Error Exponents Analysis of Dual-hop \eta-\mu and \kappa-\mu Fading Channel with Amplify-and-Forward Relaying

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Abstract: In this study, the authors investigate the Gallager's error exponents of dual-hop amplify-and-forward systems over generalised η-μ and κ-μ fading channels, two versatile channel models which encompass a number of popular fading channels such as Rayleigh, Rician, Nakagami-*m*, Hoyt and one-sided Gaussian fading channels. The authors present new analytical expressions for the probability density function of the end-to-end signal-to-noise-ratio (SNR) of the system. These analytical expressions are then applied to analyse the system performance through the study of Gallager's exponents, which are classical tight bounds of error exponents and present the tradeoff between practical information rate and the reliability of communication. Two types of Gallager's exponents, namely, random coding error exponent and expurgated error exponent, are studied. Based on the newly derived analytical expressions, the authors provide an efficient method to compute the required codeword length to achieve a predefined upper bound of error probability. In addition, the analytical expressions are derived for the cutoff rate and ergodic capacity of the system. Moreover, simplified expressions are presented at the high SNR regime. All the analytical results are verified via Monte–Carlo simulations.

1 Introduction

The dual-hop relaying technique [1, 2] has triggered enormous research interests in academia [3–5], because it can provide huge improvement on throughput, coverage and energy consumption of the wireless communication systems with low extra cost. Among various relaying protocols proposed in the literature, such as amplify-and-forward (AF), decode-and-forward and compress-and-forward protocols, the AF protocol, where an intermediate relay simply amplifies the received signal and forward the scaled signal to the destination, has gained significant interest because of its simplicity and low implementation cost.

As such, a great deal of works have appeared in the literature to study the performance of dual-hop AF relaying systems. Thus far, the performance of dual-hop AF relaying systems is in general well-understood in popular fading channels, such as Rayleigh, Rician, Nakagami-m and Weibull fading channels. For instance, the outage performance of AF relaying systems was studied in [3] for Rayleigh fading channels, and in [4] for Nakagami-m fading channels. While the bit error rate of dual-hop AF relaying transmission was analysed in [6] for Rayleigh fading channel and in [7] for Nakagami-m fading channel. The ergodic capacity of dual-hop AF relaying systems over Nakagami-m channels with different heuristic precoding schemes was analysed in [8].

Although these works have profoundly improved our understanding on the achievable performance of dual-hop AF relaying systems, a major limitation is that all these works are based on channel models which rely on the key assumption that the scatters in the propagation environment are homogeneously distributed. Therefore the results predicted by these prior works may fail to give an accurate performance account of practical scenarios where the scatters are non-homogeneous. Responding to these, versatile channel models, namely the generalised κ-μ and η-μ distributions were proposed in [9]. Since which, the performance of generalised κ-μ and η-μ fading models in various important communication systems has been examined, see [10] and references therein. Despite its importance, there are very few works have investigated the performance of dual-hop AF relaying systems over generalised κ-μ and η-μ fading channels. Only in [11] the moment-generating functions (MGF) of the dual-hop AF relaying systems over η-μ and κ-μ fading channels were derived, the outage probabilities and average bit error probabilities were then derived as the application of the MGF. And in a recent work [12], the outage probability of dual-hop AF relaying systems over mixed κ-μ and η-μ fading channels was investigated. Therefore there is an intense demand to understand the important performance measures such as ergodic capacity and error rate of dual-hop AF relaying systems in generalised κ-μ and η-μ fading channels.

Motivated by this, in this paper, we present a thorough investigation of the error exponent of dual-hop AF systems in generalised κ-μ and η-μ fading channels. The error exponent was defined by Shannon in 1959 as

$$E(R) := \limsup_{L\to\infty} \frac{-\ln P_e^{\text{opt}}(R, L)}{L} \tag{1}$$

where $P_e^{\text{opt}}(R, L)$ is the average error probability of the optimal code of length L and rate R. It provides key insight into the tradeoff between information rate and the reliability in wireless communication systems, and can also help with the evaluation of the required codeword length given a predefined P_e. Owing to the difficulty of deriving exact expression for the error exponent, Gallager introduced the random coding error exponent (RCEE) and expurgated error exponent [13] as computable tight upper bounds of the error exponent. Since which, the Gallager's exponents, as a key performance indicator, have been widely studied in literature [14–20]. In the context of dual-hop AF relaying systems, the error exponents have been studied in [21, 22] for Rayleigh and Nakagami-m fading channels, respectively.

The main contribution of the paper can be summarised as follows:

- We derive the exact analytical expressions of the probability density function (p.d.f.) of end-to-end signal-to-noise-ratio (SNR) for dual-hop AF relaying system over generalised η-μ and κ-μ fading channels.
- Based on the analytical p.d.f. expression, numerical expressions were derived for the RCEE, cutoff rate, ergodic capacity and expurgated error exponent of dual-hop AF relaying systems. Simplified analytical expressions of the RCEE, cutoff rate and expurgated error exponent are provided at high SNR regime, which significantly reduces the computation time.
- With the help of the analytical expression of the RCEE, we characterise the required codeword length to achieve the predefined upper bound of error probability.
- It is notable that our analytical expressions include the previous results on Rayleigh, Rician and Nakagami-m fading channels [22] as special cases. Moreover, the presented results can be extended to Hoyt and one-sided Gaussian fading channels as well which have not been reported elsewhere.

Following notations are adopted throughout this paper. Matrices and vectors are denoted by bold uppercase and bold lowercase letters, respectively. $\boldsymbol{I}_n$ denotes the $n \times n$ identity matrix. $(\cdot)^\dagger$ denotes the conjugate transpose of a matrix or vector. $\lceil \cdot \rceil$ denotes the smallest integer larger than or equal to an enclosed quantity. $\mathbb{C}^{n\times m}$ denotes the set of $n \times m$ complex matrices. $\mathcal{CN}(\mu, \sigma^2)$ denotes the circularly symmetric complex Gaussian distribution with mean μ and variance σ^2. $\mathrm{tr}(\cdot)$ and $\det(\cdot)$ denote the trace and determinant of a matrix. $\mathbb{E}[\cdot]$ and $\ln(\cdot)$ denote the expectation operation and natural logarithm, respectively.

The rest of this paper is organised as follows: Section 2 introduces the system model and Gallager's error exponents. Section 3 presents the derivations of RCEE, cutoff rate, expurgated error exponent and ergodic capacity of the system, the asymptotic expressions are proposed and evaluated as well. Finally, Section 4 concludes the paper.

2 System model

2.1 Dual-hop AF system

In this paper, we consider a single antenna dual-hop AF relaying system. Assuming block fading, such that the channel remains unchanged during T_c symbols. For N_b independent coherence intervals, the block codeword length is $N_b \times T_c$ symbol periods. Moreover, it is assumed that there is no direct communication link between the transmitter and receiver. Hence, the received signal over the $T_c \times N_b$ symbol periods is given by

$$\mathbf{y}_k = h_2 G(h_1 \mathbf{x}_k + \mathbf{n}_{1,k}) + \mathbf{n}_{2,k}, \quad k = 1, 2, \ldots, N_b \tag{2}$$

where $\mathbf{x}_k \in \mathbb{C}^{1\times T_c}$ and $\mathbf{y}_k \in \mathbb{C}^{1\times T_c}$ denote the transmitted and received signal sequences. h_1 and h_2 denote the fading coefficients of the first and second hops, respectively. G is the power gain of relay. $\mathbf{n}_i \sim \mathcal{CN}_{1,T_c}(0, N_0)$ $(i = 1, 2)$ are the relay and destination additive white Gaussian noise vectors.

Therefore the end-to-end SNR is given by

$$\gamma_{\mathrm{end}} = \frac{\left(|h_1|^2/N_0\right)\left(|h_2|^2/N_0\right)}{\left(|h_2|^2/N_0\right) + \left(1/(G^2 N_0)\right)} \tag{3}$$

Assuming the ideal relaying gain, that is, $G^2 = (1/|h_1|^2)$. [Please note, the adoption of the ideal relaying gain is mainly for analytical tractability. Nevertheless, it can serve as a tight upper bound for practical relaying gain such as $G^2 = (1/(|h_1|^2 + N_0))$. Owing to this, it has been widely used in the literature, for instance [3, 4, 22].], the end-to-end SNR at the destination is given by

$$\gamma_{\mathrm{end}} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \tag{4}$$

where $\gamma_1 = (|h_1|^2/N_0)$ and $\gamma_2 = (|h_2|^2/N_0)$.

2.2 Gallager's exponents

2.2.1 Random coding error exponent: In [13], the upper bound of average error probability for discrete-time memoryless channel with maximum likelihood decoding is defined as

$$P_{\mathrm{e,m}} \le \left[\frac{2\,\mathrm{e}^{r\delta}}{\xi}\right]^2 \exp\{-T_c N_b [E_r(\rho, p_{\mathbf{x}}(\mathbf{x}), r)]\} \tag{5}$$

where $p_{\boldsymbol{x}}(\boldsymbol{x})$ is the input source distribution and parameters ρ, r, δ are arbitrarily selected from regimes $0 \le \rho \le 1$, $r \ge 0$ and $\delta \ge 0$. Coefficients ξ and σ are defined as

$$\xi \simeq \frac{\delta}{\sqrt{2\pi N_b \sigma_\xi^2}} \tag{6}$$

$$\sigma_\xi^2 = \int_x [\mathrm{tr}(\mathbf{x}^\dagger \mathbf{x}) - T_c \mathcal{P}] p_{\mathbf{x}}(\mathbf{x})\,\mathrm{d}x \tag{7}$$

where $\mathcal{P}$ is the power constraint. The RCEE [13, 5.6.16] of SISO system is derived as

$$E_r(p_x(x), R, T_c) = \max_{0\le\rho\le1}\left\{\max_{r\ge0} E_0(p_{\mathbf{x}}(\mathbf{x}), \rho, r, T_c) - \rho R\right\} \tag{8}$$

The element $E_0(p_{\boldsymbol{x}}(\boldsymbol{x}), \rho, r, T_c)$ is defined by (see equation (9) at bottom of the page)

where $p_h(h)$ denotes the distribution of channel coefficient h. We denote $E_0(p_{\boldsymbol{x}}(\boldsymbol{x}), \rho, r, T_c)$ and $E_r(p_{\boldsymbol{x}}(\boldsymbol{x}), R, T_c)$ as $\tilde{E}_0(\rho, \beta, T_c)$ and $E_r(R, T_c)$, respectively. Therefore $\tilde{E}_0(\rho, \beta, T_c)$ is given by Shin and Win [23]

$$\tilde{E}_0(\rho, \beta, T_c) = \mathcal{A}(\rho, \beta) - \frac{1}{T_c}\ln\left\{\mathbb{E}\left[\left(1 + \frac{\gamma|h|^2}{\beta(1+\rho)}\right)^{-T_c\rho}\right]\right\} \tag{10}$$

where $\mathcal{A}(\rho, \beta) = (1+\rho)(1-\beta) + (1+\rho)\ln(\beta)$ and $\gamma = (\mathcal{P}/N_0)$ is the effective end-to-end SNR. With the variance of Lagrange multiplier r, the parameter $\beta = 1 - r\mathcal{P}$ is restricted to $0 \le \beta \le 1$. The RCEE can be rewritten as

$$E_r(R, T_c) = \max_{0\le\rho\le1}\left\{\max_{0\le\beta\le1} \tilde{E}_0(\rho, \beta, T_c) - \rho R\right\} \tag{11}$$

2.2.2 Ergodic capacity: According to Ahmed and McLane [24], expression (10) is a convex function of β in range $0 \le \beta \le 1$. We know that there always exits $\beta = \hat{\beta}$ for $0 \le \rho \le 1$ to maximise $E_0(\rho, \beta, T_c)$, where $\hat{\beta}$ is the result of equation $((\partial E_0(\rho, \beta, T_c))/\partial\rho) = 0$.

$$E_0(p_{\mathbf{x}}(\mathbf{x}), \rho, r, T_c) \triangleq -\frac{1}{T_c}\ln\left(\int_h p_h(h)\int_{\mathbf{y}}\left(\int_{\mathbf{x}} p_{\mathbf{x}}(\mathbf{x})\,\mathrm{e}^{r[\mathrm{tr}(\mathbf{x}\mathbf{x}^\dagger) - T_c\mathcal{P}]} p(\mathbf{y}|\mathbf{x}, h)^{(1/(1+\rho))}\mathrm{d}\mathbf{x}\right)^{(1+\rho)}\mathrm{d}\mathbf{y}\,\mathrm{d}h\right) \tag{9}$$

The information rate R in expression (11) can be derived by

$$R = \frac{\partial E_0(\rho, \beta, T_c)}{\partial \rho}|_{\beta=\hat{\beta}} \quad (12)$$

By setting $\rho=0$ and the related $\hat{\beta}|_{\rho=0} = 1$ in (12), the ergodic capacity is obtained as

$$\langle C \rangle = \frac{\partial E_0(\rho, \beta, T_c)}{\partial \rho}|_{\rho=0,\beta=1} \quad (13)$$

2.2.3 Cutoff rate: Cutoff rate is an important parameter to evaluate channel capacity. As a lower bound of Shannon capacity, cutoff rate gives the practical limit of symbol rate for sequential decodings. Owing to the low complexity of calculation and the practicality in information rate evaluation, cutoff rate is frequently applied to analyse the system performance such as different coding and modulation schemes [25]. According to Shin and Win [23], cut-off rate can be derived as an extension of RCEE from (10) by setting $\rho=1$ and $\hat{\beta} = 1$ as

$$R_0 = E_0(p_{\mathbf{x}}(\mathbf{x}), 1, 1, T_c) = -\frac{1}{T_c}\ln\left\{\mathbb{E}\left[\left(1+\frac{\gamma h^2}{2}\right)^{-T_c}\right]\right\} \quad (14)$$

2.2.4 Expurgated error exponent: In random coding regime, good and bad codes are collected unbiasedly. Therefore both of them contribute the dominant error probability. A compendious improvement is to expurgate the bad codewords from the codewords ensemble, which can improve the error exponent obviously at low rate regime.

Developed by Gallager [13], Xue *et al.*, [16], the expurgated error exponent is defined as

$$E_{ex}(R, T_c) = \max_{\rho\geq 1}\left\{\max_{0\leq\beta\leq 1} E_x(R, T_c) - \rho R\right\} \quad (15)$$

in which

$$E_x(R, T_c) = \mathcal{A}'(\rho, \beta) - \frac{1}{T_c}\ln\left\{\mathbb{E}\left[\left(1+\frac{\gamma|h|^2}{2\beta\rho}\right)^{-T_c\rho}\right]\right\} \quad (16)$$

where $\mathcal{A}'(\rho, \beta) = 2\rho(1-\beta) + 2\rho\ln(\beta)$.

3 Gallager's exponents of dual-hop AF system over η-μ and κ-μ fading channels

This section presents a detailed performance investigation of the dual-hop AF relaying systems in generalised η-μ and κ-μ fading channels through the characterisation of the Gallager's exponents. It starts under the η-μ fading channels.

3.1 η-μ channel

The generalised η-μ distribution is normally used to analyse the performance of small-scale signals in a non-line-of-sight environment. The p.d.f. of η-μ distribution is given by Yacoub [9]

$$f_{\gamma_{\eta-\mu}}(\gamma) = \frac{2\sqrt{\pi}\mu^{\mu+(1/2)}\phi^{\mu}\gamma^{\mu-(1/2)}\exp\left(-(2\mu\gamma\phi/\bar{\gamma})\right)}{\Gamma(\mu)\psi^{\mu-(1/2)}\bar{\gamma}^{\mu+(1/2)}} I_{\mu-(1/2)}\left(\frac{2\mu\gamma\psi}{\bar{\gamma}}\right) \quad (17)$$

where μ denotes the half number of multipath clusters, $\bar{\gamma}$ is the average power and $I_{\alpha}(\cdot)$ is the αth order of modified Bessel function of the first kind. Parameters ψ and ϕ are defined differently in two formats:

Format 1: η denotes the scattered-wave power ratio between the in-phase and quadrature components in each multipath cluster and $0<\eta<\infty$, then $\psi=((\eta^{-1}-\eta)/4)$ and $\phi=((2+\eta^{-1}+\eta)/4)$.

Format 2: η is the correlation coefficient between the in-phase and quadrature components in each multipath cluster and $-1<\eta<1$, then $\psi=(1/(1-\eta^2))$ and $\phi=(\eta/(1-\eta^2))$. In this paper, we use the definition of Format 1.

Theorem 1: The p.d.f. of end-to-end SNR of dual-hop AF relaying systems in η-μ fading channels is given by (see (18))

where

$$G_{p,q}^{m,n}\left[x\Big|\begin{matrix}a_1, \ldots, a_p\\ b_1, \ldots, b_q\end{matrix}\right]$$

is the Meijer's G-function [26].

Proof: Since the two channels before and after relay are i.i.d., we obtain

$$p_{\gamma_1,\gamma_2}(\gamma_1, \gamma_2) = p_{\gamma_1}(\gamma_1)p_{\gamma_2}(\gamma_2) \quad (19)$$

Letting $s=\gamma_1+\gamma_2$, $t=\gamma_1\gamma_2$, and using the first Jacobian transformation, (19) can be written as

$$p_{s,t}(s, t) = p_{\gamma_1,\gamma_2}(\gamma_1(s, t), \gamma_2(s, t))\left|\frac{\partial(\gamma_1, \gamma_2)}{\partial s, \partial t}\right| \quad (20)$$

(see (21))

After expanding modified Bessel functions of the first kind to the summations of infinite series and subsequently making two binomial expansions, $p_{s,t}(s, t)$ can be rewritten as (see equation (22) at bottom of the next page)

With another Jacobian transform, we can obtain the expression of $p_{s,x}(s, x)$ as (see equation (23) at the bottom of the next page)

where $x=(t/s)$. Using [27, Sec. 6.2], we can further avoid variable s and derive the p.d.f. of the joint channel (see equation (24) at the bottom of the next page)

where $W_{k,\lambda}(\cdot)$ is the Whittaker-W function [26, 9.222], and the integral is achieved with the aid of [26, 3.383.4].

$$f_x^{\eta-\mu}(x) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{2m}\sum_{b=0}^{2n}\frac{(-1)^b 2^{6-8\mu-4m-4n}\pi\bar{\gamma}^{-1}\mu\psi^{2m+2n}(m!n!)^{-1}}{\Gamma(\mu)^2\Gamma\left(m+\mu+(1/2)\right)\Gamma\left(n+\mu+(1/2)\right)\phi^{2\mu+2m+2n-1}}\binom{2m}{a}\binom{2n}{b}$$
$$\times\Gamma\left(m+n-\frac{a}{2}-\frac{b}{2}+\frac{1}{2}\right)G_{1,2}^{2,0}\left[\frac{8\mu\phi x}{\bar{\gamma}}\Big|\begin{matrix}2\mu+m+n-\frac{a}{2}-\frac{b}{2}-\frac{1}{2}\\ 2\mu-1, 4\mu+2m+2n-1\end{matrix}\right] \quad (18)$$

$$= \frac{4\pi\mu^{2\mu+1}\phi^{2\mu}t^{\mu-(1/2)}e^{-(2\mu\phi s/\bar{\gamma})}}{\Gamma(\mu)^2\psi^{2\mu-1}\bar{\gamma}^{2\mu+1}} I_{\mu-(1/2)}\left(\frac{\mu\psi\left(s+\sqrt{s^2-4t}\right)}{\bar{\gamma}}\right) I_{\mu-(1/2)}\left(\frac{\mu\psi\left(s-\sqrt{s^2-4t}\right)}{\bar{\gamma}}\right)\left|\frac{1}{\sqrt{s^2-4t}}\right| \quad (21)$$

With the help of [28, 13.1.33], (24) is reconfigured in the form of confluent hypergeometric function as (see equation (25) at bottom of the page)

By the end, Theorem 1 is obtained according to the relationship between confluent hypergeometric function and Meijer's G-function [29, 07.34.03.0612.01]. □

Although this analytical expression involves infinite-term summations, the actual evaluation converges fairly quick as shown in Fig. 1 [Unless otherwise noted, all the Monte-carlo simulations in our work are based on 1 million running.]. It can be observed that the series related to the p.d.f. of dual-hop $\eta-\mu$ fading converge faster than those of dual-hop $\kappa-\mu$ fading. According to our calculation, the difference between the finite-term analytical results and the simulation results with different η, μ and γ is quite small. When $m=n=5$, $f_{\text{sim}}(\gamma)-f_{m=n=5}(\gamma)<10^{-4}$ in everywhere, and when $m=n=10$, $f_{\text{sim}}(\gamma)-f_{m=n=10}(\gamma)<10^{-6}$. Similarly for the dual-hop $\kappa-\mu$ fading channel, $f_{\text{sim}}(\gamma)-f_{m=n=10}(\gamma)<10^{-4}$ and $f_{\text{sim}}(\gamma)-f_{m=n=15}(\gamma)<10^{-6}$. It is clear in Fig. 1, the whole curves of finite-term analytical results are matching the simulation results well when $m=n\geq 5$ for $\eta-\mu$ distribution and $m=n\geq 10$ for $\kappa-\mu$ distribution. In addition, the analytical p.d.f. expression is amenable to further mathematical manipulations, and enables the characterisation of the key performance indicator, that is, the Gallager's exponents.

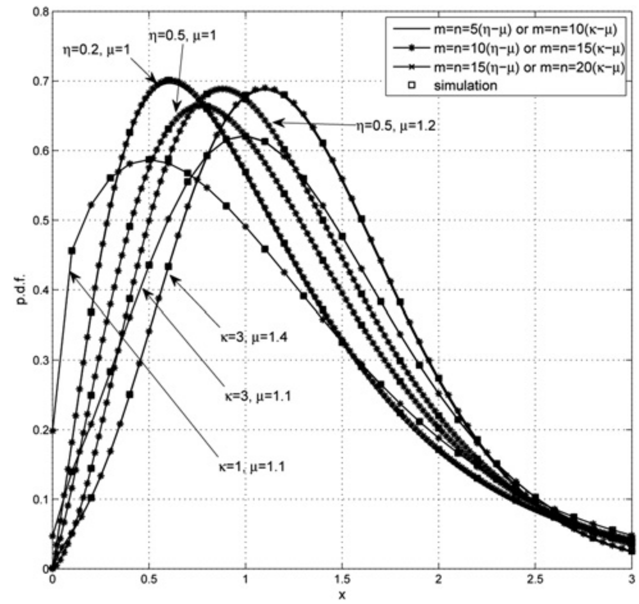

Fig. 1 *pdf of end-to-end SNR over η-μ and κ-μ fading with* $\bar{\gamma}=3$

3.1.1 Random coding error exponent: Based on the definition and mathematical model of RCEE given in the previous section, the analytical expression of RCEE of dual-hop AF relaying system over η-μ fading channel is derived as follows.

Proposition 1: The analytical expression of RCEE of dual-hop AF relaying systems in η-μ fading channels can be expressed as (see equation (26) at the bottom of the next page)

Proof: See Appendix 1.

To reduce the computation time of Meijer's G-function, we look into the high SNR regime, and present an accurate approximation consisting of the simple Gamma functions.

Corollary 1: The RCEE of dual-hop AF η-μ SISO fading channel at high SNR regime is given by (see equation (27) at bottom of the next page)

where

$$Z_1=\left(\frac{8\mu\phi\beta(1+\rho)}{\bar{\gamma}\gamma}\right)$$

$$p_{s,t}(s,t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{2m}\sum_{b=0}^{2n}\frac{(-1)^b 2^{2-2m-2n}\pi\mu^{2m+2n+4\mu}\phi^{2\mu}\psi^{2m+2n}\binom{2m}{a}\binom{2n}{b}}{\bar{\gamma}^{4\mu+2}m!n!\Gamma(\mu)^2\Gamma\big(m+\mu+(1/2)\big)\Gamma\big(n+\mu+(1/2)\big)}t^{2\mu-1}\exp\left(-\frac{2\mu\phi s}{\bar{\gamma}}\right)$$
$$\times s^{2m+2n-a-b}(s^2-4t)^{(a/2)+(b/2)-(1/2)} \qquad (22)$$

$$p_{s,x}(s,x)=p_{s,t}(s,sx)|s|=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{2m}\sum_{b=0}^{2n}\frac{(-1)^b 2^{2-2m-2n}\pi\mu^{2m+2n+4\mu}\phi^{2\mu}\psi^{2m+2n}\binom{2m}{a}\binom{2n}{b}}{\bar{\gamma}^{4\mu+2m+2n}m!n!\Gamma(\mu)^2\Gamma\big(m+\mu+(1/2)\big)\Gamma\big(n+\mu+(1/2)\big)}$$
$$\times x^{2\mu-1}\exp\left(-\frac{2\mu\phi s}{\bar{\gamma}}\right)s^{m+n+2\mu+(a/2)+(b/2)-(1/2)}(s-4x)^{m+n-(a/2)-(b/2)-(1/2)} \qquad (23)$$

$$f_x^{\eta-\mu}(x)=\int_{-\infty}^{\infty}f_{s,x}(s,x)\,\mathrm{d}s=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{2m}\sum_{b=0}^{2n}\left[\frac{(-1)^b 2^{(3/2)+\mu-m-n}\pi\bar{\gamma}^{-m-n-3\mu+(1/2)}\Gamma\big(m+n-(a/2)-(b/2)+(1/2)\big)\binom{2m}{a}\binom{2n}{b}}{\mu^{(1/2)-m-n-3\mu}\phi^{(1/2)-\mu+m+n}\psi^{-2m-2n}\Gamma(\mu)^2\Gamma\big(m+\mu+(1/2)\big)\Gamma\big(n+\mu+(1/2)\big)m!n!}\right.$$
$$\left.\times x^{3\mu+m+n-(3/2)}\exp\left(-\frac{4\mu\phi x}{\bar{\gamma}}\right)W_{\mu+(a/2)+(b/2),-m-n-\mu}\left(\frac{8\mu\phi x}{\bar{\gamma}}\right)\right] \qquad (24)$$

$$f_x^{\eta-\mu}(x)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{2m}\sum_{b=0}^{2n}\frac{(-1)^b 2^{3-4m-4n-2\mu}\pi\bar{\gamma}^{-2\mu}\mu^{2\mu}\psi^{2m+2n}\binom{2m}{a}\binom{2n}{b}\Gamma\big(m+n-(a/2)-(b/2)+(1/2)\big)}{\phi^{2m+2n}\Gamma\big(n+\mu+(1/2)\big)m!n!\Gamma(\mu)^2\Gamma\big(m+\mu+(1/2)\big)}$$
$$\times x^{2\mu-1}\exp\left(-\frac{8\mu\phi x}{\bar{\gamma}}\right)U_{(1/2)-2\mu-m-n-(a/2)-(b/2),1-2m-2n-2\mu}\left(\frac{8\mu\phi x}{\bar{\gamma}}\right) \qquad (25)$$

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Proof: See Appendix 1.

The asymptotic expression (27) consisting of a group of Gamma functions consumes fairly less time for calculating than the analytical expression (26) containing Meijer's G-function. As well known, a Meijer's G-function is a general numeric integral result for a group of Gamma functions. That means, if a general numeric integral consists of K multiplying calculation, the calculation of asymptotical expressions should spend around only $(1/K)$ time of the analytical expressions. The accuracy of the simplification (27) is presented in Fig. 3.

3.1.2 Expurgated error exponent:

Corollary 2: For the expurgated error exponent of dual-hop AF relaying systems in η-μ fading channels, the analytical expression and the simplified approximation at high-SNR regime are given by (see equation (28) at the bottom of the page)

and (see equation (29) at the bottom of the next page)

respectively, where

$$Z_2 = \left(\frac{16\mu\phi\beta\rho}{\bar{\gamma}\gamma}\right)$$

Proof: See Apendix 2.

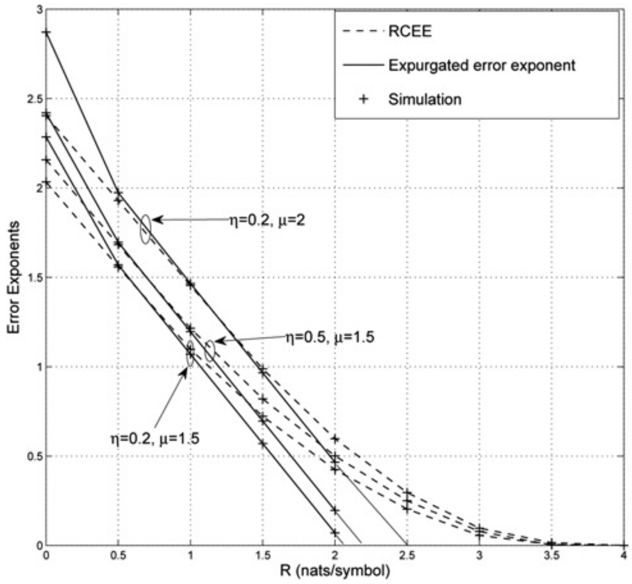

Fig. 2 *Simulation and analytical results for the RCEE and expurgated error exponent of dual-hop AF η-μ fading SISO channel for selected value of η and μ with γ = 15 dB, $\bar{\gamma}$ = 3 and T_c = 5*

Fig. 2 compares the performance of RCEE $\left(E_{\rm r}^{\eta-\mu}(R, T_{\rm c})\right)$ and expurgated error exponent $\left(E_{\rm ex}^{\eta-\mu}(R, T_{\rm c})\right)$ as a function of information rate R with different values of η and μ. Please note, unless otherwise specified, we use $\bar{\gamma} = 3$ and $T_{\rm c} = 5$ in all simulations. As expected, larger R always means lower error exponents and the expurgated error exponent shows more advantages than the RCEE at low R regime. It should be noted that both parameters η and μ improve the error exponents performance when increasing, but the impact of μ on the error exponents is more significant than η.

$$E_r(R, T_{\rm c}) = \max_{0\le\rho\le1}\left\{\max_{0\le\beta\le1}\left\{\mathcal{A}(\rho,\beta) - \frac{1}{T_{\rm c}}\ln\left\{\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{2m}\sum_{b=0}^{2n}\frac{(-1)^b\pi\mu\psi^{2m+2n}\Gamma\big(m+n-(a/2)-(b/2)+(1/2)\big)}{\phi^{2m+2n+2\mu-1}\bar{\gamma}\Gamma(T_{\rm c}\rho)\Gamma(\mu)^2\Gamma\big(m+\mu+(1/2)\big)} \times \frac{2^{6-8\mu-4m-4n}\binom{2m}{a}\binom{2n}{b}\beta(1+\rho)}{\Gamma\big(n+\mu+(1/2)\big)m!n!\gamma} G^{3,1}_{2,3}\left[\frac{8\mu\phi\beta(1+\rho)}{\bar{\gamma}\gamma}\middle| \begin{matrix} 0,\ 2\mu+m+n-\frac{a}{2}-\frac{b}{2}-\frac{1}{2} \\ 2\mu-1,\ 4\mu+2m+2n-1,\ T_{\rm c}\rho-1\end{matrix}\right]\right\}\right\} - \rho R\right\} \tag{26}$$

$$E_r^{\infty}(R, T_{\rm c}) = \max_{0\le\rho\le1}\left\{\max_{0\le\beta\le1}\left\{\mathcal{A}(\rho,\beta). - \frac{1}{T_{\rm c}}\ln\left\{\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{2m}\sum_{b=0}^{2n}\frac{(-1)^b 2^{6-8\mu-4m-4n}\pi\mu\psi^{2m+2n}}{\bar{\gamma}\phi^{2m+2n+2\mu-1}\Gamma(T_{\rm c}\rho)\Gamma(\mu)^2\Gamma\big(m+\mu+(1/2)\big)} \times \frac{\binom{2m}{a}\binom{2n}{b}\beta(1+\rho)\Gamma\big(m+n-(a/2)-(b/2)+(1/2)\big)}{m!n!\gamma\Gamma\big(n+\mu+(1/2)\big)}\left(\frac{\Gamma(2\mu+2m+2n)\Gamma(T_{\rm c}\rho-2\mu)}{\Gamma(2\mu)^{-1}\Gamma\big(m+n-(a/2)-(b/2)+(1/2)\big)}Z_1^{2\mu-1} + \frac{\Gamma(-2\mu-2m-2n)\Gamma(4\mu+2m+2n)}{\Gamma(T_{\rm c}\rho-4\mu-2m-2n)^{-1}\Gamma\big(-2\mu-m-n-(a/2)-(b/2)+(1/2)\big)}Z_1^{4\mu+2m-2n-1} + \frac{\Gamma(2\mu-T_{\rm c}\rho)\Gamma(4\mu+2m+2n-T_{\rm c}\rho)\Gamma(T_{\rm c}\rho)}{\Gamma\big(2\mu+m+n-(a/2)-(b/2)-T_{\rm c}\rho+(1/2)\big)}Z_1^{T_{\rm c}\rho-1}\right)\right\}\right\} - \rho R\right\} \tag{27}$$

$$E_{\rm ex}(R, T_{\rm c}) = \max_{\rho\ge1}\left\{\max_{0\le\beta\le1}\left\{\mathcal{A}'(\rho,\beta) - \frac{1}{T_{\rm c}}\ln\left\{\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{2m}\sum_{b=0}^{2n}\frac{(-1)^b 2^{7-8\mu-4m-4n}\pi\mu\psi^{2m+2n}}{\bar{\gamma}\phi^{2m+2n+2\mu-1}\Gamma(T_{\rm c}\rho)\Gamma(\mu)^2\Gamma\big(m+\mu+(1/2)\big)} \times \frac{\binom{2m}{a}\binom{2n}{b}\beta\rho\Gamma\big(m+n-(a/2)-(b/2)+(1/2)\big)}{m!n!\gamma\Gamma\big(n+\mu+(1/2)\big)} G^{3,1}_{2,3}\left[\frac{16\mu\phi\beta\rho}{\bar{\gamma}\gamma}\middle| \begin{matrix} 0,\ 2\mu+m+n-\frac{a}{2}-\frac{b}{2}-\frac{1}{2} \\ 2\mu-1,\ 4\mu+2m+2n-1,\ T_{\rm c}\rho-1\end{matrix}\right]\right\}\right\} - \rho R\right\} \tag{28}$$

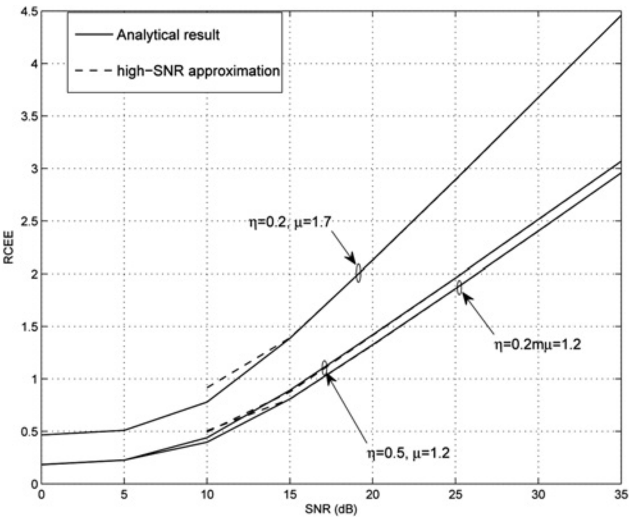

Fig. 3 *Analytical results for the RCEE and high-SNR approximation of dual-hop AF η-μ fading SISO channel for selected value of η and μ with R = 2, $\bar{\gamma} = 3$ and $T_c = 5$*

Fig. 3 illustrates analytical results and approximation at high SNR regime of RCEE with selected parameters η and μ when $R = 2$. As clear seen, the RCEE increases with SNR, η and μ as expected. Moreover the simplified approximation results work perfectly when $\gamma \geq 15$ dB.

Table 1. shows the effects of parameters η and μ on the required code length of dual-hop AF SISO systems over η-μ fading channel with different SNR. The code length is estimated to achieve the predefined upper bound of error probability $P_e^{E_r} = 10^{-6}$ at the information rate $R = 2$. The required code length $L = T_c \times \lceil N_b \rceil$ is obtained by solving the following equation of N_b [24]

$$P_e^{E_r} = \frac{8\pi e^2 (1-\beta)^2 N_b}{T_c} \times \exp\{-N_b T_c E_r(R, T_c)\} \tag{30}$$

We see that both the increasing of parameters η and μ decrease the required code length, and the effect of μ is more significant than η. For instance, when SNR = 10 dB and $\eta = 0.2$, the required codeword length decreases 44.27% as μ increases from 1.5 to 2.0. However, the required codeword length decreases 29.17% as η increases from 0.2 to 0.5 when $\mu = 1.5$.

Table 1 Required code lengths L_r for dual-hop AF η-μ fading SISO channel to achieve the predefined upper bound of error probabilities $P_e^r = 10^{-6}$ with $R = 2$, $\bar{\gamma} = 3$ and $T_c = 5$

SNR, dB	$\eta = 0.2$ $\mu = 1.5$	$\eta = 0.2$ $\mu = 2$	$\eta = 0.5$ $\mu = 1.5$	$\eta = 0.5$ $\mu = 2$
10	384	214	272	185
15	42	29	35	25
30	17	12	15	5

3.1.3 Cutoff rate

Corollary 3: The cutoff rate R_0 of dual-hop AF relaying systems in η-μ fading channels is derived as (see (31))

It should be noted that the parameter T_c in formula (31) is inversely proportional to the cutoff rate, which indicates how communication reliability impacts the information rate. This trend is presented more intuitively in Fig. 4.

Proof: See Appendix 3.

Corollary 4: The simplified approximation of cutoff rate of dual-hop AF relaying systems in η-μ fading channels at high SNR regime is given by (see equation (32) at the bottom of the next page)

where

$$Z_3 = \left(\frac{16\mu\phi}{\bar{\gamma}\gamma}\right)$$

Proof : See Appendix 3.

Fig. 4 represents the cutoff rate R_0 with selected parameters η and μ with $T_c = 5$ and $\bar{\gamma} = 3$. It shows that the cutoff rate increases with SNR, η and μ, but the effect of μ on the value of cutoff rate is more pronounced than η. The same as in Fig. 3, when $\gamma \geq 15$ dB, the asymptotic expression could describe cutoff rates accurately enough. In addition, the curves represent the tradeoff between information rate and reliability of block-coding communication by showing the cutoff rate decreases with T_c.

3.1.4 Ergodic capacity

Corollary 5: The ergodic capacity of dual-hop AF relaying systems in η-μ fading channels can be expressed as (see equation (33) at bottom of the next page)

$$\begin{aligned} E_{\text{ex}}^{\infty}(R, T_c) = \max_{\rho \geq 1}\Bigg\{ \max_{0\leq\beta\leq 1}\Bigg\{\mathcal{A}'(\rho,\beta) - \frac{1}{T_c}\ln\Bigg\{\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{2m}\sum_{b=0}^{2n} \frac{(-1)^b 2^{7-8\mu-4m-4n}\pi\mu\psi^{2m+2n}}{\bar{\gamma}\phi^{2m+2n+2\mu-1}\Gamma(T_c\rho)\Gamma(\mu)^2\Gamma\big(m+\mu+(1/2)\big)} \\ \times \frac{\binom{2m}{a}\binom{2n}{b}\beta\rho\Gamma\big(m+n-(a/2)-(b/2)+(1/2)\big)}{m!n!\gamma\Gamma\big(n+\mu+(1/2)\big)}\Bigg(\frac{\Gamma(2\mu+2m+2n)\Gamma(T_c\rho-2\mu)}{\Gamma\big(m+n-(a/2)-(b/2)+(1/2)\big)}Z_2^{2\mu-1} \\ + \frac{\Gamma(-2\mu-2m-2n)\Gamma(T_c\rho-4\mu-2m-2n)}{\big(\Gamma(4\mu+2m-2n)\big)^{-1}\Gamma\big((1/2)-2\mu-m-n-(a/2)-(b/2)\big)}Z_2^{4\mu+2m+2n-1} \\ + \frac{\Gamma(2\mu-T_c\rho)\Gamma(4\mu+2m+2n-T_c\rho)\Gamma(T_c\rho)}{\Gamma\big(2\mu+m+n-(a/2)-(b/2)-T_c\rho+(1/2)\big)}Z_2^{T_c\rho-1}\Bigg)\Bigg\}\Bigg\} - \rho R\Bigg\} \end{aligned} \tag{29}$$

$$\begin{aligned} R_0 = -\frac{1}{T_c}\ln\Bigg\{\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{2m}\sum_{b=0}^{2n}\frac{(-1)^b 2^{7-8\mu-4m-4n}\pi\bar{\gamma}^{-1}\mu}{\Gamma(T_c)\Gamma(\mu)^2\Gamma\big(m+\mu+(1/2)\big)}\frac{\phi^{1-2\mu-2m-2n}\psi^{2m+2n}}{\Gamma\big(n+\mu+(1/2)\big)m!n!\gamma}\binom{2m}{a}\binom{2n}{b} \\ \times\Gamma\left(m+n-\frac{a}{2}-\frac{b}{2}+\frac{1}{2}\right)G_{2,3}^{3,1}\left[\frac{16\mu\phi}{\bar{\gamma}\gamma}\Bigg|\begin{matrix}0, 2\mu+m+n-\frac{a}{2}-\frac{b}{2}-\frac{1}{2}\\ 2\mu-1, 4\mu+2m+2n-1, T_c-1\end{matrix}\right]\Bigg\} \end{aligned} \tag{31}$$

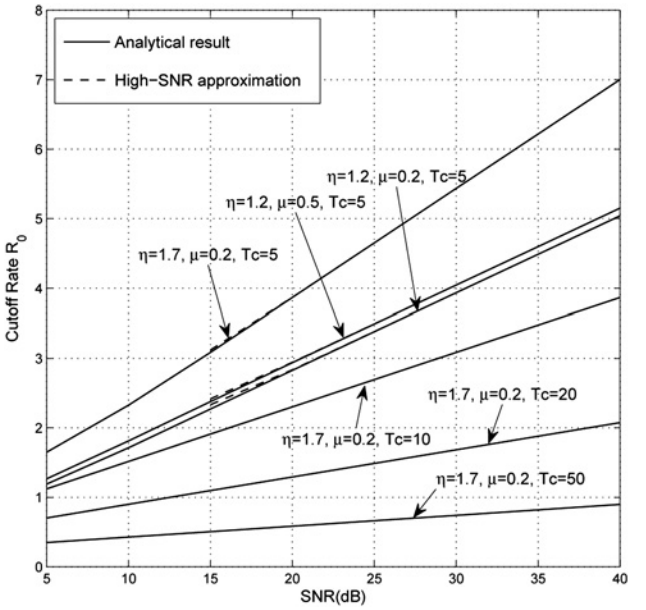

Fig. 4 *Analytical results of the cutoff rate and high-SNR approximation of dual-hop AF η-μ fading SISO channel with selected value of η and μ*

Proof: See Appendix 4.

3.2 κ-μ channel

The generalised κ-μ distribution has advantages on representing small-scale fading signal in line-of-sight environments [9]. The p.d.f. of κ-μ distribution is given by

$$f_{\gamma_{\kappa,\mu}}(\gamma)=\frac{\mu(1+\kappa)^{(\mu+1/2)}}{\kappa^{(\mu-1/2)}\mathrm{e}^{\mu\kappa}}\gamma^{(\mu-1/2)}\mathrm{e}^{-((\mu(1+\kappa)\gamma)/\bar{\gamma})}I_{\mu-1}\left(\frac{2\mu\sqrt{\kappa(1+\kappa)\gamma}}{\bar{\gamma}}\right) \tag{34}$$

where $\kappa > 0$ is the power ratio between the dominant components and the scattered waves, and μ is linked to the number of multipath clusters. When $k \to 0$, expression (34) leads to the Nakagami-m distribution and it becomes Rayleigh distribution by setting $k \to 0$, $\mu = 1$.

Theorem 2: The p.d.f. of end-to-end SNR of dual-hop AF κ-μ fading SISO channel is derived as (see equation (35) at the bottom of the page)

Proof: Based on [26, 8.445], we first replace the modified Bessel function $I_v(\cdot)$ in (34) by infinite series

$$\begin{aligned} f_{\gamma_1,\gamma_2}(\gamma_1,\gamma_2) &= f(\gamma_1)f(\gamma_2) \\ &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\mu^{2m+2n+2\mu}\kappa^{m+n}(1+\kappa)^{m+n+2\mu}\bar{\gamma}^{-2-2\mu-2m-2n}}{\mathrm{e}^{2\mu\kappa}m!n!\Gamma(m+\mu)\Gamma(n+\mu)} \\ &\quad \times (\gamma_1\gamma_2)^{\mu-1}\gamma_1^m\gamma_2^n\,\mathrm{e}^{-((\mu(1+\kappa)(\gamma_1+\gamma_2))/\bar{\gamma})} \end{aligned} \tag{36}$$

The joint p.d.f. of end-to-end SNR from two i.i.d. $\kappa-\mu$ channels is generated through a similar approach applied by the derivation for $\eta-\mu$ channels. Set $w=\gamma_1+\gamma_2$, $z=\gamma_1\gamma_2$ (see equation (37) at bottom of the next page)

Then the joint end-to-end SNR can be expressed as $x=(z/w)$, therefore we have (see equation (38) at the bottom of the next page)

and (see equation (39) at the bottom of the page)

The integral in (39) is achieved with the help of [26, 3.383.4]. Finally, we transform (39) to Theorem 2, which is in the form of Meijer's G-function, according to [28, 13.1.33] and [29, 07.34.03.0612.01]. □

As can be observed in Fig. 1, the infinite summation can converge quickly within $m=n=10$, hence it can be efficiently evaluated. In the following, this p.d.f. expression is used in the analysis of the error exponent.

$$\begin{aligned} R_0^{\infty} = -\frac{1}{T_c}\ln\Bigg\{ &\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{2m}\sum_{b=0}^{2n}\frac{(-1)^b 2^{7-8\mu-4m-4n}\pi\mu\psi^{2m+2n}\Gamma\big(m+n-(a/2)-(b/2)+(1/2)\big)\binom{2m}{a}\binom{2n}{b}}{m!n!\gamma\bar{\gamma}\phi^{2m+2n+2\mu-1}\Gamma(T_c)\Gamma(\mu)^2\Gamma\big(m+\mu+(1/2)\big)\Gamma\big(n+\mu+(1/2)\big)} \\ &\times\left(\frac{\Gamma(2\mu+2m+2n)\Gamma(T_c-2\mu)\Gamma(2\mu)}{\Gamma\big(m+n-(a/2)-(b/2)+(1/2)\big)}Z_3^{2\mu-1}+\frac{\Gamma(-2\mu-2m-2n)\Gamma(T_c-4\mu-2m-2n)}{\Gamma\big(-2\mu-m-n-(a/2)-(b/2)+(1/2)\big)}\right. \\ &\left.\times\,\Gamma(4\mu+2m+2n)Z_3^{4\mu+2m-2n-1}+\frac{\Gamma(2\mu-T_c)\Gamma(4\mu+2m+2n-T_c)\Gamma(T_c)}{\Gamma\big(2\mu+m+n-(a/2)-(b/2)-T_c\rho+(1/2)\big)}Z_3^{T_c-1}\right) \end{aligned} \tag{32}$$

$$\begin{aligned} \langle C\rangle = &\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{2m}\sum_{b=0}^{2n}\frac{(-1)^b 2^{6-8\mu-4m-4n}\pi\mu\psi^{2m+2n}\binom{2m}{a}\binom{2n}{b}}{\bar{\gamma}\phi^{2\mu+2m+2n-1}\Gamma(\mu)^2\Gamma\big(m+\mu+(1/2)\big)m!n!\gamma\Gamma\big(n+\mu+(1/2)\big)\Gamma\big(m+n-(a/2)-(b/2)+(1/2)\big)} \\ &\times G_{3,4}^{4,1}\left[\frac{8\mu\phi}{\gamma\bar{\gamma}}\,\middle|\,\begin{matrix}-1,0,2\mu+m+n-\frac{a}{2}-\frac{b}{2}-\frac{1}{2}\\ 2\mu-1,4\mu+2m+2n-1,-1,-1\end{matrix}\right] \end{aligned} \tag{33}$$

$$\begin{aligned} f_x^{\kappa-\mu}(x) = &\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{m}\sum_{b=0}^{n}\frac{(-1)^{n-b}\Gamma\big((m/2)+(n/2)-(a/2)-(b/2)+(1/2)\big)\binom{m}{a}\binom{n}{b}\mu^{1+m+n}\kappa^{m+n}(1+\kappa)}{2^{2\mu+m+n-3}\bar{\gamma}\mathrm{e}^{2\mu\kappa}m!n!\Gamma(m+\mu)\Gamma(n+\mu)} \\ &\times G_{1,2}^{2,0}\left[\frac{4\mu(1+\kappa)x}{\bar{\gamma}}\,\middle|\,\begin{matrix}\mu+\frac{m}{2}+\frac{n}{2}-\frac{a}{2}-\frac{b}{2}-\frac{1}{2}\\ \mu-1,2\mu+m+n-1\end{matrix}\right] \end{aligned} \tag{35}$$

3.2.1 Random coding error exponent

Proposition 2: The analytical expression of RCEE of dual-hop AF relaying systems in κ-μ fading channels is given by (see equation (40) at bottom of the page)

Proof: See Appendix 5.

Corollary 6: The simplified approximation of expurgated error exponent of dual-hop AF relaying systems in κ-μ fading channels at high SNR regime is represented by (see equation (41) at bottom of the page)

where

$$Z_4 = \frac{4\mu(1+\kappa)\beta(1+\rho)}{\bar{\gamma}\gamma}$$

Proof: See Appendix 5.

3.2.2 Expurgated error exponents

Corollary 7: The analytical expression and simplified approximation of expurgated error exponent of dual-hop AF relaying systems in κ-μ fading channels are given by the following expressions, respectively (see equations (42 and 43) at bottom of the next page)

$$\begin{aligned} f_{w,z}(w,z) &= f_{\gamma_1,\gamma_2}(\gamma_1(w,z),\, \gamma_2(w,z)) \left| \frac{\partial(\gamma_1,\, \gamma_2)}{\partial w,\, \partial z} \right| \\ &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{m}\sum_{b=0}^{n} \frac{(-1)^{n-b} 2^{-m-n} \kappa^{m+n} (1+\kappa)^{m+n+2\mu} \binom{m}{a}\binom{n}{b}}{m!n!\Gamma(m+\mu)\Gamma(n+\mu)\,\mathrm{e}^{2\mu\kappa}\bar{\gamma}^{2\mu+2m+2n-2}} \\ &\quad \times w^{a+b}(w^2-4z)^{(m/2)+(n/2)-(a/2)-(b/2)-(1/2)} z^{\mu-1}\,\mathrm{e}^{-\left((\mu(1+\kappa)w)/\bar{\gamma}\right)} \end{aligned} \tag{37}$$

$$\begin{aligned} f_{w,x}(w,x) &= f_{w,z}(w,xw)|w| \\ &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{m}\sum_{b=0}^{n} \frac{(-1)^{n-b} 2^{-m-n} \bar{\gamma}^{2-2m-2n-2\mu}}{m!n!\Gamma(m+\mu)\Gamma(n+\mu)} \mu^{2m+2n+2\mu} \kappa^{m+n} (1+\kappa)^{m+n+2\mu}\,\mathrm{e}^{-2\mu\kappa} \\ &\quad \times x^{\mu-1} w^{(m/2)+(n/2)+(a/2)+(b/2)+\mu-(1/2)} (w-4x)^{(m/2)+(n/2)-(a/2)-(b/2)-(1/2)} \mathrm{e}^{-\left((\mu(1+\kappa)w)/\bar{\gamma}\right)} \end{aligned} \tag{38}$$

$$\begin{aligned} f_x(x) &= \int_0^{\infty} f_{w,x}(w,x)\,\mathrm{d}w \\ &= \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{m}\sum_{b=0}^{n} \frac{(-1)^{n-b}\Gamma((m/2)+(n/2)-(a/2)-(b/2)+(1/2))\binom{m}{a}\binom{n}{b}\mu^{(3m/2)+(3n/2)+(3\mu/2)-(1/2)}\kappa^{m+n}}{2^{-\mu}\bar{\gamma}^{(3\mu/2)+(m/2)+(n/2)-(1/2)}\,\mathrm{e}^{2\mu\kappa} m!n!\Gamma(m+\mu)\Gamma(n+\mu)(1+\kappa)^{(1/2)-(3\mu/2)-(m/2)-(n/2)}} \\ &\quad \times x^{(3\mu/2)+(m/2)+(n/2)-(3/2)}\,\mathrm{e}^{-\left((2\mu(1+\kappa)x)/\bar{\gamma}\right)} W_{(n/2)+(a/2)+(b/2),-(m/2)-(n/2)-(\mu/2)}\left(\frac{4\mu(1+\kappa)x}{\bar{\gamma}}\right) \end{aligned} \tag{39}$$

$$\begin{aligned} E_r(R, T_\mathrm{c}) = \max_{0\le\rho\le 1} \Bigg\{ \max_{0\le\beta\le 1} \Bigg\{ \mathcal{A}(\rho,\beta) - \frac{1}{T_\mathrm{c}} \ln \Bigg\{ \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{m}\sum_{b=0}^{n} \frac{(-1)^{n-b}\mu^{1+m+n}\Gamma\big((m/2)+(n/2)-(a/2)-(b/2)-(1/2)\big)}{2^{2\mu+m+n-3}\kappa^{-m-n}\bar{\gamma}\Gamma(T_\mathrm{c}\rho)\Gamma(m+\mu)\,\mathrm{e}^{2\mu\kappa}} \\ \times \frac{\binom{m}{a}\binom{n}{b}(1+\kappa)\beta(1+\rho)}{m!n!\gamma\Gamma(n+\mu)} G_{2,3}^{3,1}\left[\frac{4\mu(1+\kappa)\beta(1+\rho)}{\bar{\gamma}\gamma} \middle| \begin{matrix} 0,\, \mu+\frac{m}{2}+\frac{n}{2}-\frac{a}{2}-\frac{b}{2}-\frac{1}{2} \\ 2\mu-1,\, 4\mu+2m+2n-1,\, T_\mathrm{c}\rho-1 \end{matrix} \right] \Bigg\} \Bigg\} - \rho R \Bigg\} \end{aligned} \tag{40}$$

$$\begin{aligned} E_r^{\infty}(R, T_\mathrm{c}) = \max_{0\le\rho\le 1} \Bigg\{ \max_{0\le\beta\le 1} \Bigg\{ \mathcal{A}(\rho,\beta) - \frac{1}{T_\mathrm{c}} \ln \Bigg\{ \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{m}\sum_{b=0}^{n} \frac{(-1)^{n-b}\mu^{1+m+n}\Gamma\big((m/2)+(n/2)-(a/2)-(b/2)-(1/2)\big)}{2^{2\mu+m+n-3}\kappa^{-m-n}\bar{\gamma}\Gamma(T_\mathrm{c}\rho)\Gamma(m+\mu)\,\mathrm{e}^{2\mu\kappa}} \\ \times \frac{\binom{2m}{a}\binom{n}{b}\beta(1+\rho)(1+\kappa)}{\Gamma(n+\mu)m!n!\gamma} \Bigg[\frac{\Gamma(\mu+m+n)\Gamma(T_c\rho-\mu)Z_4^{\mu-1}}{\Gamma(\mu)^{-1}\Gamma\big((m/2)+(n/2)-(a/2)-(b/2)+(1/2)\big)} \\ + \frac{\Gamma(T_c\rho-2\mu-m-n)}{\Gamma\big((1/2)-\mu-(m/2)-(n/2)-(a/2)-(b/2)\big)} \\ \times \frac{\Gamma(-\mu-m-n)Z_4^{2\mu+m+n-1}}{\Gamma(2\mu+m+n)^{-1}} + \frac{\Gamma(\mu-T_\mathrm{c}\rho)\Gamma(\rho+2\mu+m+n-T_\mathrm{c})\Gamma(T_\mathrm{c}\rho)Z_4^{2\mu+m+n-1}}{\Gamma\big((1/2)+(m/2)+(n/2)-(a/2)-(b/2)\big)+\mu-T_\mathrm{c}\rho} \Bigg] \Bigg\} \Bigg\} \Bigg\} \end{aligned} \tag{41}$$

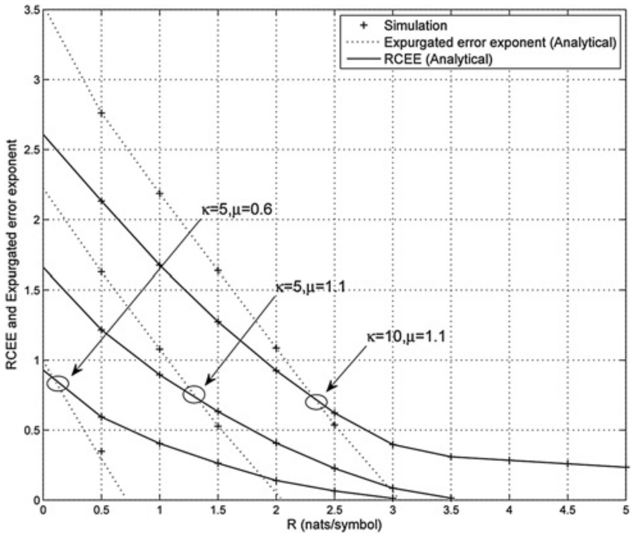

Fig. 5 *Analytical and simulation results for the RCEE and expurgated error exponent of dual-hop AF κ-μ fading SISO channel with* $\bar{\gamma} = 3$, $T_c = 5$ *and R = 1.5*

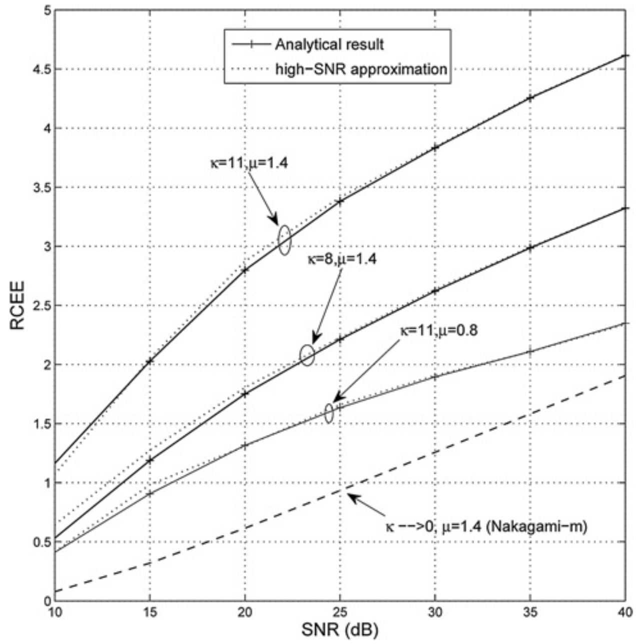

Fig. 6 *Analytical and high-SNR approximation of RCEE against SNR of dual-hop AF κ-μ fading SISO channel with* $\bar{\gamma} = 3$, $T_c = 5$ *and R = 1.5*

where

$$Z_5 = \frac{8\mu(1+\kappa)\beta\rho}{\bar{\gamma}\gamma}$$

Proof: See Appendix 6.

Fig. 5 illustrates the RCEE and expurgated error exponent against different information rate R with $T_c = 5$, $\bar{\gamma} = 3$ and $\gamma = 15$ dB. It is clear that both the RCEE and expurgated error exponent decline monotonically when R increases. The curves of RCEE and expurgated error exponent are all convex. At low information rate, the expurgated error exponent performs better than RCEE as expected. We can see that both parameters μ and κ have positive effect on the error exponents. This is because that κ denotes how the power is concentrated and μ illustrates the number of multipath clusters.

Fig. 6 demonstrates the analytical result and high SNR approximation of RCEE as functions of end-to-end SNR with different value of κ and μ. It is clear that the RCEE of dual-hop AF κ-μ fading SISO channel increases absolutely with SNR and the slope of the curves is determined by both κ and μ. From $\gamma = 10$ dB, the high SNR approximation matches the analytical result well.

Table 2 presents the required code length to achieve the predefined upper bound of error probability $P_e^{E_r} = 10^{-6}$ at transmission rate $R = 1.5$ of dual-hop AF SISO systems over κ-μ fading channel with fixed $T_c = 5$ and $\bar{\gamma} = 3$. As can be seen, the increasing of both parameters κ and μ help to reduce the required code length considerably. The reduction at low-SNR regime is more proportionally remarkable. For example, considering μ increases from 0.6 to 1.1, the required code length reduces 61.66% when SNR = 10 dB but it just decreases 55% when SNR = 30 dB. Similarly, when κ increases from 5 to 10, the required code length decreases 48.65% with SNR = 10 dB but it only decreases 33.33% when SNR = 30 dB.

3.2.3 Cutoff rate

$$E_{ex}(R, T_c) = \max_{\rho\ge 1}\left\{\max_{0\le\beta\le 1}\left\{\mathcal{A}'(\rho,\beta) - \frac{1}{T_c}\ln\left\{\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{m}\sum_{b=0}^{n}\frac{(-1)^{n-b}\mu^{1+m+n}\Gamma\big((m/2)+(n/2)-(a/2)-(b/2)-(1/2)\big)}{2^{2\mu+m+n-4}\,e^{2\mu\kappa}\Gamma(T_c\rho)\Gamma(m+\mu)\Gamma(n+\mu)}\right.\right.\right.$$
$$\left.\left.\left.\times\frac{\binom{m}{a}\binom{n}{b}\kappa^{m+n}(1+\kappa)\beta\rho}{m!n!\gamma\bar{\gamma}}G^{3,1}_{2,3}\left[\frac{8\mu(1+\kappa)\beta\rho}{\bar{\gamma}\gamma}\middle|\begin{matrix}0,\ \mu+\frac{m}{2}+\frac{n}{2}-\frac{a}{2}-\frac{b}{2}-\frac{1}{2}\\ 2\mu-1,\ 4\mu+2m+2n-1,\ T_c\rho-1\end{matrix}\right]\right\}\right\} - \rho R\right\} \tag{42}$$

$$E^{\infty}_{ex}(R, T_c) = \max_{\rho\ge 1}\left\{\max_{0\le\beta\le 1}\left\{\mathcal{A}'(\rho,\beta) - \frac{1}{T_c}\ln\left\{\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{m}\sum_{b=0}^{n}\frac{(-1)^{n-b}\mu^{1+m+n}\kappa^{m+n}\Gamma\big((m/2)+(n/2)-(a/2)-(b/2)-(1/2)\big)}{2^{2\mu+m+n-4}\,e^{2\mu\kappa}\bar{\gamma}\Gamma(T_c\rho)\Gamma(m+\mu)\Gamma(n+\mu)}\right.\right.\right.$$
$$\times\frac{\binom{2m}{a}\binom{n}{b}(1+\kappa)\beta\rho}{m!n!\gamma}\left[\frac{\Gamma(\mu+m+n)\Gamma(T_c\rho-\mu)Z_5^{\mu-1}}{\Gamma(\mu)^{-1}\Gamma\big((m/2)+(n/2)-(a/2)-(b/2)+(1/2)\big)}\right.$$
$$+\frac{\Gamma(-\mu-m-n)\Gamma(T_c\rho-2\mu-m-n)Z_5^{2\mu+m+n-1}}{\Gamma(2\mu+m+n)^{-1}\Gamma\big((1/2)-\mu-(m/2)-(n/2)-(a/2)-(b/2)\big)}$$
$$\left.\left.\left.\left.+\frac{\Gamma(\mu-T_c\rho)\Gamma(\rho+2\mu+m+n-T_c)Z_5^{2\mu+m+n-1}}{\Gamma(T_c\rho)^{-1}\Gamma\big((1/2)+(m/2)+(n/2)-(a/2)-(b/2)\big)-T_c\rho}\right]\right\}\right\}\right\} \tag{43}$$

Table 2 Required code lengths L_r for dual-hop AF κ-μ fading SISO channel to achieve the predefined upper bound of decoding error probabilities $P_e^r \le 10^{-6}$ with $R = 1.5$, $\bar{\gamma} = 3$ and $T_c = 5$

SNR, dB	$\kappa = 5$ $\mu = 0.6$	$\kappa = 5$ $\mu = 1.1$	$\kappa = 10$ $\mu = 0.6$	$\kappa = 10$ $\kappa = 1.1$
10	193	74	38	28
15	120	26	13	13
30	20	9	6	6

Corollary 8: The analytical expression of cutoff rate of dual-hop AF relaying systems in κ-μ fading channels can be expressed as (see (44))

Proof: See Appendix 7.

Corollary 9: The cutoff rate of dual-hop AF relaying systems in κ-μ fading channels at high SNR regime can be expressed as (see (45)) where

$$Z_6 = \frac{4\mu(1+\kappa)\beta(1+\rho)}{\bar{\gamma}\gamma}$$

Proof: See Appendix 7.

Fig. 7 depicts the analytical results and high SNR approximation of cutoff rate as a function of SNR with selected values of κ and μ. When block length T_c is fixed, we see that the cutoff rate monotonically increase with SNR, κ and μ, respectively. The high-SNR approximation works perfectly when SNR is >15 dB. On the other hand, the cutoff rate decreases when T_c increases. Note that Shannon capacity just offers the achievable information rate, but cut-off rate affords more insight into the tradeoff of information reliability and coding complexity in block-coding channels.

3.2.4 Ergodic capacity

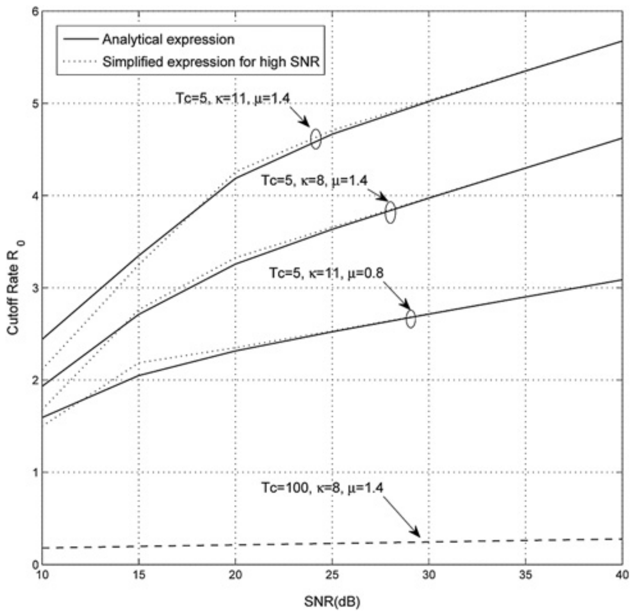

Fig. 7 *Analytical results and high-SNR approximation of cutoff rate against SNR of dual-hop AF κ-μ fading SISO channel with $\bar{\gamma} = 3$ and $T_c = 5$*

Corollary 10: The ergodic capacity of dual-hop AF relaying systems in κ-μ fading channels is given by (see (46))

Proof: See Appendix 8.

In Fig. 8, the effects of parameters κ and μ on ergodic capacity with different SNR are evaluated. The value of ergodic capacity increases with κ, μ and SNR, respectively. In addition, a special

$$R_0 = -\frac{1}{T_c}\ln\left\{\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{m}\sum_{b=0}^{n}\frac{(-1)^{n-b}\binom{m}{a}\binom{n}{b}\mu^{1+m+n}\kappa^{m+n}(1+\kappa)\Gamma\big((m/2)+(n/2)-(a/2)-(b/2)-(1/2)\big)}{2^{2\mu+m+n-4}m!n!\gamma e^{2\mu\kappa}\bar{\gamma}\Gamma(T_c\rho)\Gamma\big(m+\mu-(1/2)\big)\Gamma\big(n+\mu-(1/2)\big)}\right.$$
$$\left.\times G_{2,3}^{3,1}\left[\frac{8\mu(1+\kappa)}{\bar{\gamma}\gamma}\middle| \begin{matrix} 0,\ \mu+\frac{m}{2}+\frac{n}{2}-\frac{a}{2}-\frac{b}{2}-\frac{1}{2} \\ 2\mu-1,\ 4\mu+2m+2n-1,\ T_c\rho-1\end{matrix}\right]\right\} \quad (44)$$

$$R_0^{\infty} = -\frac{1}{T_c}\ln\left\{\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{m}\sum_{b=0}^{n}\frac{(-1)^{n-b}\binom{m}{a}\binom{n}{b}\mu^{1+m+n}\kappa^{m+n}(1+\kappa)\Gamma\big((m/2)+(n/2)-(a/2)-(b/2)-(1/2)\big)}{2^{2\mu+m+n-4}m!n!\gamma e^{2\mu\kappa}\bar{\gamma}\Gamma(T_c\rho)\Gamma\big(m+\mu-(1/2)\big)\Gamma\big(n+\mu-(1/2)\big)}\right.$$
$$\times\left[\frac{\Gamma(\mu+m+n)\Gamma(T_c-\mu)Z_6^{\mu-1}}{\Gamma(\mu)^{-1}\Gamma\big((m/2)+(n/2)-(a/2)-(b/2)+(1/2)\big)}+\frac{\Gamma(-\mu-m-n)\Gamma(T_c-2\mu-m-n)Z_6^{2\mu+m+n-1}}{\Gamma(2\mu+m+n)^{-1}\Gamma\big((1/2)-\mu-(m/2)-(n/2)-(a/2)-(b/2)\big)}\right.$$
$$\left.\left.+\frac{\Gamma(\mu-T_c)\Gamma(+2\mu+m+n-T_c)\Gamma(T_c)Z_6^{2\mu+m+n-1}}{\Gamma\big((1/2)+(m/2)+(n/2)-(a/2)-(b/2)+\mu-T_c\big)}\right]\right\} \quad (45)$$

$$\langle C\rangle = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{a=0}^{m}\sum_{b=0}^{n}\frac{(-1)^{n-b}2^{3-m-n-2\mu}\binom{m}{a}\binom{n}{b}\mu^{1+m+n}\kappa^{m+n}(1+\kappa)}{m!n!\,e^{2\mu\kappa}\bar{\gamma}\gamma\Gamma(m+\mu)\Gamma(n+\mu)}$$
$$\times G_{3,4}^{4,1}\left[\frac{4\mu(1+\kappa)x}{\gamma\bar{\gamma}}\middle| \begin{matrix} -1,\ 0,\ \mu+\frac{m}{2}+\frac{n}{2}-\frac{a}{2}-\frac{b}{2}+\frac{1}{2} \\ \mu-1,\ 2\mu+m+n-1,-1,-1\end{matrix}\right] \quad (46)$$

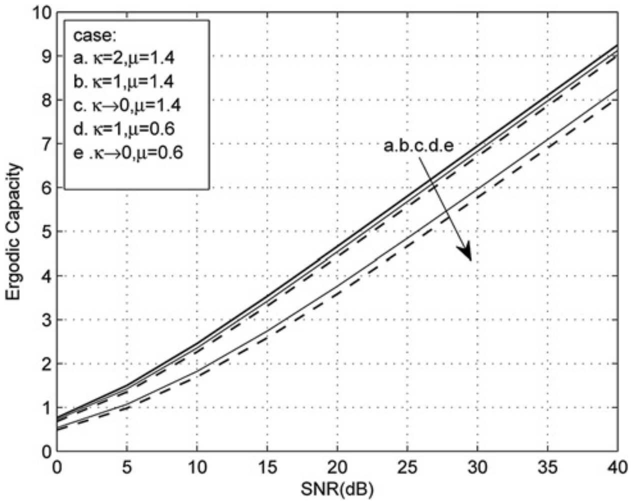

Fig. 8 *Analytical results for the Ergotic Capacity of dual-hop AF κ-μ fading SISO channel for selected parameters κ and μ with* $\bar{\gamma} = 3$ *and* $T_c = 5$

case that $\kappa \to 0$, which is related to the Nakagami-*m* distribution is shown as the dash lines.

3.3 With other distributions

As mentioned above, the general *η*-*μ* and *κ*-*μ* distributions include a series of other wildly studied statistical distributions as special cases, such as Raleigh, Nakagami-*m*, Hoyt and Rician distributions. This subsection introduces how to derive analytical expressions of RCEE, expurgated error exponent, ergodic capacity and cutoff rate for these widely used fading channels from our analytical results. Note that all the subsequent analytical results are presented without strict mathematical proofs, because the involved mathematical manipulations are simple and straightforward.

*3.3.1 Nakagami-*m *fading:* Nakagami-*m* distribution is used to formulise the fading signal which consists of a bundle of multi-path clusters without any scatter. It can be corresponded from the *η*-*μ* fading distribution with $\eta \to 1$ in Format 1 or from *κ*-*μ* distribution with $\kappa \to 0$.

Substituting $\kappa \to 0$ into Theorem 2, we can obtain the p.d.f. of end-to-end SNR of dual-hop AF Nakagami-*m* fading channel as (see (47))

$$= 2^{3-2\mu}\mu\Gamma\left(\frac{1}{2}\right)\bar{\gamma}^{-1}\Gamma(\mu)^{-2}G_{1,2}^{2,0}\left[\frac{4\mu x}{\bar{\gamma}}\middle| \begin{matrix}\mu-\frac{1}{2}\\ \mu-1,\,2\mu-1\end{matrix}\right] \quad (48)$$

By rewriting (48) in the term of hypergeometric function with the aid of [29, 07.34.03.0612.01], and replacing the parameter *μ* by *m* because of Nakagami-*m* distribution, the p.d.f. of end-to-end SNR of dual-hop AF nakagami-*m* fading channel is obtained as

$$f_x(x) = 2m^m\Gamma\left(\frac{1}{2}\right)\bar{\gamma}^{-m}\Gamma(m)^{-2}x^{m-1}U\left(\frac{1}{2}-m,\,1-m,\,\frac{4mx}{\bar{\gamma}}\right) \quad (49)$$

It is remarkable that expression (49) can also be verified by the result presented in [30].

Thereafter, the analytical expressions of RCEE, expurgated error exponent, cut-off rate and ergodic capacity of dual-hop AF relaying systems in Nakagami-*m* fading channels can be derived through the similar way by substituting $\kappa \to 0$ and $\mu = m$ into (40), (44), (46) and (42)

- RCEE (see equation (50) at bottom of the page)
- cutoff rate

$$R_0 = -\frac{1}{T_c}\ln\left\{\frac{2^{4-2m}m\Gamma(-(1/2))}{\gamma\bar{\gamma}\Gamma(T_c)\Gamma(m)^2}G_{2,3}^{3,1}\left[\frac{8m}{\bar{\gamma}\gamma}\middle| \begin{matrix}0,m-\frac{1}{2}\\ 2m-1,4m-1,T_c-1\end{matrix}\right]\right\} \quad (51)$$

- ergodic capacity

$$\langle C\rangle = \frac{2^{3-2m}m}{\bar{\gamma}\gamma\Gamma(m)^2}G_{4,1}^{3,4}\left[\frac{4mx}{\gamma\bar{\gamma}}\middle| \begin{matrix}-1,\,0,\,m+\frac{1}{2}\\ m-1,\,2m-1,\,-1,\,-1\end{matrix}\right] \quad (52)$$

- expurgated error exponent

$$E_{ex}(R,T_c) = \max_{\rho\ge 1}\left\{\max_{0\le\beta\le 1}\left\{\mathcal{A}'(\rho,\beta) - \frac{1}{T_c}\ln\left\{\frac{2^{4-2m}m\Gamma(-(1/2))\beta\rho}{\gamma\bar{\gamma}\Gamma(T_c\rho)\Gamma(m)^2}\times G_{2,3}^{3,1}\left[\frac{8m\beta\rho}{\bar{\gamma}\gamma}\middle| \begin{matrix}0,\,m-\frac{1}{2}\\ 2m-1,\,4m-1,\,T_c\rho-1\end{matrix}\right]\right\}\right\} - \rho R\right\} \quad (53)$$

Although the expressions above are derived from *κ*-*μ* fading, we should note that the same results can be also obtained from *η*-*μ* fading as an alternative way. We provide the analytical results of the RCEE (50) and the ergodic capacity (52) in Figs. 6 and 8, respectively.

3.3.2 Hoyt, Rician, Rayleigh and one-side Gaussian and fading: Discarding the consideration of the multipath clusters, our analytical results of *η*-*μ* and *κ*-*μ* fading can also degrade to error exponents for systems over Hoyt (Nakagami-*q*), Rician, Rayleigh and one-side Gaussian and fadings

$$\lim_{\kappa\to 0}f_x^{\kappa-\mu}(x) = \lim_{\kappa\to 0}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{a=0}^{i}\sum_{b=0}^{j}\frac{(-1)^{j-b}2^{3-i-j-2\mu}\binom{i}{a}\binom{j}{b}\mu^{1+i+j}\kappa^{i+j}(1+\kappa)\Gamma((i/2)+(j/2)-(i/2)-(j/2)+(1/2))}{\bar{\gamma}e^{2\mu\kappa}i!j!\Gamma(i+\mu)\Gamma(j+\mu)} \times G_{1,2}^{2,0}\left[\frac{4\mu(1+\kappa)x}{\bar{\gamma}}\middle| \begin{matrix}\mu+\frac{i}{2}+\frac{j}{2}-\frac{a}{2}-\frac{b}{2}-\frac{1}{2}\\ \mu-1,\,2\mu+i+j-1\end{matrix}\right] \quad (47)$$

$$E_r(R,T_c) = \max_{0\le\rho\le 1}\left\{\max_{0\le\beta\le 1}\left\{\mathcal{A}(\rho,\beta) - \frac{1}{T_c}\ln\left\{\frac{2^{3-2\mu}\mu\Gamma(-(1/2))\beta(1+\rho)}{\gamma\bar{\gamma}\Gamma(T_c\rho)\Gamma(\mu)^2}G_{2,3}^{3,1}\left[\frac{4\mu\beta(1+\rho)}{\bar{\gamma}\gamma}\middle| \begin{matrix}0,\,\mu-\frac{1}{2}\\ 2\mu-1,\,4\mu-1,\,T_c\rho-1\end{matrix}\right]\right\}\right\} - \rho R\right\} \quad (50)$$

• Hoyt (Nakagami-q): By setting $\mu = 0.5$ and $\eta = q^2$ in (18), (26), (28), (31) and (33), the analytical expressions of dual-hop AF relaying systems in Hoyt fading channels are obtained.
• Rician: When $\mu = 1$ and $\kappa = K$, analytical expressions (35), (40), (42), (44) and (46) degenerate to equations for the dual-hop AF relaying systems in Rician fading channels. We should note that K is the ratio between the power in the direct path and the power in the other scattered paths.
• Supposing $\kappa \to 0$ and $\mu = 1$ in expressions (35), (40), (42), (44) and (46), we can apply our results for dual-hop AF relaying systems in Rayleigh fading channels.
• By setting $\kappa \to 0$ and $\mu = 0.5$ in expressions (35), (40), (42), (44) and (46), we can obtain the analytical results for dual-hop AF relaying systems in one-sided Gaussian fading channels.

4 Conclusion

In this paper, the error exponents of dual-hop AF relaying systems in generalised η-μ and κ-μ fading channels are investicated. More specifically, we first developed the new analytical expression of p.d.f. of end-to-end SNR. Subsequently new analytical expressions of RCEE, expurgated error exponent, cutoff rate and ergodic capacity were derived. Our generalised analytical results include the other previously popular fading channels as special cases. Moreover, simplified and insightful approximations were provided, which were shown to be tight at high SNR regime. Meanwhile, the required code length to achieve a certain upper bound of error probability are estimated based on our analytical expressions. The results showed that larger in-phase to quadrature components ratio, dominant components to scattered waves ratio and number of multipath clusters could reduce the required code length and improve the communication reliability of dual-hop AF channels.

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7 Appendix

7.1 Appendix 1: Proof of Proposition 1 and Corollary 1

As it is difficult to substitute (18) into (10) directly, we represent (10) in terms of Meijer's G-function with the help of [29, 07.34.03.0271.01] as

$$\tilde{E}_0(\rho, \beta, T_c) = \mathcal{A}(\rho, \beta) - \frac{1}{T_c} \ln\left\{ \mathbb{E}\left[\Gamma(T_c\rho)^{-1} G_{1,1}^{1,1}\left(\frac{\gamma x}{\beta(1+\rho)} \Big|_{0}^{1 - T_c\rho} \right) \right] \right\} \quad (54)$$

Then, by using the formula [29, 07.34.21.0011.01], we derive Proposition 1.

In (26), we know $\left((8\mu\phi\beta(1+\rho))(/\bar{\gamma}\gamma) \right) \to 0$ when $\gamma \to \infty$. Based on this condition, we derive the approximation at high SNR regime in Corollary 1 with the help of [29, 07.34.06.0006.01].

7.2 Appendix 2: Proof of Corollary 2

For the convenience of the following integral, (16) is firstly rewritten in the form of Meijer's G-function with the aid of [29, 07.34.03.0271.01]

$$\tilde{E}_x(\rho, \beta, T_c) = \mathcal{A}'(\rho, \beta) - \frac{1}{T_c} \ln\left\{\mathbb{E}\left[\Gamma(T_c\rho)^{-1} G_{1,1}^{1,1}\left(\frac{\gamma x}{2\beta} \Big| \begin{matrix} 1 - T_c\rho \\ 0 \end{matrix}\right)\right]\right\} \quad (55)$$

Substituting (18) into (55), (28) in Corollary 2 can be derived according to Wolfram inc [29, 07.34.21.0011.01]. With the help of [29, 07.34.06.0006.01] and supposing $((16\mu\phi\beta\rho)/\bar{\gamma}\gamma) \to 0$ in (28), the simplified approximation of expurgated error exponent at high SNR regime is obtained as (29).

7.3 Appendix 3: Proof of Corollaries 3 and 4

Using expressions (18) (14) and the integration formula provided by Wolfram inc [29, 07.34.21.0011.01], cutoff rate of dual-hop AF η-μ fading SISO channel (31) can be derived with some simple algebraic manipulations. Actually, we also can obtain the expression of R_0 by substituting $\rho = 1$ and $\beta = 1$ into the corresponding part of (26).

7.4 Appendix 4: Proof of Corollary 5

Substituting (10) into (13), we can specialise the ergodic capacity as

$$\langle C \rangle \triangleq \mathbb{E}[\ln(1 + \gamma x)] \quad (56)$$

According to Prudnikow *et al.*, [31, eq. (2.24.3.1)], we represent (56) as

$$\langle C \rangle \triangleq \mathbb{E}[\ln(1 + \gamma x)] \triangleq \mathbb{E}\left(G_{2,2}^{1,2}\left[\gamma x \Big| \begin{matrix} 1, 1 \\ 1, 0 \end{matrix}\right]\right) \quad (57)$$

Combining (18) and (57) and applying the integral formula provided by Abramowitz and Stegun [28], the expression of ergodic capacity for dual-hop AF SISO system over η-μ channel is derived.

7.5 Appendix 5: Proof of Proposition 2 and Corollary 6

With the same mathematical manipulation in Appendix 2, (10) is first transformed to a Meijer's G-function (54) according to [29, 07.34.03.0271.01]. The RCEE of dual-hop $\kappa - \mu$ SISO system is then derived by submitting (35) into (54) with the aid of [29, 07.34.21.0011.01].

When $\gamma \to \infty$ at the high SNR regime, it is easy to know that $((4\mu(1+\kappa)\beta(1+\rho))(/\bar{\gamma}\gamma)) \to 0$ in (40). In this scenario, the approximation (41) can be developed from (40) with the aid of [29, 07.34.06.0018.01].

7.6 Appendix 6: Proof of Corollary 7

The derivations of (42) and (43) are of the same approach for Appendix 5. After transforming (16)–(55), the expression (42) is obtained by combining (55) and (35), the integral is made according to Wolfram inc [29, 07.34.21.0011.01]. The expression (43) is subsequently derived through the transformation guided by Wolfram inc [29, 07.34.06.0018.01].

7.7 Appendix 7: Proof of Corollaries 8 and 9

Directly submitting $\rho = 1$, $\beta = 1$ into (40) and (41), expressions (44) and (45) are generated.

7.8 Appendix 8: Proof of Corollary 10

Combining (35) and (57) in Appendix 3, we have

$$\langle C \rangle = \int_0^\infty G_{2,2}^{1,2}\left[\gamma x \Big| \begin{matrix} 1, 1 \\ 1, 0 \end{matrix}\right] f_x^{\kappa-\mu}(x)\,dx \quad (58)$$

Then, Corollary 10 can be obtained with the help of integral formula provided by Abramowitz and Stegun [28].