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Use of conventional site investigation parameters to calculate critical velocity of trains from Rayleigh waves

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1 **CALCULATING CRITICAL VELOCITY OF TRAINS FROM RAYLEIGH WAVES**
2 **USING CONVENTIONAL SITE INVESTIGATION PARAMETERS**
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2 **ABSTRACT**
3 This paper presents a practice ready approach to calculating railway track critical velocity
4 from Rayleigh waves and ground borne vibrations using conventional site investigation
5 parameters. The different types of ground wave are discussed together with equations
6 defining the wave velocities mathematically.
7 Relationships between the terms in the equations defining Rayleigh Wave velocity
8 and thus track critical velocity are established – enabling the reader to undertake these
9 calculations from a standard low cost site investigation.
10 It is suggested that the design "train critical velocity" may be restricted to 0.7 x
11 Rayleigh Wace velocity. However it is reported that the "track critical velocity" is in the
12 region of 1.1-1.3 x Rayleigh Wave velocity.
13
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16 *Keywords:* Rayleigh waves, ground borne vibrations, conventional site investigation
17 parameters, train critical velocity, track critical velocity
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19
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21

1

INTRODUCTION
There is growing demand for increased capacity on the world's railways, both for passenger traffic and freight capacity. One way forward to achieve a substantial increase in capacity is to construct a separate right of way high speed line – as in China, Korea, Taiwan, Japan and Europe. These high speed lines are almost exclusively carrying passenger traffic. Then, the reduction of traffic on the classic routes leaves more flexibility to increase local commuting traffic and introduce freight traffic (usually at a much lower speed).

An alternative to stepwise and expensive provision of new major rights of way for high speed rail lines is to incrementally increase speed on existing railway routes. This strategy is satisfactory provided that the trains do not exceed the so called "train critical velocity". Train operators and track infrastructure owners and operators are thus faced with a number of questions that have proved to be very difficult to answer:

(1) How do we define critical velocity for a train?
(2) How do we define track critical velocity?
(3) How do we assess the critical velocity of a section of track?
(4) Which geotechnical investigation techniques are required?
(5) Can we use a classical standard low cost site investigation?

The objectives of the paper are to deliver practise ready outcomes to the above questions.

WAVEFORMS
Before discussing the above questions, we need to review waveforms.

There are three main types of waves generated by the passage of a train:
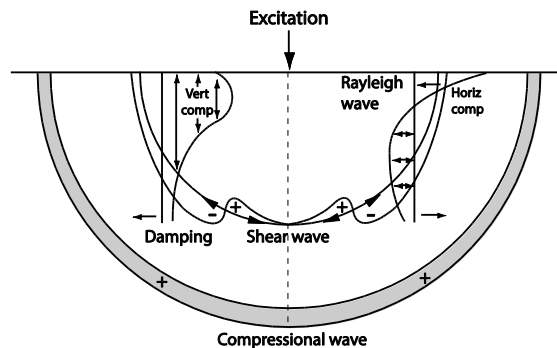
(a) Compression or P-waves
(b) Shear waves; and
(c) Rayleigh waves, often referred to as surface waves

Their relative positions and subjective amplitudes are illustrated in Figure 1 below.
.

Figure 1 – Waves generated by the passage of a train (slice view)

The equation for Compression waves (P-waves) is

$$V_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$ (1)

And for shear waves is:

$$V_s = \sqrt{\frac{\mu}{\rho}}$$ (2)

Where ρ is density, λ is bulk modulus and μ is the shear modulus (λ and μ are also known as Lame's parameters).

The relationship for Rayleigh waves is given below:

$$K = \frac{V_R}{V_S}$$ (3)

Where:

$$K = \frac{0.87 + 1.12\upsilon}{1 + \upsilon}$$

υ = Poisson's ratio
V_S = shear wave velocity
V_R = Rayleigh wave velocity

Although other types of waves are theoretically possible (e.g. Lamb waves in layers and Stoneley waves at interfaces), compressional, shear and Rayleigh waves are the most common. The importance of Rayleigh waves is that they transmit approximately two thirds of the total excitation energy from a passing train (Rayleigh waves $\approx 67\%$, shear waves $\approx 26\%$, compression waves $\approx 7\%$). Hence Rayleigh waves are most likely to cause vibration effects on both the railway track and nearby structures.

CRITICAL VELOCITY EFFECTS

Much has been written about critical velocities for trains – but relatively little information is available regarding a simple interpretation of the data. A relationship reported by Connolly [1] is given in Figure 2.
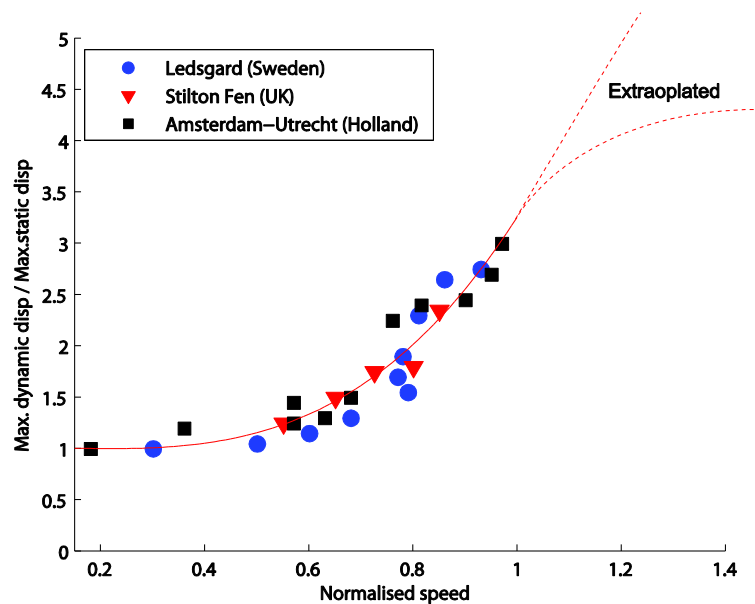
Figure 2 – Critical velocity effects [1]

This figure shows maximum vertical dynamic displacement of the railway track normalised against maximum static displacement – on the ordinate. On the abscissa it shows normalised train speed – i.e. train speed divided by the soil Rayleigh Wave velocity. In order to use this relationship, one must be able to predict the Rayleigh Wave velocity for the railway site in question. There is no simple in situ test available - the Rayleigh Wave velocity has to be calculated using the above parameters.

Two approaches to analysing train critical velocity exist:
(1) Using a 3-D fully coupled numerical model – which is expensive and very time consuming. If one were to use a 3-D model using a Finite Element package (e.g. Abaqus) on a super-computer, this might execute in 24 hours, but on a desk-top computer one would be looking at days or weeks to achieve the same outcome. The remaining weakness is the difficulty of estimating the soil input parameters for the model. Some recent work builds on the output from multiple FE analyses using Artificial Neural Networks (ANN) to give a fast analysis for a simplified set of models [2].
(2) An alternative and much simplified approach uses the classical equations given above.

Returning to the detail of Figure 2, when train speeds approach the underlying Rayleigh wave speed of the supporting soil large increases in track vibration may occur. Krylov [3] evaluated first 'critical velocity' related to the soil underlying Rayleigh wave velocity and second 'track critical velocity'. The latter is based on the minimal phase velocity of bending waves propagating in the in the track supported by the ballast. Krylov [4] presented an analysis that showed that the 'track critical velocity' is generally 10-30 per cent higher than the Rayleigh wave velocity.

Field experimental evidence of critical velocity effects has been collected on Swedish, UK and Dutch lines and is shown in Figure 2. It is clear that as the normalised speed (train velocity/Rayleigh wave velocity) increases towards a value of 1, the track displacement grows exponentially. Trains exceeding the Rayleigh wave velocity are referred to as 'trans-Rayleigh' trains. These can give rise to a so-called 'boom' and the creation of a Mach cone. This exponential growth is a function of the initial track displacement and therefore not always problematic. One has to be cautious and questioning when authors use analogies of a

1 sub-sonic airliner passing through the sound barrier – these may be over dramatizations? For
2 example, if track displacements are very low, then although an increase in train speed may
3 cause a threefold increase in vertical displacements, this value may still be below the
4 maximum safety threshold. Despite this, it is clear that under certain, and not fully
5 understood circumstances, that track deflections can become large (e.g. >10mm [5],[6]).
6   Accurate predictions from numerical modelling are difficult due to the complex
7 coupling between track and ground structure that contains many individual Rayleigh wave
8 speeds and resonant frequencies. Similarly, the train excitation also generates a wide
9 spectrum of excitation frequencies that increases problem complexity [7],[8].
10   There are many unanswered questions such as whether track deflections continue to
11 increase past the critical velocity. In practical terms many railway designers will attempt to
12 ensure that train velocity does not exceed 0.7 x Rayleigh wave velocity.
13
14 **NUMERICAL ANALYSIS INPUT DATA**
15 If one were to undertake a non-linear 3-D fully coupled Finite Element analysis, then one
16 would need to establish the following parameters:
17   Density – measured in a conventional SI - the mass divided by the unit volume of a
18 material- it typically increases with depth.
19
20   Poisson's ratio – When a material is compressed using a force in a single direction,
21 Poisson's ratio defines the degree to which the material expands in the other two directions.
22 This is the ratio of expansion to the contraction caused by the compression.
23   Increases of Poisson's ratio within a soil are often due to the presence of the water
24 table. This is particularly true for clays which when fully saturated become incompressible
25 (i.e. $\upsilon \approx 0.5$). In this case the P-wave speed increases dramatically because the wave speed
26 becomes more representative of the water rather than the soil. On the other hand the S-wave
27 velocity remains unchanged because water has no shear strength and thus the wave speed
28 remains representative of the soil. Changes in wave speed with respect to Poisson's ratio are
29 shown in Figure 3. It can be noticed that Poisson's ratio also has an effect on Rayleigh wave
30 speed. This effect is minor because the Rayleigh wave speed can never exceed the shear
31 wave speed. Therefore Rayleigh wave speed is usually located in the range of 85-95% of the
32 S-wave velocity.
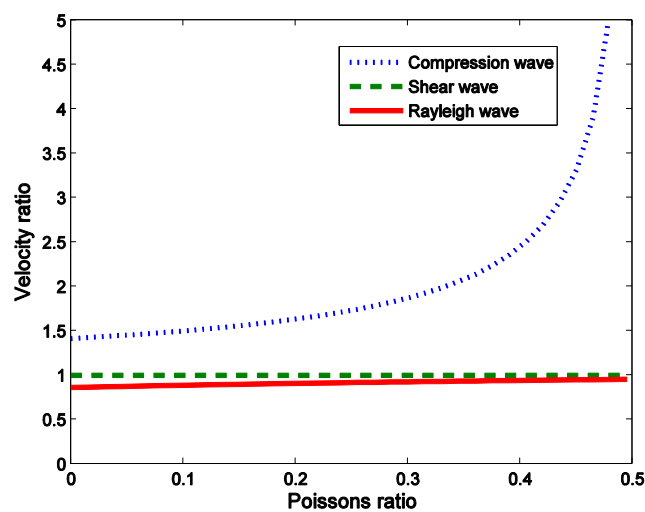
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34
35 Figure 3 - The effect of Poisson's ration on seismic wave speeds [1]
36

Connolly & Forde

1   Young's modulus – is a measure of the stiffness of a material. It is calculated using
2   the tangent modulus of the initial, linear portion of the stress-strain curve. As stiffness (of
3   both track and subgrade) is the main criteria used for quality control during construction,
4   Young's modulus is an influential parameter in the generation and propagation of railway
5   vibration.
6   At large strains soils behave non-linearly because shear modulus depends highly on
7   strain. Although large strains may occur in geotechnical engineering applications such as pile
8   driving, blasting or on off-shore oil rigs, in the case of ground vibration from railways, soil
9   particle deformation is typically very small in comparison to its dimensions. The magnitude
10 of strain experienced by the soil during train passage is therefore low (10-5 %) and can be
11 modelled using 'small strain' theory. This allows for the soil to be considered as a linear
12 elastic material and for the shear modulus to be considered to be equal to the 'maximum
13 shear modulus'.
14   Damping – A measure of the rate at which energy is reduced as it disperses and passes
15 through a material. The total damping ratio is composed of geometrical and material
16 damping and has a non-linear relationship with frequency. This frequency dependence
17 makes damping modelling more complex for time domain modelling in comparison to
18 frequency domain modelling. Regarding in-situ soils, material damping is typically greatest
19 in the upper layers and reduces with depth. This is because the soil particles in the upper
20 layers are less compacted, meaning the wave loses greater energy as it passes through the air
21 voids. Furthermore, if a soil is saturated then it may exhibit elevated viscous damping at high
22 frequencies. Regarding the track, damping is caused by the ballast and, if present, by a
23 combination of rail pads, under-sleeper pads and ballast mats.
24   Only one of the four key input parameters for a non-linear fully coupled finite element
25 numerical model discussed above would be measures in a conventional site investigation –
26 density.
27
28 **CONVENTIONAL SITE INVESTIGATIONS**
29 Table 1 lists some of the parameters needed to predict ground borne vibrations and their
30 context, where S.I. is Site Investigation:
31

Soil parameter	Soil type	Purpose or application	Obtained in traditional S.I.
Liquid limit	Clay	Classification	Yes
Plastic limit	Clay	Classification	Yes
Moisture content	Granular & clay	Classification	Yes
Density	Granular & clay	Classification + numerical analysis	Yes
Particle size distribution	Granular & clay	Classification	Yes
Shear strength	Granular & clay	Design: foundation & slope stability	Yes
Over-consolidation ratio	Granular & clay		No
Poisson's ratio	Granular & clay	Numerical analysis	No
Young's modulus	Granular & clay	Numerical analysis	No
Damping	Granular & clay	Numerical analysis	No

32   Table 1 – Material properties

For the numerical analysis of wave propagation four main material properties are required: Density, Poisson's ratio, Young's modulus and damping. Although more traditionally measured soil characteristics such as moisture content, particle size distribution, liquid and plastic limits, consolidation ratio, etc. affect material characteristics (and thus wave propagation), their effect is usually included in Density, Poisson's ratio, Young's modulus and damping.

The real challenge is that whilst a conventional site investigation measures Density, - it does not measure Poisson's ratio, Young's modulus and damping.

In order to relate traditional low cost Site Investigations to the parameters in the Rayleigh Wave velocity occasion given above, one can make use of Jamiolkowski et al (1979) [9] proposed a relationship between undrained shear strength c_u and Young's modulus, E

$$E = K_c . c_u \qquad (4)$$

Where the correlation factor K_c is within certain ranges as below, for an over consolidation ratio of between 1 and 2:

Plasticity Index, I_p (%)	Correlation factor, K_c
< 30	600 – 1,500
30 – 50	300 – 600
➢ 50	175

Table 2 (based on Jamiolkowski et al, 1979 [9])

If one assumes for design purposes that the "design critical velocity" is limited to 0.7 x Rayleigh wave velocity, then one can compute Table 3 below, based on saturated clays:

Soil	C_u (kN/m^2)	I_p	Kc	E (MPa)	Density (Mg/m^3)	Poisson's ratio	Rayleigh wave velocity (Km/h)	Design critical velocity (Km/h)
Soft	40	25	1,500	60	1.80	0.5	362	252
Stiff	400	40	500	200	2.13	0.5	608	426
Subgrade	85	25	1,500	128	2.10	0.5	488	342
Layer 1	85	25	1,500	128	1.60	0.5	559	392
Layer 2	150	40	500	75	2.00	0.5	384	269
Layer 3	400	60	175	70	2.00	0.5	371	259

Table 3: Calculations of "Design Critical Velocity"

One could argue that Table 3 is conservative by setting the "design critical velocity" at 0.7 x Rayleigh wave velocity? There are 2 interesting things that emerge from the analysis in Table 3:

- ➢ Higher plasticity soils could be more problematical than low plasticity soils. This aspect requires further evaluation.

- ➢ Further consideration needs to be given to soft/weak soils. Krylov [3] reported unexpectedly poor performance of the weak soils at Ledsgaard. This could be due a build up in positive pore water pressure in the saturated clay – giving rise to a reduction in the effective stress and consequently a short term reduction in the encountered undrained shear strength. The latter would then reduce the Rayleigh velocity and thus the "design critical velocity". This latter mechanism, well known in highway construction circles, has not been discussed in the railway environment.

Note that ground improvement techniques used for critical velocity mitigation are similar to the subgrade stiffening described for common vibration abatement, but placed beneath the track, rather than at soil locations outwith the track. The purpose of this is to increase the underlying Rayleigh wave speed. At Ledsgard (see Figure 2), lime/cement columns were placed to depths of between 7m and 13m below the track. This solution was found to significantly reduce the track deflections. Alternative solutions include stone columns, piles and the application of polyurethane,

CONCLUSIONS

A method of estimating Train Critical Velocity has been proposed, although it requires to be validated in the field. It has been demonstrated that Conventional Site Investigations do not yield data appropriate for input parameters for non-linear fully coupled finite element models of ground borne vibrations from high speed and classic railways.

However by employing empirical analyses by of Jamiolkowski et al, it is possible to estimate the input parameters for the Rayleigh Wave velocity from data obtained in a Conventional Site Investigation.

It is suggested a "train critical velocity" of 0.7 x Rayleigh Wave velocity may be a conservative starting point for route analysis. If the above statement is adopted than there will be no requirement to analyse "track critical velocity" as the latter is 10-30 per cent greater than the Rayleigh Wave velocity.

REFERENCES

[1] Connolly, D. P., Kouroussis, G,. Laghrouche, O., Ho, C. and M. C. Forde, Benchmarking railway vibrations – Track, vehicle, ground and building effects, *Construction and Building Materials*, Elsevier, in press, doi: 10.1016/j.conbuildmat.2014.07.042

[2] Connolly, D. P., Kouroussis, G,. Giannopoulos, A, Verlinden, O, Woodward, P. K. and M. C. Forde, Assessment of railway vibrations using an efficient scoping model, *Soil Dyn. Earthq. Eng*., 2014, 58 (March): 37-47.

[3] Krylov, V. V "Vibrational impact of high-speed trains. I. Effect of track dynamics," *J.Acoust. Soc. Am.*, vol. 100, no. 5, p. 3121, 1996.

[4] Krylov, V. V. (2001). *Noise and Vibration from High-Speed Trains*. (V. V Krylov, Ed.) (pp. 1–435). Thomas Telford Ltd. doi:10.1680/navfht.29637

[5] Krylov, V. (1995). Generation of ground vibrations by superfast trains. Applied Acoustics, 44(2), 149–164. doi:10.1016/0003-682X(95)91370-I

[6] Krylov, V., Dawson, A., Heelis, M., & Collop, A. (2000). Rail movement and ground waves caused by high-speed trains approaching track-soil critical velocities. Proc of the Inst of Mech Eng, Part F: Jnl of Rail and Rapid Transit, 214(2), 107–116. doi:10.1243/0954409001531379

[7] Connolly, D, Giannopoulos, A, Fan, W. Woodward, P. K. and M. C. Forde, Optimising low acoustic impedance back-fill material wave barrier dimensions to shield structures from ground borne high speed rail vibrations, *Constr. Build. Mater*., vol. 44, pp. 557–564, Jul. 201

[8] Kouroussis, G Connolly, D. P., Forde, M. C. and O. Verlinden, "Train speed calculation using ground vibrations", Proceedings of the Institution of Mechanical Engineers, Proc. IMechE Part F: Journal of Rail and Rapid Transit, DOI: 10.1177/0954409713515649.3.

[9] Jamiolkowski, M. Lanellotia, R. Marchetti, S., Nova, R. and E. Pasqualini, Design parameters for soft clays, in Proc. 7th European Conference: Soil Mechanics and Foundation Engineering, 1979, pp. 27–57.