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*When Do We Learn to Cooperate? The Role
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When Do We Learn to Cooperate? The Role of Social Learning in Social Dilemmas

James A Best*

June 7, 2011

Abstract

In this paper, I look at the interaction between social learning and cooperative behavior. I model this using a social dilemma game with publicly observed sequential actions and asymmetric information about payoffs. I find that some informed agents in this model act, individually and without collusion, to conceal the privately optimal action. Because the privately optimal action is socially costly the behavior of informed agents can lead to a Pareto improvement in a social dilemma. In my model I show that it is possible to get cooperative behavior if information is restricted to a small but non-zero proportion of the population. Moreover, such cooperative behavior occurs in a finite setting where it is public knowledge which agent will act last. The proportion of cooperative agents within the population can be made arbitrarily close to 1 by increasing the finite number of agents playing the game. Finally, I show that under a broad set of conditions that it is a Pareto improvement on a corner value, in the ex-ante welfare sense, for an interior proportion of the population to be informed. **JEL Codes:** C72, D62, D82, D83.

Keywords: Asymmetric information, cooperation, efficiency, social learning, social dilemmas.

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1 Introduction

Social learning allows agents to infer information about the payoffs to actions by observing the actions of other agents. In standard models of social learning agents do not care about whether their action reveals information to others or not.¹ However, when there is social learning within a social dilemma agents do care about what information their actions reveal to others. In social dilemmas the privately optimal action is not the socially optimal action: either because it has negative externalities or because some other action has positive externalities. In which case there is a potential cost to choosing the privately optimal action: doing so reveals information that causes others to choose the privately optimal action. Hence, there is an incentive for agents to forgo the privately optimal action. This suggests two possibilities when there is social learning within the context of a social dilemma; 1) if actions have externalities agents will act to purposefully influence other agents' information sets; 2) agents may be able to induce other agents to act cooperatively in social dilemmas if there is observational learning.

In this paper I examine these two possibilities using a model that I call an 'Example Setting Game'. In an Example Setting Game a finite number of agents act sequentially in a social dilemma with asymmetric information about the payoffs of actions. A proportion of the population, informed agents, have perfect information about the payoffs to actions. The rest, uninformed agents, do not know which actions yield which payoffs. In the action set there are two actions which correspond to *cooperate* and *defect* in a Prisoners' Dilemma: it is individually optimal, under perfect information, to defect but it is a Pareto improvement if everyone cooperates. There are other actions in the action set which yield a lower individual payoff than *cooperate* and *defect*. Uninformed agents risk getting one of these lower payoffs if they act without the guidance of the informed agents' actions. Hence, the uninformed observe the actions of the informed agents in order that they can avoid getting these lower payoffs. The informed agents know that their actions influence the behavior of the uninformed. Therefore, the informed have an incentive to forgo the higher private payoff from *defect*: to conceal from the uninformed which action corresponds to *defect*.

Using the Example Setting Game I show three main results. First, in social dilemmas with social learning, better informed agents may choose socially optimal actions to influence the information of other agents. This behavior can cause the majority of uninformed agents in the population to never learn which action corresponds to the privately optimal action. Moreover, it can induce cooperative behavior in the majority of the population: the proportion of agents cooperating can be made arbitrarily close to one by choosing a large enough population. Second, if too many agents, not necessarily all, have better information they will choose the privately optimal action and the uninformed agents will learn what this action is. This results in no one behaving cooperatively. Finally, information being held by an interior proportion of the popu-

¹This is the issue of informational externalities that play a central role in Banerjee (1992) and Bikhchandani *et al.* (1992). Agents copy the actions of others too much, from a social welfare perspective, because they don't consider the effect of their action on the information of other agents.

lation can be *ex-ante* Pareto superior to both corner cases where all or no agents are informed.

When an informed agent defects this may reveal information about payoffs that causes a portion of the population to play defect. Consequently, the informed agent bears some proportion of the social cost of the action: exactly that proportion of the population that defects as a result of the informed agent's action. Informed agents then internalize the social cost of an action to the extent which they expect it to cause other agents to defect who would have not defected otherwise. Informed agents then, weigh the difference in the private payoffs of *cooperate* and *defect* against their relative effect on the externalities generated by the actions of other agents. If they internalize a large enough proportion of the social cost relative to the difference in private payoffs then they will cooperate rather than defect. The informed agents cooperate in order that they may influence the information of uninformed agents, this is the intuition behind the first result. In some cases nearly all the Leaders will cooperate: implying nearly all of the population cooperates.

The intuition behind the second result is that, if the proportion of informed is high then each informed agent expects to influence the actions of fewer agents; because uninformed agents have the opportunity to observe the actions of many informed agents. Hence, the more informed agents there are the less they internalize the social cost of an action: because they crowd out each others influence. When all agents are informed they don't internalize any of the social cost of an action and they all defect. However, it is also the case that when a positive proportion are uninformed the influence of each individual informed agent can still be too small to make them cooperate.

The third result follows because there are two determinants of welfare that are determined by the proportion of informed agents. Aggregate welfare is determined by both the externalities and the private payoffs that actions generate. A higher proportion of informed agents leads to a higher number of agents defecting in expectation: causing lower welfare to accrue from externalities. However, a higher proportion of informed agents leads to fewer uninformed agents acting without guidance. This implies a smaller number of agents choosing a strictly dominated action, an action other than *cooperate* or *defect*, thus reducing the welfare that accrues from private payoffs.

The informed agent in my paper plays a comparable role to the Leader in Hermalin (1998) and Andreoni (2005). In these papers there is a single Leader with information about the value of contributing to a public good. The Leader acts before his Followers who do not have this information. The Leader signals, to the Followers, the true value of contribution to the public good through the Leader's own contribution. There is a unique equilibrium where the Leader contributes more than the second-best level to the public good while all Followers contribute at the second-best level. The total contribution to the public good is higher than in a perfect information game.

The informed agents in my paper are different from the Leader in Hermalin (1998) in four ways. First, there is no signalling in my paper. The informed agents may pay

a cost to conceal certain information from uninformed agents: they do not pay a cost to give credibility to some private information that they wish to reveal. Second, the equilibrium action of informed agents does not reveal all information about payoffs to the uninformed agents: as is the case in the signalling equilibrium of Hermalin (1998) and Andreoni (2005). This second result implies that the uninformed agents choose the socially optimal action in my paper: in a signalling framework it is the Leader alone who contributes above second best. Third, there are many informed agents with a stochastic order of action: implying informed agents are uncertain about the effect of their action on overall externalities. Fourth, informed agents do not necessarily act first. Therefore, there is a partially positive welfare effect from increasing the number of informed agents as noted above. If an informed agent acts first, like the Leader in the Hermalin and Andreoni papers, there are no positive welfare effect from increasing the number of informed agents.

This paper uses a framework similar to that expounded in the social learning models of Banerjee (1992) and Bikchandani *et al.* (1992). However, in my model the information of the informed is perfect. Hence, I do not examine the main concerns of social learning literature: the aggregation of imperfect information by groups and the issue of *informational externalities*. I abstract away from these issues in order that the effect of social learning on the generation of non-informational externalities can be examined with greater clarity. Therefore, the only learning in my model is the process of the uninformed learning from the informed.²

This paper also differs from other social learning literature by having externalities to the particular actions of agents. There are two other papers, (Dasgupta, 1999; Bhalla, 2007), that examine non-informational externalities in social learning models. The results in these papers pertain to the effect on social learning of coordination externalities. In contrast my paper examines a different kind of externality. The actions in my model are good or bad for society in themselves; irrespective of how many other agents choose the action. Moreover, I focus on the effect of social learning on the externalities generated rather than the effect of non-informational externalities on informational externalities. The only effect of externalities on the learning process that I examine is the manner in which the externalities can cause the informed to manipulate the information sets of the uninformed.

The folk theorem states that cooperative behavior can be supported in a subgame perfect equilibrium of an infinitely repeated prisoner's dilemma. Likewise, cooperative behavior can be supported in a subgame perfect equilibrium of a prisoner's dilemma where agents play sequentially and the population is infinite. In finite games, however, these equilibria break down. The mechanism in this paper for generating cooperative behavior with a finite population is dependent on asymmetric information. This has some similarity to Kreps *et al.* (1982) where asymmetric information over an agent's

²Preliminary work on a model with imperfect precision of information suggests that cooperative equilibria are still sustainable but that sometimes agents will herd on the wrong action: one that is believed to be cooperative but is in fact Pareto dominated by cooperate. Moreover, the welfare optimizing proportion of informed agents will be greater the higher the imprecision of the information.

type can yield partial cooperation in a finitely repeated prisoner's dilemma. In my model it is learning and the uncertainty about payoffs, not the fact that a game is repeated, that induces the possibility of cooperative behavior. Furthermore, the results in my paper are driven by uncertainty over other agents' information sets rather than uncertainty about other agents' rationality.

Finally, this paper implies that it may be desirable to restrict the public availability of information. There is other work that finds gains from restricting information such as Morris and Shin (2002). However, their welfare results are driven by public information leading to inferior decision-making. This is not the driving factor in my paper.

The structure of this paper is as follows. Section 2 gives a simplified description of how the Example Setting Game works that illustrates some of the main results. Section 3 sets up a model of an Example Setting Game with a specific payoff structure. Section 4 analyzes the nature of the equilibria in the Example Setting Game described in Section 3. Section 5 looks at how the proportion of informed agents determines the extent of cooperation in an Example Setting Game. Section 6 looks at welfare: I give conditions under which it is optimal, in the ex-ante welfare sense, to have a small non-zero proportion of the population informed. Section 7 describes a much larger set of payoff, externality and information structures under which the equilibrium results of Sections 4 and 5 still hold. Section 8 summarizes the results and discusses some issues not dealt with in the formal model. All proofs are left to the appendix.

2 A Simple Illustration

There is a tribe of islanders just beginning to interact with a globalized world. The children of the tribe inherit their own portion of beach-front property when they turn twenty-one. When they inherit their land they must decide what to do with it immediately. There are three options for their beach; training in traditional fishing; letting a consortium build a large hotel; or building low impact eco-resorts. Letting the consortium build a large hotel will be the most profitable should no one else do so. However, if everybody builds hotels they will all be worse off than if they build eco-resorts because an ugly coast line caused by everyone building big hotels will drive away tourism. Fishing is a very bad option: the payoff from fishing has the lowest individual payoff. Moreover, choosing Fishing when no one has chosen Hotel is worse than choosing Hotel when everyone has chosen Hotel.

Only one of the islanders, the Chief's son, knows which actions yield which payoffs. The other islanders only know that there is an individually worst action; an individually optimal action that is socially costly; and there is an action with no social cost and an intermediate individual payoff. These other islanders do not know, however, which action corresponds to which payoff. It is common knowledge that the Chief's son has this information. The islanders do not care about the well being of the others: each would like to choose the most individually profitable action.

It so happens that the Chief's son turns twenty-one before the rest of the islanders: he makes his investment decision first. The other islanders can infer that the son will not choose the individually worst option. Moreover, because they fear choosing the worst option³ they will copy him, even if they know he has not chosen the most privately profitable action. The Chief's son knows all this and chooses to build the eco-resort rather than the hotel. He does this because he doesn't want to suffer the negative externalities generated by all the islanders that will copy him. Hence, the Chief's son sets a good example because he has internalized the social cost of his action; and the islanders get the first best outcome.⁴

However, there is an alternative scenario where the son has a sister with exactly the same information as himself.⁵ It is common knowledge that she has this information. She turns twenty-one after exactly half of the islanders have turned twenty-one. The payoffs from building a hotel when only half of the islanders have built a hotel is larger than the payoffs of building an eco-resort when no islander has built a hotel. The worst that can happen to the sister if she builds the hotel will be if all the islanders after her build a hotel. This is better than building an eco-resort: therefore she will build a hotel. The islanders after her will all copy her; because they know that her action will be at least as selfish as her brothers. Hence, in equilibrium, the sister and the younger half of the islanders will all choose to build a hotel.

The effect of the sister on the equilibrium outcome causes more damage than just inducing the younger half of the islanders to build a hotel. In this equilibrium the son knows that only half of the islanders will copy him if he builds an eco-resort. However, as with his sister, determining half of the islanders actions is not a large enough incentive to make him build an eco-resort instead of a hotel. Consequently, the son decides to build a hotel; as does everybody else in the village, yielding a worse outcome for all. The future bad influence of his sister crowds out the influence of the son; and with it his incentive to set a good example. If only the first person were informed they would all be better off. When there are too many potential informed agents the informed agents will not attempt to set good examples and the outcome is not first best.

In this example all the villagers would prefer the sister to have no information. This is because when the informed agent always acts first no welfare benefits accrue from subsequent informed agents. Extra informed agents provide no extra information and crowd out the first informed agent's incentive to set a good example. However, it is not

³Which, from their perspective, could be either of the two options that the Chief's son has not chosen. Note, this requires that the intermediate payoff is greater than the average of the other two payoffs.

⁴The fact that the Chief's son's action is the action that will be chosen by all implies that it is rational, unlike in the perfect information case, for the Chief's son to act according to Kant's categorical imperative.

⁵This illustration captures some of the key elements of the model proper. However, it differs in that I have assumed that agents know when subsequent informed agents will act. In the model developed later in the paper agents do not know when subsequent informed agents will act. In this illustration equilibrium play is determined, in part, by this precise knowledge of when future informed agents act. In the model proper it will be based upon expectations of when future informed agents act.

necessarily the case that the Chief’s son would turn twenty-one first. If he didn’t then, without his action to guide them, the islanders may choose fishing: a worse outcome than everyone choosing to build a hotel. Thus, an advantage of more islanders being informed is that there is a greater probability that an informed child acts early. If an informed islander acts early then less people act without ‘guidance’; meaning less people risk choosing the very bad option of fishing. Hence, the proportion of informed agents has two welfare effects, cutting in opposite directions, when there is a stochastic component to the order of action. The social benefit of more information is that less agents choose very poor actions with low individual payoffs. The social cost is that more agents will be uncooperative in expectation.

3 The Example Setting Game

An Example Setting Game is a sequentially played social dilemma with asymmetric information about payoffs. A finite population of n agents are drawn sequentially from an infinite general population N . Each agent acts after being drawn from the general population and before the next agent is drawn. The action of agent i is a_i and is chosen from the action space $A = [0, 1]$. The profile of all actions up to and including a_i is denoted as A_i . Two elements $c, d \in [0, 1]$ are chosen by ‘Nature’ with uniform probability before the first agent is drawn. c and d are the only elements in the action space with positive instant payoffs. The instant payoff from d is larger than the instant payoff from c . However, there is a negative externality generated by d . After the n th agent has acted all agents receive a negative ‘externality payoff’ proportional to the number of agents who have chosen d .⁶ This gives the payoff to agent i as a function of the action profile of the population, A_n , as:

$$U_i(A_n) = u(a_i) - \sum_{j=1}^n \kappa_j \tag{1}$$

where the instant payoff from action a_i is given by:

$$u(a_i) = \begin{cases} z & \text{if } a_i = c \\ v & \text{if } a_i = d \\ 0 & \text{if } a_i \notin \{c, d\} \end{cases} \tag{2}$$

and :

⁶The pertinent equilibrium results also hold for finite action sets of three or more actions and for richer distributions of instant payoffs and externalities. This is examined further in the penultimate section ‘Alternative Payoff and Externality Distributions’.

$$\kappa_j = \begin{cases} \kappa > 0 & \text{if } a_j = d \\ 0 & \text{if } a_j \neq d \end{cases} \quad (3)$$

and the following conditions on the payoffs hold:

$$v - z > \kappa > 0 \quad (4)$$

$$(v - \kappa) - z = \Delta > 0 \quad (5)$$

$$L = n \cdot \kappa - v + z > 0 \quad (6)$$

κ is the negative payoff to other agents from playing d and will be referred to as an externality. ‘Own-action payoff’ of action a_i is used to refer to the instant payoff less any negative end of game payoff to i from action a_i . The own-action payoff is equal to an action’s instant payoff for all actions except d . d has an own-action payoff of $v - \kappa$.⁷ Condition (5) implies that this is larger than the own action payoff of choosing c . The difference between the two own-action payoffs of c and d , Δ , is referred to as the ‘defection incentive’. Hence, c and d are akin to *cooperate* and *defect* in a prisoners’ dilemma.⁸ Throughout this paper c and d shall be referred to as ‘*cooperate*’ and ‘*defect*’. The difference in the instant payoffs to c and d must be less than the product of the externality and the population size for the game to be a social dilemma. Hence, condition (6) implies the game is a social dilemma. L denotes the magnitude of the Pareto loss from the population choosing an all *defect* rather than an all *cooperate* action profile. L can be thought of as a measure of the size of the social dilemma that the population faces. Consequently, *cooperate* is always Pareto superior to all *defect*.

Information in the game is asymmetric. There are two types of agents: Informed and Uninformed. Proportion α of the general population are Informed and proportion $1 - \alpha$ are Uninformed where $0 \leq \alpha \leq 1$. Informed agents observe Nature’s move at the beginning of the game and Uninformed agents do not. Therefore, only the Informed have direct knowledge of the own action payoffs to all actions: crucially which actions correspond to c and d . The type of agents and their actions are public knowledge subsequent to having chosen an action⁹. Note, that Uninformed agents only observe the value of the action that Informed agents choose and do not observe whether this number corresponds to c , d or neither. Thus the information set h_i of agent i contains the action profile A_{i-1} and the types of all agents up to and including i . There is a set of Uninformed agents that act before the first Informed agent: I call these Uninformed agents ‘the Ignorant’. They are called Ignorant because this set of Uninformed agents have not even had the opportunity to learn about payoffs through observing the action

⁷It makes for easier exposition to have the externality from choosing d also affect the agent who chooses d . It makes no difference to the results as d is still individually optimal.

⁸To see this consider the game where $n = 2$. In this case $a_1 = a_2 = c$ is Pareto superior to $a_1 = a_2 = d$ but d is the dominant strategy under perfect information.

⁹It is not necessary for agent types to be known for a cooperative equilibrium to be sustained but it does make for a simpler proof.

of an Informed agent. All agents know the value of α and the payoffs attached to an agent correctly choosing c or d . However, agents do not know the exact number of Informed or Uninformed agents that will succeed them.

4 Equilibrium Play

Agents' payoffs are determined by the own-action payoff to their action and the externalities generated by other agents. An agent's choice of action affects the expectation of the end of game externality payoff in two ways. The first way is the externality generated by their action. The second way is the effect of the agent's action on the actions of subsequent agents. To examine this second effect on the end of game externality payoff I introduce the concept of an actions' 'externality impact'. The externality impact of some action a_i is the number of agents subsequent to i that play d , given that choice of a_i , multiplied by κ .

The externality impact of an action is determined by the order of the types of subsequent agents. Agents do not know if subsequent agents will be Informed agents or Uninformed agents. Consequently, agent i 's choice is determined by an expectation of the externality impact of action a_i . The expected externality impact of a_i is the expectation, conditional on h_i , of externalities generated by all agents subsequent to i given the particular choice of a_i . For example, let i compare two actions; $a_i = c$ and $a_i = d$. If i expects more subsequent agents to play d when $a_i = d$ than when $a_i = c$ then $a_i = d$ has a larger expected externality impact than $a_i = c$. Agent i 's decision problem is to choose a_i such that it maximizes the expectation of own-action payoff net of the externality impact of a_i . In this model these expectations and actions will be determined within a perfect Bayesian equilibrium; all uses of 'equilibrium' in this paper refer to perfect Bayesian equilibrium.

Equilibrium play is examined in four parts. The first part defines and discusses an Informed agent's 'Following'. In the second part I examine the game with perfect information. I show that on the equilibrium path all agents play d if $\alpha = 1$. I use equilibrium play in the perfect information case to introduce a proposition that I call the 'Pandora Effect': if it is a pure strategy equilibrium for an agent to play d then all subsequent agents play d for all values of α . The third part establishes the conditions under which Informed agents will definitely play d , or definitely not play d , for a given set of parameters. It is found that Informed agents will defect within a finite distance from the end of the game in all equilibria. They will never defect before this point in any pure strategy equilibria. Finally, a pure strategy equilibrium will be shown to exist in which Informed agents and their Followings cooperate up to a point and defect thereafter.

4.1 The Following

Agents maximize their own-action payoff net of its externality impact. In the equilibria discussed in this paper I use the concept of a ‘Following’ to analyze the expected externality impact of an Informed agent’s action. I define a ‘Following’ below.

Definition 1 (Following) *Let k be the first Informed agent to act subsequent to agent i : then the Following of agent i is all the Uninformed agents acting after i and before k .*

Note, if Uninformed agent j acts after i and before k then j is in i ’s Following irrespective of the action chosen by j .¹⁰

The Following of i have all seen the same Informed agents act. Also, the actions of other Uninformed agents contain no information. Hence, all the Uninformed agents in i ’s Following will have, in equilibrium, the same beliefs about payoffs. These beliefs will be determined by the actions of all previous Informed agents including Informed agent i . An Informed agent’s action will affect the beliefs of agents in their Following but not of subsequent Informed agents, because Informed agents have perfect information about payoffs.

The expected effect of an Informed agent’s choice of action on the actions and beliefs of their own Following is different from the expected effect on subsequent Informed agents’ Followings. The Informed agent knows all the actions of previous Informed agents and therefore knows the exact information set available to their Following. Where as, an Informed agent does not know how subsequent Informed agents will act. Therefore, an Informed agent can choose an action conditioned on exact knowledge of their Followings information sets; but cannot choose an action conditioned on exact knowledge of the information sets of subsequent Informed agents’ Followings. This implies that Informed agent i knows the effect of their action on the actions of their Following; but they do not necessarily know the effect on the actions of later Informed agents’ Followings.

In this paper there will ultimately be two factors that determine the expected externality impact of an Informed agent’s action. The first is the expectations that Informed agents have about the effect of their actions on the actions of their Followings. In the equilibria examined in this paper the actions of an Informed agent’s Following are uniquely determined by the Informed agent’s action. The second factor is the expected number of agents in their Following. Lemma 1 gives agent i ’s expectation of the size of i ’s Following.

¹⁰A further note, I generally only talk about Informed agents’ Followings as Uninformed agents’ actions do not have an equilibrium effect on beliefs. I would have restricted the concept of a Following to Informed agents if not for the fact that to do so complicates later notation.

Lemma 1 *Agent i 's expectation of the size of i 's Following is:*

$$E[F_i] = \frac{(1 - \alpha) - (1 - \alpha)^{n+1-i}}{\alpha}. \quad (7)$$

4.2 The Pandora Effect

I now examine a game with perfect information; $\alpha = 1$. Where $\alpha = 1$ all agents play d . This follows from backwards induction. The expected difference in the externality impacts of any two actions is 0 for the n th agent: because there are no agents acting after the n th agent. Therefore, $a_n = d$ independently of history as it maximizes own-action payoff. Consequently, the expected difference in the externality impacts of any two actions is 0 for agent $n - 1$: because n will choose $a_n = d$ for any action of $n - 1$. Therefore, $a_{n-1} = d$ independently of history also. The same argument then applies for $n - 2$ and so on back to the first agent. This outcome is Pareto inferior to all agents choosing c .

In general, where $\alpha \in [0, 1]$, equilibrium play will not be same as for $\alpha = 1$. However, if it is revealed by agent i to all subsequent agents that some particular action is d then the equilibrium of the subgame, beginning at agent $i + 1$, will be for all agents to play d . This follows from the same backward induction argument used to establish equilibrium play in the case of $\alpha = 1$. This implies the Proposition below, which I call ‘The Pandora Effect’.

Proposition 1 (The Pandora Effect) *If $a_i = d$ for Informed agent i in any pure strategy equilibrium then all subsequent agents will play $a_j = a_i = d \forall j > i$.*

If an action is played in a pure strategy equilibrium then everybody knows what that action is. Consequently, an Informed agent reveals which action corresponds to d when they play d as a pure strategy. The revelation of this information is irreversible on the equilibrium path. Hence, all subsequent agents will defect after an Informed agent has defected as part of their pure strategy on the equilibrium path. This is because all subsequent agents know *how* to defect: the knowledge of how to do bad cannot be put back in the box.

4.3 Defection

Informed agent i will play *defect* when own-action payoff net of expected externality impact is greater for d than c or some $a_i \notin \{c, d\}$. I will show that this is the case for i in any equilibrium where the following inequality holds: i 's expected Following is

smaller than the ratio of the defection incentive¹¹ to the magnitude of the externality. I define the ‘defection condition’, D , as holding for agent i where this inequality weakly holds. D strictly holds when this inequality strictly holds. More formally:

$$D \text{ holds if and only if } E[F_i] \leq \frac{\Delta}{\kappa}.$$

$$D \text{ strictly holds if and only if } E[F_i] < \frac{\Delta}{\kappa}.^{12}$$

One property of the ‘defection condition’, which will be useful later on, is that if D holds for agent i then it also holds for agent $i + 1$. If D doesn’t hold for agent i then it doesn’t hold for agent $i - 1$. Therefore, if the number of agents for which D holds in the population is Ψ there is a critical agent $\theta = n - \Psi$ for which the following Lemma holds:¹³

Lemma 2 *D holds for an agent i if and only if $i > \theta = n - \Psi$.*

Further intuition about what condition D implies can be gained from considering the case where an Informed agent’s action is copied by their Following and only their Following. If their expected Following is large enough they will not choose d because the externalities generated by their Following will be large in expectation: this is the case when D does not hold. If, however, their expected Following is very small the Informed agent will choose to play d : because the benefit from the higher own-action payoff dominates the externalities that will be generated by the Informed agent’s expected Following. The higher own-action payoff strictly dominates the externalities when D strictly holds.

I show that in all equilibria the action of any Informed agent $i > \theta$ will never affect the actions of agents outside their Following. Hence, we can treat agents $i > \theta$ as if they only need to think about their expected Following. Using this consideration, and condition D , I am able to derive Proposition 2 below. Proposition 2 gives sufficient conditions for defection by Informed agents and Uninformed agents in all equilibria.

Proposition 2 *On the equilibrium path in all equilibria each of the following are sufficient conditions to imply $a_j = d$:*

¹¹Recall that the defection incentive is the difference in the own-action payoffs of *cooperate* and *defect*. Which is the difference in payoffs of the two actions should these actions not affect the actions of any other agent.

¹²Recall that $E[F_i]$, the expected Following of i , is the expected number of Uninformed agents acting after i but before the next Informed agent. Therefore, a more intuitive way of arranging this inequality may be $\kappa \cdot E[F_i] < \Delta$. The expected externality generated by i ’s Following, if they all choose d , is less than the difference in the own-action payoffs of c and d . Which implies that the Informed agent i ’s expected payoff from d is greater than c if condition D strictly holds and only i ’s Following is influenced by i ’s action.

¹³It may be the case that $\theta = 0$.

1. j is an Informed agent for whom D strictly holds.
2. j is subsequent to some i who is an Informed agent for whom D strictly holds.

This proposition states that in all equilibria the first Informed agent for which D strictly holds will play d and so will all subsequent agents. To see why this holds, suppose the first Informed agent $i > \theta$ plays c instead and all subsequent Informed agents continue playing d . In this case the first Informed agent could only stop their Following, at best, from playing d ; not a sufficient incentive to play c instead of d when D strictly holds. Hence, the first Informed agent for which D holds will forgo playing d only if doing so causes the next Informed agent, at least, to forgo playing d too. However, the subsequent Informed agent will also require that the Informed agent subsequent to them is influenced by their action too, and so on. This chain of Informed agent to Informed agent influence cannot go on indefinitely: because n will always play d if n is an Informed agent. Hence, an Informed agent just before n can only expect to influence their own Following. Backwards induction then reveals that any Informed agent $i > \theta$ can only ever expect to influence their own Following. Which, given the implications of the Defection condition, implies that the first Informed agent $i > \theta$ will always play d as a pure strategy. All subsequent agents then play d by the Pandora Effect.

The first Informed agent for which D holds and those agents acting subsequently are referred to hereafter as ‘Defectors’. This is because they are the only agents which will defect in a pure strategy equilibrium. For illustration purposes, suppose a pure strategy equilibrium existed where all Informed agents played d ; including those for which D did not hold. This couldn’t be an equilibrium because the first Informed agent for which D did not hold would have an incentive to cooperate. The Following of the first Informed agent would all cooperate if the first Informed agent cooperated: as they would all believe the Informed agent had played *defect*. However, by the fact that D does not hold, the lower externality impact of playing c would dominate the difference in the own-action payoffs. So the first Informed agent would never play d as a pure strategy if D didn’t hold. Its straightforward to extend this to all the other Informed agents subsequent to the first for which D does not hold. This is summarized in Proposition 3 below.

Proposition 3 *There is no pure strategy equilibrium where some Informed agent, for whom D does not hold, plays d on the equilibrium path.*

4.4 An Equilibrium

There is an equilibrium where Informed agents up to and including the critical agent θ cooperate and then defect thereafter. In this equilibrium each Informed agent is copied by their Following. This is formalised in the proposition below.

Proposition 4 *There is a pure strategy equilibrium consisting of the following strategies and out of equilibrium beliefs.*

Informed agent i 's strategy is:

$$a_i = \begin{cases} c & \text{if } i \leq \theta \\ d & \text{if } i > \theta. \end{cases} \quad (8)$$

Uninformed agent j 's strategy is to play $a_j = a_i$ if they are in the Following of Informed agent i and to play $a_j = 0$ if there is no Informed agent $i < j$.

Off the equilibrium path beliefs are:

1. *All Uninformed agents in Informed agent i 's Following believe that $a_i = d$ regardless of the actions of previous Informed agents.*
2. *All Uninformed agents who deviate from the equilibrium path are believed to be uninformed.*

This equilibrium follows in a straightforward fashion from Proposition 1 through 3. Informed agent $i > \theta$ can never affect the actions of more agents than their Following in this equilibrium. Hence, it is optimal for any Informed agent $i > \theta$ to play d . Informed agent $i \leq \theta$ will cause at least their Following to play d if i plays d : this is sufficient to make c optimal for $i \leq \theta$. From the Pandora Effect it is optimal for the Following of Informed agent $i > \theta$ to copy $i > \theta$. The expected payoff for agents in the Following of $i \leq \theta$ if they don't copy is 0. Hence, it is optimal for the Following of $i \leq \theta$ to copy i .

I have shown in Proposition 2 that all equilibria will imply the same ex-ante play after agent θ as in the equilibrium above. I have not shown that all equilibria imply all Informed agents cooperate before and up to agent θ . There are two putative sets of equilibria that would not imply such equilibrium play. The first set are pure strategy equilibria in which Informed agent $i \leq \theta$ plays $a_i \notin \{c, d\}$. This may be sustained if an Informed agent prior to θ is punished by subsequent agents for playing c . It is conceivable that some set of off the equilibrium path beliefs make such a punishment strategy incentive compatible. Such equilibria would be Pareto inferior to the equilibrium above as expected externalities are the same but average own-action payoffs are lower. The second set are possible equilibria in which Informed agents up to and including θ play mixed strategies. In this paper I do not show whether these putative equilibria exist or not.

5 The Conditions for Cooperation

In this section I look at what conditions on the parameters imply that a given game is expected to yield cooperative behavior. Moreover, I look at how much cooperative behavior can exist in such a game. I find two main results for the equilibrium defined above. The first is that there exists a critical proportion of informed agents above which there will be no cooperative agents in a population of any size. The second result, is that below this critical level the number of agents that will choose to defect has an upper bound that holds for any population size.

Theorem 1 *In the equilibrium defined in Proposition 4 there will be no cooperative agents for finite n if the proportion of informed agents is at or above a critical level $\bar{\alpha}$, where:*

$$\bar{\alpha} = \frac{\kappa}{\kappa + \Delta} \quad (9)$$

This result follows from the fact that the expected Following of i is the sum of a geometric series that converges to 0. Hence, the expected Following of i as a function of $n - i$ is bounded above by some finite number for all values of $n - i$. This upper bound monotonically decreases in the value of α : as α tends to 1 the expected Following tends to 0 for any population size. Hence, when too large a proportion of agents are informed i 's expected Following is always too small to dominate the defection incentive¹⁴, no matter how many agents are acting subsequent to i .

Note that Theorem 1 does not imply that $\alpha < \bar{\alpha}$ is a sufficient condition for some agent to cooperate. $\alpha < \bar{\alpha}$ implies only that there exists a finite upper bound on the total number of potential Defectors, Ψ , that holds for all values of n . Recall that the Defectors, are the first Informed agent for which D holds and the set of all subsequent agents. This is summarized in Theorem 2 below.

Theorem 2 *If $0 < \alpha < \bar{\alpha}$ then the number of Defectors, Ψ , is bounded above: for large enough n there is some agent for which D does not hold.*

For example, there is some $\alpha < \bar{\alpha}$ for which there can never be more than twenty defectors. This holds whether the population size is ten or ten thousand. This upper bound only binds when the population size, n , is larger than the upper bound. So, given $\alpha < \bar{\alpha}$, there will be some agents prepared to cooperate if n is large enough. Moreover, Theorem 2 implies that as n becomes very large the expected number of agents who choose to defect remains constant. Therefore, the proportion of agents who are Defectors will tend to 0 as n tends to infinity. This yields the following corollary to Theorem 2.

¹⁴Recall that the defection incentive is the difference in own-action payoffs

Corollary 1 *If $0 < \alpha < \bar{\alpha}$ then as $n \rightarrow \infty$ the expected proportion of the population that cooperates tends to one.*

6 Welfare and the Proportion of Informed Agents

I show in this section that it can be better, in the ex-ante welfare sense, to have neither perfect information nor complete ignorance. There are sufficient conditions that imply expected welfare is greater when the proportion of informed agents, α , has an interior value rather than a corner value.

Lemma 3 below gives the average expected welfare.

Lemma 3 *The expected average welfare as a function of α is:*

$$E[W(\alpha)] = P \frac{\tilde{z}}{n} (n - \Psi - E[I_{\theta-1} + F_{\theta}]) + \left(\frac{v}{n} - \kappa\right) (\Psi - E[F_{\theta}]) \quad (10)$$

$P = 1 - (1 - \alpha)^{\theta}$ is the probability of some agent being an Informed agent who is not a Defector. $E[I_{\theta-1}]$ is the expected number of Ignorant agents conditional on some agent being an Informed agent who is not a Defector.

The expected average welfare in the corner cases of $\alpha = 0$ and $\alpha = 1$, $E[W(0)]$ and $E[W(1)]$, are:

$$E[W(0)] = 0 \quad (11)$$

$$E[W(1)] = v - \kappa \cdot n \quad (12)$$

The relative sizes of v , κ and n determine whether expected welfare is higher in the case of perfect information or of complete ignorance. Expected welfare is higher in a game of complete ignorance than a game of perfect information where $v - \kappa \cdot n < 0$. Expected welfare is higher in a game of perfect information than a game of complete ignorance where $v - \kappa \cdot n > 0$. In the first case, the loss due to externalities dominates the private gain from agents being able to choose d . In the second case, the loss of positive private payoffs from having no information dominates the cost of externalities generated by agents choosing d . These two cases are called ‘externality dominated games’ and ‘information dominated games’ respectively. There are sufficient conditions for an interior α to yield higher ex-ante expected welfare in both externality dominated games and information dominated games.

In an externality dominated game the cost to society of some agent defecting is very high. This cost will be incurred with positive probability if some portion of the general population is informed as some agent will defect with positive probability. However, a small number of Informed agents may still induce a better outcome through their good example, c , than no Informed agents at all. For this to be the case, the expected cost of externalities generated by defectors must be dominated by the welfare gains from the majority being able to get a positive instant payoff of z . Proposition 5 gives sufficient conditions for an interior α to yield greater welfare than a corner solution in an externality dominated game.

Proposition 5 *There is always some $\alpha \in (0, 1)$ for which $E[W(\alpha)] > E[W(0)] > E[W(1)]$ when:*

$$v - \kappa \cdot n < 0 \tag{13}$$

and condition (14) holds:

$$z > ((n - 1)\kappa - \Delta)E \left[\frac{\underline{X} - F_{n-\underline{X}}}{P(n - \underline{X} - I_{n-\underline{X}-1} + F_{n-\underline{X}}) + \underline{X} - F_{n-\underline{X}}} \right] \tag{14}$$

There always exists a large finite n^* such that condition (14) holds when condition (15) holds:

$$n > n^* \text{ and } z > \kappa. \tag{15}$$

The right hand side of condition (14) in Proposition 5 is always finite for small enough $\alpha > 0$. This implies that if the payoff from *cooperate* is sufficiently large, relative to the extent that the externality dominates the defection incentive, then I prefer to have some informed agents in the population. The private payoff z can be attained by Informed agents without causing negative externalities. Therefore, z can be seen as the pure benefit of information. It is that component which can be realised in an individuals payoff without causing lower payoffs for others. While $L = (n - 1)\kappa - \Delta$, the size of the social dilemma, can be interpreted as the cost of information. If the pure benefit of information is large enough, that is if z is large relative to L , then having some information in the population is always preferred to having none. Condition (15) implies that this is always the case in a large population when the externality inflicted on society by one person playing *defect* is less than the value of all agents being able to choose *cooperate*.

Now I consider the case of an information dominated game. The relevant point of comparison for an information dominated game is the case of perfect information where the whole population is informed, $\alpha = 1$. In this case a reduction in the proportion of informed agents implies a positive expected number of Ignorant agents who will not

choose d . This, relative to the case of a fully informed population, yields an average loss of $v - \kappa \cdot n > 0$ for each Ignorant agent. Take the case where the reduction is small enough that condition D still holds for all agents. In this case expected average welfare is strictly lower. However, if D no longer holds for some agents then the the loss from the Ignorant has some compensation. The loss is offset by gains from agents playing *cooperate* where they would have previously played *defect*. Each agent who plays c instead of d implies a gain of $(n - 1)\kappa + \Delta = L > 0$.

The overall effect of reducing α to a level that induces some cooperation is ambiguous. I would like a general solution for when this will be welfare improving or not; such a solution has been elusive. I only look at the specific case where the proportion of informed agents is such that D holds with equality for the second agent. In this case D does not hold for the first agent who, if an Informed agent, will play *cooperate*. The proportion of informed agents that implies D holds with equality for the second agent is defined as α_{n-1} . I am able to derive the following proposition for the case where $\alpha = \alpha_{n-1}$.

Proposition 6 *Where $v - \kappa \cdot n > 0$ there is some $0 < \alpha < 1$ for which $E[W(\alpha)] > E[W(1)] > E[W(0)]$ if the following inequality holds:*

$$L - 1 - \alpha_{n-1}z > 0 \tag{16}$$

where α_{n-1} is decreasing in L .

Hence, the larger the size of the social dilemma, L , the larger the value of z that implies imperfect information is preferred to perfect information. As noted earlier z can be seen as a measure of the value of information net of externalities. Therefore, the larger this value of information the larger the social dilemma needs to be to imply that I would rather not have all members of the population informed in an information dominated game.

7 Alternative Action Spaces, Payoff Distributions and Externalities

The action set, instant payoff distribution and the nature of the externalities in the model outlined above make for easy exposition of the equilibrium results but are restrictive. In this section I show that the equilibrium results hold for a much less restrictive set of assumptions. I do not relax any other aspect of the game: the number of agents, the proportion of Informed agents in the general population, the order of action and the information sets of agents remain the same.

In the model above I have the action set A , the instant payoff function $u(\cdot)$ and the externality function $\kappa(\cdot)$. Here, I replace them with A' , $u'(\cdot)$ and $\kappa'(\cdot)$ respectively. Let

the action set A' be any measurable subset of the real line. In the action set there are still two singletons c and d analogous to *cooperate* and *defect*. I now allow the instant payoffs given by $u'(\cdot)$ and the externalities given by $\kappa'(\cdot)$ to be such that actions in A' other than c or d can have non-zero payoffs and/or have externalities. Also, I now allow the externalities given by $\kappa'(\cdot)$ for c and d to be positive or negative.¹⁵ Informed agents still know the exact mapping from the action space to the payoff and externality space while Uninformed agents do not. Uninformed agents still know what the instant payoffs and externalities are even though they do not know the mapping. It may be the case that the instant payoff or externality of an action is correlated, in a non-trivial sense, with the instant payoffs or externalities of other actions. This allows Uninformed agents to learn more from the actions of Informed agents than they were previously able¹⁶.

In the following proposition I outline sufficient conditions on A' , $u'(\cdot)$ and $\kappa'(\cdot)$ under which the equilibrium results of the paper still hold.

Proposition 7 *Propositions 1 to 4 and Theorems 1 and 1 hold for any action set A' , instant payoff function $u'(\cdot)$ and externality function $\kappa'(\cdot)$ if all the following conditions hold:*

$$E_j[u'(a_j = a) - \kappa'(a_j = a) | h_j, a_i = c] < u'(c) - \kappa'(c) \quad \forall a \in A'_c, \quad \forall i \leq \theta \quad \text{and} \quad \forall j \in F_i, \quad (17)$$

$$u'(d) - \kappa'(d) > u'(a) - \kappa'(a) \quad \forall a \in A', \quad (18)$$

$$u'(c) - \kappa'(c) \geq u'(a) - \kappa'(a) \quad \forall a \in A'_d, \quad (19)$$

$$\kappa'(c) < \kappa'(a) \quad \forall a \in A'. \quad (20)$$

Condition (17) states that the expected own-action payoff of an action other than c is strictly less than the own-action payoff of c for any Uninformed agent who has played subsequent to Informed agents who have only played c . For condition (17) to hold it must be the case that the action set must admit a minimum of three possible actions. Moreover, there must be at least one action that yields a worse payoff than

¹⁵Note that in the set up that a positive value given by $\kappa'(\cdot)$ implies a negative externality and a negative value given by $\kappa'(\cdot)$ implies a positive externality.

¹⁶For example, with the action set $A' = \{1, 2, 3\}$ an Uninformed agent may believe that 1 is d with probability p if $3 = c$ and believe it is d with probability $q \neq p$ if $2 = c$. The Uninformed agent would be able to infer which element was c from equilibrium play and therefore update his expectations for the payoffs to playing $a = 1$ accordingly.

cooperate. Furthermore, it also implies that the payoff function cannot be invertible from equilibrium play and Informed agents' strategy functions.

Conditions (18) and (19) imply that *defect* and *cooperate* still have the first and second highest own action payoffs. The first of these conditions is a matter of definition. If d does not yield the highest own-action payoff then agents, in a perfect information case, will want to choose some action other than d . I would then wish to call this other action d instead. Conditions (18) and (19) also imply that the difference in own-action payoffs for *cooperate* and *defect* is positive. This implies there is an incentive to play defect, the defection incentive, which is equivalent to Δ in the standard model. Consequently, the relevant consideration for Informed agents deciding whether to play *cooperate* or *defect*, conditional on being copied by just their Following, is still the Defection Condition. Condition (20) states that *cooperate* generates the most positive externality of all actions and is therefore the socially optimal action ¹⁷.

Finally, the combination of conditions (19) and (20) warrants further discussion. In conjunction they imply that the socially best action is necessarily the individually second best. This does not allow for cases where the actions that have the second to n th best own-action payoffs are socially worse than c . It seems reasonable to expect a spectrum of actions socially worse than c that yield higher own-action payoffs. If *both* (19) and (20) must hold for the equilibrium results to go through then the results seem a little uninteresting as they will rarely apply to any particular social dilemma. However, in such cases I still ought to get results which are very much in line with the nature of the results derived for the standard model above. This is not shown here formally but it is easy to intuitively see why the results will be much the same.

Consider the case where there is a set of actions $\{d_1, d_2, \dots, d_n\}$ such that the own-action payoffs are monotonically increasing in the index on d but all have a higher own-action payoff than c . Also, the externality generated by these actions is worse than the externality generated by c and monotonically increasing in the index on d .¹⁸ Instead of Informed agents' strategies being to jump from c to d_n immediately after some cut-off point they gradually progress from c to d_1 and upwards through the index to d_n instead. This is because the lower indexed d s have a lower externality impact and own-action payoffs than the higher indexed d s. Thus, by exactly the same logic as with c and d in the standard model I can see that in this case the agents will cooperate till a point and then cooperate a bit less; and then a bit less; and so on. This is a slow deterioration in cooperative behavior rather than an instant collapse. However, it is still the case that completely cooperative behavior will exist up to a finite distance from the end and that the population will tend towards being completely cooperative

¹⁷In order for this to be a social dilemma it also needs to be the case that the following condition holds:

$$u'(d) - u'(c) < n(\kappa'(d) - \kappa'(c)). \quad (21)$$

¹⁸I only consider the case where the externalities and the own-action payoffs are both monotonically increasing because there is never any reason to choose an action with a lower own-action payoff and larger externality.

for arbitrarily large populations.

8 Summary and Implications

In social dilemmas with risky action sets the opportunity for good leadership exists if information is restricted to a small proportion of the population. This can induce results, in expectation, that are arbitrarily close to Pareto efficient. Under perfect information the outcome is inefficient. This cooperative behavior happens within the context of a finite game: distinguishing these results from cooperation in infinitely repeated games. Moreover, if too many agents have access to information, the incentive for informed agents to choose socially optimal actions is crowded out and the population fails to improve on the second-best outcome.

In this model the order of action is exogenous. This reflects a stochastic element in the factors that govern when an agent has to make any decision. However, it is reasonable to suppose that the type of agents will affect when they want to act. While this model does not endogenize the order of action it does allow the examination of agents' preferences over when to act, given the order of everyone else's action and perfectly patient agents. When the social dilemma is caused by negative externalities everybody wants to act last. Informed agents can choose the privately optimal action and not be followed. Uninformed agents have the chance of copying a late Informed agent and getting the higher private payoff.

When the dilemma is due to positive externalities the positive externalities only accrue if an Informed agent shows Uninformed agents what to do. Consequently, if there is an Informed agent acting early in the game all the other Informed agents prefer to act last. The Informed agents acting last accrue the gains from the contributions of others while choosing to play *defect* themselves. However, if all the Informed agents are acting late then each Informed agent prefers to act at the beginning of the game rather than the end. Acting early initiates cooperative behavior and allows the Informed agent to benefit from the positive externalities generated by subsequent Uninformed agents. Obviously, each Informed agent prefers it if some other Informed agent acts First while they wait till the end and play *defect*. Finally, uninformed agents always prefer to act after some Informed agent has acted, as they get a low expected payoff when acting without the guidance of an Informed agent. Moreover, the Uninformed agents prefer to act after an Informed agent who has played *defect*.

One prediction of this model is that cooperative behavior occurs more frequently amongst particular kinds of groups. Namely, groups where there are a few clearly well informed people and a large number of poorly informed people who fear the consequences of independent action.¹⁹ Therefore, it may be desirable to restrict information

¹⁹For example, it might imply that large groups of children with a few adults ought to demonstrate more cooperative behavior than large groups of adults with a few children. I think that the relative

to a small number of agents and introduce greater uncertainty about the payoffs to particular actions.

Such policies may improve social outcomes in my model but only on the assumptions of perfect knowledge amongst Informed agents and a degree of homogeneity in agents utility functions. This constrains Informed agents' choices to be both accurate and, when setting a good example, in line with the interests of everyone else. Whether this is an accurate characterization of the game would be, in general, very difficult for Uninformed agents to verify. Hence, Uninformed agents without this information ought to be sceptical of policies of information restriction grounded on the results of this model.²⁰

A further implication of this model is that Informed agents may want to hide their actions or hide the fact that they are an Informed agent. When there are negative externalities all Informed agents would pay a premium to choose the bad action and hide their action from subsequent agents. For example, a manager may go to great lengths to shirk in a fashion that cannot be observed by his employees. In a game with positive externalities, all but the first Informed agent would pay to choose the privately optimal action and hide it from subsequent agents.

When Informed agents are able to hide their identity a good example setting equilibria still exists. This equilibria is almost identical to that in Proposition 4. This equilibrium is maintained by the beliefs of Uninformed agents about the 'true' identity of Uninformed agents who do not copy the action of prior agents on and off the equilibrium path. The equilibrium path belief is that any agent not copying the agent immediately preceding them is an Informed agent. This belief results in an equilibrium where the first Informed agents to play *cooperate* and *defect* are recognized as Informed agents. Between the first cooperating and first defecting Informed agent all agents will cooperate. All the agents after the first defecting Informed agent will defect. Hence, the Uninformed agents and Informed agents in these two groups will be indistinguishable. Off the equilibrium path beliefs are such that the last agent to act differently from the previous agent is considered an Informed agent playing *defect*. This belief insures that Informed agents never have an incentive to play off the equilibrium path by the same logic as in the proof of Proposition 4.

ignorance of children makes them good subjects for examining this model as the strategies that adults use to deal with social circumstances are often adopted by children and these strategies have a great deal of persistence into later life. Hence, adults when being observed by children may think it wise to ensure that these children learn strategies that are beneficial to the adults. Practising honesty, fairness and concern for the elderly may be wise if this is a strategy that will be imitated by those younger than yourself when you are elderly.

²⁰Arguments are often had over the extent to which information available to departments of government is available to the general public. A common defence is that this information is kept secret for the good of the people. A common complaint against this argument is that the people do not know that the government has their best interests at heart or are competent at dealing with this information. The argument for secrecy here is different to my own but the argument against secrecy is similar to the one outlined above.

Appendix

PROOF of Lemma 1:

The expected number of agents in an Informed agent's Following is given by:

$$E[F_i] = (1 - \alpha)^{x+1}(x + 1) + \alpha \sum_{j=1}^x j(1 - \alpha)^j \quad (22)$$

where

$$n - (i + 1) = x$$

The summation term on the right hand side of (22) gives:

$$\begin{aligned} \sum_{j=1}^x j(1 - \alpha)^j &= (1 - \alpha) + 2(1 - \alpha)^2 + \dots + x(1 - \alpha)^x \\ &= \sum_{j=0}^{x-1} (1 - \alpha)^{x-j} + \sum_{j=0}^{x-2} (1 - \alpha)^{x-j} + \dots + \sum_{j=0}^{x-k} (1 - \alpha)^{x-j} + \dots + \sum_{j=0}^0 (1 - \alpha)^{x-j} \\ &= \sum_{k=1}^x \sum_{j=0}^{x-k} (1 - \alpha)^{x-j} \\ &= \sum_{k=1}^x \left(\frac{(1 - \alpha)^k - (1 - \alpha)^{x+1}}{\alpha} \right) \\ &= \frac{(1 - \alpha)^1 - (1 - \alpha)^{x+1}}{\alpha} + \frac{(1 - \alpha)^2 - (1 - \alpha)^{x+1}}{\alpha} + \dots + \frac{(1 - \alpha)^x - (1 - \alpha)^{x+1}}{\alpha} \\ &= \frac{\sum_{j=1}^x (1 - \alpha)^j - x(1 - \alpha)^{x+1}}{\alpha} \end{aligned}$$

Substituting this back into (22) gives:

$$\begin{aligned}
E[F_i] &= (1 - \alpha)^{x+1}(x + 1) + \alpha \left(\frac{\sum_{j=1}^x (1 - \alpha)^j - x(1 - \alpha)^{x+1}}{\alpha} \right) \\
&= (1 - \alpha)^{x+1}(x + 1) + \sum_{j=1}^x (1 - \alpha)^j - x(1 - \alpha)^{x+1} \\
&= (1 - \alpha)^{x+1} + \sum_{j=1}^x (1 - \alpha)^j \\
&= \sum_{j=1}^{x+1} (1 - \alpha)^j = \sum_{j=1}^{n-i} (1 - \alpha)^j
\end{aligned}$$

□

PROOF of Proposition 1:

In any perfect Bayesian equilibrium, agents beliefs on the equilibrium path are determined by Bayes' rule and the players' equilibrium strategies. Let it be the case that $a_i = d$ is a pure strategy on the equilibrium path of a perfect Bayesian equilibrium. Informed agent i plays d as a pure strategy; this implies that i plays $a_i = d$ with probability 1. From Bayes' rule subsequent agents on the equilibrium path, both Followers and Informed agents, believe $a_i = d$ with probability 1.

The expected externality impact of the n th agents' action is always 0 as n is the last agent to act. Therefore, maximizing n 's own-action payoff trivially maximizes n 's expected own-action payoff net of externality impact. On the equilibrium path, whether n is an Informed agent or a Follower, n believes $a_i = d$ and plays $a_n = a_i = d$. On the equilibrium path the action of $n - 1$ cannot change n 's true belief that, with probability 1, $a_i = d$.²¹ Therefore, the externality impact of a_{n-1} is also 0 as $a_n = d$ independent of a_{n-1} . $n - 1$ maximizes own-action payoff and plays $a_{n-1} = a_i = d$ whether $n - 1$ is an Informed agent or a Follower. The same argument holds for agent $n - 2$ and by backwards induction for all agents back to the $i + 1$ th agent. □

PROOF of Proposition 2:

In the following proof of Proposition 2 any statement about the actions of Informed agents is conditioned on D strictly holding for that Informed agent.

Consider an equilibrium where Informed agents' actions can only affect the actions of their own Following. Let j be the first Informed agent subsequent to Informed agent i . The Pandora effect implies all agents subsequent to j will play $a_j = a_i = d$ if $a_i = d$ is a pure strategy for i . In such an equilibrium the maximum difference in the expected

²¹The only case in which $n - 1$'s action could affect n 's beliefs such that n believes $a_i \neq d$ with positive probability is if a_{n-1} is off the equilibrium path.

payoffs of *defect* and *cooperate* for i is if all i 's Following copy and play a_i . This would yield an expected difference in utility of:

$$E[U_i(d) - U_i(c)] = \Delta - \kappa \cdot E[F_i] \quad (23)$$

This is strictly positive where D strictly holds. Therefore if D strictly holds for i in such an equilibrium then i would prefer d to c and, *a fortiori*, all other actions. In equilibria where j 's strategy is the same but the Following of i have a strategy other than play a_i then the value of equation (23) will be even larger and the reason to play *defect* will be greater. This result can now be used to derive Proposition 2.

If the last agent is an Informed agent then $a_n = d$ is the optimal action for n in all equilibria for any history of actions. Therefore, from the above argument, if $n - 1$ is an Informed agent and D holds then $a_{n-1} = d$ in all equilibria. From the Pandora effect this implies $a_n = a_{n-1} = d$ in all equilibria where $n - 1$ is an Informed agent for which D strictly holds. By backward induction this holds for $n - 2$ if D strictly holds and so on till the first Informed agent for which D strictly holds. Thus, if D strictly holds for Informed agent i on the equilibrium path of any equilibrium then $a_i = d$. The Pandora effect then implies that $a_j = a_i = d$ for all $j > i$. This concludes the proof of Proposition 2. \square

PROOF of Proposition 3 If it is a pure strategy for the first Informed agent, i , to play d then the Pandora Effect implies that all subsequent agents play d on the equilibrium path. If i chooses an action other than d then i 's Following still play a_i as they are unaware that they are off the equilibrium path. In such a case the minimum that i gains from playing c is if only i 's Following copy a_i and all subsequent agents play d . In this case the minimum expected gain from cooperating instead of defecting is $\kappa \cdot E[F_i] - \Delta$. If condition D doesn't hold for i this gain is larger than zero. Therefore it cannot be an equilibrium for the first Informed agent to play d if D doesn't hold. If it is a pure strategy for the second Informed agent, j , to play d then j has a minimum expected gain of $\kappa \cdot E[F_i] - \Delta$ from playing c . This follows as Informed agent j 's Following will not play d because they will not know which option is d on the equilibrium path as the first Informed agent can't have played d either. Proposition 3 then follows from forward induction. \square

PROOF of Proposition 4 The off-path beliefs of Followers about Informed agents implies that it is rational for Followers to copy the last Informed agent off the equilibrium path. Therefore, in this equilibrium, Informed agent i 's action will affect the actions of i 's Following alone. Therefore, where D holds it will be optimal to defect and where D does not hold it will be optimal to cooperate. Thus the decision rules in Proposition 4 maximize Informed agents expected utility. The equilibrium play by the Informed agents implies that the actions of Followers subsequent to the first Informed agent maximizes expected own-action payoff as agents acting before the first Informed agent

subsequent to θ cannot choose d . Those acting after the first Informed agent subsequent to θ play d from the Pandora Effect; which is what adopting the above strategy entails.

Off the equilibrium path beliefs imply that any deviation by a Follower will not change the number of other agents playing d . Therefore the expected externality impact is at the same as when they play the equilibrium strategy. Therefore copying the previous Informed agent's action maximizes their expected utility as it maximizes their own action payoff. Before the first Informed agent acts then Uninformed agent's expected own action payoffs will be 0 from any action. Deviation from the equilibrium path will not change the expected externalities. Therefore, choosing action 0 is weakly preferred to any other action by Followers acting prior to the first Informed agent. This concludes the proof of Proposition 4. \square

PROOF of Theorem 1:

If D holds for the first agent then D holds for all subsequent agents. D holds for the first agent if:

$$E[F_1] = \frac{(1 - \alpha) - (1 - \alpha)^n}{\alpha} < \frac{\Delta}{\kappa} \quad (24)$$

$E[F_1]$ is monotonic and increasing in n which gives the unclosed upper bound of $E[F_1]$ as $\frac{1 - \alpha}{\alpha}$. Therefore, there is no finite population size for which D will hold for any agent when the following equation holds.

$$\frac{1 - \alpha}{\alpha} \leq \frac{\Delta}{\kappa} \quad (25)$$

Which always holds where $\alpha \geq \bar{\alpha}$. \square

PROOF of Theorem 2 $\alpha < \bar{\alpha}$ implies that $\frac{1 - \alpha}{\alpha} > \frac{\Delta}{\kappa}$. Therefore, there exists some finite integer X such that:

$$E[F_1] = \frac{(1 - \alpha) - (1 - \alpha)^n}{\alpha} \leq \frac{\Delta}{\kappa} \text{ for } n = X \quad (26)$$

and

$$E[F_1] = \frac{(1 - \alpha) - (1 - \alpha)^n}{\alpha} > \frac{\Delta}{\kappa} \text{ for } n = X + 1. \quad (27)$$

If $n = X + 1$ condition D does not hold for the first agent. Ψ is bounded above by finite X . If $n > X$ then there exists some agent for which D does not hold.

\square

PROOF of Corollary 1 There are two sets of agents that will not choose *cooperate*: the Ignorant²² and the Defectors. The Defectors are the set of agents for which D holds and are either an Informed agent or acting subsequent to an Informed agent for which D holds.

The expected size of the first set, the Ignorant, is bounded above by the expected size of the Following of the first agent in a with a population size of $n + 1$. This is increasing in n and bounded above by $\frac{1 - \alpha}{\alpha}$ which is finite for $1 > \alpha > 0$.

The number of defectors is bounded above by finite X .

Hence, the expected proportion of agents playing c in equilibrium is bounded below by:

$$\frac{n - (X + \frac{1 - \alpha}{\alpha})}{n} \quad (28)$$

The upper bounds for the Ignorant and the Defectors are invariant in n : equation (28) tends to one as $n \rightarrow \infty$. \square

PROOF of Lemma 3:

There are two mutually exclusive and exhaustive sets of possible outcomes. The first set is where some agent $i \leq \theta$ is an Informed agent. The second set is where all $i \leq \theta$ are Followers. The first set happens with probability $P = 1 - (1 - \alpha)^\theta$ and the second set with probability $1 - P = (1 - \alpha)^\theta$.

In the first set of outcomes the expected number of agents acting before an Informed agent or the expected number of Ignorant agents, $E[I_{\theta-1}]$, is given by:

$$E[I_{\theta-1}] \equiv E[F_0^{\theta-1}] = \sum_{i=1}^{\theta-1} (1 - \alpha)^i = \frac{(1 - \alpha) - (1 - \alpha)^\theta}{\alpha} \quad (29)$$

Where $F_0^{\theta-1}$ is the Following of the zeroth (hypothetical) agent in an Example Setting Game where $n = \theta - 1$. The expected number of agents acting subsequent to θ who choose c and not d due to acting before the first uncooperative Informed agent is the Following of θ will be²³:

$$E[F_\theta] = \sum_{i=1}^{\Psi} (1 - \alpha)^i = \frac{(1 - \alpha) - (1 - \alpha)^{\Psi+1}}{\alpha} \quad (30)$$

²²Recall that these are the agents acting before the first Informed agent.

²³Of course θ may not be an Informed agent in which case it is the hypothetical Following of θ .

The expectation of the total externality generated is the product of the externality and the number of uncooperative agents less the expected number of uncooperative agents acting before the first Informed agent. This is the same in both sets of worlds and is given by:

$$\kappa(\Psi - E[F_\theta]) \quad (31)$$

Therefore the expected average welfare conditional on the first set of outcomes is:

$$\frac{z}{n}(n - \Psi - E[I_{\theta-1} - F_\theta]) + \left(\frac{v}{n} - \kappa\right)(\Psi - E[F_\theta]) \quad (32)$$

Conditional on the second set of outcomes expected average welfare is:

$$\left(\frac{v}{n} - \kappa\right)(\Psi - E[F_\theta]) \quad (33)$$

This gives an unconditional expected average welfare of:

$$W^e = P \frac{z}{n}(n - \Psi - E[I_{\theta-1} - F_\theta]) + \left(\frac{v}{n} - \kappa\right)(\Psi - E[F_\theta]) \quad (34)$$

□

PROOF of Proposition 5:

Define \underline{X} as the number of agents in the population for which D holds when $\alpha = 0$. In which case D doesn't hold for agent $n - \underline{X}$ and does hold for agent $n + 1 - \underline{X}$. In the case of $\alpha = 0$ the expected Following of agent i is $n - i$. As D holds for agent $n + 1 - \underline{X}$ then $E[F_{n+1-\underline{X}}|\alpha = 0] = (\underline{X} - 1)\kappa \leq \Delta$. As D doesn't hold for agent $n - \underline{X}$ then $E[F_{n-\underline{X}}|\alpha = 0] = \underline{X}\kappa > \Delta$. Therefore, there must be some $\alpha = \underline{\alpha} > 0$ for which:

$$\kappa \cdot E[F_{n-\underline{X}}|\alpha = \underline{\alpha}] = \kappa \left(\frac{(1 - \underline{\alpha}) - (1 - \underline{\alpha})^{\underline{X}+1}}{\underline{\alpha}} \right) = \Delta \quad (35)$$

This implies that if $\alpha \in [0, \underline{\alpha}]$ if $\underline{X} \in \left[\frac{\Delta}{\kappa}, \frac{\Delta + \kappa}{\kappa}\right]$. Hence, expected welfare is greater for $\alpha \in (0, \underline{\alpha})$ than $\alpha = 0$ when the following inequality holds.

$$(E[W(\alpha)|\alpha \in (0, \underline{\alpha})]) = P \frac{z}{n}(n - \underline{X} - E[I_{n-\underline{X}-1} - F_{n-\underline{X}}]) + \left(\frac{v}{n} - \kappa\right)(\underline{X} - E[F_{n-\underline{X}}]) > 0 \quad (36)$$

This rearranges to give:

$$\frac{z}{n} > \left(\kappa - \frac{v}{n}\right) E \left[\frac{(\underline{X} - F_{n-\underline{X}})}{P(n - \underline{X} - I_{n-\underline{X}-1} + F_{n-\underline{X}})} \right] \quad (37)$$

Let:

$$K = E \left[\frac{\underline{X} - F_{n-\underline{X}}}{P(n - \underline{X} - I_{n-\underline{X}-1} + F_{n-\underline{X}})} \right] \quad (38)$$

Inserting this into equation (37) and using the fact that $v = z + \Delta + \kappa$ I get:

$$\frac{z}{n} > \left(\kappa - \frac{z + \Delta + \kappa}{n}\right) K \quad (39)$$

Which rearranges to give:

$$z > ((n - 1)\kappa - \Delta) \frac{K}{1 + K} \quad (40)$$

which yields condition (14) in Proposition 5.

The Following of agent $n - \underline{X}$ is invariant in the size of n for $n > \underline{X}$. The expected number of Ignorant agents is bounded above by the finite value $\frac{1 - \alpha}{\alpha}$. Also, as $n \rightarrow \infty$ then $P \rightarrow 1$. Hence, as n becomes arbitrarily large condition (14) becomes:

$$z > \kappa \quad (41)$$

This yields condition (15) in Proposition 5. \square

PROOF of Proposition 6:

The welfare of a game with $\alpha = \alpha_{n-1}$ is:

$$E[W(\alpha_{n-1})] = \alpha_{n-1} \frac{z}{n} (1 + E[F_1]) + \left(\frac{v}{n} - \kappa\right) (n - 1 - E[F_1]) \quad (42)$$

Welfare for α_{n-1} is greater than for perfect information when $E[W(\alpha_{n-1})] - E[W(1)] > 0$. This gives us:

$$E[W(\alpha_{n-1})] - E[W(1)] = \alpha_{n-1} \frac{z}{n} (1 + E[F_1]) + \left(\frac{v}{n} - \kappa\right)(n - 1 - E[F_1]) - \left(\frac{v}{n} - \kappa\right)n \quad \text{\textcircled{X}430}$$

$$\alpha_{n-1} \frac{z}{n} (1 + E[F_1]) - \left(\frac{v}{n} - \kappa\right)(1 + E[F_1]) \quad \text{\textcircled{X}440}$$

$$\alpha_{n-1} z (1 + E[F_1]) - (v - n\kappa)(1 + E[F_1]) \quad \text{\textcircled{X}450}$$

$$\alpha_{n-1} z - (z + \kappa + \Delta - n\kappa) \quad \text{\textcircled{X}460}$$

$$\alpha_{n-1} z - (z + \Delta - (n - 1)\kappa) \quad \text{\textcircled{X}470}$$

$$(n - 1)\kappa - \Delta - 1 - \alpha_{n-1} z = L - 1 - \alpha_{n-1} z \quad \text{\textcircled{X}480}$$

This is the inequality in equation (16).

The relationship between α_{n-1} and L : if $\alpha = \alpha_{n-1}$ then $E[F_2] = \sum_{i=1}^{n-1} (1 - \alpha_{n-1})^i = \frac{\Delta}{\kappa}$. If n is held constant $\frac{\kappa}{\Delta}$ increases then L increases and $\frac{\Delta}{\kappa}$ decreases. This implies that α_{n-1} increases to maintain the above equality. Likewise, if $\frac{\kappa}{\Delta}$ is held constant and n increases then α_{n-1} increases to maintain the above equality and L increases. Hence, if L increases α_{n-1} increases. \square

PROOF of Proposition 7:

The argument in the proof of The Pandora Effect relies only on d yielding the highest own-action payoff of all actions in A . Hence, Proposition 1 holds if condition (18) holds.

The argument in the proof of Proposition 2 relies on the relationship between the own-action payoffs of c and d defined in condition (5). Condition (5) states only that the own-action payoff from d is greater than the own-action payoff of c : this is equivalent to conditions (18) and (19). It also relies on the expected Following size of an Informed agent; this is independent of A' , $u'(\cdot)$ and $\kappa'(\cdot)$. Finally, it relies on the Pandora Effect which has been shown to hold under condition (18). Therefore, Proposition 1 holds when conditions (18) and (19) hold.

The arguments in the proofs of Propositions 3 and 4 rely on Informed agents' Followings copying their Informed agents. This occurs in equilibrium where copying yields the highest expected own-action payoff for Followers. This is necessarily the case for Followers in the Following of an Informed agent playing defect. It is also the case for Followers in the Following of an Informed agent playing cooperate as a pure strategy if condition (17) holds. Conditions (18), (19) and (20) imply it cannot be optimal for Informed agents to play any action other than c or d . Therefore, it must be optimal, conditional on being copied by their Following, for Informed agents for whom D does not hold to play c . Hence, Propositions 3 and 4 hold if conditions (17), (18), (19) and (20) hold.

Finally, the arguments for Theorems 1 and 1 depend only on Propositions 1 to 4. Hence, Theorems 1 and 1 hold where conditions (17), (18), (19) and (20) hold. \square

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