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Modelling wind direction with an application

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Abstract: In this paper we investigate various approaches to model directional data. The motivating example relates to a wind direction dataset collected at the University of Edinburgh. Of particular interest is the need for a flexible model to determine appropriate summaries of the extensive data and thus to gain a better understanding of features of weather patterns. A simulation study was used to evaluate the properties of a numerical estimation procedure.

Keywords: Directional data analysis; Likelihood inference; Weather patterns.

1 Introduction

We consider methods for modelling direction data. In particular, we investigate modelling extensive data collected by the School of Geosciences at the University of Edinburgh. Datasets for the weather station are available at <http://www.geos.ed.ac.uk/abs/Weathercam/station/data.html> over several years. In this paper we consider a subset of the wind direction data in our analyses; the dataset relates to wind directions with wind speeds greater than 5ms^{-1} and recorded at 12 noon in the year of 2013.

2 The WFGSN model

We investigate the Wrapped Flexible Generalized Skew Normal (WFGSN) model proposed by Hernández-Sánchez and Scarpa (2012) for analysis of the wind direction data. The WFGSN density may be expressed as

$$f(\theta) = \frac{2}{\omega} \sum_{r=-\infty}^{\infty} \phi\left(\frac{\theta + 2\pi r - \xi}{\omega}\right) \Phi\left\{\alpha\left(\frac{\theta + 2\pi r - \xi}{\omega}\right) + \beta\left(\frac{\theta + 2\pi r - \xi}{\omega}\right)^3\right\}$$

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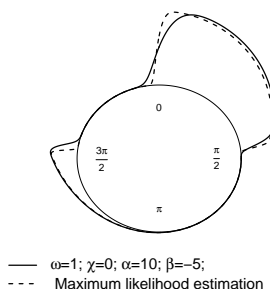


FIGURE 1. Comparison of densities: true and maximum likelihood estimated using an artificial dataset with $n = 50$.

for $0 \leq \theta \leq 2\pi$, $\xi \in \mathbb{R}$, $\omega > 0$, $\alpha \in \mathbb{R}$ and $\beta \in \mathbb{R}$ and WFGSN is a bimodal and asymmetrical distribution. In our modelling we also considered a more general version of the WFGSN distribution of degree $2k-1$ with $k = 1, 2, \dots$ defined by

$$f(\theta; \xi, \omega, \boldsymbol{\kappa}) = \frac{2}{\omega} \sum_{r=-\infty}^{\infty} \phi\left(\frac{\theta + 2\pi r - \xi}{\omega}\right) \Phi\left\{\sum_{i=1}^k \kappa_i \left(\frac{\theta + 2\pi r - \xi}{\omega}\right)^{2i-1}\right\},$$

for $0 \leq \theta \leq 2\pi$, $\xi \in \mathbb{R}$, $\omega > 0$ and $\boldsymbol{\kappa} = (\kappa_1, \dots, \kappa_k)^T \in \mathbb{R}^k$.

3 Simulation study

We investigate the ML estimation of the parameters from the WFGSN model. The log likelihood function, based on observations $\theta_1, \dots, \theta_n$, for the first version of WFGSN is given by

$$l(\xi, \omega, \alpha, \beta) = n \log(2) - n \log(\omega) + \sum_{i=1}^n \log \left[\sum_{r=-\infty}^{\infty} \phi\left(\frac{\theta_i + 2\pi r - \xi}{\omega}\right) \Phi\left\{\alpha \left(\frac{\theta_i + 2\pi r - \xi}{\omega}\right) + \beta \left(\frac{\theta_i + 2\pi r - \xi}{\omega}\right)^3\right\}\right].$$

We used the function `nllminb` in R to maximize the log likelihood, and this seemed to work well for both the simulation study and the application to real data which is given in the following section. Figure 1 gives a graphical comparison of a true model and a fitted model to an artificial dataset with sample size $n = 50$. This provides an illustration that ML estimation can work well. Table 1 gives the results of a simulation study, which is part of a larger study. In some cases the ML estimators were biased but generally they seemed satisfactory.

4 Application to wind direction dataset

We now consider application of the WFGSN model to the wind direction dataset, to investigate the nature of the weather patterns. Figure 2 shows densities of fitted models for WFGSN distributions, $k = 1, 2$. For comparison, Figure 3 gives a fitted mixture of von Mises distributions (Mardia and Jupp, 2000; Jammalamadaka and SenGupta, 2001).

5 Discussion

We have seen that the WFGSN can produce fits which are flexible. Our initial modelling involved using finite mixture models, and we had some issues with possibly poor fits, but the WFGSN approach seemed to provide an attractive alternative model which suited the data well. The simulation study illustrated that the WFGSN model seemed to have some nice features with regard to the estimation of the parameters.

Acknowledgments: We are particularly grateful to the School of Geosciences at the University of Edinburgh for making the wind direction dataset available.

TABLE 1. Simulation study results.

		Parameters			
		ξ	ω	α	β
	True value	0.0000	1.0000	10.0000	-5.0000
$n = 100$	Mean	-0.0055	1.0025	17.0381	-8.5205
	Bias	-0.0055	0.0025	7.0381	-3.5205
	SD	0.0247	0.0723	11.0265	5.1575
	RMSE	0.0253	0.0723	13.0813	6.2445
	CI Lower	-0.0104	0.9881	14.8502	-9.5438
	CI Upper	-0.0006	1.0168	19.2260	-7.4971
$n = 500$	Mean	-0.0022	0.9987	10.6971	-5.3579
	Bias	-0.0022	-0.0013	0.6971	-0.3579
	SD	0.0108	0.0313	1.6519	0.9740
	RMSE	0.0110	0.0313	1.7930	1.0377
	CI Lower	-0.0044	0.9925	10.3694	-5.5512
	CI Upper	-0.0001	1.0049	11.0249	-5.1647

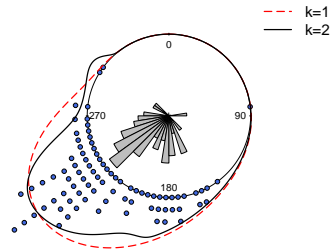


FIGURE 2. Rose diagram with densities of fitted WFGSN distributions, $k = 1, 2$.

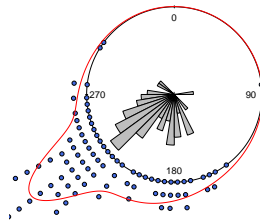


FIGURE 3. Rose diagram with density of fitted mixture of von Mises distributions.

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