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Causal Sets: Quantum gravity from a fundamentally discrete spacetime

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Abstract.

In order to construct a quantum theory of gravity, we may have to abandon certain assumptions we were making. In particular, the concept of spacetime as a continuum substratum is questioned. Causal Sets is an attempt to construct a quantum theory of gravity starting with a fundamentally discrete spacetime. In this contribution we review the whole approach, focusing on some recent developments in the kinematics and dynamics of the approach.

1. Motivation

The challenge to construct a quantum theory of gravity is great. There exist many approaches, each of them having their merits, however there is no widely accepted and confirmed theory. The difficulty of the problem suggests, similarly with other paradigms in the history of physics, that we may need to abandon some of our a-priori assumptions about reality. In particular the continuity of spacetime that we assume, is questioned in several places. Approaches to quantum gravity that start from a continuum theory, observe a (possibly effective) discreteness¹. Another field that we may get hints about quantum gravity, is the black holes thermodynamics. Infinities that arise in this case, could be avoided if space was discrete as it is observed in [1]². The infinities we face in quantum field theory (though we have learnt to live with them), and singularities in general relativity are two other reasons why discreteness could simplify our physical picture. All the above, would be resolved, if one was willing to abandon spacetime

¹ Discreteness of volume in Loop Quantum Gravity, dualities between very small and large scales in string theory, etc.

² Research in causal sets, involves some treatment of black hole thermodynamics. In this contribution we will not deal with it, but we give the following references for the interested reader [2].

continuity³. It is therefore a natural starting point, for building a new quantum theory of gravity, to take spacetime fundamentally discrete.

The second element in causal sets, is *causality*. Here again, it seems more intuitive to speak of causal relations (that have the natural interpretation of cause and effect), rather than some standard spacetime where one needs to assume topology, differential structure and metric. After all, as we will see below causal relations encode most of the information of the metric.

We therefore, come to conclude that it may be better starting a quantum gravity theory from a discrete spacetime that comes along with the causal relations of the elements (points) of this spacetime. This is precisely the starting point of the causal set approach to quantum gravity. It was originally introduced in [3].

2. Definition

Mathematically a causal set (or causet) is a set C endowed with a partial order relation \prec which is irreflexive ($x \not\prec x$), transitive ($x \prec y \prec z \Rightarrow x \prec z$), and locally finite [$x, y] \equiv (|\{z | x \prec z \prec y\}| < \infty \forall x, z \in C)$ (where $|A|$ indicates cardinality of the set A). The local finiteness condition imposes the requirement of discreteness, because it requires that there is only a finite number of elements between every pair in the causet. The partial order relation, represent the causal relation between elements of C , so if $x \prec y$ it means that x is at the past of y .

The next thing to do is to see why a causal set (as defined above) could possibly replace continuum spacetime. This is based on a theorem by Malament [4]:

The metric of a globally hyperbolic spacetime can be reconstructed uniquely from its causal relations up to a conformal factor.

The theorem suggests that the causal structure encodes most of the information of the metric. What we lack to obtain the full metric is a way to fix the conformal factor, in other words to fix the volume. This is precisely done by considering a discrete spacetime. In causal set we make the correspondence Volume= Number of elements⁴. By having only a finite number of causal set elements between any pair of causally related spacetime points, we can use that number to fix the volume. We thus have the “slogan” Order+Number=Geometry. We are now in position to make the central conjecture of causal sets (also called the Hauptvermutung):

Central Conjecture: Two distinct, non-isometric spacetimes cannot arise from a single causal set.

The conjecture, while it still remains a conjecture has very solid theoretical and numerical evidence in its support⁵. The central conjecture guarantees that there can be only one (essentially) spacetime approximating a given causal set. Here we must stress the fact that a causal set behaves as a continuum manifold, under several conditions, and in particular, when the number of causal set elements is very big. In this case we speak of the *continuum approximation*⁶.

³ There are also models for modifying gravity that would resolve the cosmological constant problem, that favor discreteness.

⁴ In particular each element of the causal set corresponds to 1 Plank unit Volume.

⁵ Most of the work on the kinematics of causal sets, regarding the dimension (e.g. [5]), timelike and spacelike distances ([6, 7]), topology ([8]), provide support to the conjecture.

⁶ It is the analogue of continuum limit of other approaches, however it is not really a limit, since we never take the discreteness scale going to zero, as it is done for example in CDT [9].

We define the concept of *faithful embedding* to make precise when a continuum spacetime approximates a causal set:

A faithful embedding is a map ϕ from a causal set \mathcal{P} to a spacetime \mathcal{M} that:

- (i) preserves the causal relation (*i.e.*, $x \prec y \iff \phi(x) \prec \phi(y)$) and
- (ii) is “volume preserving”, meaning that the number of elements mapped to every spacetime region is Poisson distributed, with mean the volume of the spacetime region in fundamental units, and
- (iii) \mathcal{M} does not possess curvature at scales smaller than that defined by the “intermolecular spacing” of the embedding (discreteness scale).

The central conjecture therefore reads as “a causal set cannot be faithfully embedded in two non isometric spacetimes”. Taking a closer look at the definition of faithful embedding, we see that a regular lattice cannot correspond to a faithful embedding of Minkowski spacetime. It fails because, in a regular lattice, considering a very boosted frame, we will end up with big volumes having no elements at all and fail to satisfy the condition (ii) of faithful embedding. Instead, a *random* lattice would do this job. For example one generated by sprinkling elements in spacetime randomly with probability $P(n) = \frac{(\rho v)^n \exp^{-\rho V}}{n!}$, in other words a Poisson sprinkling⁷.

Causal sets are constructed in such a way, that they respect Lorentz invariance at the kinematic level already⁸. These two observations are analyzed in detail in [10].

3. Kinematics

In all discrete/combinatoric approaches to quantum gravity one important step is the so called “inverse problem” [11]. It consists of two parts, the first is how do continuum like structure arise from the fundamentally discrete base. The second part, is how shall we recognize that we do have something continuum like (and what) when we actually have it.

In our case, we want to address this issue for a causal set. In particular we want to obtain continuum notions from the partial order. A typical causal set (if taken by pure counting and attributing equal weights to all causal sets), it is a Kleitman-Rothschild order [12]. These orders have only three layers (*i.e.* longest chain is a 3-element chain) and 1/4 of elements sit at the first layer, 1/2 in the second and 1/4 in the third. This is certainly not manifold-like (it contains only 3-moments of time). We thus need the dynamics to select for us a causal set that looks like a continuum manifold. For now we will deal with properties of causal sets that in the case there exist a manifold that faithfully embeds to it, correspond to continuum properties such as lengths of curves.

Some definitions are in order:

Definition 1: A pair of elements x, y is a *link* if $x \prec y$ and $\nexists z | x \prec z \prec y$. In other words is the most basic relation, that cannot be implied by transitivity.

Definition 2: Chain C is a collection of elements such that $\forall x, y \in C$ either $x \prec y$ or $y \prec x$. In other words it is linearly ordered⁹.

Definition 3: We define a *path* to be a chain, where each pair of consecutive elements are links.

Definition 4: We define a *2-link*, given a pair of unrelated elements x, y , an element z that x, z and y, z are both links.

⁷ And the expected variance is of course \sqrt{V} .

⁸ Unlike for example spin networks, where Lorentz invariance is expected to arise due to some properties of the superimposed Lorentz-violating spin networks

⁹ this intuitively corresponds to a timelike curve at the continuum

The concept of timelike curve is represented by a chain. Moreover, it can be shown that we can define the timelike distance between two related elements $x \prec y$ of the causal set. In the continuum it corresponds to the maximum length of timelike curve that joins x and y . In the causal set this is simply the chain of maximum cardinality starting from x and ending at y [6]¹⁰.

$$d_t(x, y) = \max |C| \text{ where } C \text{ start at } x \text{ and ends at } y \quad (1)$$

It has been tested theoretically but also numerically [6, 7], that in the case that the causal set is faithfully embedded at a manifold¹¹ the above distance agrees with the continuum spacelike distance, in the limit of a sufficiently big causal set.

To get spacelike distance for a causal set that corresponds to Minkowski spacetime is considerably more difficult. Lee Smolin for example has claimed that what lacks from causal sets is some spatial information. The first naive attempt to get the spatial distance would be the following. Consider x, y two unrelated (spacelike) elements of the causal set. Then let $u \in J^+(x) \cap J^+(y)$ and $v \in J^-(x) \cap J^-(y)$.

$$d_s(x, y) = \min d_t(v, u) \quad (2)$$

Unfortunately this fails as was already noted in [6] and confirmed numerically in [7]. A more refined treatment is required where $u \in J^+(x) \cap J^+(y)$ but also u is a 2-link (w.r.t. x, y). The reader is referred too [7] for further details.

It can be also shown, that using this notion of spacelike distance, one can define closest spatial neighbors (timelike closest neighbors are simply the links). Using this concept we can measure the length of any curve in the causal set (not only on chains). More importantly this generalizes in curved spacetimes, provided that there is no curvature at very small scales. This is a reasonable assumption made also by Gibbons in [13].

The dimension of a causal set is also examined in [5]. Assume we have two related elements $x \prec y$, with $d_t(x, y) = n$ and the number of elements between them being N ($|V_{xy}| = N$ where $z \in V_{xy}$ if $x \prec z \prec y$). We then see how the volume scale as we double the timelike distance¹². This and other dimension measures that work for causal sets embedable in continuum spacetimes as well as extensions of the notion of dimension for non-manifold like causal sets can be found in [5].

Finally, the spatial topology has been analyzed (e.g. in [8]). The concept of spatial slice is replaced by the following: We consider a *maximal anti-chain* a collection of elements $\{e_i\}$ such that $e_i \not\prec e_j$ and does not exist any other element being unrelated to all the elements of our anti-chain¹³. However, this is not a good analogue for a spatial slice. We need to “thicken” by considering all elements with distance a number n from the anti-chain. There is a nice way to recover the topology of this slice, using simplicial complexes. The results are in very good agreement with continuum results for causal sets that embed in continuum spacetimes [8].

4. Dynamics

While a lot can be said from the kinematic sector of a theory (and as we will see later, some phenomenological effects), the true physical content of a theory lies in the dynamics. There are two ways to try and adopt the dynamics. The first¹⁴ takes as fundamental ingredient the

¹⁰ To be more precise it is proportional and the proportionality constant in general depends on the dimension

¹¹ Basically, it was tested that causal sets that arose from sprinkling to some continuum manifold

¹² in other words take an element z such that $d_t(x, z) = 2n$ and find $|V_{xz}|$

¹³ Adding any other element makes our set stop being an anti-chain

¹⁴ called initially by Sorkin as the “principled” approach

causal set itself and attempts to generate the full causal set from some basic principles that are in some sense natural to the causal set approach, in other words simply related with the partial order. The second approach¹⁵, is an attempt to guess some dynamics (possibly by rephrasing the continuum Einstein action, in terms of the order relation) and get “correct” result¹⁶. Finally, of interest is to consider, quantum matter (or fields) on one classical causal set (an analogue of QFT in curved spacetime). There has been considerable progress in the latter recently. Here we will first consider some classical (alas stochastic dynamics). Then we will discuss about quantum dynamics. This task has not been fully accomplished, and we will only mention some toy models, attempts and directions for future research. Finally we will deal with quantum matter on a causal set.

4.1. Classical Dynamics for Causal Sets

The stochastic classical dynamics have been analyzed in depth (see [14]). One can construct a partial order that consists of all the causal sets (a partial order of partial orders), where two elements are related if the one is subset (when viewed as a partial order) of the other. It is called poscau (see Fig. 4.1). This is the arena for causal set dynamics and is an analogue of the superspace. It is the set of all possible spacetimes. We can imagine a process that generates all the causal sets, by growing the causal set element by element. This process is characterized by some probability, that each new-born element will be at the future or spacelike (unrelated) from each of the already existing elements. These processes are known as (*classical*) *growth dynamics* (CGD)¹⁷. We impose some physical conditions that these transitions probabilities must obey. In particular the requirements are:

- (i) **Internal temporality.** Each new element is born to the future or spacelike to all existing elements (never at the past).
- (ii) **Discrete general covariance.** The probability of arriving at a particular causal set does not depend on the order the elements are born. In other words the probability does not depend on the path chosen on the Figure.
- (iii) **“Bell’s causality”.** This condition states that the probability of new-born elements depends on the past of the new-born element and thus is not affected by births in spacelike regions. The name comes from the fact that it is related with the paradoxes that arise in quantum theory due to non-locality. It is believed that this condition may need to be relaxed (or transformed) if we were to consider a quantum theory of causal sets¹⁸.
- (iv) **The Markov Sum rule.** This means that the sum of all probabilities starting from a particular causal set (element of the poscau), sum up to one.

These conditions turn out to be very restrictive, and there is only one free parameter left for each level¹⁹. One particular choice of these parameters results in the transitive percolation that has been studied in depth (see references in [14]). The different choices give rise to very different causal sets. While some of these have striking similarities with cosmological models [15], the resulting causal sets are not in general manifold like and thus we would need to resort to quantum dynamics to explain the continuum like behavior we see.

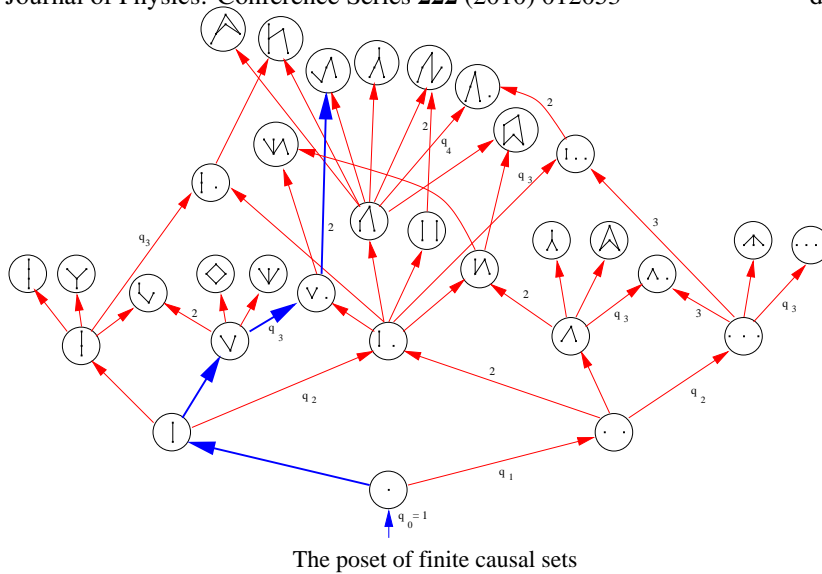
¹⁵ called the “opportunistic” approach

¹⁶ It is opportunistic, since the Einstein action has a very good motivation in terms of continuum spacetime, but the analogue on a causal set is not something intuitive.

¹⁷ They can be thought as generalizations of random walks

¹⁸ However, it is not necessary since non-local effects on a causal set, may arise even if the dynamics of the causal set itself are in some sense local.

¹⁹ By level, we mean the number of total elements of the causal set.



The poset of finite causal sets

4.2. Quantum Causal Sets

A first attempt to quantize the above dynamics, would be to use quantum amplitudes instead of probabilities. In other words to allow the transition probabilities to be complex numbers²⁰. This would be a generalization of what is known as quantum random walk (QRW). The whole causal sets program is based on spacetime rather than space. It is not possible to do a canonical quantization, since there is no well defined way to break up the full causal set to spatial slices with an external time parameter²¹. It is generally believed that in order to deal with quantum causal sets, one is forced to resort to sum over histories quantization. The mathematical ingredient one needs to use is the *quantum measure* [16]²². To interpret it properly, one needs to address the basic problems of the foundations of quantum theory. Consistent (or decoherent) histories is one approach, while there is considerable recent activity in a novel interpretation the “Pimbinò” interpretation (also called the co-event or anhomomorphic logic interpretation) [17].

The most interesting result however comes from considering 2-dimensional partial orders [18]. Here the dimension is not to be understood as spacetime dimension, it is rather a technical term for partial orders. In particular it means the following: Assume that we have a set $\mathcal{P} = \{e_1, e_2, \dots\}$ and we have two linear orderings of this set. Then we take the intersection of this linear orderings, in other words an element $e_1 \prec e_2$ iff e_1 is before e_2 in both linear order we have. The resulting partial order is called 2-dimensional, if it can be generated from the intersection of 2 linear orderings²³. For 2-dimensional partial orders it turns out, that they do correspond to 2-dimensional spacetimes, however this analogy breaks down for higher dimensions.

Brightwell et al in [18] considered 2-d orders and attempted to find what the typical 2-

²⁰ Tentative results show that the typical causal set still is not one manifold-like

²¹ Despite the fact that as we have seen there exist an analogue of a spacelike surface for causal sets that correspond to spacetimes that allow such a foliation. However is not fundamental for the approach.

²² In classical stochastic theories such as the Brownian system, one uses again histories, and a measure (the Wiener measure) on the set of possible histories. In a sense we believe that quantum theory comes as a natural generalization of classical stochastic theories rather than deterministic theories.

²³ By analogy it is n-dimensional if it is the intersection of n linear orderings.

d order would look like. Note, that as we have already mentioned a typical causal set is not manifold-like (it is a Kleitman-Rothschild order). This problem is also known as the entropy problem and is faced in other approaches such as the DT (dynamical triangulation) and a possible solution is to restrict to sum over sum particular class rather than all possible configurations (as was done in CDT (causal dynamical triangulations) [9]). Interestingly, in 2-d orders the major contribution comes from causal sets that are manifold-like and moreover there are faithfully embeddable in 2-d Minkowski. This can be seen as a toy model to see how manifold-like behavior (and indeed flat!) arises from simple considerations on a causal set.

Finally, recently Bombelli and Svedrlov in [19] have taken the ‘‘opportunist’’ approach. They have written the Einstein action in terms of causal set elements, and have used this weight to make a sum over all possible causal sets. However, this work is at the beginning. One has to see what is a typical causal set, if we were to use this weight and also what consequences that would have. The reader is referred to the original papers for further details.

4.3. Quantum Matter on Causal Set

One thing of great interest, would be to see how matter and fields behave, on a single fixed (and thus classical) causal set. This question corresponds to fixing the background to some (generally) curved spacetime and allowing quantum fields on it. For causal sets, we need to firstly see how we recover standard results, and then see whether the assumption of discrete background may have some effect that could be possible to test and thus confirm or falsify the causal set assumption²⁴.

The first attempt would be to consider point (massive) particles moving along a chain of the causal set that faithfully embeds at Minkowski spacetime. There are several models considered [20, 21], that in all of those, when taking the continuum approximation, the particle follows approximately timelike geodesic, but deviating (swerving) slightly. In other words it is like having some drift, and all models result in a diffusion equation depending on a single parameter the diffusion strength k . Let us here briefly describe the first model ([20]). The particles trajectory is a chain $\{e_1, e_2, \dots\}$ in the causal set. we define a forgetting time t_f , which is the time below which the causal set may behave non-locally, but above which it behaves normally. If the particle has position e_n with four momentum p_n the next element is chosen

- e_{n+1} is at the causal future of e_n and at proper time t_f
- the momentum change $|p_n - p_{n+1}|$ is minimized (where p_{n+1} is on the mass shell and proportional to the vector between e_n and e_{n+1}).

The resulting motion, heuristically, stays as close to straight as possible, having however some random fluctuations in the particles momentum. The diffusion equation resulting in the continuum approximation was (see [20])

$$\frac{\partial \rho}{\partial \tau} = k \nabla^2 \rho - \frac{1}{m} p^\mu \partial_\mu \rho \quad (3)$$

Following similar lines, one may attempt to see what happens for massless particles. The added difficulty being that the trajectory is no longer a chain (which by definition is a timelike curve). The resulting equation has two parameters (diffusion and drift parameters) k_1, k_2 and there is again some non-locality scale. The reader is referred to [21].

²⁴ Of course the observed effect, typically would depend on some parameters and perhaps on the full quantum dynamics.

This particular feature, of some non-locality scale, appears also from consideration of particles propagators. In order to do this one needs to define the D'Alembertian operator \square . In [22], the d'Alembertian is introduced. We will follow briefly [23], to obtain the retarded propagator for a particle on a causal set.

To get the propagator $K(x, y)$ from an element x to y in its future, we need to sum over all the chains (or paths) on a causal set from x to y with a particular amplitude (c.f. quantum mechanics where one adds the quantum amplitudes to go from one point to another and sums over all the possible trajectories). We need to make a choice of which trajectories to sum over (either all chains or all paths), and what weight to put on each trajectory. These choices have to be made in such a way, that when a causal set is faithfully embedable in Minkowski spacetime, to give us back a propagator of the Klein-Gordon equation (in particular, with the choices made we shall get the retarded propagator). In particular the amplitude depends on two parameters a being the probability that the particle would 'hop' once along the trajectory from one element to another, and b being the probability that would stop at an element of the trajectory. For a chain of length n (where we have n hops and $n - 1$ intermediate stops) the amplitude would be $a^n b^{n-1}$. For causal set in 1+1 Minkowski and 3+1 Minkowski we fix the values of a and b to be respectively²⁵:

$$a = \frac{1}{2}, \quad b = -\frac{m^2}{\rho} \quad (4)$$

$$a = \frac{\sqrt{\rho}}{2\pi\sqrt{6}}, \quad b = -\frac{m^2}{\rho} \quad (5)$$

With these choices the retarded Klein-Gordon propagator is obtained as shown in [23].

Building up on this work, using the retarded Klein-Gordon propagator, Johnston in [24] proceeded and considered scalar quantum field on a causal set, and he computed the Feynman propagator²⁶.

5. A Solution to the Cosmological Constant Problem

The biggest probably success of the causal sets approach to date, in terms of concrete result, is the prediction of the correct order of magnitude of the cosmological constant [26]. The argument is heuristic, but conclusive, in the sense that a zero cosmological constant (or of different magnitude) is ruled out by causal sets (or else a different result rules out causal sets). Another interesting point of this prediction is that it was first made in 1990, when at the time it was believed that its value is zero, and only much later was measured to be of that order of magnitude. Here we will present briefly the argument, and the reader is referred to [26] where it first appeared and [27] where it is analyzed in some depth along with possible problems and open questions.

The cosmological constant problem arises from the following observation. The dark energy is the 70% of effective energy density of the universe. Moreover it has negative pressure. This is usually explained by the use of a cosmological constant Λ . However, it appears to be very small, since the natural value we would expect from the vacuum expectation value is $\rho_{vac} \simeq m_{pl}^4$ which is $\simeq 10^{120} \rho_{obs}$ the observed value. Most mechanism that makes it small brings it to identically

²⁵ The values depend only on the mass of the particle and the volume corresponding to each causal set element, i.e. the density.

²⁶ Of crucial importance was the use of Pauli-Jordan function [25] $\Delta(x) := G_R(x) - G_A(x)$ and its analogue for a causal set. Note that this could also generalize for causal sets embedable in curved spacetimes.

zero (e.g. supersymmetry). The other strange feature is that the cosmological constant is of the order of magnitude of the ambient density, but only *now* (i.e. in our epoch). This also seems as an unnatural fine-tuning. The causal set proposal will suggest why the latter fine tuning appears and explain how the cosmological constant is non-zero (and of the desired magnitude) given a mechanism that would otherwise bring it to zero.

We need to introduce here an alternative formulation of general relativity the so-called *unimodular* gravity. It is a path integral formulation where the total volume V is kept fixed. The equation of motions are identically the same as in ordinary GR. The cosmological constant term $\Lambda = \Lambda_0 + \lambda$ appears in the action along with V in a $-\Lambda V$ term:

$$\delta\left(\int\left(\frac{1}{2\kappa}R - \Lambda_0\right)dV - \lambda V\right) = 0 \quad (6)$$

In unimodular gravity, the cosmological constant and the volume are conjugate pairs very much like Energy-Time. It obeys a similar uncertainty principle where $\Delta V \cdot \Delta\Lambda \sim \hbar$. In standard unimodular gravity, the volume is fixed, and the cosmological constant is completely undetermined from the parameters of the theory as expected.

However, in causal set things change. The role of time is replaced by the number of elements N (e.g. growth dynamics) and in QT we do not sum over different times. It is natural to causal sets to consider fixed N , which is *almost* unimodular gravity. The difference being that we have fixed N instead of V . N is proportional to V *up to poisson fluctuations*, i.e. $\pm\sqrt{V}$. We thus have some “kinematic fluctuations”. Taking the conjugate of the volume we get $\Delta\Lambda \sim 1/\Delta V \sim 1/\sqrt{V}$. Assuming that there is a mechanism that brings the cosmological constant to zero, then applying this on a causal set we get $\Lambda \sim V^{-1/2} \sim H^2 \sim \rho_{\text{critical}}$. This cosmological constant is thus of the correct magnitude and is *always* of the order of magnitude of the critical density. This happens by simple considerations of the kinematics of the causal set (being a random partial order). The V is the volume of the past and thus result in an everpresent Λ of varying magnitude and sign. Particular models/implimantations of dynamics are needed to check whether this assumption change the standard picture of cosmology (as e.g. the Big Bang Nucleosynthesis). In [27] such a model is considered.

6. Summary and Conclusion

We have briefly reviewed the causal sets approach to quantum gravity, giving the original references for deeper study. In particular we have seen what a causal set is and how it succeeds in being both discrete and Lorentz invariant. The continuum concept arise from solely the partial (causal) order, and it has been shown that these do agree with the continuum notions for large enough causal set that have a continuum spacetime that faithfully embeds in it. Further we have seen how with some very simple and natural requirements for the (classical) dynamics we get a family of solutions that is quite restrictive. Getting the full quantum dynamics of causal sets, is the major open issue for causal sets, even though there are considerable recent developments. Firstly a 2-dimensional full gravity model that gives as the typical causal set Minkowski spacetime (which is certainly manifold-like!). Secondly, transforming the Einstein action in terms of causal relations has been done and it remains to be examined. Thirdly, there is progress in the conceptual part, and the development of a histories quantum theory of closed systems, the Piombino interpretation. A semi-classical description, where spacetime is a single causal set, but matter (and or fields) lying on it are quantum has been developed recently in a satisfactory degree and what remains now, is to explore possible deviations from standard QFT on curved background physics, in order to test the assumption of causal sets. Finally,

we reviewed the proposed explanation, using causal sets, for the small non-zero cosmological constant observed.

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