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Stochastic Geometric Models for Green Networking

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***Abstract*—In this work, we use a stochastic geometric approach in order to study the impact on energy consumption when base stations are switched off independently of each other. We present here both the uplink and downlink analysis based on the assumption that base stations are distributed according to an independent stationary Poisson point process. This type of modeling allows us to make use of the property that the spatial distribution of the base stations after thinning (switching-off) is still a Poisson process. This implies that the probability distribution of the SINR can be kept unchanged when switching-off base stations provided that we scale up the transmission power of the remaining base stations. We then solve the problem of optimally selecting the switch-off probabilities so as to minimize the energy consumptions while keeping unchanged the SINR probability distribution. We then study the trade-off in the uplink performance involved in switching-off base stations. These include energy consumption, the coverage and capacity, and the impact on amount of radiation absorbed by the transmitting user.**

I. Introduction

Green networking is often referred to as the practice of selecting energy-efficient networking technologies and products, and minimizing resource use whenever possible [1]. The energy consumed for mobile networks causes around 2% of total carbon emission [2]. Moreover, more than 50% of the energy consumption is directly attributed to base station (BS) equipment and 30% more to mobiles switching and core transmission equipment [3].

Green networking covers all aspects of the network (personal computers, peripherals, switches, routers, and communication media). Energy efficiencies of all network components must be optimized to have a significant impact on the overall energy consumption by these components. Consequently, these energy savings through "green networking" will reduce CO_2 emissions and thus will help mitigate global warming [4].

Growing awareness to dangers related to large scale energy consumption, international agreements as well as legislation have reduced energy consumption in several sectors [5]. There is also a growing willingness to reduce energy consumption in wireless networks. One simple method for a network operator to reduce energy consumption, which is also easy to implement, is to reduce the number of active base stations (BSs). However, this can reduce the network quality of service (QoS) and also alter the amount of power emitted by the mobile nodes– if a BS goes inactive or switched-off, then mobile nodes served by this BS reconnect to their next nearest BS and are required to transmit more uplink power to maintain connectivity. In this paper we study the effect of switching-off a fraction of BSs on the uplink and downlink power and the network quality of service (QoS).

The amount of power transmitted by the BSs (downlink power) or the amount of power transmitted by the mobiles (uplink power) depend on relative position with each other and also on the required QoS. We use stochastic geometric approach and model the locations of BSs and mobiles as homogeneous independently marked Poisson point processes. This approach, introduced in [6]–[8], has been demonstrated to predict the network performance well while being more tractable than the traditional grid-based models. We use this modeling approach for both uplink and downlink and use signal to interference and noise ratio (SINR) based QoS that is expressed in terms of the spatial parameters.

In our analysis we use the property that the spatial distribution of the BSs after thinning (switching-off) is still a Poisson process. In the downlink analysis this implies that the probability distribution of the SINR can be kept unchanged when switching-off BSs provided that we scale up the transmission power of the remaining BSs. We aim to obtain a switch-off probability that minimizes the energy consumptions in the BSs while keeping the SINR probability distribution unchanged. In the uplink analysis, we study the effect of switching-off BSs on the amount of power transmitted by the mobiles and the coverage. This includes another aspect of what we consider as green networking, that of minimizing the average uplink transmitted power, as the latter is proportional to the amount of energy that our body is exposed in communications by wireless terminals. Standards on the maximum amount of permitted radiation to the humans exist (see [9]) due to the awareness that the radiation can cause health problems [10]. We compute the impact of these standardized uplink radiation bounds on the system coverage.

II. Related work

Several works focus on the BS deployment in order to reduce power while taking into account the Capital Expenditure (CapEX) and Operational Expenditure (OpEx) [11], [12]. Other references deal with improving the energy efficiency in order to accomplish the same task with less energy. Several solutions aiming at reducing power from BS may be divided into different types as following

- Increasing the number of cells in order to reduce the cell size leading to a reduction in the average transmitted power. This approach is more efficient for indoor network [13], [14].
- Femto-cells and indoor distributed antenna systems using MIMO channel: This architecture is used to reduce co-channel interference introduced by frequency reuse among the femto cells and maintain high spectral efficiency [15].
- Cooperation at the BSs level: In [16], the authors show how the degree of redundancy of a network may reduce the power. The authors propose an approach based on cooperation between BSs in order to minimize the active number of BSs while satisfying the minimum required quality of service and minimum coverage. Cooperation is also possible when using C-RAN (Cloud Radio Access Network) via a backhaul, see [17] for a short timely tutorial.

This paper summarizes results that appeared in several conference proceedings [5], [18] (as well as some material from [19], [20]).

III. Point Processes Preliminaries

Stochastic geometry is a rich branch of applied probability which allows the study of random phenomena on the plane or in higher dimensions. It is intrinsically related to the theory of point processes [6]. A point process (p.p.) Φ can be depicted as a *random collection of points* in space. More formally, Φ is a random, finite or countably-infinite collection of points in the space $\mathbb{R}_d$, without accumulation points [7]. A point measure is a measure which is locally finite and which takes only integer values on some space E. Each such measure can be represented as a discrete sum of Dirac measures on E

$$\Phi = \sum_i \delta_{X_i}. \tag{1}$$

The random variables $\{X_i\}$ taking values in E are the points of Φ. The *intensity measure* Λ of Φ on B is defined as $\Lambda(B) = \mathbf{E}[\Phi(B)]$ denoting the expected number of points in $\Phi \cap B$. For some $\mathrm{d}x$, if $\Lambda(\mathrm{d}x) = \lambda \mathrm{d}x$ is multiple of Lebesgue measure, we call Φ a *homogeneous p.p.* and λ is its intensity parameter [7].

A. Poisson Point Processes

A p.p. on some metric space E with intensity measure Λ is Poisson if for all disjoint subsets $A_1, \ldots, A_n$ on E, the random variables $\Phi(A_i)$ are independent and Poisson.

B. Marked Point Processes

In a marked point process (m.p.p.), a mark belonging to some measurable space and carrying some information is attached to each point. In our context, the points are the BSs and the marks are considered to be the transmitted power by each BS.

IV. The Downlink

A. The Model

We consider a homogeneous independently marked Poisson point process (i.m.P.p.p.) of BSs. Assume that each of these BSs transmits with power P. We show by Φ the i.m.P.p.p with intensity measure λ.

Consider a mobile at an arbitrary point on the plane, say the origin. Let p_0 denote the point in Φ which is the closest to it, represents the BS to which it is connected. Let $|x_i|$ be the distance of p_i to the origin. We assume attenuation due to a path-loss. The power of the transmission received from p_0 is thus given by $P|x_0|^{-\alpha}$. The total interference from other BSs is $\sum_{i>0} |x_i|^{-\alpha}$. Thus, the SINR at the mobile is

$$\text{SINR} = \frac{P|x_0|^{-\alpha}}{P\sum_{i>0} |x_i|^{-\alpha} + \sigma^2} \tag{2}$$

where α and σ^2 stand for path loss and additive noise variance, respectively.

B. Switching-off Base Stations

The aim of the network operator is to minimize the total power spent in the system. We consider a scenario in which network operator tries to achieve this goal by turning-off a fraction of BSs. For example, turning-off those BSs when a number of mobiles served by them is small. However, the load on the BS can change dynamically and the duration for which the BSs are turned-off can be much longer than the duration over which the load on the BSs can change. So, the operator can decide to switch them off randomly. We denote by q the probability with which the operator decides to keep a BS turned-on. Let Φ^q denote the homogeneous i.m.P.p.p. obtained from the original one by deleting independently points with probability $1-q$. Deleted points correspond to BSs that are switched off. The intensity measure of Φ^q is $\lambda_q = q\lambda$. This is called the thinning property of the Poisson p.p. [7].

Define $w(q) = q$ when considering the problem on the line, and $w(q) = \sqrt{q}$ on the plane [7]. Now, the point process $\{y_i\}$ where $y_i = x_i/w(q)$ is obviously a homogeneous i.m.P.p.p with parameter $q\lambda$. Therefore, if we replace all x_i in (2) by y_i and replace P by P' then we can interpret the SINR that is obtained as one corresponding to a network where BSs are located according to a homogeneous i.m.P.p.p with intensity $q\lambda P$, BSs are the switched off with probability $1-q$ independently, and the power of each BS is increased replaced by P'.

Now if we choose $P' = Pw(q)^{-\alpha}$ then the SINR is seen to remain unchanged. We conclude that if BSs are switched off with probability $1-q$ then the transmission power of the BS has to increase by $w(q)^{-\alpha}$ in order for the distribution of the SINR to remain unchanged.

C. Optimal Switching-off Probabilities

Assume that the power used by a BS that transmits at a power P is given by $P_0 + \beta P$. We consider some operational costs which arise from overhead assumed to be constant. Here, the power consumed due to operational costs is represented by P_0. We also suppose that $\beta \geq 1$. Then the power consumption density of the original network is $\lambda(P_0 + \beta P)$.

1) The line: We are interested to see what is the gain in energy by switching-off BSs (independently) with probability $(1-q)$, given that at the same time we increase the transmission energy to compensate for decreasing the resources in a way that the probability distribution of the SINR are unchanged.

After switching-off BSs, the power consumption density of the network is

$$q\lambda(P_0 + \beta P') = q\lambda(P_0 + \beta P q^{-\alpha}). \tag{3}$$

So that the gain in power consumption density is

$$\begin{aligned} G(q) &= \lambda(P_0 + \beta P) - \lambda q(P_0 + \beta P') \\ &= \lambda(P_0(1-q) + \beta P[1 - q^{1-\alpha}]). \end{aligned} \tag{4}$$

The switching probabilities that maximize this gain are obtained by solving

$$\frac{dG(q)}{dq} = -P_0 - (1-\alpha)\beta P q^{-\alpha} = 0 \tag{5}$$

which gives

$$1 - q^* = \max\left\{1 - \left(\frac{\beta P(\alpha-1)}{P_0}\right)^{\frac{1}{\alpha}}, 0\right\}. \tag{6}$$

2) The plane: We calculate by the same way the power consumption density of the network after switching-off BSs

$$q\lambda(P_0 + \beta P') = q\lambda(P_0 + \beta P q^{-\alpha/2}). \tag{7}$$

The gain in power consumption density is given by

$$\begin{aligned} G(q) &= \lambda(P_0 + \beta P) - \lambda q(P_0 + \beta P') \\ &= \lambda(P_0(1-q) + \beta P[1 - q^{1-\alpha/2}]). \end{aligned} \tag{8}$$

The switching probabilities that maximize this gain are obtained by solving

$$\frac{dG(q)}{dq} = -P_0 - \beta P(1-\alpha/2)q^{-(\alpha/2)} = 0 \tag{9}$$

which gives

$$1 - q^* = \max\left\{1 - \left(\frac{\beta P(\alpha/2-1)}{P_0}\right)^{\frac{1}{\alpha/2}}, 0\right\}. \tag{10}$$

D. Simulation Results

In this section, we compare the optimal switching-off probabilities with respect to path loss α for different operational costs P_0, and we also match the optimal switching-off probabilities in terms of β for some α. Moreover, gain in power consumption is compared with respect to α for different P_0.

In Figure 1 and 2 [PP 1 and 2], we depict the change of switching-off probabilities in terms of path loss α. From the figures, we observe that for higher path loss values, the number of switched off BSs is decreased. In other words, we need to keep more BSs switched on. Also, for the same path loss value optimum switching off probability is higher for higher P_0. That means, the switching-off strategy tells us to remove BSs with a higher probability for higher P_0. On the other hand, if we compare the optimal switching-off probabilities with respect to the dimension (line or plane), we remark that it is necessary to switch on more BSs.

We depict in Figure 3 and 4, [PP 3 and 4] the comparison of switching-off probabilities in terms of β for $\alpha = (2.5, 4, 6)$. We interpret that for higher values of β the number of switched on BSs is increased. Furthermore, in case of plane the used switched on BSs is higher than that of line.

In Figure 5 and 6, [PP 5 and 6], the comparison of gain in power consumption $G(q^*)$ with respect to α is given. We calculate $G(q^*)$ in terms of optimal switching-off probabilities. It is assumed to be unit intensity parameter λ. We observe that as long as P_0 increases, the obtained $G(q^*)$ increases. This means that for high operational costs the gain in power consumption by switching-off BSs is also high.

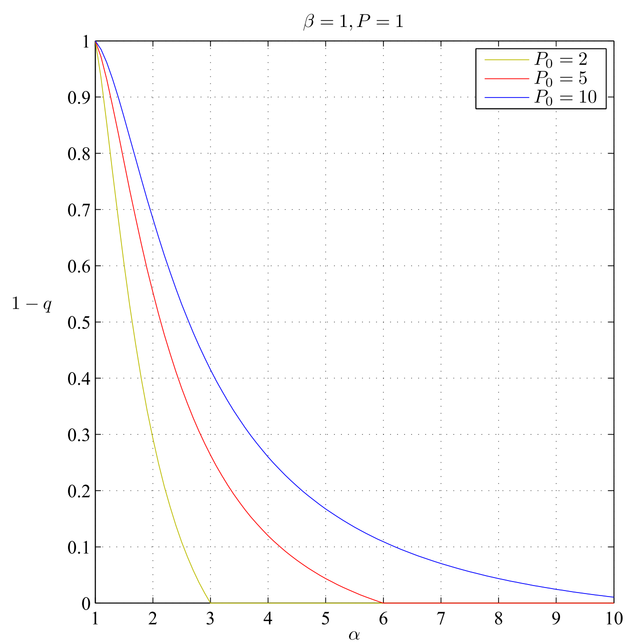

Fig. 1. The change of optimal switching-off probabilities with respect to path loss in case of line

V. The Uplink

A. Model & Performance Analysis

We consider a cellular network in which OFDMA is the access method. The study shall be performed on a particular resource unit consisting of one time slot and one frequency band. The mobiles are Poisson distributed with intensity β as well as BSs are Poisson distributed with intensity λ, both of which are assumed to be independent. We assume that each mobile connects to its nearest BS, and the random variable L represents the distance between a mobile and its closest

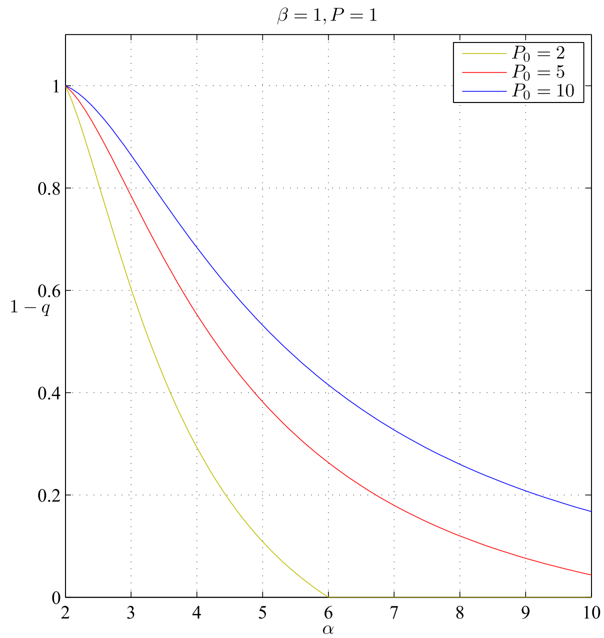

Fig. 2. The change of optimal switching-off probabilities with respect to path loss in case of plane

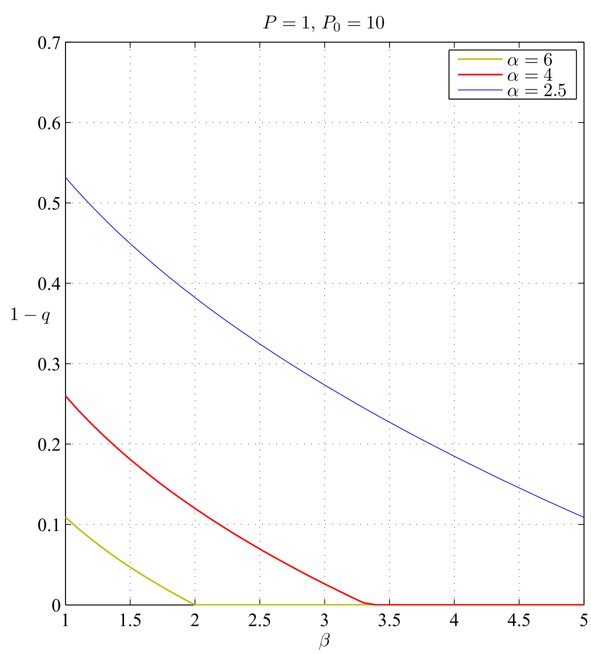

Fig. 3. The change of optimal switching-off probabilities with respect to β in case of line

BS. Initially, we consider that the mobiles are highly rare which might be interpreted as the traffic is quite low. Thus, the interference can be neglected. Afterwards, we study a more complex model that takes into account the interference.

Assume that the network operator keeps only a fraction of $0 < q < 1$ BSs switched-on during which the traffic is low and

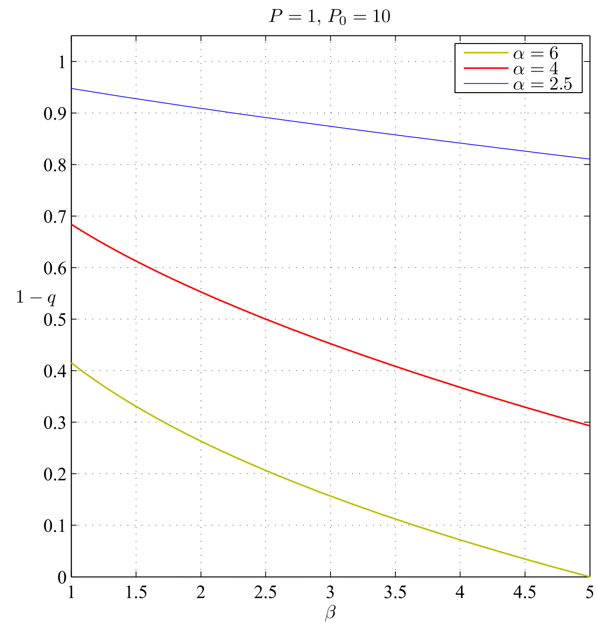

Fig. 4. The change of optimal switching-off probabilities with respect to β in case of plane

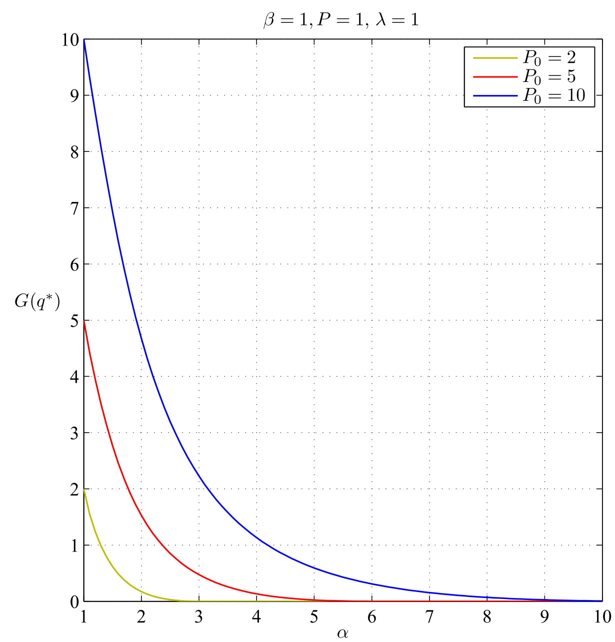

Fig. 5. The gain in power consumption with respect to path loss in case of line

switches-off the remaining. We analyze the cases where the a call-duration is relatively much shorter than the time during which the BS is switched-off, or the network load changes quickly. In such a situation, using the following mechanisms might not be useful: a mechanism looking at the present state of the network for determining the station that has to turn to sleep mode and the network operator selects a BS to switch-off

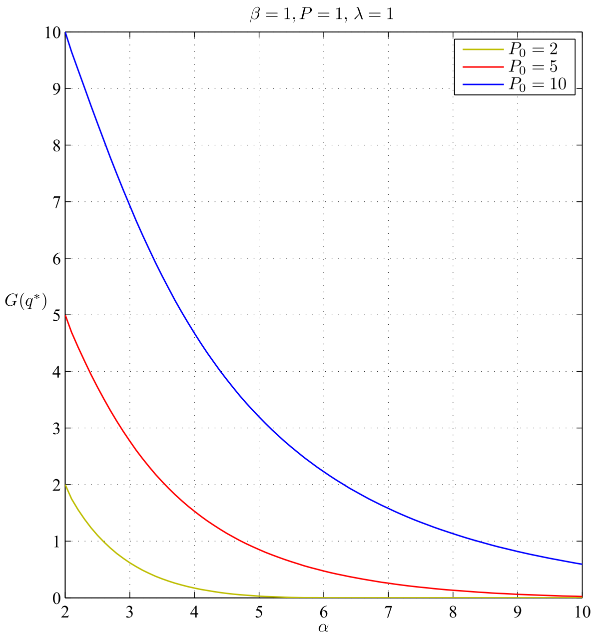

Fig. 6. The gain in power consumption with respect to path loss in case of plane

independently of the others.

Assume that the distance between a mobile and its nearest BS is l. It is well known that the probability of having no a BS within an d-dimensional ball of radius l is $\exp(-\lambda V(l))$ where $V(l)$ is the volume of a d-dimensional ball with radius l. Particularly, in case of a line, i.e., $(d = 1)$, then $V(l) = 2l$; and in case of plane, i.e., $d = 2$ it becomes $V(l) = \pi l^2$.

Let us represent by $p \le p_m$ the transmission power of a mobile, where we set a maximal power limit given by p_m. Let us call it as *green limit threshold*; its value is decided by health considerations: it is a power limit under which human body will not be in health related risks. We assume a power control mechanism where $p = p(l)$ is controlled in the following way: a target SNR, η, is reached at the nearest BS which is in the distance l. Let $p(l)$ be the lowest transmission power that assures the required SNR.

We only consider the attenuation due to the path loss:

$$\frac{p(l)l^{-a}}{\sigma^2} = \eta, \tag{11}$$

where σ^2 is the noise variance and a is the path loss exponent. If $p(l) > p_m$ then, we assume that an outage occurs. In case of the line, we assume that $a > 1$, and in case of the plane, $a > 2$. Inverting equation (11), we obtain $p(l) = \sigma^2 \eta l^a$. We denote by l_m being the distance at which p_m is reached. It is given by

$$l_m = \left(\frac{p_m}{\sigma^2 \eta}\right)^{\frac{1}{a}}. \tag{12}$$

We consider the following frameworks to react to outage:

- **(i) No transmission (NT):** there is no transmission when a mobile is not covered, or

$$p_{nt}(l) = \begin{cases} \sigma^2 \eta l^a & \text{if } l \le l_m \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

- **(ii) Always transmit (AT):** Transmission occurs at the maximum power when $l > l_m$ resulting in bad quality of service. Thus

$$p_{at}(l) = \begin{cases} \sigma^2 \eta l^a & \text{if } l \le l_m \\ p_m & \text{otherwise} \end{cases} \tag{14}$$

This is equivalent to $p_{at}(l) = \min(\sigma^2 \eta l^a, p_m)$.

B. Uplink Power and Coverage Probability

Let us represent by $\Delta(\lambda, l_m)$ the expected uplink power, i.e.,

$$\Delta(\lambda, l_m) := \mathbf{E}[p(L)] = \int_{B(l_m)} p(s) dP(s), \tag{15}$$

where $B(l)$ is the ball of radius l at the origin. In the following, we calculate the expected power transmission of a mobile.

Proposition 1. *In the NT framework, the expected power transmission of a mobile, on a line, is given by*

$$\begin{aligned}\Delta_{nt}(\lambda, l_m) &= 2\lambda \int_0^{l_m} \sigma^2 \eta l^a \exp\{-2\lambda l\} dl \\ &= \frac{\sigma^2 \eta 2^{-\frac{a}{2}} l_m^a (\lambda l_m)^{-\frac{a}{2}} \exp\{-\lambda l_m\} WM\left(\frac{a}{2}, \frac{a}{2} + \frac{1}{2}, 2\lambda l_m\right)}{a+1},\end{aligned}$$

where WM$(\cdot,\cdot,\cdot)$ denotes the WhittakerM function. In case of the two dimensional plane, it can be given by

$$\begin{aligned}&\Delta_{nt}(\lambda, l_m) \\ &= \frac{\sigma^2 \eta \pi^{-\frac{a}{4}} l_m^a (\lambda l_m^2)^{-\frac{a}{4}} \exp\{-\frac{\pi \lambda l_m^2}{2}\} WM\left(\frac{a}{4}, \frac{a}{4} + \frac{1}{2}, \pi \lambda l_m^2\right)}{\frac{a}{2}+1}\end{aligned}$$

In the AT framework, the expected transmission power is given by

$$\begin{aligned}\mathbf{E}[p(L)] &= \Delta_{nt}(\lambda, l_m) + P(L > l_m) p_m \\ &= \Delta_{nt}(\lambda, l_m) + p_m \exp\Big(-\lambda V(l_m)\Big).\end{aligned} \tag{16}$$

The proof of the above proposition and that of the next corollary is direct, except for the expressions for $\Delta(\lambda, l_m)$ for which we thank Maple.

Assuming no bound on the transmission power of mobiles, and denoting the expected power in this regime as $\overline{\Delta}(\lambda)$, we have the following corollary.

Corollary 1. *As $p_m \to \infty$, we have for the line:*

$$\overline{\Delta}(\lambda) := \lim_{l \to \infty} \Delta(\lambda, l) = \sigma^2 \eta (2\lambda)^{-a} \Gamma(a+1)$$

For the plane we get:

$$\overline{\Delta}(\lambda) = \sigma^2 \eta (\pi \lambda)^{\frac{-a}{2}} \Gamma\left(\frac{a}{2} + 1\right)$$

Let us suppose that a mobile is covered, if it is connected to a BS within a distance of l_m from any BS, otherwise it will not be covered. The expression for coverage probability is given in the following proposition.

Proposition 2. *The coverage probability at the target SNR is given by*

$$c(\lambda, l_m) = 1 - \Pr\{L > l_m\} = 1 - \exp(-\lambda V(l_m))$$

in both regimes.

C. Effect of Base Station Deactivation

Recall that q is the probability that a given BS switched-on (Subsection IV-B). Not that the expected power transmitted by any mobile can be given by $\Delta(q\lambda, l_m)$.

The Figure 7 depicts the change of $\Delta(q\lambda, l_m)$ with respect to q for the "no transmit" (NT) scenario in a plane. Note that if the number of switched-off BSs increases, i.e., $q \ll 1$, meaningfully, the probability that a mobile connects to a BS decreases. Thus, as expected, the mobiles transmits no any power in the NT case. This leads to decrease in the expected power near origin in the above plot. However, the coverage is very poor in this region. This is also shown in Figure 7.

The term $\Delta(\lambda, l_m)$ is average of all potential calls with regards to the distance to the BS, including those that are in outage conditions. We shall be more interested in measures that characterizes successful calls. To this end, we define the following for the NT framework: $J_{nt}(\lambda) = \frac{\Delta_{nt}(\lambda, l_m)}{c_{nt}(\lambda, l_m)}$. $J_{nt}(\lambda)$ which represents the average uplink power per successful call while the density of the BSs is λ. For the AT framework, it becomes $J_{at}(\lambda) = \Delta_{at}(\lambda, l_m)$. The change of $J_{nt}(q\lambda)$ as function in q is shown in Figure 7. Note that $J_{nt}(q\lambda)$ decreases with respect to q meaning that less uplink power is transmitted per successful call when the density of the BSs increases.

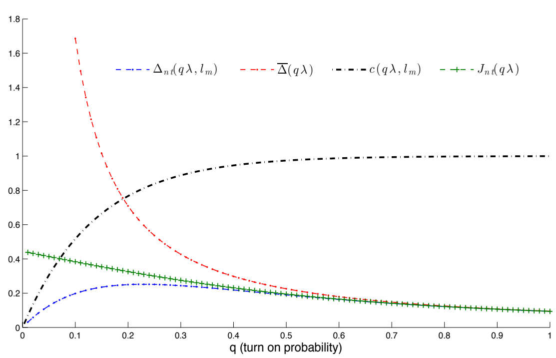

Fig. 7. Expected Uplink Power, Coverage and Successful calls: $\sigma^2 = 0.01, \eta = 35, \lambda = 1, a = 2.5, p_m = 1$

D. Exponential Attenuation

In this subsection, we relax the assumption that mobile nodes are sparsely distributed. Unfortunately, in this case, the analytic expressions considering the interference are not tractable when the attenuation is expressed using the path loss model. To get better insights, we consider the absorbing channel model instead of path loss model as former is more amenable to analysis when interference is non-negligible. As we will see later, absorption channel model allows to express interference through a recursion and to apply techniques from *stochastic difference equations* to derive tractable expressions. Specifically, we assume that the power received at a distance of D from a transmitter is given as $\exp(-\xi D)$ times the transmitted power, where $\xi > 0$ is the attenuation factor. (e.g., under humid air conditions, the attenuation is exponential in distance). As earlier, we continue to assume that each mobile connects to a nearest BS and adjusts its power to maintain SNR of η, i.e., a mobile at a distance l from BS transmit power $p(l)$ such that

$$\frac{p(l)\exp\{-l\xi\}}{\sigma^2} = \eta.$$

If p_m is the maximum power that each mobile can transmit, then inverting the above equation, we get the corresponding maximum distance is $l_m = (1/\xi)\log\left(p_m/(\sigma^2\eta)\right)$. Repeating the calculation for expected uplink power we have the following. On the line

$$\begin{aligned}\Delta(\lambda, l_m) : &= \mathbb{E}[p(L)] = 2\lambda \int_0^{l_m} \sigma^2\eta \exp\{\xi l\}\exp\{-2\lambda l\}dl \\ &= \frac{2\lambda\sigma^2\eta}{\xi - 2\lambda}\Big\{\exp\{(\xi - 2\lambda)l_{max}\} - 1\Big\}, \qquad (17)\end{aligned}$$

and on the plane

$$\begin{aligned}\Delta(\lambda, l_m) &= 2\pi\lambda \int_0^{l_m} \sigma^2\eta l \exp\{\xi l\}\exp\{-\lambda\pi l^2\}dl \\ &= \sigma^2\eta\Big\{1 - e^{(\xi l_m - \lambda\pi l_m^2)} + \frac{\xi\exp\{\xi^2/(4\lambda\pi)\}}{2\sqrt{\lambda}} \cdot \\ &\quad \left(erf\left\{\frac{\xi}{2\sqrt{\lambda\pi}}\right\} + erf\left\{\frac{2\pi\lambda l_m - \xi}{2\sqrt{\lambda\pi}}\right\}\right)\Big\}. \qquad (18)\end{aligned}$$

Note that the coverage probability remains unchanged for a given l_m. The behavior of expected transmitted power when a fraction of BSs are switched-off is same as in the case of path loss model.

E. Accounting for the Interference

We next take into account the total interference in analyzing the power emitted by the mobiles. For analytic tractability we focus on a simple linear model and make the following assumptions:

- The BSs and mobiles are distributed on a line at locations that form Poisson process with parameter β and λ respectively, and are independent of each other.
- Mobiles are not power constrained. Each mobile transmits at a power that guarantees a target SINR of η. Thus for a mobile that has a nearest BS at distance y where the interference level is I, transmits at power

$$p(y) = (\sigma^2 + I)\eta\exp(\xi y) \qquad (19)$$

  where σ^2 is the noise level at the receiver.
- BSs have directional antenna. All the antennas transmit towards the east. (e.g., to communicate with vehicles going in that direction.) Then it is also natural to consider

directional receiving antennas at the BSs that would receive signals from west.

- A BS is restricted to receive one call at a time on a given resource (frequency or time). Therefore if a BS is at the same time closest to two mobiles, then some fair criteria will decide which one will be chosen to transmit first.
- We focus on one resource (in time/frequency). Once the resource is assigned to a mobile by a BS then it is reserved so that no other mobile within radius R of the BS will get this resource. (We allow for $R = 0$ in which case there is no resource reservation). Such reservation is useful in pico-cells as it facilitates fast switching between neighboring pico-cells.

Blocking Rate. Consider an active mobile at location d_n. Let y_n denote its distance to the nearest BS. By the resource reservation assumption, the BS at $d_n + y_n$ reserve the resource for the active mobile at d_n, and all the mobiles between d_n and $d_n + y_n + R$ are blocked and thus cannot transmit. Location of the next mobile that can transmit is then given $d_{n+1} = d_n + y_n + R + x_n$, where $\{y_n\}_{n\geq 1}$ and $\{x_n\}_{n\geq 1}$ are homogeneous Poisson processes of intensity λ and β, respectively, and are independent of each other. Then it is easy to note that the expected distance between two consecutive transmitting mobiles is $\mathbf{E}[y_n] + \mathbf{E}[x_n] + R = 1/\lambda + 1/\beta + R$, and the density of mobiles that transmit (not blocked) is

$$\gamma = \frac{1}{\frac{1}{\lambda} + \frac{1}{\beta} + R} \tag{20}$$

which is the harmonic mean of λ and β (when $R = 0$). The blocking rate is then $\beta - \gamma$.

The interference. The total interference I_n for n-th active mobile is sum of the received power at location $d_n + y_n$ from all the active mobiles at locations $\{d_i, i < n\}$. The absorption model allows to expression the interference recursively as follows:

$$I_n = \Big(I_{n-1} + p_{n-1}(\exp(-\xi y_{n-1}))\Big)\exp(-\xi(x_{n-1} + y_n + R)).$$

Substituting equation (19) we get for any n we get

$$\begin{aligned} I_n &= \Big(I_{n-1} + \eta(\sigma^2 + I_{n-1})\Big)\exp(-\xi(x_{n-1} + y_n + R)) \\ &= A_{n-1} I_{n-1} + B_{n-1}, \end{aligned} \tag{21}$$

where

$$A_{n-1} = (1+\eta)\exp(-\xi(x_{n-1} + y_n + R)), \text{ and}$$
$$B_{n-1} = \eta\sigma^2 \exp(-\xi(x_{n-1} + y_n + R)).$$

We note that the two component random vectors (A_n, B_n) are i.i.d. and their expected value is the same for all n given as

$$\begin{aligned} \mathbf{E}[A] &= (1+\eta)\exp(-\xi R)\frac{\beta\lambda}{(\xi+\beta)(\xi+\lambda)} \\ \mathbf{E}[B] &= (\eta\sigma^2)\exp(-\xi R)\frac{\beta\lambda}{(\xi+\beta)(\xi+\lambda)}. \end{aligned}$$

Theorem 1. *The stationary solution of (21) satisfies the following iteration*

$$I_n = \sum_{j=0}^{n-1}\Big(\prod_{i=n-j}^{n-1} A_i\Big)B_{n-j-i} + \Big(\prod_{i=0}^{n-1} A_i\Big)I_0 \tag{22}$$

Further, if the following condition holds

$$1 + \eta < \exp(\xi R)\frac{(\xi+\beta)(\xi+\lambda)}{\beta\lambda}, \tag{23}$$

then

$$I_n^* = \sum_{j=0}^{\infty}\Big(\prod_{i=n-j}^{n-1} A_i\Big)B_{n-j-1} \text{ for all } \quad n \in \mathbb{Z} \tag{24}$$

is the finite stationary solution of (21).

Proof. (A_n, B_n) are a sequence of i.i.d, non negative and finite random variables. Under the assumption (23), we obtain

$$\log \mathbf{E}[A] = \log(1+\eta)\exp(-\xi R)\frac{\beta\lambda}{(\xi+\beta)(\xi+\lambda)} < 0.$$

Further Jensen's inequality yields $\mathbf{E}[\log A] \leq \log \mathbf{E}[A] < 0$. Also $\mathbf{E}[\log B] < \infty$. Hence the condition (6) in [21][Thm 2A] holds and the result follows. □

Corollary 2. *Assume (23) holds, we have in stationary regime*

$$\mathbf{E}[I^*] = \frac{\mathbf{E}[B]}{(1 - \mathbf{E}[A])} = \frac{\eta\sigma^2}{\exp(\xi R)(\frac{(\xi+\beta)(\xi+\lambda)}{\beta\lambda}) - (1+\eta)}$$

For the case where there is no resource reservation, i.e., $R = 0$, the condition (23) simplifies to

$$\eta < \frac{\xi^2}{\lambda\beta} + \frac{\xi}{\lambda} + \frac{\xi}{\beta} \tag{25}$$

and the expected of interference is given by

$$\mathbf{E}[I^*] = \frac{\sigma^2\eta}{\frac{\xi^2}{\lambda\beta} + \frac{\xi}{\lambda} + \frac{\xi}{\beta} - \eta} \tag{26}$$

Next we obtain the expression for expected power as stated in the following corollary.

Corollary 3. *The expected power is given by*

$$\begin{aligned} \mathbf{E}[p_n] &= \eta\Big(\frac{\sigma^2\lambda}{\lambda - \xi} + \mathbf{E}[I_{n-1}](1+\eta)\frac{\beta}{\beta+\xi}\exp(-\xi R) \\ &\quad + \eta\frac{\sigma^2\beta}{\xi+\beta}\exp(-\xi R)\Big) \end{aligned} \tag{27}$$

where $\mathbf{E}[I_{n-1}]$ *is given by (2).*

Proof. For any given I_n and y_n from eq. (19) we have

$$\begin{aligned} p_n &= \eta(\sigma^2 + I_n)\exp\{\xi y_n\} \\ &= \eta(\sigma^2 + A_{n-1}I_{n-1} + B_{n-1})\exp\{\xi y_n\} \qquad (28) \\ &= \eta(\sigma^2\exp\{\xi y_n\} + (1+\eta)I_{n-1}\exp\{-\xi(x_{n-1} + R)\} \\ &\quad + \sigma^2\eta\exp\{-\xi(x_{n-1} + R)\}) \qquad (29) \end{aligned}$$

where (28) follows from (21), and (29) follows from (V-E). Note that I_{n-1} does not depend on x_{n-1}. Taking expectation on both sides in (29) and recalling that y_n and x_n are independent and are exponentially distributed with parameter λ and β respectively, we get the desired result. □

We are now ready to study the effect of switching-off a fraction of BSs. In Figure 8, we plot the expected power as a function of turn-on probability q, for different values of β. It is interesting to note that the expected power is increasing when more and more BSs are turned on while in the case of no interference (see Figure 7) it is decreasing. However, as see from equation (20), the blocking rate improves (i.e., lesser mobiles are blocked) as q increases.

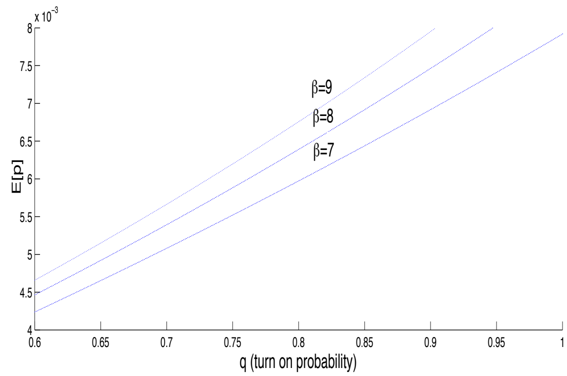

Fig. 8. Expected power as function of q: with $\lambda = 3, \eta = 0.2315, \xi = 1 \sigma = 0.01, R = 0$

VI. Conclusion and Discussion

In this paper, we studied the effect of energy saving mechanisms in networks through deactivation of base stations. We considered a specific scenario where the network operator randomly deactivates each of the BSs and provided some insight on the effect of switching-off BSs on both the uplink and downlink energy transmissions. We presented both an uplink and a downlink analysis, based on a stochastic geometric modeling. The optimal switch-off probability was computed so as to minimize some cost function. Our analysis included some aspects that are usually not accounted for, such as the exposure of the user to power radiation in the uplink, and the impact of limiting this radiation on the system coverage.

There are various ways in which the analysis in this paper can be applied. Indeed, many of the results that were obtained for a cellular network context will also hold in a context of Internet of Things (IoT) in which sensors or actuators replace the cellular terminals. We shall study in the future (i) the adaptation of these ideas to IoT and (ii) the impact of Radio-Units switching-off in C-RAN networks. Note that the performance impact is exactly the same as happens if the fronthaul/backhaul link is not available. See [22].

References

[1] http://searchnetworking.techtarget.com/

[2] McKinsey & Company, "The impact of ICT on global emissions on behalf of the global esustainability initiative (GeSI)", tech. rep., November 2007.

[3] V. Lee and C. Wheelock, "Green telecom networks," *Research report, PikeResearch.*, 2008, Available at http://www.pikeresearch.com/research/green-telecom-networks

[4] N. Chilamkurti, S. Zeadally, and F. Mentiplay, "Green networking for major components of information communication technology systems," *EURASIP Journal on Wireless Communications and Networking*, vol. 2009.

[5] E. Altman, M. K. Hanawal, R. El-Azouzi, and S. Shamai, "Tradeoffs in green cellular networks," *GreenMetrics Workshop, held in conjunction with SIGMETRICS*, San Jose, USA, June 2011.

[6] M. Haenggi, J. G. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 27, pp. 1029–1046, Sept. 2009.

[7] François Baccelli, and Bartłomiej Błaszczyszyn. "Stochastic geometry and wireless networks: volume I theory", *Foundations and Trends in Networking*, vol. 3, pp. 249–449, 2009.

[8] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Trans. Commun.*, vol. 59, no. 11, pp. 3122-3134, Nov. 2011.

[9] J. C. Lin. *"Biological bases of current guidelines for human exposure to radio-frequency radiation"*., 45(3):182–184, June 2005.

[10] E. Cardis et al. "The interphone study: design, epidemiological methods, and description of the study population". *Eur. J. Epidemiology.*, 22:647–664, July 2007.

[11] K. Johansson. *"Cost effective deployment strategies for heterogeneous wireless networks"*. dissertation, KTH Information and Communication Technology, Stockholm, Sweden, November 2007.

[12] H. Claussen, L. T. W. Ho. and L. G. Samuel. "Financial analysis of a pico-cellular home network deployment". *in the Proc. IEEE International Communications Conference (ICC), Glasgow, Scotland*, June 2007.

[13] E. F. T. Martijn and M. H. A. J. Herben. "Characterization of radio wave propagation into buildings at 1800 mhz". *Ph.D. dissertation.*, 2:122–125, 2003.

[14] G. W. R. Hoppe and F. M. Landstorfer. "Measurement of building penetration loss and propagation models for radio transmission into buildings". *IEEE Veh. Technol. Conf.*, pages 2298-2302, Sept. 1999.

[15] V. Chandrasekhar, J. G. Andrews, and A. Gatherer. "FemtoCell networks: A Servey". *IEEE Comm. Magazine.*, 46(9):59-67, September 2008.

[16] V. Mancuso and S. Alouf. "Reducing costs and pollution in cellular networks". *To appear in IEEE Communications Magazine, special issue on Green Communications.*, June 2010.

[17] S.-H. Park, O. Simeone, O. Sahin and S. Shamai (Shitz), "Fronthaul compression for cloud radio access networks," *IEEE Sig. Processing Mag., Special Issue on Signal Processing for the 5G Revolution*, vol. 31, no. 6, pp. 69-79, Nov. 2014.

[18] E. Altman, C. Hasan, J.-M. Gorce and L. Roullet, "Green networking: Downlink considerations," *5th International Conference on Network Games, Control and Optimization (NetGCooP)*, pp. 1-4, Paris, 12-14 Oct. 2011.

[19] D. Tsilimantos, J.-M. Gorce, E. Altman, "Stochastic analysis of energy savings with sleep mode in OFDMA wireless networks," *INFOCOM 2013*.

[20] C. Hasan, E. Altman, E. and J.-M. Gorce, "The coalitional switch-off game of service providers," *in IEEE 9th International Conference on Wireless and Mobile Computing, Networking and Communications (WiMob) 2013*, pp. 223-230, 7-9 Oct. 2013.

[21] P. Glasserman and D. D. Yao. "Stochastic vector difference equations with stationary coefficients". *Journal of Applied Probability.*, 32(4):851–866, December 1995.

[22] R. Karasik, O. Simeone and S. Shamai (Shitz), "Robust uplink communications over fading channels with variable backhaul connectivity," *IEEE Trans. Wireless Comm.*, vol. 12, no. 11, pp. 5788-5799, Nov. 2013.

Appendix A: WhittakerM Function

WhittakerM function arises as one of the solution to the following differential equation

$$W'' + \left(\frac{-1}{4} + \frac{k}{z} + \frac{\frac{1}{4} - m^2}{z^2} \right) W = 0,$$

where $W' := dW/dz$, and it is defined as the following hypergeometric series

$$\text{WhittakerM}(k, m, z) = z^{m+1/2} \exp\{-z/2\} \sum_{n=0}^{\infty} \frac{(m-k+1/2)_n}{n!(2m+1)_n} z^n,$$

where $(m)_n := m(m+1) \cdots (m+n-1)$.

Appendix B: LommelS2 Function

LommelS2 function arises as one of the solution to the following differential equation

$$z^2 W'' + zW' + (z^2 - \nu^2) z = z^{\mu+1},$$

where $W' := dW/dz$.