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Complex Predication and the Metaphysics of Properties

Abstract: The existence of *complex predicates* seems to support an abundant conception of properties. Specifically, the application conditions for complex predicates seem to be *explained by* the distribution of a *sparser* base of predicates. This explanatory link might suggest that the existence and distribution of properties expressed by complex predicates are explained by the existence and distribution of a sparser base of properties. Thus, complex predicates seem to legitimize the assumption of a wide array of properties. The additional properties are *no explanatory addition* to the sparse base. I argue, however, that construing complex predicates as expressing properties undermines the explanatory links between simple and complex predications. The argument explores a variety of accounts of complex predicates currently on offer and develops a number of themes from the work of Herbert Hochberg.

According to an influential tradition, a predicate of one's preferred theory ought to correspond to something in the world, a feature shared by the various entities that satisfy the predicate.[1] If the predicate 'is a kangaroo' occurs in one's preferred theory, then one ought to posit a *property* which all of the various things that satisfy 'is a kangaroo' have in common.[2]

Philosophers of this persuasion divide into two camps. Some endorse *sparse* conceptions of properties on which some collections of entities lack a common property. These philosophers hesitate to deploy additional predicates in their preferred description of the world. The predicates of day-to-day language and the special sciences likely do not correspond to properties. These predicates must ultimately give way to the predicates of a more austere schema. Other property theorists endorse *abundant* conceptions: every arbitrary grouping of individuals shares some property. Therefore, a theory would be *no worse* for having a predicate corresponding to any such grouping.

The existence of *complex predicates* seems to support an abundant conception of properties. Complex predicates are formed by coordinating predicates using operators such as 'and' and 'or'. For example, one may shorten sentences (1) and (2) to (1*) and (2*), respectively.

(1) Joe is young and Joe is a kangaroo.
(1*) Joe *is young and is a kangaroo.*

[1] Bergmann (1952: 430) says, "Who admits a single primitive predicate admits properties among the building stones of his world."
[2] This does not entail that properties are in any way *linguistic* or *language dependent*. Rather, according to this framework, one should formulate one's preferred theory so as to reflect which properties one takes to be in the world. Thus, a philosopher who genuinely believes that there is a property of being a kangaroo should see fit to include the predicate 'is a kangaroo' in her preferred theory. Nor does this view require one to endorse a deviant (non-Quinean) criterion of ontological commitment. What is required is the presupposition that in a perspicuous theory, predicates correspond to all and only the properties.

(2) Joe is young or Joe is a kangaroo.
(2*) Joe *is young or is a kangaroo*.

(1) and (2) are composed of sub-sentences connected by the sentential operators 'and' and 'or', respectively. (1*) and (2*) are predications in which 'and' and 'or', respectively, connect simpler predicates to form more complex ones. Thus, (1*) contains the complex predicate 'is young and is a kangaroo' and (2*) contains the complex predicate 'is young or is a kangaroo'.

Additional complex predicates can be formed using the expression 'such that'. Thus, a relational statement such as (3) may be contracted into a predication such as (3*).

(3) Joe saw Joe.
(3*) Joe *is such that he saw himself*.

More generally, one can transform any sentence in which 'Joe' figures in transparent position into a predication on Joe using 'such that'. In this way, a sparse base of predicates can generate a wide array of complex predicates.

Moreover, the application conditions for complex predicates seem to be *explained by* the distribution of a *sparser* base of predicates. This position purports to *legitimize* abundant theories of properties, since they are seen as raising no new explanatory requirements. It has even been held that the facts about the complex predicate *logically follow* from facts about the components.[3]

I will investigate this *explanatory claim*: that the distribution of properties expressed by complex predicates is explained by the distribution of properties expressed by the predicates that figure in them. Put differently, I ask whether the ingredients that go into the content of a complex statement—a conjunction such as (1) or a disjunction such as (2)—suffice to explain what is required in the world for a corresponding complex predication to be true. The statements (1) and (2) surely require something of the world. Do (1*) and (2*)—construed as ascriptions of properties—require anything more? I will argue that they do, and that these additional requirements stand in need of explanation. This paper therefore aims to illuminate how *complexity in language* relates to *complexity in the world*.

In §1, I explore purported applications of construing complex predicates as expressing properties. In §2, I set aside currently fashionable explanatory strategies that appeal to grounding. In §§3-7, I explore various accounts of complex predicates, arguing that these are insufficient to deliver the explanatory benefits without a background property theory. In §8, I discuss approaches to complex predicates that do not posit complex properties. The arguments explore some interconnected themes developed by Herbert Hochberg over his long career.

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[3] Salmon (2010: 451).

1. The Applications of Complex Predicates

Complex predicates have been held to play a variety of roles. I focus on their roles in logic and in developing a notion of reduction as property identification. These purported applications illustrate why the distribution of properties expressed by complex predicates must be explained by the distribution of properties expressed by the simpler predicates composing them. I express these explanatory demands using the following schema.

Explanation:	The truth of **Φa** explains the truth of the claim **a is an x such that Φ*x***.[4]
Logicality:	**Φa** is equivalent to the claim **a is an x such that Φ*x*** by logic plus the semantic account of complex predication. No appeal needs to be made to a background property theory.

The latter constraint is stronger than the former. If there is a sense in which complex predications are logically entailed by the distribution of the predicates that they contain, then their behavior can be explained in terms of that of their constituents predicates.

A reason for rejecting these explanatory claims will correspondingly provide a reason for rejecting these applications of complex predication. My aim in the paper is to show that if we construe complex predications as genuine property ascriptions, then the explanatory claims are unsupportable.

1.1 Quantification and Identity

Complex predicates figure centrally in a dominant understanding of the development of the logic of quantification and identity. According to this story, Frege and Russell showed that the quantifiers 'something' and 'everything' are higher-level predicates that take first-level predicates as arguments.[5] A sentence such as 'something is a kangaroo' attributes the 2nd-level property of being instantiated to the property expressed by 'is a kangaroo', just as 'Joe is a kangaroo' attributes the 1st-level property of being a kangaroo to Joe.

This story is apocryphal. Nonetheless, there is a grain of truth to it since both Frege and Russell spoke of quantifiers as combining with expressions similar in role to complex predicates. Consider Russell's (1905, 480) characterization of sentences of the form 'everything is C' and 'something is C'—written **C(everything)** and **C(something)**, respectively—in terms of propositional functions:

(A) **C(everything)** means **C(*x*) is always true**
(B) **C(something)** means **C(*x*) is sometimes true**.

[4] I use bold text instead of corner quotes.
[5] This story figures in Cresswell (1973: 6), Sainsbury (1979: 10-11), Dummett (1982: chapter 2), and Soames (2010: 119-129).

In these formulations, $\mathbf{C(}\boldsymbol{x}\mathbf{)}$ expresses a propositional function, which—very roughly—is the worldly correlate of a complex predicate in Russell's early ontology.

In an account of this sort, complex predicates are required to analyze generalities such as (4) and (5).

(4) Something is such that it is young and it is a kangaroo.
(5) Everything is such that if it is a kangaroo, then it is young.

In order to treat the quantifiers 'something' and 'everything' as predicates of predicates, they must operate on predicates in (4) and (5). Using italics to represent a propositional function, one may regiment these as (4*) and (5*):

(4*) *x is a young and x is a kangaroo* is sometimes true.
(5*) *if x is a young, then x is a kangaroo* is always true.

(4*) may be read as: the propositional function, *x is young and x is a kangaroo,* is sometimes true. (5*) may be read as: the propositional function, *x such that if x is young, then x is a kangaroo,* is always true. Thus, complex predicates—under the guise of propositional functions—are required to analyze even very simple quantified sentences on this account.[6]

To streamline presentation, I use the notation of the λ-calculus. A complex predicate of the form **is an *x* such that** $\boldsymbol{\phi}(\boldsymbol{x})$ will be regimented using the λ-abstract $\boldsymbol{\lambda x \phi}(\boldsymbol{x})$.[7] The view that quantifiers operate on complex predicates can then be expressed by treating the quantifiers '∀' and '∃' as operating on λ-abstracts such as $\boldsymbol{\lambda x \phi}(\boldsymbol{x})$. Thus, sentences of the form **everything is such that it is** $\boldsymbol{\phi}$ will be represented as $\forall\boldsymbol{\lambda x \phi}(\boldsymbol{x})$, while sentences of the form **something is such that it is** $\boldsymbol{\phi}$ will be represented as $\exists\boldsymbol{\lambda x \phi}(\boldsymbol{x})$.

Quantification and identity are intimately related. So it is natural that complex predicates are attractive to those theorizing about identity. Specifically, one of the defining characteristics of identity is Leibniz's Law: if *a* and *b* are identical, then what holds of *a* also holds of *b.* A traditional way of regimenting this idea allows for universal substitution of '*a*' for '*b*'.

Leibniz's Law 1 [LL1]:

$$\frac{\begin{array}{c} a=b \\ \phi \end{array}}{\phi[a/b]}$$

[6] For related reasons, Russell (1918: 237) worries that positing general facts will force him to reintroduce worldly correlates of truth functional connectives such as 'and' and 'or' into his ontology.
[7] This calls for two comments. First, I am not yet imposing any interpretation on λ-abstracts beyond their role in regimenting English expressions. Second, λ-abstracts *can* occur in predicate position, but I remain neutral as to whether they can also occur in subject position.

This understanding of Leibniz's Law works most cleanly in extensional contexts. For instance, it allows one to infer 'Phosphorous is bright' from 'Hesperus is bright' and 'Hesperus = Phosphorous'.

However, this regimentation faces difficulties if we admit *intensional contexts* such as modal or epistemic operators.[8] Thus, many would resist the inference from the premises 'Hesperus = Phosphorus' and 'the Babylonians believed that Hesperus was bright' to the claim 'the Babylonians believed that Phosphorus was bright'. This might lead one to adopt another form of Leibniz's Law according to which anything truly predicated of *a* is also truly predicated of anything identical to *a*.

Leibniz's Law 2 [LL2]:

$$\frac{\begin{array}{c} a=b \\ \lambda x\phi x(a) \end{array}}{\lambda x\phi x(b)}$$

This line of reasoning allows one to resist the above inference, if one denies that 'the Babylonians believed that Hesperus was bright' entails 'λ*x*(the Babylonians believed that *x* was bright)(Hesperus)', which says that Hesperus is such that the Babylonians believed that *it* was bright.

One important consequence of this conceptualization of quantification and Leibniz's Law is that one cannot directly apply a rule of universal instantiation or Leibniz's Law 1 in arbitrary formulas. This can be illustrated in the case of quantification by considering the inference from (6) to (7).

(6) Everything is such that it is young and it a kangaroo.
(7) Joe is young and Joe a kangaroo.

On the view under consideration, one cannot directly infer (7) from (6). Rather, the inference must proceed via the intermediate step (7*).

(6)	Everything is such that it is young and it a kangaroo.	
(7*)	λx(*x* is young and *x* is a kangaroo)(Joe)	∀-Instantiation
(7)	Joe is young and a kangaroo.	Conversion

Thus, the logicality of the inference from (6) to (7) depends on the logicality of the "conversion" from (7*) to (7). Analogous results hold for applications of Leibniz's Law.

So in order for the standard inferences to count as logical on this construal, the link between a basic sentence and the predication of the corresponding λ-abstract must be logical. At very least, this serves to highlight that there must be some intimate link between a statement in which a name occurs **Φ(a)** and a corresponding complex predication **λ*x*Φ(x)(a)**.

[8] See discussion in Stalnaker (1977).

1.2 Property Reductions

It has been argued that the properties expressed by complex predicates play a role in inter-theoretic reduction. This purported application provides another illustration of the requisite explanatory connection between a complex predicate and its component expressions. Imagine that a traveller in Australia learns the predicate ‘is a joey’ by immersion. The traveller will eventually come to accept (8).

(8) For any *x*, *x* is a joey *iff x* is young and *x* is a kangaroo.

If one takes the predicates in this sentence to express properties, then (8) is striking. For, it surely demands explanation that the property expressed by ‘is a joey’ is always coincident with the pair of properties expressed by ‘is young’ and ‘is a kangaroo’.

The predicate ‘is a joey’ is a toy example. But there are serious issues at stake. The predicates of the special sciences—chemistry, biology, psychology, and so on—are sometimes coincident with open sentences of the languages of physics. Such coincidence demands explanation. One of the aims of science is to provide such explanations.

Kim and others argued that these explanations issue from the fact that special sciences can be deduced from the facts stated by the underlying science together with *statements identifying the properties of the special sciences with properties of the underlying science.*[9] On this proposal, one first constructs a complex predicate from the open sentence, ‘λx(*x* is young and *x* is a kangaroo)’. One then identifies the property expressed by a target special science predicate with the property expressed by the complex predicate. This is meant to provide an explanation, since—as Kim (2000, 98) remarks -- “Identity takes away the logical space in which explanatory questions can be formulated.” The property of being a joey and the property of being young and a kangaroo are “coinstantiated because they are in fact one and the same property”.

Whether or not identities are so explanatory, they are insufficient on their own to explain a truth such as (8). Sentence (8) follows by logic from the relevant property identity only if it is a matter of logic that anything satisfying **λ*x*(*x* is young and *x* is a kangaroo)** also satisfies the open sentence ***x* is young and *x* is a kangaroo**. Hochberg (1978: 137) has remarked that even if one takes a sentence such as (8) as an analysis of ‘is a joey’, this does not answer “the question of whether there is an additional property or whether the property [being a joey] is ‘reduced’ to [being young] and [being a kangaroo]”. As Kim (103-4) seemingly acknowledges, to fully explain (8), the link between a complex predication such as ‘Joe is young and a kangaroo’ and corresponding complex sentence such as ‘Joe is young and Joe is a kangaroo’ must also take away the logical space in which explanatory questions can be formulated.

[9] See Armstrong (1978: chapters 13 and 14) and Causey (1988).

2. Grounding

A popular recent strategy for dealing with explanatory demands appeals to *grounding*. Proponents of this strategy attempt to satisfy the demand to explain one phenomenon in terms of another by suggesting that the former are *grounded* in the latter. It is often held that if A-facts are grounded in B-facts, then there is an explanation of A-facts by B-facts. Thus, one might hold that if facts reported by complex predications are grounded in facts reported by atomic predications, then there is an explanation of the former by the latter.

I will not address this strategy in detail, but note only that it bifurcates. Some proponents of grounding, *the modest ones*, appeal to it only as a *placeholder*. The grounding claim reports that *some strategy for metaphysical explanation* of A-facts by B-facts is available.[10] A grounding claim acts as a promissory note for a forthcoming explanation. One might hope that a satisfactory account of complex predicates would figure in this promised explanation. This paper aims to dash that hope.

Other grounding theorists, *the ambitious ones*, argue that a grounding claim *is* an explanation.[11] If the A-facts ground the B-facts, then the A-facts explain the B-facts. Nothing further is required. These grounding theorists would see the task of explaining complex properties as an instance of a broader task in metaphysical explanation. Thus, on this view, the equivalence between 'Joe is young and a kangaroo' and 'Joe is young and Joe is a kangaroo' is not ultimately explained by any facts peculiar to complex predication, but in terms of a primitive notion of grounding. For this reason, I put these more ambitious strategies aside.

3. Set-Theoretic and Second-Order Approaches

The simplest models for complex predicates are set-theoretic. Stalnaker (1977: 328-9) models $\boldsymbol{\lambda x \Phi}$ as designating the set of objects satisfying $\boldsymbol{\Phi}$. Similarly, Lewis (1986) argues that complex predicates are better explained using class-theoretic resources than a theory of universals.

This is all fine as model-theory. But it is difficult to accept as metaphysics. There are familiar difficulties with class-nominalism such as those arising from the possibility of co-instantiated (or even necessarily co-instantiated) properties.

More importantly, construing properties as classes does not in itself explain the distribution of properties expressed by complex predicates in terms of the distribution of properties expressed by simpler predicates. In other words, it does not show that the resources used to explain the simple predications

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[10] Wilson (unpublished) and Jenkins (2011) are at least open to the possibility that grounding or dependence claims may issue from underlying explanatory connections such as identity. See also MacBride (2013: §§3.6, 3.7).
[11] Perhaps, Fine (2001).

suffice for the complex ones. The existence of, say, the intersection of two sets does not follow from the existence of the sets, except by supposing a background theory of classes. Thus, holding that the conjunction of two predicates expresses the intersection of the classes expressed by the predicates requires a background property theory in the form of a theory of classes.

Moreover, classes seem inapt to play the roles described above for the properties expressed by complex predicates. For instance, if one wants to model the claim that something is self-identical as the claim that the property of being self-identical is instantiated, then one must hold that there is a property of being self-identical. If the property of being self-identical is the set of self-identical things, then there is a universal class. This is problematic.

It might be tempting to take complex predicates as disguised second-order descriptions of properties. The most simple-minded approach would take a complex predicate such as 'is young and a kangaroo' ['λx(Yx&Kx)'] as a definite description of the unique property *f* possessed by anything that is young and is a kangaroo, so that $\Phi(\lambda x(Yx\&Kx)) =_{def} (\exists! f)((fx \equiv (Yx\&Kx))\&\Phi(f))$. Hochberg (1977: 193) shows why this simple-minded approach is unsatisfactory. Namely, the schema "implies that there is one and only one property such that it is had by those things and only those things" that are young and a kangaroo. But on the assumption that we are dealing with properties and not classes, we want to leave open that the possibility that there are many properties had by all and only those things that are both young and a kangaroo.

To leave room for the possibility that there are co-extensive properties possessed by anything that satisfies some given complex predicate, complex predicates must be disguised *indefinite* and not *definite* descriptions.

$$\Phi(\lambda x(Yx\&Kx)) =_{def} (\exists f)((fx \equiv (Yx\&Kx))\&\Phi(f)).$$

There is nothing intrinsically objectionable about such an account, as there is with construing complex predicates as disguised definite descriptions. But such an account sits ill with the explanatory project. Rather than explaining the distribution of properties specified in terms of complex predicates in terms the distribution of simpler properties, it simply presupposes a background property theory. As Hochberg (2011: 69) remarks, "To specify something as the same as something else without saying what that something else is or knowing what it is on other grounds—by acquaintance, say—is problematic". It is especially problematic if the specification is meant to meet explanatory demands.

4. Primitive Complex Predications

I noted two ways to form a complex predicate. One way appeals to *combinatory* operators such as 'and', 'or', and 'not'. These combine with predicates to form new predicates. 'Not' takes a predicate such as 'is young' as an argument and yields its negation, 'is not young'. Similarly, 'and' and 'or' take two predicates such as 'is young' and 'is a kangaroo' and yield a conjunction and disjunction of these predicates, respectively: 'is young and is a kangaroo' and 'is young or is a kangaroo'. Sentences predicating these negative, conjunctive, and disjunctive predicates are *not* themselves negative, conjunctive, or disjunctive. They are simply predications.

The other way to form a predicate appeals to *abstraction*; complex predicates are the result of attaching the expression 'is such that' to an open sentence. This approach is embodied in the λ-calculus notation that I made use of above, as well as Russell's circumflex notation.

Given that each approach corresponds to a well worked out mathematical theory, it might be tempting to rest content with one or the other methods for forming complex predicates as *primitive.* I will argue, however, that this sort of primitivism does not secure the desired explanations of claims involving complex predications in terms of more basic predations.

4.1 Primitive Combinators

A proponent of the combinator approach must explain the relationship between the predicate-forming combinators 'not', 'and', and 'or' and the more familiar sentence-forming operators with the same spelling and pronunciation. As Partee and Rooth (2002: 336) say, it is "no accident" that these are the same words.[12]

The underlying reason for this is that sentences containing corresponding sentential and predicate conjunction seem logically equivalent. Thus, (2) 'Joe is young or Joe is a kangaroo' seems to entail (2*) 'Joe *is young or is a kangaroo'.* Hochberg (1978, 253) remarks,

> [T]o have "complex predicates" we require additional "logical" signs to form any such predicates. We would, thus, require the construction of a calculus for properties that would [...] specify the connection between '$\vee$ ' as the standard sign for disjunction and, say, '$^\vee$' as used to form predicates. Thus, we would need to provide a basis for sentences like
>
> $$(x)[(F_1x \vee F_2x) \equiv F_1{}^{\vee}F_2(x)]$$
>
> […] In short, we would have to construct a "logic" of properties.

On this approach, the signs for predicate forming operators would need to be differentiated from the signs for sentence forming operators and the entailment between them would have to be explained. But if such predicate forming operators are *primitive*, then it is difficult to see what an explanation could consist in. We have instead a brute assertion of the equivalences such as $(x)[(F_1x \vee F_2x) \equiv F_1{}^{\vee}F_2(x)]$.

The task becomes even more difficult if one views predicates as standing for properties. As Hochberg (2003: 74) says: "Suppose one takes the pair of

[12] Evans (1985c) calls it "absurd" to suppose that 'and' is ambiguous.

predicates 'is **Φ**' and 'is not-**Φ**' to indicate two properties […] that are the 'negations' of each other. If one then distinguishes 'it is not the case that *x* is **Φ**' and '*x* is not-**Φ**', it is not at all clear in what sense the predicate 'is not-**Φ**' represents a property that is a negative property – the negation of **Φ**." In particular, if properties simply are entities, then there is no explanation of what features of the property of being not Φ would make it logically incompatible with the property of being Φ.

At any rate, if one takes predicate conjunction as fundamental, then one certainly cannot derive the facts about conjunctive predication *from the logical rules that govern sentential conjunction*. One needs to add a new logical rule connecting conjunctive predications to sentential conjunctions. In this sense, one cannot extract an account of conjunctive, disjunctive, and negative predication from a disquotational semantic characterization of these operators plus one's account of sentential conjunction, disjunction, and negation, respectively.

There is a lacuna here that the proponent of such combinatory approaches could exploit. Thoroughgoing advocates of combinatory approaches such as Curry and Feys (1958/1968) might simply refrain from deploying sentential connectives, taking the operations of negation, conjunction, and disjunction to operate on a single type of object. Lewis (1983b)—following Frege (1879/1970: 3-4)—imagines that sentences consist in applying an unvoiced predicate to this sort of object. In other words, all sentences are predications.

Fully exploring this strategy would take us into complicated territory. So I merely note two unwelcome features. The first is that a pure combinatory approach avoids the need to explain certain equivalences only by positing other equivalences in need of explanation. In order to reconstruct first-order logic, combinatory logic must supplement the predicate forming operators 'not', 'and', and 'or' with other combinators. In particular, to differentiate the monadic predicate **λx(Fx&Gx)** from the dyadic predicate **λxλy(Fx&Gy)**, the combinatorial approach typically appeals to a new operator, **W**, that takes a dyadic predicate and fills both argument positions.[13] The introduction of such operators raises new logical connections. For instance, **(WF)a** is the result of applying **W** to a relation term **F** (or **λxλy(Fxy)**) and then applying the result to a term **a**. Thus, **(WF)a** must be *logically* equivalent to **Faa**. This is not a problem *per se* for the advocate of combinatory logic. But it does darken the prospects of doing so without providing a more worked out background theory of the objects designated by the "combinated" predicates.

The other unwelcome feature of the combinatory approach is related to an objection from Hochberg (2003: §1g) discussed in §5 below. The general worry is that combinators destroy type distinctions. The operators will need to be able to apply to predicates of any *adicity*. One might be concerned that this makes it more difficult to resist the paradoxes. But a more direct concern is that predicates are identified partially by contrasting them with other types of expression. If the operators may take purportedly 0-adic, monadic, and dyadic

[13] The notation '**W**' is taken from Curry and Feys (1958/1968: 152).

expressions indiscriminately, then it is less plausible that any of the expressions of the language are actually predicates.

4.2 Primitive Abstraction

Those who appeal to primitive λ-abstraction need not posit two notions of negation, conjunction, and so on. Sentential operations and λ-abstraction suffice. But still, conjunctions such as (1) 'Joe is young and Joe is a kangaroo' are different, at least syntactically, from corresponding ascriptions of conjunctive properties as in (1*) 'Joe is young and is a kangaroo'. Similarly, a statement that an object stands in a relation to itself as in (3) 'Joe saw Joe' [or: '*S*(Joe,Joe)'] differs from an equivalent statement ascribing the corresponding reflexive property to that object as in (3*) 'Joe is such that he saw himself.' [or: 'λ*xS*(*x,x*)(Joe)']. This equivalence demands explanation.

For now, suppose that the truth conditions of sentence (3) will be specified disquotationally: 'S(Joe,Joe)' is true iff *Joe saw Joe.* The truth conditions of sentence (3*) will be specified as follows: 'λ*xS*(*x,x*)(Joe)' is true iff λ*xS*(*x,x*)(Joe). From this semantic characterization, one cannot derive that (3) is true merely from the fact that (3*) is true without employing conversion. Thus, the equivalence of (3) and (3*) is not explained by their disquotational truth conditions alone. One must suppose that conversion is an additional logical law that blocks the demand for further explanation.

Another version of this strategy derives from what Church (1951, 1973) calls alternative (0) and is developed by contemporary proponents of "structured" propositions. On this approach, every sentence has a *content*. We may think of this as the *proposition expressed by a sentence*. I represent the relevant structured entity using brackets '{…}'. Thus, '{A,B}' will designate the structured whole composed of the designate of 'A' and 'B'. In the case of an atomic sentence such as (3), this content is determined by the meanings of 'Joe', 'saw', and 'Joe', taken in that order.[14] A proponent of structured propositions might say that the content of (3) is a structured proposition containing the relation of seeing, S, and two occurrences of the individual Joe, represented as: {S, Joe, Joe}.

Complex sentences raise complications, but—as a concession to the advocate of complex properties—I will assume that they too have a single content. Thus, the content of a sentence of the form 'Φ&Ψ' derives from the contents of 'Φ' and of 'Ψ', and the content of '&'. Specifically, I assume that it is the structured fact whose first component is the content of 'Φ', whose second component is the content of '&' represented as '^', and whose third component is the content of 'Ψ'. We may then represent the content of (1) as {{Y,Joe},^,{K,Joe}}.[15]

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[14] I am ignoring the many complications that arise concerning relational predication.

[15] Here I am following Soames's (2003: 101-6) pseudo-Russellian account of propositions. Once again, I will ignore the many complications with taking logical

According to the proponent of primitive λ-conversion, the content of a complex predication such as (1*) and (3*) will be a simple predication, composed only of the property and the individual to which it is ascribed. Thus, the content of these sentences will be {λx(Yx&Kx),Joe} and {λxS(x,x),Joe}, respectively.[16]

What is the connection between the proposition expressed by a sentence such as (1) or (3) and the proposition expressed by corresponding complex predication (1*) or (3*)? Salmon (2010, 448) offers an answer by contrasting the content of two sentences analogous to (3) and (3*).

> These sentences therefore do not express the same proposition. The λ-abstract in [(3*)] expresses a property or concept not expressed in [(3)]: that of [self-seeing]. Yet [(3*)] and [(3)] are logically equivalent, by the rules of λ-expansion (which licenses the inference from [(3*)] to [(3)]) and λ-contraction (which licenses the reverse inference).[17]

So Salmon (210: 451) thinks that the propositions expressed by sentences that are λ-conversions of each other are distinct but *logically equivalent.*

> It is essential to my view that λ-converts like [(3*)] and [(3)] are logically equivalent but nonsynonymous.

Thus, the logical connection between the content of a complex predication and the content of its conversion is secured merely by *positing* a logical equivalence.

Kripke (2005: 1025, footnote 45) warns Salmon that the lack of an explanation for this logical equivalence might undermine arguments that Salmon himself would want to offer. The point isn't that conversion is an invalid step in a derivation. Rather, it's that the legitimacy doesn't follow from the logical moves required to understand simple predications and sentential operation them. Nor does it follow from these taken together with our account of what complex predication does. Indeed, it must be assumed as an additional rule in order to make our account of complex predication work.

Thinking of predicates as expressing properties adds to this perplexity. For why should standing in a relation to one's self as is asserted to obtain in (3) 'S(Joe,Joe)' entail that one has a different, "reflexive", property as is asserted to obtain in (3*) 'λxS(x,x)(Joe)'? This question is especially difficult to answer when assuming "structured" accounts of content outlined above. The proposition expressed by (3*) is composed simply of the property expressed by 'λxS(x,x)' and an individual, Joe. The internal structure of the predicate 'λxS(x,x)' is not

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constants to represent. I do so only as a concession to advocates of complex predication.

[16] This account essentially follows Soames (2003: 101-6), but I believe also underlies the view of Salmon (2010).

[17] The example has been changed for continuity.

reflected in the proposition expressed. As Russell (1905, 482) remarks in a related context, the connection is "merely linguistic" and "through the phrase". What is wanted is a "logical relation" between the proposition expressed by a complex predication and the proposition expressed by its conversion. Without additional considerations, that relation has gone missing.[18]

Hochberg (1987: 90) brings this out by contrasting the difference between an individual α standing in a relation to β, α standing in that relation to α, and α standing in that relation to itself.

> It should then be obvious that to assert (1) that α has R to β, (2) that α does not have R to itself, and (3) that β does not have R to α must be understood as claiming that exactly what is asserted to obtain between α and β is asserted not to obtain between a and itself and between β and α.

Hochberg's view, as I understand it, is that the intimate connection between holding that $R(\alpha,\alpha)$ and $\lambda xR(x,x)(\alpha)$ suggests that these state one and the same fact. Asserting that $R(\alpha,\beta)$ ascribes to the pair $<\alpha,\beta>$ exactly what asserting $\neg R(\alpha,\alpha)$ or $\neg$R(α, itself) denies of the pair $<\alpha,\alpha>$. In §5 and §6, I turn to a Russellian and a Fregean attempt to secure this result.

5. Complex Predicates as Propositional Functions

Suppose that a sentence **S(n)** contains a proper name **n** but is free of λ-abstracts. Suppose further that the sentence has a content: it expresses a proposition or states a fact. It would be helpful if a sentence of the form **λxS(x)(n)** expresses the very same proposition, or states the same fact. The goal is to secure against the objection that:

> It is […] absurd to take the fact that *a* has the property of standing in R to *b* to be distinct from the fact that *a* stands in *R* to *b*. (Hochberg 2003: 80)

On this view, the λ-abstract expresses a *propositional function,* "abstracted" from a whole proposition. The proposition is a value of the function for a given argument. Thus, the content of **Rab** is identical to the content of **λxRxb(a)**. This

[18] Salmon (2010: 465) allows that the λ-abstract in (3*) 'λxS(x,x)(Joe)' has a propositional function—in the sense described below—as its semantic value, "in some sense", but he denies that (3*) expresses the proposition which is the value of this function when Joe is the argument. Thus, (3*) expresses the proposition {λxS(x,x),Joe}, where λxS(x,x) is a propositional function. The value of this function for argument Joe is the proposition {S, Joe, Joe}, which is different from {λxS(x,x),Joe}. It would follow that the link between these two propositions is more than "purely verbal". But not by much. For we have no explanation of why the result of predicating a propositional function of an individual is always equivalent to the proposition that is the value of the propositional function for that individual as an argument.

is meant to explain the equivalence between a complex predication and its conversion.

But what is a propositional function? In his early work, Russell is a realist about propositional functions. After finding a number of reductions unsatisfactory, Russell (1903: §81) takes propositional functions to be "indefinable" structured wholes containing "the variable". Replacing the variable by a corresponding individual in this complex results in a *proposition*. The propositional function *denotes* such propositions.[19]

The immediate problem with such a view is that there is no "backward road" from a proposition to a propositional function that takes it as a value. That is, even assuming that 'Joe sees Joe' expresses a proposition, representable as {Joe, seeing, Joe}, it does not follow that there is a corresponding structured complex {*x*, seeing, *x*} that denotes it. The propositional function—so construed—is something over and above than the proposition.

Another problem is inferring the existence of the proposition from the propositional function. Russell suggests that there is a "logical connection" between the two, "in virtue of which such [propositional functions] inherently and logically denote such [propositions]".[20] But it is unclear what such a logical relation might be.[21] For this reason, Russell (1903: §81; 1906) tries various strategies to do without propositional functions.

It is worth noting that there are other *realist* accounts of propositional functions. For instance, Soames (2003: 102-106; 2008) offers a reconstruction of Russell's logic, according to which propositional functions are simply ordinary functions from individuals into propositions.

Viewing complex predicates as denoting functions from individuals into propositions in this way *presupposes* a background theory of functions. So it's hard to see how it could explain the logical relationship between a complex statement and a corresponding complex predication.

More importantly, functions are best understood as single-valued relations. A relation R is a function just in case any object *a* is related by R to exactly one object *b*. In order to generate enough functions, one already needs an abundant supply of relations in the background. So rather than explaining the distribution of abundant properties in terms of a sparse base, this conception of complex predicates simply assumes a background property theory. At least, it

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[19] See discussion in Hylton (2005b) and Klement (2003; §7).

[20] Russell (1903: §56) is talking about the relation of a denoting concept to its denotation, of which the relation of propositional functions to propositions seems to be a special case. Russell (1905: 486) also says that the relation between a denoting concept and its denotation must be "logical".

[21] Obviously, these considerations relate to Russell's infamously complicated Gray's Elegy argument in "On Denoting".

assumes a property theory generous enough to provide abundant single-valued relations between objects and propositions.[22]

Hochberg (2003: §1g) points out another problem with taking the properties specified by complex predicates to be functions in this way: it is questionable whether the strategy generalizes to relations. In particular, if one supposes that the complex predicate 'λx(Sxx)' expresses a function from individuals to conjunctive propositions, then it is natural to suppose that 'λxλy(Sxy)' is a relation term that expresses a function from an individual to a proposition. This function takes an individual *a* to yield the propositional function *λy(Say)*. This new function takes an individual *b* to yield the proposition that *Sab*. This identification of relations with functions from individuals into new functions—known as Schönfinkelization—is a perfectly legitimate way to model relations. Yet as metaphysics, it seems implausible as it creates an asymmetry between properties (which take an individual into a proposition) and relations (which take an individual to yield a function).

6. Complex Predicates as Fregean Functions

The task is to show that the truth of a complex predication such as 'Joe is young and is a kangaroo' is logically entailed by—or at least explained by—the truth of a corresponding complex sentence such as 'Joe is young and Joe is a kangaroo'. The Fregean account of complex predicates purports to secure this result. For Frege, the result of removing some number of occurrences of a name from a sentence leaves an expression that takes the semantic value of the name as an argument.

> Suppose that a simple or complex symbol occurs in one or more places of an expression[…]If we imagine this symbol as replaceable by another (the same one each time) at one or more if its occurrences, then the part of the expression that shows itself invariant under such replacement is called the function; and the replaceable part, the argument of the function. (Frege 1879: 13)

The sequence '…is young and…is a kangaroo' results from removing 'Joe' from 'Joe is young and Joe is a kangaroo'. On Frege's view, this sequence is a complex predicate, denoting a function mapping an individual to the true just in case that individual is both young and a kangaroo. Function terms that result from the removal of some occurrences of a name from a sentence are the complex predicates of Frege's system.

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[22] Russell (1903: §482) himself argues that ordinary mathematical functions presuppose background propositional functions: "If *f(x)* is not a propositional function, its value for a given value of *x* […] is the term *y* satisfying the propositional function *y=f(x), i.e.* satisfying, for the given value of *x,* some relational proposition; this relational proposition is involved in the definition of *f(x),* and some such propositional function is required in the definition of any function which is not propositional."

Frege (1893: §1) introduces the Greek letters 'ξ' and 'ζ' to mark the argument places. Thus, the *monadic predicate* '(ξ) is young and (ξ) is a kangaroo', results from the removal of both occurrences of 'Joe' from (1), while the *dyadic predicate* '(ξ) is young and (ζ) is a kangaroo' relates two individuals just in case one is young and the other is a kangaroo.

This approach can account for the logicality of the inference from a complex predication to a corresponding complex or relational sentence because, at some level, they are the very same sentence. As Evans (1985c: 204) remarks, a statement attributing a complex predicate such as '(ξ) sees (ξ)' to Joe is the same as the statement attributing a relational predicate such as '(ξ) sees Joe' to Joe.

> Consider for a moment the properties determined by the following two monadic predicate expressions: ['(ξ) sees Joe' and '(ξ) sees (ξ)']. These are certainly different properties, in that there are objects which satisfy the second but which do not satisfy the first. But it is not correct to infer from that in all cases, the ascription of one property yields a statement with a different content from that which results from the ascription of the other property to that individual. When the two properties are ascribed to [Joe], the results are the same, namely: ['Joe sees Joe'].

Similarly, a statement attributing the complex predicate '(ξ) is young and (ξ) is a kangaroo' to Joe just is the complex sentence (1) 'Joe is young and Joe is a kangaroo'. The sentences in each pair are logically equivalent, because they are the very same sentence.

Another advantage is that one need not posit two types of conjunction, one between predicates and the other between sentences. As Dummett (1981, 15) says,

> [I]t is of great importance that the predicate itself is not thought of as having been built up out of its component parts: we do not need to invoke the conception of the conjunction of two predicates, ['(ξ) is young'] and ['(ξ) is a kangaroo'] to explain the formation of the predicate ['(ξ) is young and (ξ) is a kangaroo'].

So unlike approaches appealing to primitive combinators, everything is handled through the sentential connective 'and' together with the abstraction operation.

At its heart, the strategy rests on the Fregean idea that a single "content" may be "carved" into different function and argument structures.

> The same conceptual content may be regarded as a function of this or that argument, so long as function and argument are completely determinate. (Frege 1879, 14)

But, it also rests on the idea that a *single sentence* has many different function and argument structures. The sentence 'Joe is young and Joe is a kangaroo' is *both a conjunction and a predication.* This cuts against the grain of modern syntactic theories according to which a sentence such as (1) 'Joe is young and Joe is a kangaroo' has a single compositional structure.

I see two difficulties with this approach. One problem is that the very formulation of the approach requires one to be able to identify the constituents of a sentence. One must be able to see that 'Joe' is a constituent of 'Joe is young and Joe is a kangaroo'. Yet, on the Fregean approach, it is difficult to see where these syntactic facts come from. For instance, (9) results from applying the quantifier 'someone' to the predicate '(ξ) is young and Joe is a kangaroo'.

(9) Someone is young and Joe is a kangaroo.

One should be able to form a predicate by removing 'Joe' from (9). But it is unclear what allows us to infer that 'Joe' is a constituent of (9). As Dummett would say, the complex predicate '(ξ) is young and Joe is a kangaroo' is not "built up" from the constituent predicate '(ξ) is young'. So if '(ξ) is young' is not a constituent, it is unclear why 'Joe' should be.[23]

Another problem posed by Russell concerns the coordination of argument places in a Fregean function term. Specifically, Frege's informal notation distinguishes the argument places of a function term using marks of incompleteness 'ξ' and 'ζ', but he is quite clear that these do not belong in his official notation (1893: §1). Nor does he give them any interpretation. They merely mark the gaps where a name has been removed.

But, if Frege has no genuine notation to mark the gaps in a function term, then what requires that two gaps be filled by the same term? Russell (1903, §482) objects on precisely these grounds.

> Frege wishes to have the empty places where the argument is to be inserted indicated in some way; thus he says that in $2x^3$+x the function is 2( )^3 +( ). But here his requirement that the two empty places are to be filled by the same letter cannot be indicated: there is no way of distinguishing what we mean from the function involved in $2x^3$+y.

As Russell is thinking, unless one offers some interpretation of the marks of incompleteness, the difference in marks of incompleteness does not correspond to anything. And so, we cannot distinguish a relational predicate such as 'ξ sees ζ' from a monadic predicate such as 'ξ sees ξ'.

7. Complex Predicates as Fragments of Sentential Contents

Russell (1903) considered a metaphysical version of the Fregean approach to complex predication, a variant of which has recently been defended by Armstrong (1997). Both Russell and Armstrong believe that some sentences have structured wholes as contents. Thus, Russell thought that sentences

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[23] See Pickel (2010) for a more sustained criticism.

expressed structured propositions, usually composed of the objects and properties that they are about. Armstrong holds that true atomic sentences and their conjunctions are *made true* by states of affairs, which are structured wholes composed of individuals and universals.

It might be tempting to think that complex predicates can be "carved out" of these structured wholes. Russell (1903: §§43, 44) considered the possibility that propositional functions could be replaced by "carvings" of these structured wholes that he called *assertions.*

> We may say, broadly, that every proposition may be divided, some in only one way, some in several ways, into a term (the subject) and something which is said about the subject, which something I shall call the *assertion.* Thus "Socrates is a man" may be divided into *Socrates* and is *a man.* (Russell 1903: §43)

Suppose that {Joe, sees, Joe} is the structured whole expressed by 'Joe sees Joe'. The idea then would be that a complex property such as seeing oneself, could be treated as the result of removing both occurrences of the individual Joe from the structured whole {Joe, sees, Joe}, leaving {( ), sees, ( )}.

Armstrong's (1997: 35-6) similar idea was to view a complex property as a *state of affairs type.* One may picture a complex property by picturing the state-of-affairs, but leaving blanks where the objects should be. In this way, both Russell and Armstrong considered explaining some of their complex properties in terms of simpler ones. Such additional properties would be "no ontological addition" to the simple ones.

Russell quickly rejects this analysis because it does not allow one to distinguish between a complex predicate attributed to a single subject and one that relates distinct subjects. Contrasting this approach with his own, which appeals to propositional functions containing "the variable", Russell (1903: §44) says:

> An assertion was to be obtained from a proposition by simply omitting one of the terms occurring in the proposition. But when we omit [Joe], we obtain [{( ), sees, ( )}]. [...] [I]t is essential that, in restoring the proposition, the *same* term should be substituted in the two places where dots indicate the necessity of a term. [...] Of this requisite, however, no trace whatever appears in the would-be assertion, and no trace can appear, since all mention of the term to be inserted is necessarily omitted. When an *x* is inserted to stand for the variable, the identity of the term to be inserted is indicated by the repetition of the letter *x;* but in the assertional form no such method is available.

Thus, Russell argued that propositional functions could not be reduced to assertions, or—in our terminology—complex predicates do not express fragments of propositions got at by knocking out individual constituents.

Hochberg (2001: 99) offers a related criticism of Armstrong, centered around Armstrong's idea that universals must be instantiated.

> [W]hile Armstrong can hold that given two facts Fa and Gb, there is a conjunctive fact Fa&Gb […], he cannot hold that given the two universals F and G, there is a conjunctive universal (F&G)[…]. For he holds that all universals must be instantiated.

Suppose that 'Fa&Ga', if true, expresses {{F, a}, ∧, {G, a}}. Armstrong would like to be able to abstract the property of being F and G from this fact and to represent it by leaving blanks for both occurrences of 'a': {F, ( ), ∧, G ( )}. The problem is that 'Fa&Gb' might express a fact representable as {{F, a}, ∧, {G, b}}. Removing occurrences of both 'a' and 'b' would seemingly yield the same representation: {F, ( ), ∧, G ( )}. But Armstrong would not want to infer the existence of a complex property being F and G from the existence of something that is F and something else that is G. For there might be nothing that has both F and G, in which case the property of being F and G would be an uninstatiated universal, which Armstrong seeks to avoid. For this reason, Armstrong himself (1997: 37) concedes that there is something "more" to a complex property than the state of affairs, but he does not explain what this is.

8. Complex Predicates as Incomplete Symbols

We have seen that there are grave difficulties in securing a logical connection between a complex predication—construed as ascribing a property to an individual—and a corresponding complex statement. The natural solution is to find some way to do without complex properties in our austere schema, our preferred theory of the world.

The applications of complex properties will then have to be dealt by other means. For instance, the treatment of the quantifiers in our preferred schema may ultimately have to do without them attaching to complex predicates, but directly to open sentences as in Tarski (1936). If we wish to allow for intensional contexts that do not permit the substitution of identicals, then we will have to be more careful in formulating our rules for substitution.

Finally, as Kim seemingly concedes, inter-theoretic reductions will need to be grounded by something other than identity.

> Say, there are three […] first-order properties, P1, P2 and P3. For something to have M, then, is for it to have P1 or have P2 or have P3. Here there is a disjunctive *proposition* or *fact* that the object has one or another of the three first-order properties; that is exactly what the fact that it has M amounts to. There is no need here to think of M itself as a property in its own right – not even a disjunctive property with the *P*s as disjuncts."
> (Kim 2000, 103-4)

The idea seems to be that certain predications such as 'Joe is a joey' may be reduced to complex statements such as 'Joe is young and Joe is a kangaroo' without that reduction proceeding by way of property identification, but by holophrastic paraphrase, truth-making, grounding or some other reductive device.

What of complex predicates as they occur in ordinary language? A revisionist might find reason to simply jettison them. Those who want to preserve the truth of such complex predications, however, might take refuge in a strategy of treating them as incomplete symbols, expressions which contribute to the truth conditions of sentences that contain them but which don't correspond to any element of the reality represented by these statements. This seems to be the proposal sketched by Hochberg (1978: 255).

> Complex predicates do not stand for entities in this sense. Rather, complex predicates are meaningful signs of the system in that they are formed, according to general syntactic rules, from primitive predicates which are so interpreted. One may then suggest that we speak of complex properties as a result of our linguistic apparatus having certain features […] By contrast , primitive predicates require properties in order for such predicates to be legitimate signs of the schema.

Construing complex predicates in this way carries significant explanatory burdens. In particular, we must specify exactly how the attribution of a complex predicate depends on the distribution of properties expressed by simpler predicates.

But regardless of the ultimate assessment of complex predicates, they cannot be construed as expressing properties without some cost. Either, that cost is paid by positing a primitive new logical connection between a complex predication and its conversion. Or, that cost is to be paid by an additional non-logical connection. Thus, complex predications do not in themselves legitimize abundant theories of properties.

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