THE UNIVERSITY *of* EDINBURGH

Edinburgh Research Explorer

Analysis of the Maximum Efficiency of Kite-Power Systems

Link:
Link to publication record in Edinburgh Research Explorer

Document Version:
Peer reviewed version

OPEN ACCESS

Analysis of the maximum efficiency of kite-power systems

S. Costello, C. Costello, G. François, and D. Bonvin

Articles you may be interested in
Coordinated operation of wind power and other resources considering power system requirements
J. Renewable Sustainable Energy **7**, 023121 (2015); 10.1063/1.4918287

Fast security risk assessment of the power system with wind power penetration
J. Renewable Sustainable Energy **6**, 053101 (2014); 10.1063/1.4893571

Analysis of a model of fuel cell - gas turbine hybrid power system for enhanced energy efficiency
AIP Conf. Proc. **1497**, 84 (2012); 10.1063/1.4766770

Towers for Offshore Wind Turbines
AIP Conf. Proc. **1225**, 475 (2010); 10.1063/1.3464894

Current methods for the dynamic analysis of horizontal axis wind turbines
AIP Conf. Proc. **135**, 472 (1985); 10.1063/1.35468

Analysis of the maximum efficiency of kite-power systems

S. Costello,[1,a)] C. Costello,[2,b)] G. François,[1,c)] and D. Bonvin[1,d)]

[1]*Laboratoire d'Automatique, Ecole Polytechnique Fédérale de Lausanne,*
CH-1015 Lausanne, Switzerland
[2]*Department of Mechanical Engineering, Institute of Technology Tallaght, Dublin, Ireland*

This paper analyzes the maximum power that a kite, or system of kites, can extract from the wind. First, a number of existing results on kite system efficiency are reviewed. The results that are generally applicable require significant simplifying assumptions, usually neglecting the effects of inertia and gravity. On the other hand, the more precise analyses are usually only applicable to a particular type of kite-power system. Second, a novel result is derived that relates the maximum power output of a kite system to the angle of the *average* aerodynamic force produced by the system. This result essentially requires no limiting assumptions, and as such it is generally applicable. As it considers average forces that must be balanced, inertial forces are implicitly accounted for. In order to derive practically useful results, the maximum power output is expressed in terms of the system overall strength-to-weight ratio, the tether angle, and the tether drag through an efficiency factor. The result is a simple analytic expression that can be used to calculate the maximum power-producing potential for a system of wings, flying either dynamically or statically, supported by a tether. As an example, the analysis is applied to two systems currently under development, namely, pumping-cycle generators and jet-stream wind power. © 2015 AIP Publishing LLC.
[http://dx.doi.org/10.1063/1.4931111]

I. INTRODUCTION

This paper develops a simple method to evaluate the efficiency of a projected kite-power system. The concept of using kites, or airplanes on tethers, to harness energy from the wind has received much attention in recent years. A number of startups are working to commercialize kite power. A good review of the developments in this area is given by Fagiano and Milanese[16] and Ahrens *et al.*[1] Kites are wings, just like the blades of a turbine, the wings of an airplane, or the sails of a yacht. The particularity of a kite is that it is connected to the ground using a flexible tether. The advantage of kites with respect to wind turbines is that the aerodynamic forces acting on the kite are transmitted directly to the ground by the tether. As a result, a large and costly tower is not required, and kites can access much higher altitudes than the wind turbines. Thus, in theory, kite-power systems could have significantly lower material costs than conventional wind turbines. In addition, kites can easily access altitudes of several hundred meters, where the wind is stronger and less turbulent.[32,33] Some very ambitious projects even aim to harness winds at altitudes of several kilometers, where the wind is predicted to be almost an order of magnitude stronger than near the surface.[28] A number of different concepts exist for exploiting the aerodynamic force acting on an airborne wing. Perhaps the most popular is the pumping-cycle approach, where the tether is used to drive a ground-based generator (see, for

a)Electronic mail: sean.c.costello@gmail.com
b)Electronic mail: colm.costello@ittdublin.ie
c)Also at Faculty of Engineering and Architecture, University of Applied Sciences and Arts, Western Switzerland. Electronic mail: gregory.francois@epfl.ch.
d)Electronic mail: dominique.bonvin@epfl.ch

    7, 053108-1

example, Ref. 25). Another promising technique, referred to as drag power, uses small turbines mounted on the kite itself to generate electricity.[34] More ambitious proposals include two interconnected kites,[18] several kites connected to a ground-based merry-go-round,[10] and large quad copters hovering at an angle to the wind.[28]

This paper is concerned with the efficiency of kite-power systems. As an emerging technology, kite power has yet to prove its economic merit. Practitioners working on a particular design concept must surmount numerous practical obstacles related to structures, generator systems, sensing, automatic control, and take-off and landing. The aim of this article is to develop a simple theory that can be used to perform a quick, early-stage efficiency analysis of kite-power systems. Notions of efficiency are extremely important in energy generation. Carnot efficiency is fundamental to the study of heat engines, while wind turbines cannot exceed Betz's limit.[7] An understanding of efficiency is based on an upper bound. The efficiency of a wind turbine is defined as the ratio of the derived mechanical power to the kinetic energy of the wind passing through the disc covered by the turbine blades. This is a very useful measure because the diameter of the turbine and its cost are closely related. Hence, the Betz efficiency of a turbine is one of the quantities that can be used to predict the return on investment. Unfortunately, the Betz limit cannot meaningfully be applied to kites as the area a kite moves in is generally very large and the kite will only remove a small fraction of the wind energy passing through that area; thus, a kite system would have a very low Betz efficiency.

Previous work on the efficiency of kite-power systems has usually focused on analyzing, and finding ways to maximize, the power output for a particular setup.[3,5,15,16,21] In a seminal paper, Loyd[24] derived expressions for the surprisingly high power that a tethered kite can produce flying in a crosswind direction. Much of the contemporary development is based on Loyd's analysis, and it is often assumed that Loyd derived an upper bound for the power a wing can produce. In fact, Loyd derived upper bounds for the power production for three particular configurations, the so-called "lift power," "drag power," and "simple kite" configurations. Many authors have extended Loyd's analysis of the lift-power configuration, studying the power a single kite flying crosswind can generate. Horn *et al.*,[19] Houska and Diehl[21,22] used numerical simulations to calculate the optimal path the kite should fly for power generation if the tether is attached to a generator in the pumping-cycle approach. Dadd *et al.*[12] and Williams *et al.*[36] studied the optimal path the kite should fly, and the resulting power extracted from the wind, when the tether is attached to a vehicle, such as a boat. Argatov and Silvennoinen[3] and Argatov *et al.*[5] used the assumption of force equilibrium, neglecting the kite inertia, to derive simple analytic expressions for the instantaneous power generated by a pumping-cycle generator during the traction phase. Importantly, this analysis shows that, for a single kite on a tether, the instantaneous power extracted from the wind is proportional to the *cube of the cosine of the angle between the wind and the force it exerts on the kite*. Both Schmehl *et al.*[30] and Luchsinger[25] performed similar analyses, with inertia taken into account, and used the results to analyze the effect of mass and elevation angle on the efficiency of a single kite on a tether. While most efficiency analysis has focused on a single kite on a tether, and is tailored to pumping-cycle generators, one could imagine flying a kite in many more configurations. For example, Fagiano *et al.*[17] used relatively simple models to calculate the optimized power output from a system of kites tethered to a rotating carousel that drives a generator, a system that is currently under development.[10,23] A system of electricity-producing railway cars driven by kites is also under development.[27] It has also been proposed that a system of two interconnected kites could be more efficient than single kites.[18,37] First, in order to encompass these more radical concepts, it is desirable to generalize Loyd's efficiency analysis to encompass systems of several kites. Second, for a system based on a single kite, it would be useful to be able to analyze the maximum efficiency of the system using simple formulae, without resorting to numerical simulations that consider factors such as the dynamic maneuvers performed by the kite. This paper aims to address these two points.

This is a continuation of the work by Diehl.[14] Diehl extended Loyd's analysis to derive a general upper bound for the power a kite can produce, regardless of the system it is attached to, pointing out that a misalignment between the wind direction and the aerodynamic force

exerted on the kite will result in what he called "cosine losses." The main contribution of this paper is to quantify these cosine losses. We show that the cosine losses can be evaluated by assuming the *average* forces on the kite are balanced. This is even true for a dynamically flying kite, *without* knowing the dynamic maneuvers the kite performs. The focus on average forces is significantly different from previous approaches, which assume that the instantaneous forces on the kite are balanced. The use of average forces allows us to keep the analysis generally applicable, yet nonetheless obtain a restrictive (and hence informative) upper bound on power generation.

Our analysis is based on two bounds. First, in Section III, we derive an upper bound, referred to as P_{max}, for the power any given wing may generate with a given wind speed (this is the bound derived by Diehl[14]). However, this bound has a limited use in analyzing a system as, just like Betz's limit for turbines, practical systems cannot reach it. For this reason, we derive a more restrictive bound, $\tilde{P}$, in Section IV; its application requires more information about the system and in return it is more accurate. Although deriving this second bound necessitates an abstract mathematical formulation, its potential becomes apparent in Section V. This bound is used to investigate the effect of the main parameters of a generic high-altitude wind-power system on its overall efficiency. This yields a simple analytic expression for the maximum derivable power, applicable to almost any airborne system of wings. The efficiency calculation is applied to two systems currently under development, namely, pumping-cycle generators[2,31] and very high altitude (jet-stream) wind power.[28]

II. PROBLEM FORMULATION

A. Aerodynamic power

Consider a wing moving in a 3-D space with velocity $\vec{V}_k$ as shown in Figure 1. The flow of the apparent wind over the wing, $\vec{V}_a$, results in a net aerodynamic force acting upon the wing, which we call $\vec{F}_{aero}$. The velocity of the apparent wind is given by $\vec{V}_a = \vec{V}_w - \vec{V}_k$, where $\vec{V}_w$ is the wind velocity. This aerodynamic force can be separated into two components: the drag $\vec{F}_D$, which acts in the direction of $\vec{V}_a$, and the lift $\vec{F}_L$, which acts perpendicular to $\vec{V}_a$,

$$|\vec{F}_D| = \frac{1}{2}\rho A C_D |\vec{V}_a|^2, \tag{2.1}$$

$$|\vec{F}_L| = \frac{1}{2}\rho A C_L |\vec{V}_a|^2, \tag{2.2}$$

where ρ is the air density, A is the area of the wing, and C_L and C_D are the wing-specific lift and drag coefficients, and $\vec{F}_{aero} = \vec{F}_D + \vec{F}_L$. We define *aerodynamic power* as

$$P_{aero} = \vec{V}_k \cdot \vec{F}_{aero}. \tag{2.3}$$

This is the instantaneous power being transferred to the kite by the aerodynamic force. This power can be used to raise/lower the potential or kinetic energy of the kite, or it can be used to do work on an external object, for example, pull a boat, drive a generator or even drag a turbine through the air. It is logical that, for a given wing and wind conditions, there will be an
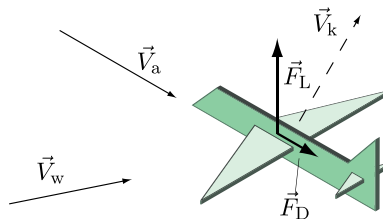

FIG. 1. A wing moving in 3-D space.

upper bound for P_{aero}; we will call this P_{max}. Regardless of the manner in which the aerodynamic power is harnessed, the average power produced (taking into account the net change in the system potential/kinetic energy) cannot exceed P_{max}. We will first derive a simple expression for P_{max}. Next, in Section IV we will derive the more restrictive bound $\tilde{P}$, which varies depending on the angle that the average aerodynamic force makes with the wind. This shows that most kite systems will derive considerably less power than P_{max}.

B. Loyd's analysis

As, for a given kite and wind speed, the aerodynamic forces are decided by $\vec{V}_{\text{k}}$, the expression for P_{aero} in Equation (2.3) can be maximized with respect to $\vec{V}_{\text{k}}$. This will give an upper bound for the power the wing can generate. It is also possible to include constraints in the formulation. Loyd solved this optimization problem with three different constraints:

(1) "Simple kite" configuration: the velocity of the wing is parallel with the total aerodynamic force, $\vec{V}_{\text{k}} \parallel \vec{F}$.
(2) "Lift power" configuration: the total aerodynamic force is parallel with the wind, $\vec{V}_{\text{w}} \parallel \vec{F}$.
(3) "Drag power" configuration: the velocity of the wing is perpendicular to the wind, $\vec{V}_{\text{k}} \perp \vec{V}_{\text{w}}$.

The reason Loyd addressed these three particular cases is because, from a practical viewpoint, they are the most intuitive configurations in which a kite would be used to generate power. His main result was that almost equally high powers can be generated in the "lift" and "drag" power configurations. These are certainly the most popular configurations under investigation today. Loyd only considered the three most likely kite-power configurations; his analysis cannot be applied to alternative schemes. For example, what is the maximum power output for the "lift power" and the "drag power" modes combined? In order to answer this question, we will now calculate the kite velocity that maximizes Equation (2.3) *without constraints*.

III. A GENERAL UPPER BOUND FOR INSTANTANEOUS POWER

We now re-derive the upper bound from Ref. 14. In doing so, we show that the maximum power a wing can generate occurs when it is operated in Loyd's "lift power" mode. We use the fact that P_{aero} can be shown to only depend on two parameters: the angle between the aerodynamic force and the wind, γ, and the velocity of the kite in the direction of the aerodynamic force, v_1.

Consider the kite shown in Figure 2. We decompose the apparent wind vector into a component that is aligned with, and a component that is perpendicular to, the aerodynamic force. The component of the apparent wind in the direction of $\vec{F}_{\text{aero}}$ is given by the difference between the component of the wind speed in that direction and the component of the kite's velocity in that direction, v_1,

$$v_{\parallel} = |\vec{V}_{\text{w}}| \cos \gamma - v_1. \tag{3.1}$$

v_2: kite velocity on plane $\perp$ to $\vec{F}_{\text{aero}}$

v_1: kite velocity in direction of $\vec{F}_{\text{aero}}$

γ: angle between $\vec{F}_{\text{aero}}$ and the wind

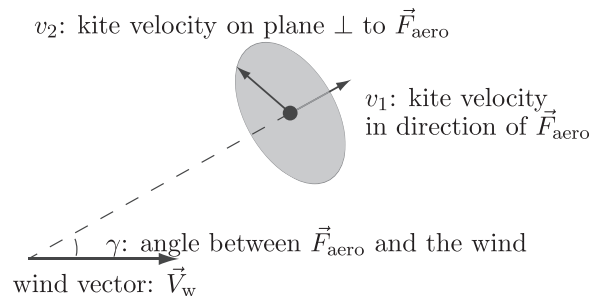wind vector: $\vec{V}_{\text{w}}$

FIG. 2. Decomposition of the kite velocity into the two perpendicular components v_1 and v_2.

The decomposed apparent wind vector and the resulting aerodynamic force vector are plotted in Figure 3, with $v_\perp$ denoting the component of the apparent wind vector that is perpendicular to $\vec{F}_{\text{aero}}$. It can be observed that

$$\frac{v_\perp}{v_\parallel} = \frac{\vec{F}_{\text{L}}}{\vec{F}_{\text{D}}} = \frac{C_{\text{L}}}{C_{\text{D}}}, \tag{3.2}$$

$$\Rightarrow \quad v_\perp = \frac{C_{\text{L}}}{C_{\text{D}}} v_\parallel . \tag{3.3}$$

This was the key observation made by both Argatov *et al.*[5] and Dadd *et al.*,[11] who derived this relationship in the context of a "zero-mass" kite model. Note that we make no assumption of zero-mass here. Also, it is important to note that v_2, the component of the kite's velocity that is perpendicular to the aerodynamic force, is *determined* by v_1 and γ. Although we do not need to calculate it for the following analysis, it will be such that $v_\perp$ satisfies Equation (3.3).

The magnitude of the apparent wind is given by

$$|\vec{V}_{\text{a}}|^2 = v_\perp^2 + v_\parallel^2 = v_\parallel^2 \left(1 + \left(\frac{C_{\text{L}}}{C_{\text{D}}} \right)^2 \right). \tag{3.4}$$

As the lift and drag forces are by definition perpendicular to each other, the magnitude of the total aerodynamic force is

$$|\vec{F}_{\text{aero}}| = \sqrt{|\vec{F}_{\text{L}}|^2 + |\vec{F}_{\text{D}}|^2}, \tag{3.5}$$

and using the expressions for the drag and lift forces from Equations (2.1) and (2.2) and inserting Equation (3.4) yield

$$|\vec{F}_{\text{aero}}| = \frac{1}{2} \rho A |\vec{V}_{\text{a}}|^2 \sqrt{C_{\text{L}}^2 + C_{\text{D}}^2}$$

$$= \frac{1}{2} \rho A N v_\parallel^2, \quad \text{with } N = C_{\text{D}} \left(1 + \left(\frac{C_{\text{L}}}{C_{\text{D}}} \right)^2 \right)^{\frac{3}{2}} \simeq C_{\text{L}} \left(\frac{C_{\text{L}}}{C_{\text{D}}} \right)^2 , \tag{3.6}$$

as $\left(\frac{C_{\text{L}}}{C_{\text{D}}} \right)^2 \gg 1$ in general. The maximal value of N corresponds to a particular orientation of the wing, upon which the lift and drag coefficients depend. Substituting in the expression (3.1) for $v_\parallel$, we obtain

$$|\vec{F}_{\text{aero}}| = \frac{1}{2} \rho A N \left(|\vec{V}_{\text{w}}| \cos \gamma - v_1 \right)^2 . \tag{3.7}$$
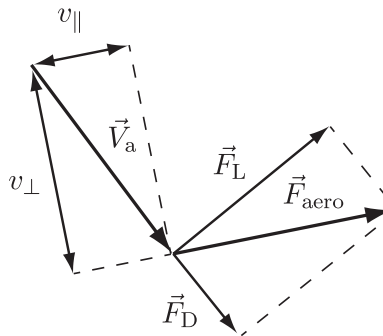

FIG. 3. The apparent wind and the resulting aerodynamic force.

We can now write the aerodynamic power in terms of v_1,

$$P_{\text{aero}} = |\vec{F}_{\text{aero}}|v_1 = \frac{1}{2}\rho AN \left(|\vec{V}_{\text{w}}|\cos\gamma - v_1\right)^2 v_1. \tag{3.8}$$

It is straightforward to maximize this expression with respect to v_1. For a given γ, the speed $v_1^* = \frac{1}{3}|\vec{V}_{\text{w}}|\cos\gamma$, which is the classic optimal reel-out speed when a kite is connected to a generator, yields the maximum power

$$P_{\text{aero}}^*(\gamma) = \left(\frac{1}{2}\rho AN\right)\frac{4}{27}|\vec{V}_{\text{w}}|^3\cos^3\gamma. \tag{3.9}$$

In turn, this power is maximized when $\gamma = 0$, yielding

$$P_{\text{max}} = \left(\frac{1}{2}\rho A|\vec{V}_{\text{w}}|^3\right)\zeta, \quad \text{with } \zeta = \frac{4N}{27}. \tag{3.10}$$

Note that the term in brackets is the power (kinetic energy per unit time) of the wind passing through a cross-sectional area A. ζ is the maximum *power harvesting factor*. $\zeta = 5$ is a typical value for both standard flexible power kites and for modern wind turbines.[14] However, more efficient rigid-wing kites, with greater $\frac{C_L}{C_D}$, could easily have power harvesting factors of about 30. For example, a Boeing 747 wing has $\zeta \simeq 34$. So far, the maximum reported power harvesting factor that has been achieved in practice is 8.[34] It is important to note that the power harvesting factor is not comparable to the power *coefficient* for wind turbines, which is always between 0 and 1, because the area used to calculate the reference power is the wing area, not the swept area! The power coefficient of a kite is a relatively meaningless quantity, due to the very large swept area of the kite. The simple analysis in this section implies that kites could potentially have far greater power harvesting factors than traditional wind turbines. Indeed, wind turbines do not make particularly efficient use of their "wing" (blade) area. This is because the portion of a wind turbine blade closest to the point of rotation is constrained to move at a fraction of the blade-tip speed, and hence experiences a low aerodynamic force.

It is worth emphasizing that P_{max} is the *absolute limit* for the amount of power that can be derived by a given wing at a given wind speed, regardless of the configuration of tethers or generators being used. Loyd also obtained this bound for the "lift power" configuration (in fact Houska[20] derived this exact expression, Loyd made some slight simplifications). So the "lift power" configuration is in fact optimal, which means that, in this configuration, a wing generates its maximum aerodynamic power. The difference is that here we derive the bound independently of the configuration.

IV. AN UPPER BOUND FOR THE AVERAGE POWER

This section presents a novel formula for the maximum *average* power that a kite can produce. The analysis is based on the average forces acting upon the kite. Logically, if the kite is not accelerating *on average*, the average external forces acting upon the kite must balance. The advantage of this approach is that it can take into account the inertial forces due to the dynamic maneuvers performed by the kite without requiring them to be explicitly calculated, as these are not external forces.

To harness the difference in speed between the ground and the air (wind), we must use the ground to push against the air through the intermediary of the wing. Conceptually, the wing creates resistance between the ground and the air, which slows down the air and some of the loss in kinetic energy is converted into aerodynamic power. Of primal importance is the force the ground exerts on the wing, this must be transmitted through a system of flexible tethers. If we could design a configuration such that this force is aligned with the wind, it would be easy to operate the wing optimally at all times. However, the only way to exert a force in line with the wind would be if the wing was flying just above the ground, and the whole point of kite

power is to harness winds at altitudes of at least several hundred meters. This imposes an angle between the restraining force and the wind. When considering a single weightless kite on a weightless tether, the aerodynamic force can reasonably be assumed to be aligned with the tether. In this case, it is apparent that the angle between the tether and the wind will decide the maximum aerodynamic power, thus Equation (3.9) can be applied to establish an upper bound on P_{aero}. However, the tether is no longer necessarily aligned with the aerodynamic force once factors such as tether drag, the mass of the kite, or the mass of the tether are taken into account. In this case, the angle of the instantaneous aerodynamic force may differ considerably from the tether angle, depending on the maneuvers performed. The analysis becomes even more complex if several kites are attached together, as has been proposed by Houska.[20] Argatov *et al.*[5] and Argatov and Silvennoinen[3] established that, for the case of a single kite attached to a generator, both the mean and the maximum achievable mechanical power depend on the cosine of the average tether angle *cubed*. We now derive a very simple upper bound for P_{aero} that can be applied to any system of kites, regardless of configuration or mass. As suggested by Argatov *et al.*,[5] it transpires that the cosine of the angle of the *average* aerodynamic force *cubed* determines the maximum power that can be derived from the wind.

Theorem 1. (Maximum Aerodynamic Power for a Force Angle)

Consider the period of time T, and let γ_0 be the angle that the average aerodynamic force $\frac{1}{T}\int_0^T \vec{F}_{aero}dt$ makes with the wind. The average aerodynamic power is upper bounded as follows:

$$\frac{1}{T}\int_0^T P_{aero}dt \leq P_{max}\cos^3\gamma_0 \quad \forall \, \gamma_0 \in \,]-\frac{\pi}{2},\frac{\pi}{2}[. \tag{4.1}$$

Proof. The idea is to show that, for a given γ_0, the average power is largest when both the direction of the aerodynamic force and the kite velocity in this direction are constant. To do so, we solve a path optimization problem. We use the spherical co-ordinate system shown in Figure 4; the zenith is aligned with the wind, γ is the polar angle, and θ is the azimuth angle.

In these co-ordinates, the aerodynamic force is

$$\vec{F}_{aero} = -r\sin(\gamma)\sin(\theta)\hat{x} + r\cos(\gamma)\hat{y} + r\sin(\gamma)\cos(\theta)\hat{z}, \quad \text{with} \quad r = |\vec{F}_{aero}|. \tag{4.2}$$

Let the average aerodynamic force point in the direction

$$\hat{n}_1 = \cos(\gamma_0)\hat{y} + \sin(\gamma_0)\hat{z}, \tag{4.3}$$

which forms an orthonormal basis along with

$$\hat{n}_2 = -\sin(\gamma_0)\hat{y} + \cos(\gamma_0)\hat{z} \quad \text{and} \quad \hat{n}_3 = \hat{x}. \tag{4.4}$$

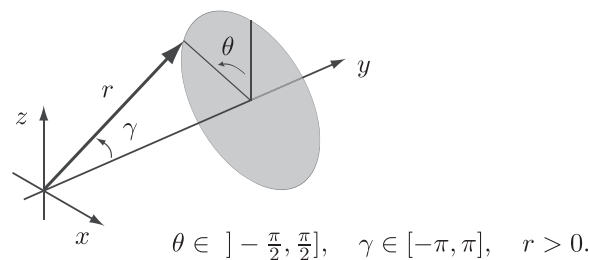The path optimization problem to be solved is

$$\theta \in \,]-\frac{\pi}{2},\frac{\pi}{2}], \quad \gamma \in [-\pi,\pi], \quad r > 0.$$

FIG. 4. Spherical co-ordinate system with the wind vector as the zenith. The wind is in the *y* direction.

$$\max_{v_1(t),\gamma(t),\theta(t)} \quad \int_0^T P_{\text{aero}}(v_1(t),\gamma(t))dt$$

$$\text{s.t. } \exists\, c \in \mathbb{R} \text{ satisfying} \quad \int_0^T \vec{F}_{\text{aero}}(v_1(t),\gamma(t),\theta(t))dt = c\,\hat{n}_1. \tag{4.5}$$

In the following, the argument t will generally be omitted, for example, $\gamma(t)$ will be referred to as γ. The expressions in this optimization problem can be simplified without modifying the problem itself. Normalizing v_1 with respect to $|\vec{V}_{\text{w}}|$ and removing in (3.7) and (3.8) the constant terms that do not affect the optimization problem, we can write the scaled aerodynamic power and force as

$$P(v,\gamma) = (\cos\gamma - v)^2 v, \quad |\vec{F}(v,\gamma,\theta)| = (\cos\gamma - v)^2, \quad \text{with } v = \frac{v_1}{|\vec{V}_{\text{w}}|}. \tag{4.6}$$

Note that the constraint in Problem 4.5 can be reformulated as

$$\dot{x} = f(u) = \begin{bmatrix} \vec{F}\hat{n}_2 \\ \vec{F}\hat{n}_3 \end{bmatrix} = (\cos\gamma - v)^2 \begin{bmatrix} \cos\theta\sin\gamma\cos\gamma_0 - \cos\gamma\sin\gamma_0 \\ -\sin\gamma\sin\theta \end{bmatrix}, \quad x(0) = 0. \tag{4.7}$$

Hence, f is the instantaneous force perpendicular to $\hat{n}_1$, expressed in the $\hat{n}_2,\hat{n}_3$ basis. Using the notation $u = [v,\gamma,\theta]^T$, the optimization problem can be re-written as

$$\max_{u(t)} \quad \int_0^T P(u)dt, \tag{4.8}$$

$$\text{s.t.} \quad \dot{x} = f(u), \quad x(0) = 0$$
$$\psi(x(T)) = x(T) = 0. \tag{4.9}$$

This is an optimal control problem that can be solved using Pontryagins Maximum Principle (PMP).[9] We form the Hamiltonian

$$H = -P + \lambda^T f \quad \text{with } \dot{\lambda}^T = -\frac{\partial H}{\partial x}, \quad \lambda^T(T) = \nu^T \frac{\partial \psi}{\partial x(T)}, \quad \nu \in \mathbb{R}^2. \tag{4.10}$$

PMP states that a necessary condition for $u^*(t)$ to be a maximizer is that $\exists\,\nu$ s.t. H is maximized by $u^*(t)\,\forall\,t$. This implies that $\frac{\partial H}{\partial u} = \left[\frac{\partial H}{\partial v}\,\frac{\partial H}{\partial \gamma}\,\frac{\partial H}{\partial \theta}\right] = [000]$, $\forall\,t$. All possible maximizers are found by solving this equation. Notice that H does not depend on x, and thus $\dot{\lambda} = 0$; also $\frac{\partial \psi}{\partial x} = I_2$ and so $\lambda(t) = \nu$, i.e., $\lambda \in \mathbb{R}^2$ is constant. The full expression for the Hamiltonian is

$$H = (\cos\gamma - v)^2(\lambda_1(\cos\theta\sin\gamma\cos\gamma_0 - \cos\gamma\sin\gamma_0) - \lambda_2\sin\gamma\sin\theta - v). \tag{4.11}$$

Let us first consider the equation $\frac{\partial H}{\partial \theta} = 0$,

$$\frac{\partial H}{\partial \theta} = -(\cos\gamma - v)^2\sin\gamma(\lambda_2\cos\theta + \lambda_1\cos\gamma_0\sin\theta) = 0. \tag{4.12}$$

We discard the solution $v = \cos\gamma$ as this yields $P = 0$ and $F = 0$. This is clearly not part of a maximizing solution because other values of u, e.g., $u = \left[\frac{1}{3}\cos\gamma_0,\gamma_0,0\right]$ yield $P > 0$ and $F = 0$. Therefore,

$$\text{either } \sin\gamma = 0 \quad \text{or} \quad \tan\theta = -\frac{\lambda_2}{\lambda_1\cos\gamma_0}. \tag{4.13}$$

These two conditions combined mean that $\vec{F}$ must lie in one plane for all t (θ is constant except at the singular point $\gamma = 0$). As $\hat{n}_1$, the required direction of the average $\vec{F}$, must lie in this plane in order to satisfy the constraint in Problem 4.5, we can conclude that this plane is characterized by $\theta = 0$, $\forall t$. Consequently $f_2 = 0$, $\forall t$,

$$f(u) = (\cos \gamma - v)^2 \begin{bmatrix} \sin(\gamma - \gamma_0) \\ 0 \end{bmatrix},$$ (4.14)

and the Hamiltonian becomes

$$H = (\cos \gamma - v)^2 (\lambda_1 \sin(\gamma - \gamma_0) - v).$$ (4.15)

The condition $\frac{\partial H}{\partial v} = 0$ gives

$$v = \frac{1}{3} \left(\cos \gamma + 2\lambda_1 \sin(\gamma - \gamma_0) \right).$$ (4.16)

Combining this with $\frac{\partial H}{\partial \gamma} = 0$ and solving for λ_1 gives

$$\lambda_1 = \frac{\cos \gamma}{\sin(\gamma - \gamma_0)} \quad \text{or} \quad \lambda_1 = -\frac{\sin \gamma}{\cos(\gamma - \gamma_0)}.$$ (4.17)

Note that the first solution implies that $v = \cos \gamma$ so, once again, we discard this possibility. Therefore, the second possibility must hold $\forall t$. At this point we must consider the values of γ that can occur during an optimal profile. If a profile is optimal, for a given value f_1 (4.14) gives

$$v(\gamma, f_1) = \cos \gamma - \sqrt{\frac{f_1}{\sin(\gamma - \gamma_0)}},$$ (4.18)

$$\text{and so} \quad P(\gamma, f_1) = \frac{f_1}{\sin(\gamma - \gamma_0)} \left(\cos \gamma - \sqrt{\frac{f_1}{\sin(\gamma - \gamma_0)}} \right).$$ (4.19)

For $\gamma_0 \in \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[$, it is straightforward to show that

$$\cos \left(\gamma_0 \pm \frac{\pi}{2} \pm \delta \right) \leq \cos \left(\gamma_0 \pm \frac{\pi}{2} \mp \delta \right) \qquad \forall \delta \in \left] 0, \frac{\pi}{2} \right[.$$ (4.20)

Using this, and noting that $\sin(\gamma - \gamma_0)$ is symmetric about $\gamma_0 \pm \frac{\pi}{2}$, we can conclude that $\forall \gamma_1 \in \left] \gamma_0 + \frac{\pi}{2}, \gamma_0 - \frac{\pi}{2} \right[$, there exists a $\gamma_2 \in \left[\gamma_0 - \frac{\pi}{2}, \gamma_0 + \frac{\pi}{2} \right]$ s.t. $P^*(\gamma_2, f_1) > P^*(\gamma_1, f_1)$. Hence, in the search for a maximum, we can restrict our attention to the interval

$$\gamma \in \left[\gamma_0 - \frac{\pi}{2}, \gamma_0 + \frac{\pi}{2} \right].$$ (4.21)

Now, returning to the main argument, from (4.17), λ_1 is given by

$$\lambda_1 = -\frac{\sin \gamma}{\cos(\gamma - \gamma_0)} \quad \text{and} \quad \dot{\lambda}_1 = -\dot{\gamma} \frac{\cos \gamma_0}{\cos^2(\gamma - \gamma_0)}.$$ (4.22)

We already determined that $\dot{\lambda}_1 = 0$, thus either $\dot{\gamma} = 0$, or $\gamma_0 = \pm \frac{\pi}{2}$ (which we exclude as it lies outside the range for γ_0 that we consider in this proof). The only constant γ satisfying the constraint in 4.5 is $\gamma = \gamma_0$ (if the angle of the aerodynamic force is constant, and the angle of the average aerodynamic force is γ_0, the aerodynamic force must always have angle γ_0). This produces the optimal aerodynamic power

$$\tilde{P}(\gamma_0) = P_{\max} \cos^3 \gamma_0. \tag{4.23}$$

Hence, any other trajectory with the same direction of average aerodynamic force γ_0 will produce less power. □

A few remarks can be made about this Theorem. First, note that N may be varied, for example, by modifying the angle of attack, or reducing the surface area of the kite. However, as it affects $\vec{F}_{\text{aero}}$ and P_{aero} proportionally, this does not affect the theorem. The maximal value of N should be used when calculating the upper bound. This will correspond to a particular angle of attack. Second, any deviation of the aerodynamic force from its average direction decreases the average aerodynamic power. Such deviations typically occur when turning the kite during dynamic flight, as the aerodynamic force vector must provide the centripetal force for the kite to turn. A detailed analysis of the effect of instantaneous inertial forces on efficiency was performed by Schmehl *et al.*[30] and Luchsinger.[25] Third, the theorem can easily be extended to a system of kites by considering the sum of their power-producing potentials as expressed in the following corollary.

Corollary 1. (Maximum Power for Multiple Wings) *Consider a system of n_{w} wings, each with area A_i and power harvesting factor ζ_i, $i = 1, \ldots, n_{\text{w}}$. Let γ_0 be the angle the average total aerodynamic force makes with the wind. Then, over the period of time T, the average aerodynamic power generated by the system is upper bounded as follows:*

$$\frac{1}{T} \int_0^T P_{\text{aero}} dt \leq \tilde{P}(\gamma_0), \tag{4.24}$$

where $\tilde{P}$ is calculated, as for a single wing, from (3.10) and (4.23) using the overall wing parameters

$$A = \sum_{i=1}^{n_{\text{w}}} A_i, \tag{4.25}$$

$$\zeta = \frac{\displaystyle\sum_{i=1}^{n_{\text{w}}} A_i \zeta_i}{A}. \tag{4.26}$$

Proof. A proof by contradiction can be constructed using Theorem 1. Let the aerodynamic force angle for each kite be $\gamma_i(t)$ and its velocity in this direction $v_i(t)$. Assume that, for a unit of time, the total aerodynamic power violates the bound, that is,

$$\frac{1}{T} \sum_{i=1}^{n_{\text{w}}} \int_0^T P_{\text{aero},i} dt > \left(\frac{1}{2} \rho |\vec{V}_{\text{w}}|^3 \right) A \zeta \cos^3 \gamma_0, \tag{4.27}$$

where γ_0 is the direction of the sum of the average aerodynamic forces $\frac{1}{T} \sum_{i=1}^{n_{\text{w}}} \int_0^T F_{\text{aero},i} \, dt$. Then, the same average power and the same average aerodynamic force can be produced using one wing with power harvesting factor ζ and area A by operating with $\gamma_i(t \frac{T}{T_i})$ and $v_i(t \frac{T}{T_i})$ for amounts of time $T_i = \frac{A_i \zeta_i}{A \zeta}$ for the total time $\sum_{i=1}^{n_{\text{w}}} T_i = 1$. The average power generated is then

$$\int_0^1 P_{\text{aero}} dt = \sum_{i=1}^{n_{\text{w}}} T_i \frac{A \zeta}{A_i \zeta_i} \left(\frac{1}{T} \int_0^T P_{\text{aero},i} \, dt \right), \tag{4.28}$$

which by (4.27) is greater than

$$\left(\frac{1}{2} \rho |\vec{V}_{\text{w}}|^3 \right) A \zeta \cos^3 \gamma_0. \tag{4.29}$$

Since by Theorem 1 this is impossible, statement (4.27) is false. □

V. A PRACTICAL UPPER BOUND FOR A GENERIC KITE SYSTEM

A. Effect of system parameters on efficiency

The upper bound for instantaneous power P_{max} (Equation (3.10)) gives the maximum power a wing with given properties can generate at a given wind speed. The bound on average power is more restrictive, i.e., $\tilde{P} \leq P_{max}$. This bound implies that, for a practical kite-power system, the power produced by the wing(s) is likely to be much lower than P_{max}. This is because, in general, the direction of the restraining forces (transmitted using tethers) acting on the wing(s) is not aligned with the wind. An intuitive explanation is that the kinetic energy of an object (in this case the air) can only be fully removed by exerting a force directly opposing its motion. If the force is misaligned with the direction of motion, only a fraction of this energy can be removed. This can be quantified by a decrease in the upper bound, or in other words, a loss of efficiency. This section quantifies how much the key parameters for a kite system affect efficiency. Linking these parameters to the angle of the average aerodynamic force γ_0 allows us to apply Theorem 1. As we are dealing with the *average* aerodynamic force, the result can be applied to both statically and dynamically flying systems of kites.

We will consider the generic airborne wind-power system shown in Figure 5. The system is composed of two parts: a static tether and a "kite system." The part designed to remove useful energy from the wind, i.e., the kite system, may be any combination of wings, tethers, weights, or balloons. They may be static or in motion. Note that the static tether can be removed if not present in the system being analyzed. The parameters required for the following analysis are: the aerodynamic characteristics of the wings and the tether, the magnitude of the time-averaged aerodynamic force produced by the kite system $|\vec{F}_{aero}|$, the angle of the time-averaged force the tether exerts on the ground ϕ_{TG}, the total mass of all airborne components M, and the static tether drag $\vec{F}_{drag}$. In the following the tether drag is assumed to be horizontal. If the vertical component of the tether drag is significant (which is unlikely), it can be added to the total weight. These basic parameters will generally be known for a projected kite-power system. They may be to some extent chosen during the design phase. The manner in which they are interrelated has been studied by Argatov and Silvennoinen,[4] including an interesting study of their affect on the efficiency of a pumping-cycle generator. As the examples will show, even if the system parameters are not exactly known, good approximate values should be available. For example, ϕ_{TG} can be approximated by the average ground tether angle in most cases. Based on these parameters, we now derive an efficiency factor e that, when multiplied by P_{max}, gives the more accurate $\tilde{P}$ upper bound for the system.

The average aerodynamic force must balance the weight of all airborne components, Mg, and the tension in the tether at the ground (assuming that on average the system is not accelerating)

$$|\vec{F}_{aero}| \begin{bmatrix} \cos \gamma_0 \\ \sin \gamma_0 \end{bmatrix} - |\vec{F}_{TG}| \begin{bmatrix} \cos \phi_{TG} \\ \sin \phi_{TG} \end{bmatrix} + \begin{bmatrix} |\vec{F}_{drag}| \\ Mg \end{bmatrix} = 0 \,. \tag{5.1}$$
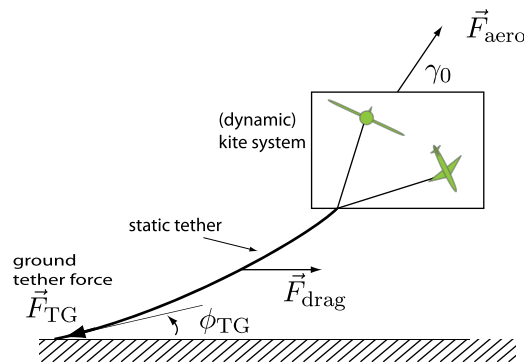

FIG. 5. Generic airborne wind-power system.

Multiplying this equation by $[\sin\phi_{TG} - \cos\phi_{TG}]$ gives

$$|\vec{F}_{\text{aero}}|(\cos\gamma_0\sin\phi_{TG} - \sin\gamma_0\cos\phi_{TG}) + |\vec{F}_{\text{drag}}|\sin\phi_{TG} + Mg\cos\phi_{TG} = 0$$

$$\Rightarrow\quad \sin(\gamma_0 - \phi_{TG}) = \frac{|\vec{F}_{\text{drag}}|}{|\vec{F}_{\text{aero}}|}\sin\phi_{TG} - \frac{Mg}{|\vec{F}_{\text{aero}}|}\cos\phi_{TG}. \tag{5.2}$$

As $\tilde{P}(\gamma_0) = \cos^3(\gamma_0)P_{\max}$, it follows that:

$$e := \frac{\tilde{P}(\gamma_0)}{P_{\max}} = \cos^3\left(\phi_{TG} + \sin^{-1}\left(\frac{|\vec{F}_{\text{drag}}|}{|\vec{F}_{\text{aero}}|}\sin\phi_{TG} + \frac{Mg}{|\vec{F}_{\text{aero}}|}\cos\phi_{TG}\right)\right). \tag{5.3}$$

The maximum power the generic kite system can generate is therefore

$$\tilde{P} = eA\zeta\left(\frac{1}{2}\rho|\vec{V}_{\text{w}}|^3\right), \tag{5.4}$$

where A and ζ can be calculated from the wings characteristics using (4.25) and (4.26). Note that effects such as *dynamic* tether drag should already be accounted for when calculating ζ. Houska and Diehl[21] derive a simple formula for incorporating dynamic tether drag into the kite lift-to-drag ratio, yielding an "effective" lift-to-drag ratio. This analysis has been considerably extended by Argatov *et al.*,[6] which allows the effect of both tether drag and tether sag on the lift-to-drag ratio to be accounted for during dynamic flight. In Sec. VI, this efficiency factor will be applied to practical systems, but we first make some observations about the expression for e:

- $|\vec{F}_{\text{aero}}|$ will increase with wind speed, yet M remains constant. This means the efficiency factor will improve as the wind speed increases. However, the maximum load will be attained at the rated wind speed, and $|\vec{F}_{\text{aero}}|$ can increase no further. Hence, the minimum (best, in terms of efficiency) value of $\frac{Mg}{|\vec{F}_{\text{aero}}|}$ is determined by the system strength-to-weight ratio, i.e., the ratio between the maximum aerodynamic force the system will tolerate and the mass of all airborne components.
- Decreasing ϕ_{TG} will improve efficiency, if all other terms remain unchanged. However, decreasing ϕ_{TG} means a longer tether is required to access winds at a given height. A longer tether weighs more, and will generate more drag.
- The influence of the static tether drag is modulated by $\sin\phi_{TG}$. As ϕ_{TG} decreases, so does the influence of tether drag on the efficiency factor.

Hence, to maximize efficiency, the system strength-to-weight ratio should be maximized and a compromise must be found between having a low ground tether angle, and minimizing tether weight and tether drag. Figures 6–8 plot the efficiency factor vs. $|\vec{F}_{\text{drag}}|/|\vec{F}_{\text{aero}}|$ and $Mg/|\vec{F}_{\text{aero}}|$ for three different values of ϕ_{TG}. Examples of wing weight-to-strength ratios are 0.01 for a surf kite,[11,35] 0.06 for a Boeing 747 with no payload,[8] and 0.33 for a DG-808C glider.[13] It can be seen that ϕ_{TG} strongly influences the efficiency, an angle of 45° resulting in a maximum efficiency of about 30%!

B. Example: Computing bounds for kite-power systems

We now apply the preceding analysis to two systems currently being developed by companies. The parameters for the Skysails and Ampyx systems are based on the data provided by these companies.

1. Rigid wings vs. flexible wings for pumping-cycle generators

The pumping cycle was first suggested by Loyd.[24] Several companies are currently developing this concept; here we study the systems proposed by Ampyx[2] and Skysails.[31] Both
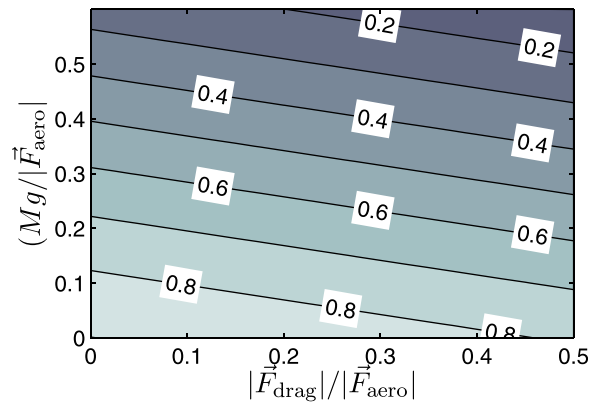
FIG. 6. The efficiency factor e for $\phi_{TG} = 15°$.
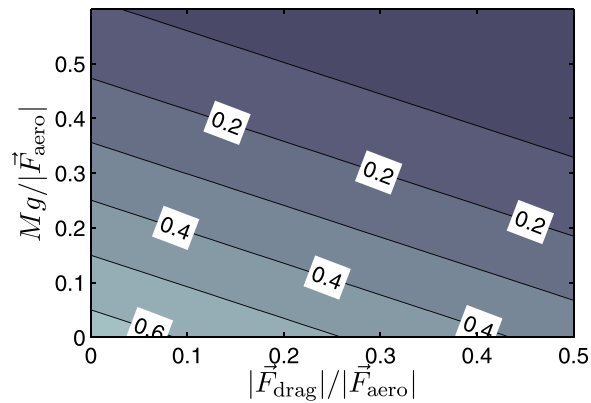
FIG. 7. The efficiency factor e for $\phi_{TG} = 30°$.
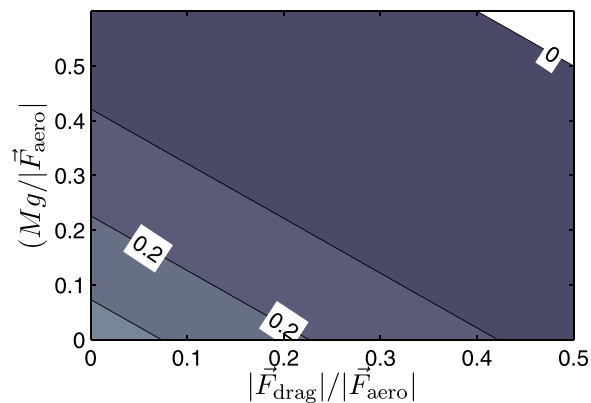
FIG. 8. The efficiency factor e for $\phi_{TG} = 45°$.

companies envisage flying a single wing, at heights of several hundred meters. A generator on the ground unwinds the tether to generate electricity. The main difference between the two systems lies in the wing. Skysails used a flexible wing, similar to a surf kite. Ampyx used a rigid wing, similar to a glider. Skysails' flexible wing does not have a high lift-to-drag ratio, but it has a very high strength-to-weight ratio. Ampyx wing has a high lift-to-drag ratio, but a far lower strength-to-weight ratio, and is undoubtedly more costly to manufacture per square meter of wing. Brief descriptions of both systems can be found on the company websites.[2,31] The parameters are given in Table I. Data provided by both companies[26] are shaded and were used to

TABLE I. Characteristics of the Ampyx and Skysails systems.

Description	Parameter	Skysails	Ampyx
Weight/strength system	$\left(\frac{Mg}{|\vec{F}_{\text{aero}}|}\right)_{\min}$	.02	.08
Average ground tether angle	ϕ_{TG}	30°	30°
Effective lift-to-drag ratio (including dynamic tether drag)	$(C_{\text{L}}/C_{\text{D}})_{\text{eff}}$	5	8.2
Area of wing	A	400 m²	3 m²
Mass of wing		320 kg	28 kg
Maximum aerodynamic force	$|\vec{F}_{\text{aero}}|_{\max}$	320 kN	3.5 kN
Average tether (Dyneema) length	l_t	420 m	425 m
Average wing height	h	250 m	200 m
Tether diameter	d_t	30 mm	2.2 mm
Average tether mass		380 kg	1.6 kg
Average airborne mass	M	700 kg	29.6 kg

make the following comparison as accurate as possible. With the exception of ϕ_{TG}, non-shaded parameters were calculated using the shaded ones. To ensure a fair comparison between the two systems, the average angle of the ground tether force was assumed to be $\phi_{\text{TG}} = 30°$, which means they would be accessing wind at the same altitude.

In order to express the average aerodynamic force as a function of the maximum force the system can tolerate (given in Table I), we introduce the load factor

$$\text{LF} = \frac{|\vec{F}_{\text{aero}}|}{|\vec{F}_{\text{aero}}|_{\max}} . \tag{5.5}$$

Figure 9 shows how the maximum capacities ζ are modulated by e for different values of LF. Note that the limiting value of $e \times \zeta = 2.1$ obtained for the Skysails system with LF = 100% is in agreement with the value calculated by Schmehl *et al.*,[30] Figure 2.13, who also considered a lift-to-drag ratio of 5. It is apparent from Figure 9 that, at full load, the Ampyx system has the potential to be more "efficient," due to its higher effective $C_{\text{L}}/C_{\text{D}}$. However, as the load factor decreases, Ampyx efficiency decreases, whereas the Skysails system can maintain its efficiency even at very low load factors. As aerodynamic force is proportional to effective wind speed squared, 50% of the full-load wind speed will result in a load factor of approximately 25%. Which wing is a better choice should thus depend not only on the cost per square meter of wing but also on the overall efficiency factor that can be expected, given the wind statistics at a particular site.
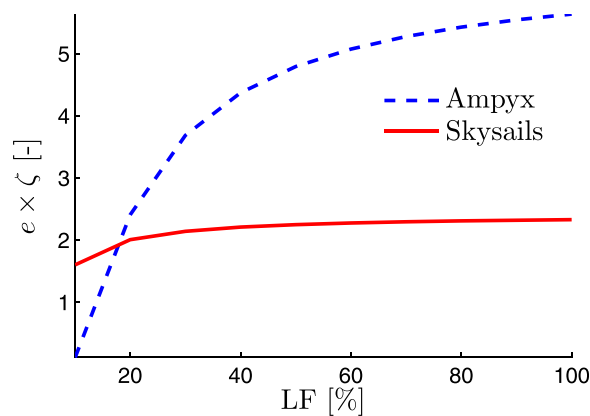

FIG. 9. The number of times the wind power density that can be produced per square meter of wing, for the Ampyx and Skysails systems.

2. Very high altitude wind power

The idea of accessing winds at altitudes of approximately 10 km is also currently under investigation.[28] To get a rough idea of the efficiency factor that such a system might have, we begin by assessing the weight-to-strength ratio of the tether

$$\left(\frac{M_t g}{|\vec{F}_{\text{aero}}|}\right)_{\text{min}} = \text{SF} \times \frac{g \rho_t l_t}{\sigma_t} = 0.29 \,, \tag{5.6}$$

where M_t is the approximate tether mass, l_t is the approximate tether length of 15 km, $\rho_t = 970 \, \text{kg/m}^3$ and $\sigma_t = 9 \times 10^9 \, \text{N/m}^3$ are the density and tensile strength of ultra-high-molecular-weight polyethylene (UHMWPE) fiber such as Dyneema® (the strongest tether material currently available). SF is the tether safety factor of 6 (to give some perspective the Royal Netherlands Navy uses a safety factor of 7 for mooring vessels[29]). Assuming that the system of wings (for example, a turbine) has a weight-to-strength ratio of about 0.05 (this is in between that of Skysail and Ampyx systems), we obtain an overall system strength-to-weight ratio

$$\left(\frac{Mg}{|\vec{F}_{\text{aero}}|}\right)_{\text{min}} \simeq 0.35. \tag{5.7}$$

So, even without taking tether-drag into account, we can see from Figure 7 that, at a tether angle of 30°, the *maximum* efficiency factor is 0.3. Additionally, this will deteriorate rapidly if the system is not operating at full load. For example, at half load (or 70% of the full-load wind speed), the maximum efficiency factor is 0.1. The system proposed by Roberts *et al.*[28] relies on having a generator in the air. This can be expected to significantly increase the system weight, further reducing the efficiency. Based on this analysis, it is questionable whether such a system would be commercially viable.

VI. CONCLUSIONS

This paper reviewed existing results for evaluating the efficiency of a general kite system, and proposed a new efficiency analysis based on average forces. The starting point is an existing result for the maximum *instantaneous* power that a wing can extract from the wind, P_{max}. A novel upper bound for the maximum *average* power was derived, $\tilde{P}$. This bound is in general much more restrictive (and hence accurate) than P_{max}. Its calculation requires the angle of the average external forces acting on the kite system to be known. The use of *average* forces allows the bound to be applied to dynamically flying kite systems, something that was previously not possible. This is because the inertial forces occurring due to momentary acceleration of the kite (or kites) must sum to zero on average, as each kite is not accelerating on average.

It was shown how to use basic system parameters to compute this bound for a generic high-altitude wind-power system. In particular, both the load factor the system operates at and the system weight were shown to strongly affect efficiency. As an example, the analysis was applied to two concrete scenarios using industrial data, namely, pumping-cycle generators and very high altitude (jet-stream) power. For a single wing on a tether, the results show that, while rigid wings are generally more efficient at full load due to their superior lift-to-drag ratio, flexible wings are much lighter and conserve their efficiency as the load factor decreases. In the case of very high altitude wind power, our conclusion is rather sobering: only very low efficiencies are achievable due to the weight of the long tether required to reach such altitudes.

[1]*Airborne Wind Energy*, edited by U. Ahrens, M. Diehl, and R. Schmehl (Springer, Berlin, 2013).
[2]See http://www.ampyxpower.com for Ampyx Power.
[3]I. Argatov and R. Silvennoinen, "Energy conversion efficiency of the pumping kite wind generator," Renewable Energy **35**(5), 1052–1060 (2010).
[4]I. Argatov and R. Silvennoinen, "Structural optimization of the pumping kite wind generator," Struct. Multidiscip. Optim. **40**(1–6), 585–595 (2010).

[5]I. Argatov, P. Rautakorpi, and R. Silvennoinen, "Estimation of the mechanical energy output of the kite wind generator," Renewable Energy **34**(6), 1525–1532 (2009).

[6]I. Argatov, P. Rautakorpi, and R. Silvennoinen, "Apparent wind load effects on the tether of a kite power generator," J. Wind Eng. Ind. Aerodyn. **99**(10), 1079–1088 (2011).

[7]A. Betz, "Das maximum der theoretisch möglichen Ausnützung des Windes durch Windmotoren," Z. Gesamte Turbinenwesen **26**, 307–309 (1920).

[8]See http://www.boeing.com for Boeing.

[9]A. E. Bryson and Y.-C. Ho, *Applied Optimal Control: Optimization, Estimation and Control* (Hemisphere Publishing Corporation, 1975).

[10]M. Canale, L. Fagiano, and M. Milanese, "High altitude wind energy generation using controlled power kites," IEEE Trans. Control Syst. Technol. **18**(2), 279–293 (2010).

[11]G. M. Dadd, D. A. Hudson, and R. A. Shenoi, "Comparison of two kite force models with experiment," J. Aircr. **47**, 212–224 (2010).

[12]G. M. Dadd, D. A. Hudson, and R. A. Shenoi, "Determination of kite forces using three-dimensional flight trajectories for ship propulsion," Renewable Energy **36**(10), 2667–2678 (2011).

[13]See http://www.dg-flugzeugbau.de/ for DG Flugzeugbau GmbH.

[14]M. Diehl, "Airborne wind energy: Basic concepts and physical foundations," in *Airborne Wind Energy*, edited by U. Ahrens, M. Diehl, and R. Schmehl (Springer, Berlin, 2013), pp. 3–22; see Ref. 1.

[15]M. Erhard and H. Strauch, "Flight control of tethered kites in autonomous pumping cycles for airborne wind energy," Control Eng. Pract. **40**(0), 13–26 (2015).

[16]L. Fagiano and M. Milanese, "Airborne wind energy: An overview," in *Proceedings of the American Control Conference (ACC)*, 2012, pp. 3132–3143.

[17]L. Fagiano, M. Milanese, and D. Piga, "Optimization of airborne wind energy generators," Int. J. Robust Nonlinear Control **22**(18), 2055–2083 (2012).

[18]L. Goldstein, "Airborne wind energy conversion systems with ultra high speed mechanical power," in *Airborne Wind Energy*, edited by U. Ahrens, M. Diehl, and R. Schmehl (Springer, Berlin, 2013), pp. 235–247; see Ref. 1.

[19]G. Horn, S. Gros, and M. Diehl, "Numerical trajectory optimization for airborne wind energy systems described by high fidelity aircraft models," in *Airborne Wind Energy*, edited by U. Ahrens, M. Diehl, and R. Schmehl (Springer, Berlin, 2013), pp. 205–218; see Ref. 1.

[20]B. Houska, "Robustness and stability optimization of open-loop controlled power generating kites," M.S. thesis, Ruprecht-Karls-Universität, Heidelberg, 2007.

[21]B. Houska and M. Diehl, "Optimal control of towing kites," in *Proceedings of the 45th IEEE Conference on Decision and Control (CDC)*, 2006, pp. 2693–2697.

[22]B. Houska and M. Diehl, "Optimal control for power generating kites," in *Proceedings of the 9th European Control Conference*, 2007, pp. 3560–3567.

[23]See http://www.kitegen.com for KiteGen.

[24]M. L. Loyd, "Crosswind kite power," J. Energy **4**(3), 106–111 (1980).

[25]R. H. Luchsinger, "Pumping cycle kite power," in *Airborne Wind Energy*, edited by U. Ahrens, M. Diehl, and R. Schmehl (Springer, Berlin, 2013), pp. 47–64; see Ref. 1.

[26]Note. This data was provided in 2013 through personal correspondence with Michael Erhard and Richard Ruiterkamp.

[27]See https://www.x-wind.de for NTS Energie- und Transportsysteme GmbH.

[28]B. W. Roberts, D. H. Shepard, K. Caldeira, M. E. Cannon, D. G. Eccles, A. J. Grenier, and J. F. Freidin, "Harnessing high-altitude wind power," IEEE Trans. Energy Convers. **22**(1), 136–144 (2007).

[29]See http://www.dyneema.com/applications/ropes-and-lines/maritime/mooring.aspx for Royal DSM N.V. Dyneema-mooring, 2014.

[30]R. Schmehl, M. Noom, and R. van der Vlugt, "Traction power generation with tethered wings," in *Airborne Wind Energy*, edited by U. Ahrens, M. Diehl, and R. Schmehl (Springer, Berlin, 2013), pp. 23–45; see Ref. 1.

[31]See http://www.skysails.info for Skysails GmBH.

[32]R. H. Thuillier and U. O. Lappe, "Wind and temperature profile characteristics from observations on a 1400 ft tower," J. Appl. Meteorol. **3**, 299–306 (1964).

[33]G. P. Van den Berg, "Wind gradient statistics up to 200 m altitude over flat ground," in Proceedings of the 1st International Meeting on Wind Turbine Noise, 2005.

[34]D. Vander Lind, "Analysis and flight test validation of high performance airborne wind turbines," in *Airborne Wind Energy*, edited by U. Ahrens, M. Diehl, and R. Schmehl (Springer, Berlin, 2013), pp. 473–490; see Ref. 1.

[35]R. Vlugt, "Aero- and hydrodynamic performance analysis of a speed kiteboarder," M.S. thesis, Delft University of Technology, 2009.

[36]P. Williams, B. Lansdorp, and W. Ockesl, "Optimal crosswind towing and power generation with tethered kites," J. Guid., Control, Dyn. **31**(1), 81–93 (2008).

[37]M. Zanon, S. Gros, J. Andersson, and M. Diehl, "Airborne wind energy based on dual airfoils," IEEE Trans. Control Syst. Technol. **21**(4), 1215–1222 (2013).