The reason why it has been claimed that the irrelevance result does not hold with "utility maximizing" clubs is that in those models demand is not determined by marginal revenue, but the average revenue. However, with average revenues the effects of a transfer would not be equal on both teams' demand functions as

$$\frac{R^{3-i}}{t_i} \neq -\frac{R^{3-i}}{t_{3-i}}.$$

The invariance result of Proposition 3 relies on the assumption of inelastic supply. When supply is elastic, the same forces that drive wages down with fixed supply are still operational and, as a result, the overall amount of talent hired in equilibrium decreases with revenue sharing. It is easy to show that under plausible conditions, similar to the equilibria with $w^*(\beta) \leq r$, both teams will hire the less talent the higher proportion of revenues are shared. However, competitive balance might go up or down, depending on the specifications of the revenue functions. Therefore, while it often does, revenue sharing does not necessarily reduce the "quality" of the league, as the effect of revenue sharing on competitive balance may compensate for the lower aggregate talent level. In order to be able to carry out appropriate welfare comparisons, we need to do a bit more work in the following section.

6 Benchmarking

In the previous section, we have obtained that revenue sharing will either not affect the talent distribution or – under plausible conditions – reduce the talent both clubs hire. Additionally, revenue sharing always has an effect on the distribution of revenues between clubs and players. In this section we investigate the (utilitarian) welfare consequences of these effects.

Our approach makes it possible to measure any non-pecuniary effects on members and club owners in terms of dollars, $\frac{U^i}{V'}$. Also, in partial equilibrium, the players' preferences are captured by their reservation wage. As a benchmark we posit that "viewers" and other customers purchase their league-related goods (TV broadcasts, stadium tickets, merchandise,...) at their valuation, or equivalently, the planner does not care about consumer surplus. This is sensible, as it is obvious that a large (weight on) the consumer surplus will lead to more talent hired at the social optimum. In sum, a useful welfare benchmark is the (dollar) sum of "revenues" and the reservation wages earned by non-employed players elsewhere.

Thus, we study the talent distribution $\left(\widehat{t}_1, \widehat{t}_2\right)$ that maximizes this total surplus,

$$Z^1(t_1, t_2) + Z^2(t_2, t_1) + r(T - t_2 - t_1). \tag{8}$$

We obtain crisp results when the following assumption is satisfied.

Assumption 3 *At the social optimum (without consumer surplus) talent allocation* $\left(\widehat{t}_1, \widehat{t}_2\right)$*, the aggregate externality of marginally increasing the rivals' talent level is negative:*[24] $Z_2^1\left(\widehat{t}_1, \widehat{t}_2\right) + Z_2^2\left(\widehat{t}_2, \widehat{t}_1\right) < 0.$

The derivatives of (8) with respect to t_1 and t_2 are

$$\begin{aligned} &Z_1^1(t_1, t_2) + Z_2^2(t_2, t_1) - r, \qquad (9)\\ &Z_2^1(t_1, t_2) + Z_1^2(t_2, t_1) - r. \end{aligned}$$

Consider the case that it is socially optimal that all players are hired. Then, the two expressions above are both non-negative. Next, note that an alternative way to characterize the solution in this case is

$$t_1 = \arg\max_t \left\{Z^1(t, T-t) + Z^2(T-t, t)\right\}.$$

The first-order condition for this problem can be written as $Z_1^1(t, T-t) - Z_2^1(t, T-t)$ $= Z_1^2(T-t, t) - Z_2^2(T-t, t)$. Taking into account that both expressions in (9) are non-negative, we obtain that

$$Z_1^1(t,T-t)-Z_2^1(t,T-t) = Z_1^2(T-t,t)-Z_2^2(T-t,t) \geq r-Z_2^2(T-t,t)-Z_2^1(t,T-t). \qquad (10)$$

Note that the left-hand side of the inequality is (3), the equation that characterizes the market equilibrium when $w^* \geq r$. The right-hand side of (10) is in fact larger than r if and only if $Z_2^1(t, T-t) + Z_2^2(T-t, t)$ is negative, that is, when Assumption 3 is satisfied. Thus, we have shown that:

Proposition 4 *Under Assumption 3, when it is efficient to hire all the talent (even without considering consumer surplus), the free market leads to the efficient allocation.*

[24] Assumption 1 already requires that both terms cannot be positive at the same talent distribution. Additionally, note that, while it is possible that – at very skewed levels of competitive balance – this aggregate externality is positive, it is practically impossible for it to be the case at the efficient talent distribution.

Thus, under the conditions of the proposition, no intervention in the market – like revenue sharing – can improve welfare.[25] As we mentioned before, the only effect is either a – (utilitarian) welfare neutral – redistribution of the proceeds, with no change in aggregate surplus, or a reduction in the latter, when revenue sharing is sufficiently strong as to reduce the amount of talent hired in the league.

Importantly, the market pressure for hiring talent can actually exceed the efficient level. This can be seen when the socially optimal talent level is less than T. Indeed, in this case we have that both expressions (9) are equal to zero at values $\widehat{t}_1, \widehat{t}_2$ such that $\widehat{t}_1 + \widehat{t}_2 < T$. Therefore, by Assumption 3

$$Z_1^1(\widehat{t}_1, \widehat{t}_2) - Z_2^1(\widehat{t}_1, \widehat{t}_2) = Z_1^2(\widehat{t}_2, \widehat{t}_1) - Z_2^2(\widehat{t}_2, \widehat{t}_1) = r - Z_2^2(\widehat{t}_2, \widehat{t}_1) - Z_2^1(\widehat{t}_1, \widehat{t}_2) > r,$$

so that, at the optimal talent distribution the willingness to pay of both clubs exceeds r. As each of the sides of the first equation above are continuous and downward sloping in own talent, this means that there exists a lower wage, w' and higher talent levels, (t'_1, t'_2) , where $Z_1^1(t'_1, \widehat{t}_2) - Z_2^1(t'_1, \widehat{t}_2) = Z_1^2(t'_2, \widehat{t}_1) - Z_2^2(t'_2, \widehat{t}_1) = w'$. Of course, this does not characterize an equilibrium. But a sufficient condition for the talent levels to increase is that they are strategic complements: an increase in the opponent's talent level increases the demand of any club – $Z_{i,3-i}^i - Z_{3-i,3-i}^i > 0$, $i = 1, 2$. In that case, incorporating the dependence of the marginal revenue curves on their second argument will increase them, leading to even higher talent levels (and wages).[26] This is what the next proposition states.

Proposition 5 *Under Assumption 3 and* $Z_{i,3-i}^i(t_i, t_{3-i}) - Z_{3-i,3-i}^i(t_i, t_{3-i}) > 0$ *for* $t_i \in [\widehat{t}_i, t'_i]$, $i = 1, 2$, *when the socially optimal talent level (without consumer surplus) is less than* T, *the free market leads to excess hiring.*

Then, revenue sharing may be beneficial, exactly because it can lower the talent level. An obvious situation where this happens is when $w^* > r$, but the efficient

[25] Note that this result does not depend on r being positive, so it also holds if the planner does not care about the unemployed.

[26] Of course this is very strong: it would be sufficient that the loss of talent level in the second step is less than the gain in the first one.

talent level is below T.[27] The intuition is that, in the unregulated equilibrium, clubs hire too much talent in order to gain a competitive edge over the rival club. It is the internalization of this externality that revenue sharing allows clubs to accomplish.

Alternatively, we could also think of the "excess" hiring implied by Proposition 5 as capturing (part of) the talent level enhancing effect of the ignored consumer surplus.

7 Conclusion

We have proposed a new modelling framework for the analysis of labor markets in professional sports. We have concentrated on three ingredients, the clubs' and the social planner's objectives and the microstructure of the market itself. There are a number of considerations that we have not addressed, despite their importance. One of them is that the result that initial ownership does not matter (Corollary 1) crucially depends on the Coasian nature of bargaining. If there are frictions, like switching costs[28] or asymmetric information, then they introduce a wedge which needs careful analysis (c.f. Burguet et al., 2002).

We have also abstracted away from credit market imperfections and asymmetries. To the extent that revenue sharing works in practice, it is likely to operate through the alleviation of those.

An additional concern could be raised with the incorporation of an increasing supply curve. The market would lead to the low reserve wage units of talent to be hired first. Would the clubs then coordinate on hiring the players with low talent? Reassuringly, the answer is no. High talent players are the ones with low per unit reseravtion wage. Just think of Lionel Messi (whose base salary is over $30 million):

[27] More generally, keeping the rival's talent level constant, revenue sharing would clearly decrease the marginal revenues (point by point). Talent levels being strategic complements in this range, would imply that incorporating the fact that the opponent's talent level has actually decreased, gives a further reason to decrease the talent hired. So the solution to (4) would occur at lower ts.

[28] Note that these are distinct from transfer fees, which are just the price.

his reservation wage per (efficiency) unit of talent is practically zero – the minimum wage divided by (nearly) infinity.

References

[1] Burguet, R., Caminal, R. and C. Matutes (2002) "Golden cages for showy birds: Optimal switching costs in labor contracts", *European Economic Review*, 46, 1153–1185.

[2] Burguet, R. and J. Sákovics (2016) "Bidding for input in oligopoly", *Edinburgh School of Economics Discussion Paper #266.*

[3] De Fraja, G. and J. Sákovics (2001) "Walras retrouvé: Decentralized trading mechanisms and the competitive price", *Journal of Political Economy*, 109(4), 842-863.

[4] El-Hodiri, M. and J. Quirk (1971) "An economic model of a professional sports league", *Journal of Political Economy*, 79, 1302-1319.

[5] Falconieri, S., Palomino, F. and J. Sákovics (2004) "Collective versus individual sale of television rights in league sports", *Journal of the European Economic Association*, 2(5), 833-862.

[6] Fort, R. and J. Quirk (1995) "Cross-subsidization, incentives and outcomes in professional team sports leagues", *Journal of Economic Literature*, 33, 1265-1299.

[7] Friedman, D. and J. Sákovics (2015) "Tractable consumer choice", *Theory and Decision*, 79(2), 333-358.

[8] Madden, P. (2015b) ""Walrasian fixed supply conjecture" versus "contest-Nash" solutions to sports league models: Game over?", *Journal of Sports Economics*, 16(5), 540-551.

[9] Madden, P. (2015a) ""Welfare economics of "financial fair play" in a sports league with benefactor owners", *Journal of Sports Economics*, 16(2), 159-184.

[10] Madden, P. (2011) "Game theoretic analysis of basic sports leagues", *Journal of Sports Economics*, 12, 407-431.

[11] Madden, P. (2010) "The regulation of a large sports league", University of Manchester Economics Discussion Paper EDP-1007.

[12] Neale, W.C. (1964) "The peculiar economics of professional sports: A contribution to the theory of the firm in sporting competition and in market competition", *Quarterly Journal of Economics*, 78, 1-14.

[13] Palomino, F. and J. Sákovics (2004) "Inter-league competition for talent vs. competitive balance", *International Journal of Industrial Organization*, 22, 783-797.

[14] Rosen S. and A. Sanderson (2001) "Labour markets in professional sports", *Economic Journal*, 111, F47-F68.

[15] Rottenberg, S. (1956) "The baseball players' labor market", *Journal of Political Economy*, 64, 242-258.

[16] Sloane, P.J. (1971) "The economics of professional football: The football club as a utility maximiser", *Scottish Journal of Political Economy*, 18(2), 121-146.

[17] Sport England (2013) "Economic value of sport in England", www.sportengland.org/research/benefits...sport/economic-value-of-sport/

Appendix

Proof of Proposition 1:

Proof. We proceed in two steps. First, we show that, in equilibrium, there are at most two wage levels paid; and if there are two, one of them must be the reservation wage.

Assume, by way of contradiction, that in equilibrium a measure $\alpha > 0$ of players accept offers in $[a, b]$ and a positive measure of players accept offers in $[c, d]$ for some $r < a \leq b < c \leq d$. Note that (almost) all players receiving these offers must be receiving the same offer from both clubs, as in the absence of competition for a player the club hiring the player will benefit from deviating and offer instead r. Take a club that hires a positive measure – $\beta > 0$ – of players for wage(s) in $[c, d]$. Similarly, denote by $\delta \in [0, \alpha]$ the measure of players this same club hires for wages in $[a, b]$. Assume first that $\delta < \alpha$, and $\beta > \alpha - \delta$. Suppose that our club deviates and increases by ε all its offers in $[a, b]$, and at the same time withdraws offers in $[c, d]$ so as to reduce by $\alpha - \delta$ the hires at those wages. Note that a set of offers that leads to this result can always be identified given the rival offers and each player's probability of acceptance. This deviation has no effect on the amount of players hired by the either club, but it reduces the wage bill by at least $(c - b)(\alpha - \delta) - \alpha\varepsilon$. Given b, c, δ, and α, we can always find an $\varepsilon > 0$ so that this reduction in the wage bill is indeed positive, and so the deviation profitable. Assume now that $\delta < \alpha$ and $\beta < \alpha - \delta$, and consider a similar deviation, where our club withdraws all its offers in $[c, d]$ and increases by ε a measure of its offers in $[a, b]$ so as to hire β more of these players. Again, note that a set of these offers can always be determined given the rival's offers and the player's strategies. This deviation has no effect on the amount of players hired by either club, but it represents a reduction in the wage bill of at least $(c - b)\beta - \alpha\varepsilon$. Again, we can always find an $\varepsilon > 0$ that makes this number positive. Finally, if $\delta = \alpha$, then we can consider the other club, and repeating the same argument find a profitable deviation for that club. This proves that, indeed, there could be at most one wage other than r paid to hired players in equilibrium.

Next, we show that there cannot be two different wages paid in equilibrium. We have shown that one of these wages has to be r. Moreover, repeating the argument in the previous paragraph, this also requires that one club's hires at r are uncontested – whereas all hires at the larger wage are contested –. That would imply that the club not hiring these players would not prefer to outbid the club that does hire them. But we know that its marginal valuation for contested players is at least the other wage paid and that it is strictly above r, so, by the continuity of the revenue function, this cannot be. ■

Proof of Proposition 2:

Proof. Suppose $w^* > r$, and suppose that Club 2 offers every player a wage w^*. Club 1's best response amounts to choosing how much talent t_1 to hire at wage w^* (or perhaps infinitesimally above w^*) letting the rest of talent $T - t_1$ go to the rival. The optimal choice satisfies the first-order condition $Z_1^1(t^*, T - t^*) - Z_2^1(t^*, T - t^*) = w^*$, that we already know that has a (unique) solution, confirming that we indeed have an equilibrium. Now consider any other pair of potential equilibrium strategies, such that all hired talent receive a – by Proposition 1 common – wage $w < w^*$. This cannot involve full employment: taking into account that $Z_1^i(t_i, T - t_i) - Z_2^i(t_i, T - t_i)$ is decreasing in t_i, for at least one of the two clubs, $Z_1^i(t_i, T - t_i) - Z_2^i(t_i, T - t_i) > w^*$, and so the club is better off slightly increasing the wage in some of its offers (so as to increase its talent and reduce the rival's). If total employment is reduced from T to E, then – by Assumption 2 – $Z_1^i(t_i, T - t_i) - Z_2^i(t_i, T - t_i) < Z_1^i(t_i, E - t_i) - Z_2^i(t_i, E - t_i)$, $i = 1, 2$. Since the equilibrium wage still has to equal marginal revenue, it must be higher than w^*.[29]

Now suppose that $w^* \leq r$ and there exist t_1, t_2, both strictly positive,[30] that satisfy $\max\{Z_1^i(t_i, t_{3-i}) - Z_2^i(t_i, t_{3-i}), Z_1^i(t_i, t_{3-i})\} = r$ for $i = 1, 2$. Suppose that $Z_2^i(t_i, t_{3-i}) \leq 0$ at this solution, for $i = 1, 2$. Consider the strategy profile where both clubs make employment offers at wage r to the same $t_1 + t_2$ players, and each of the players who receive offers accept Club i's one with probability $\frac{t_i}{t_1+t_2}$. This is an equilibrium: making offers – at wage at least r – for uncontested talent does not increase profits, and neither does making fewer offers or making offers with higher wages for contested talent increase profits. Now assume that $Z_2^1(t_1, t_2) > 0$ (and so $Z_2^2(t_2, t_1) < 0$ by Assumption 1), and consider the strategy profile where Club 1 uses the same strategy, Club 2 only makes offers for t_2 of that talent, and every player that receives two offers – all with wage r – accepts the one from Club 2. Once again, neither club – or players – profits from any deviation, and so indeed an equilibrium

[29] For completeness' sake: note that we cannot have uncontested offers accepted in equilibrium as the aggregate contested demand exceeds E.

[30] If $t_i = 0$, then in equilibrium t_{3-i} must also be zero, sin revenues are zero for any value of t_{3-i}.

with wage r exists. Finally, note that no equilibrium with (accepted) offers below r exists, since a player's acceptance of such offers is a strictly dominated strategy. So, the equilibrium with wage r is the one with lowest wage.

A sufficient condition for (4) to have a solution is that $r < \min_i [\max \{Z_1^i(0,T) - Z_2^i(0,T), Z_1^i(0,T)$ Indeed, when the condition is satisfied there always exists a solution in $[0,T]$ to the system $\max \{Z_1^i(t_i, t_{3-i}) - Z_2^i(t_i, t_{3-i}), Z_1^i(t_i, t_{3-i})\} = r$: each equation defines a continuous curve $t_i(t_{3-i})$ and $t_{3-i}(t_i)$ with support $[0,T]$ and image in $[0,T]$. Thus, the two curves cross. ■

Proof of Proposition 3:

Proof. Substituting the value of S^1 and S^2 into (6), we have

$$\begin{aligned} & \frac{1}{V_1'}\frac{dU^1(t_1, T-t_1)}{dt_1} + \beta\frac{dR^1(t_1, T-t_1)}{dt_1} + (1-\beta)\frac{dR^2(T-t_1, t_1)}{dt_1} \qquad (11) \\ = \; & -\frac{1}{V_2'}\frac{dU^2(T-t_1, t_1)}{dt_1} - \beta\frac{dR^2(T-t_1, t_1)}{dt_1} - (1-\beta)\frac{dR^1(t_1, T-t_1)}{dt_1} = w^*(\beta). \end{aligned}$$

Collecting terms,

$$\begin{aligned} & \frac{1}{V_1'}\frac{dU^1(t_1, T-t_1)}{dt_1} + \frac{dR^1(t_1, T-t_1)}{dt_1} = -\frac{1}{V_2'}\frac{dU^2(T-t_1, t_1)}{dt_1} - \frac{dR^2(T-t_1, t_1)}{dt_1} \qquad (12) \\ = \; & w^*(\beta) + (1-\beta)\left(\frac{dR^1(t_1, T-t_1)}{dt_1} - \frac{dR^2(T-t_1, t_1)}{dt_1}\right), \end{aligned}$$

and recalling that $Z_j^i(t_i, t_{3-i}) = \frac{1}{V_i'}\frac{\partial U^i(t_i, t_{3-i})}{\partial t_j} + \frac{\partial R^i(t_i, t_{3-i})}{\partial t_j}$, yields

$$Z_1^1(t_1, T-t_1) - Z_2^1(t_1, T-t_1) = Z_1^2(t_1, T-t_1) - Z_2^2(t_1, T-t_1), \qquad (13)$$

for any β. This equation is (3), so the equilibrium talent distribution, $(t^*, T-t^*)$ is unchanged by revenue sharing. That is, the first line of (12) is independent of β. The last equality in (12) then characterizes the equilibrium wage, $w^*(\beta)$. Finally note that, when $w^* > r$, $\frac{dR^1(t_1^*, T-t_1^*)}{dt_1} > 0$ and $\frac{dR^2(T-t_1^*, t_1^*)}{dt_1} < 0$, so $w^*(\beta)$ is indeed increasing. ■