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Performance analysis of spatial modulation and space-shift keying with imperfect channel estimation over generalized - Fading channels

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Performance Analysis of Spatial Modulation and Space-Shift Keying With Imperfect Channel Estimation Over Generalized $\eta{-}\mu$ Fading Channels

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Abstract—**Novel performance analysis of spatial modulation (SM) and space-shift keying (SSK) over generalized $\eta{-}\mu$ fading channels in the presence of Gaussian imperfect channel estimation at the receiver is presented in this paper. A general expression for the pairwise error probability (PEP) is derived along with an asymptotic expression at high signal-to-noise ratio (SNR). The $\eta{-}\mu$ fading channel has Nakagami-m, Rayleigh, one-sided Gaussian, and Nakagami-q (Hoyt) channels as special cases. The derived expression for the PEP is valid for integer and noninteger values of the fading parameter μ. The impact of channel estimation errors with different η and μ values is investigated. Analytical results are sustained through Monte Carlo simulation results, and a close match is reported for a wide range of SNR and for different system parameters.**

Index Terms—**Imperfect channel knowledge, performance analysis, space-shift keying (SSK), spatial modulation (SM), $\eta{-}\mu$ fading.**

I. INTRODUCTION

WIRELESS signals do not travel directly from transmitter to receiver but are subject to multipath propagation. In the past, the ultimate goal of wireless communication was to combat the distortion caused by multipath propagation in order to approach the theoretical limit of capacity for a band-limited channel. With the advent of new technologies such as multiple-input–multiple-output (MIMO) systems, location-dependent multipath propagation is exploited constructively. In fact, multipath propagation can be considered as multiple channels between transmitter and receiver, which can be utilized to provide higher total capacity than the theoretical limit for a conventional channel [1].

Space modulation techniques, such as spatial modulation (SM) [2] and space-shift keying (SSK) [3], are promising MIMO techniques for future wireless communication systems. In such techniques, location-dependent spatial information is utilized to carry additional information bits to boost the overall spectral efficiency. At the same time, only a single transmit antenna among the set of existing antennas is active at each particular time instant. Thereby, typical MIMO problems such as interchannel interference, which requires high receiver complexity, transmit antenna synchronization, and the need for multiple RF chains, are entirely avoided [2], [4].

The basic fundamental idea of these techniques may be traced back to [5], which was further developed into SM in [2], [4], and [6]. It has been shown that such techniques achieve lower complexity and enhanced error performance with moderate number of transmit antennas, as compared with other conventional MIMO techniques [2], [3], [7]–[9]. In addition and unlike what was anticipated, these techniques are shown to be more robust to channel estimation errors, as compared with other MIMO techniques, since the error probability depends on the Euclidean difference between different channel paths associated with different transmit antennas rather than the actual channel realizations themselves [10]–[12]. Hence, these techniques have been intensively studied over the past few years (see [13] and references therein).

Performance analyses of SSK in Rayleigh, Nakagami-m, and Rician fading channels have been reported in [14]–[17], respectively. In addition, SSK performance analysis with imperfect channel knowledge at the receiver is conducted in [11], [18], and [19] and, with practical channel estimates, is reported in [20] and [21].

SM performance analysis over Rayleigh fading channels with perfect channel knowledge has been reported in [4] and with imperfect channel knowledge in [10] and [11]. In addition, practical implementation of SM system with detailed performance analysis highlighting the impact of several practical system impairments is reported in [21]. The performance of SM over Nakagami-m fading channels is studied in [16] and [22]. The authors introduced a comprehensive analytical framework to compute the average bit error probability for any MIMO setup with arbitrary correlated fading channels, as well as for generic modulation schemes. An adaptive SM transmission scheme to achieve better system performance under a fixed data rate is presented in [23], and a simplified maximum-likelihood (ML)-based scheme for M-ary quadrature amplitude modulation (QAM) SM that is computationally less complex than the conventional ML scheme is proposed in [7].

In all previous literature that considers SM/SSK and their performance analysis, Rayleigh, Rice, and Nakagami-m fading channels are only considered. The considered Nakagami-m fading channel in previous studies, as in [16], assumes a Nakagami envelope distribution and a uniform-phase distribution of the fading channel. However, it has been shown in [24] that the Nakagami channel phase is not uniform and a distribution of the phase has been derived. The phase distribution can be shown to have insignificant impact on systems with maximum ratio combining receiver, such as space–time coding (STC) MIMO systems. However, considering the joint ML detector of SM/SSK systems, the phase distribution assumption impacts the performance of these systems significantly. For instance, considering a uniform-phase Nakagami channel, it has been shown that increasing the Nakagami-m parameter enhances the performance of the SSK system [16]. Meanwhile, it is shown in this paper that the performance of SM/SSK systems degrades as the μ parameter of the $\eta-\mu$ channel, which is directly related to Nakagami-m parameter, increases. This different conclusion can be supported by noting that as $m \to \infty$, the Nakagami envelope distribution becomes Gaussian and MIMO paths would not be resolvable. Hence, SM/SSK systems' performance enhances for large m values only if power imbalance among antennas is considered. Otherwise, major performance degradation is anticipated [22], [25].

In this paper, a general framework for SM/SSK performance analysis with perfect and imperfect channel knowledge at the receiver over $\eta-\mu$ fading channel is presented. $\eta-\mu$ fading channel is a general fading distribution proposed in [26], where well-known distributions such as Rayleigh, Nakagami-m, and Hoyt (Nakagami-q) can be derived as special cases. The effect of imperfect channel knowledge and the impact of changing the channel parameters η and μ on the performance of these systems will be analyzed. Similar works for other MIMO techniques have been proposed in literature recently. For instance, analytical expressions for the exact random coding exponent of MIMO systems employing space–time block code and operating over $\eta-\mu$ fading channels are derived in [27]. SM performance over $\eta-\mu$ fading channel is presented in [28]. However, [28] considers perfect channel knowledge at the receiver and assumes the envelope of the fading channel to follow $\eta-\mu$ while the phase is uniformly distributed. These simplified assumptions lead to entirely different analysis and results, as compared with the reported results in this paper. An upper bound expression for the capacity of MIMO systems under the general $\eta-\mu$ fading model is proposed in [29]. It is shown that the theoretical bound depends only on the first and the second moments of the random elements in the channel matrix [29].

Thereby and with reference to current literature, our contributions in this paper are threefold. 1) The performance of SM/SSK systems operating over generalized $\eta-\mu$ fading channels with imperfect channel knowledge at the receiver is studied, and a general closed-form expression for the pairwise error probability (PEP) is derived. 2) The derived PEP is used to obtain an upper bound of the average bit error rate (BER). 3) An asymptotic expression that is simple and accurate for the PEP at pragmatic signal-to-noise ratio (SNR) is obtained.
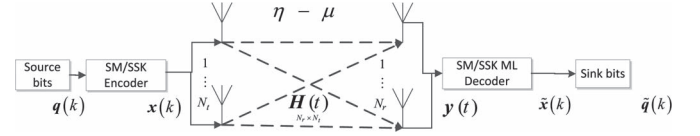

Fig. 1. SM/SSK system model over $\eta-\mu$ MIMO fading channel.

The remainder of this paper is organized as follows. In Section II, the system and channel models are introduced. Section III presents the derivation of the PEP. Some representative plots for our analytical results, along with their interpretations, are illustrated in Section IV. Section V draws the conclusions for this paper.

Mathematical Notations and Functions: Matrices are shown in boldface uppercase letters (e.g., $\mathbf{A}$), vectors are shown in boldface lowercase letters (e.g., $\mathbf{x}$), $\mathbb{E}\{\cdot\}$ denotes expectation, $\mathrm{Var}\{\cdot\}$ represents the variance, $[\cdot]^T$ is for transpose, $\|\cdot\|_F$ is the Frobenius norm, $f_X(\cdot)$ represents the probability density function (PDF), and $\Gamma(\cdot)$ is the gamma function. The Kronecker product of $\mathbf{A}$ and $\mathbf{B}$ is shown as $\mathbf{A} \otimes \mathbf{B}$, $\mathcal{M}_X(\cdot)$ is the moment-generating function (MGF) of random variable X, $\mathrm{vec}(\mathbf{A})$ denotes the vectorization operator that stacks the columns of $\mathbf{A}$ in a column vector, $\det(\mathbf{A})$ is the determinant, $\mathrm{PEP}(\cdot)$ and $\mathrm{PEP}^{\mathrm{Asym}}(\cdot)$ are the PEP and asymptotic PEP, and $Q(\cdot)$ is the Q-function.

II. SYSTEM AND CHANNEL MODELS

A. System Model

A general $N_t \times N_r$ SM/SSK MIMO system is shown in Fig. 1, where N_t and N_r are the numbers of transmit and receive antennas, respectively. A block of k-bits ($k = \log_2(M) + \log_2(N_t)$) is mapped into a constellation vector $\mathbf{x} \in \mathbb{C}^{N_t \times 1}$, i.e., $\mathbf{x} = [x_1\ x_2\ \ldots\ x_{N_t}]^T$.

Note that $\log_2(M)$ bits are used to represent an M-QAM symbol x_i from the signal constellation, whereas $\log_2(N_t)$ represents the number of bits required to identify the active transmit antenna index j in the antenna array.[1] That is

$$\mathbf{x}_{j,i} = \begin{bmatrix} 0 & \cdots & \overbrace{x_i}^{j^{\text{th position}}} & 0 \end{bmatrix}^T \tag{1}$$

where $x_i = 1$ for the SSK system. Signal vector $\mathbf{x}_{j,i} \in \mathbb{C}^{N_t \times 1}$ is then transmitted over the wireless channel $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$. A fading channel matrix $\mathbf{H}$ with $\eta-\mu$ fading entries h_i ($i = 1, 2, \ldots, N_t N_r$) is assumed in this paper. The received signal experiences complex zero-mean additive white Gaussian noise (AWGN) vector $\mathbf{w} \in \mathbb{C}^{N_r \times 1}$ with the variance of its elements being σ_n^2, i.e., $\mathbf{w} = [w_1\ w_2\ \cdots\ w_{N_r}]^T$. Thus, the received signal vector $\mathbf{y}$ is given by

$$\mathbf{y} = \sqrt{E_s}\mathbf{H}\mathbf{x}_{j,i} + \mathbf{w} \tag{2}$$

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[1]Note that SSK is a special case from SM where only spatial constellation symbols are utilized and where no signal constellation symbol is transmitted. Hence, and for the sake of brevity, we will discuss the SM system here and refer to SSK when appropriate.

where $\mathbf{y} \in \mathbb{C}^{N_r \times 1}$, and E_s denotes the transmitted signal energy. Without loss of generality, the average transmitted power in SM/SSK systems is normalized to unity $E[\mathbf{x}_{j,i}^H \mathbf{x}_{j,i}] = E_s = 1$.

At the receiver, the estimate of $\mathbf{H}$ channel is $\tilde{\mathbf{H}}$, which is obtained through transmitting orthogonal Hadamard sequences at the beginning of each frame and is assumed to be static over one frame. We assume that $\mathbf{H}$ and $\tilde{\mathbf{H}}$ are jointly ergodic and stationary processes. Furthermore, assuming orthogonality between the channel estimate and the estimation error, we have

$$\tilde{\mathbf{H}} = \mathbf{H} - \mathbf{e}_h \tag{3}$$

where $\mathbf{e}_h \in \mathbb{C}^{N_r \times N_t}$ is the channel estimation error matrix, with complex Gaussian entries having zero mean and σ_e^2 variance. Note that σ_e^2 is a parameter that captures the quality of the channel estimation and can be appropriately chosen depending on the channel dynamics and estimation schemes. The estimation error linearly reduces by increasing the number of pilots. In addition, the PDF of $\tilde{\mathbf{H}}$ no longer follows an $\eta-\mu$ distribution, and calculating such PDF is mathematically involved. However, it will be shown later that the PDF of $\tilde{\mathbf{H}}$ is not required to analyze the performance of SM/SSK systems, and only statistical properties such as mean and covariance need to be known.

At the receiver, the transmitted data and spatial symbols are jointly detected using the ML optimal detector [4] as

$$[\hat{j} \quad \hat{i}] = \arg \min_{\substack{j \in \{1:N_t\} \\ i \in \{1:M\}}} \|\mathbf{y} - \tilde{\mathbf{H}}\mathbf{x}_{j,i}\|_F^2. \tag{4}$$

It is important to note that $\mathbf{x}_{j,i} \in \mathcal{S}$, where $\mathcal{S}$ is a 2^k space containing all possible combinations of spatial and signal constellation symbols. This is unlike the ML equation in [4] and other commonly used equations in the literature, where the spatial symbol determines the channel vector, and x is an element from the signal constellation space. Although both definitions are equivalent, the considered definition in (4) simplifies and generalizes the performance analysis of SM/SSK systems, as will be shown later.

B. Channel Model

The $\eta-\mu$ distribution represents the small-scale variation of the fading signal in a non-line-of-sight environment and is originally proposed in two formats [30]. Format I considers a signal composed of clusters of multipath waves propagating in a non-homogeneous environment. The in-phase and quadrature components of the fading signal within each cluster are assumed to be independent from each other and have different powers. Format II differs from format I in assuming that the in-phase and quadrature components of the fading signal within each cluster have identical powers and are correlated with each other. The joint phase and envelope $\eta-\mu$ PDF is given by [30]

$$f_{R,\Theta}(r,\theta) = \frac{2\mu^{2\mu} h^{2\mu} r^{4\mu-1} |\sin(2\theta)|^{2\mu-1}}{(h^2 - H^2)^{4\mu^2} \Gamma^2(\mu)}$$
$$\times \exp\left(\frac{2\mu h r^2}{(h^2 - H^2)}(h + H\cos(2\theta))\right). \tag{5}$$

The physical parameter $-1 < \eta < 1$ represents the correlation coefficient between the scattered in-phase and quadrature components of each cluster of multipath. As such, $\eta = H/h$, with $H = \eta/(1 - \eta^2)$ and $h = 1/(1 - \eta^2)$, whereas the physical parameter μ is related to the number of multipath clusters in the environment. Based on this definition, the $\eta-\mu$ distribution includes other distributions as special cases. The Hoyt (Nakagami-q) distribution can be obtained by setting $(1 - \eta)/(1 + \eta) = q^2$ and $\mu = (1/2)$, the Nakagami-m distribution is obtained when $\eta = 1$ and $\mu = m$, the Rayleigh is realized when $\eta = 0$ and $\mu = (1/2)$, and the one-sided Gaussian distribution is obtained by setting $\eta = 1$ and $\mu = (1/4)$.

The $N_r \times N_t$ $\eta-\mu$ MIMO channel is obtained as

$$h_{zv} = r\exp(j\theta), \quad \forall z = 1:N_r \ \& \ v = 1:N_t \tag{6}$$

where h_{zv} is the $\eta-\mu$ channel complex fading gain between receive antenna z and transmit antenna v, and r is the envelope defined as

$$r^2 = x^2 + y^2 \tag{7}$$

where $x^2 = \sum_{i=1}^{2\mu} x_i^2$, and $y^2 = \sum_{i=1}^{2\mu} y_i^2$, with x_i and y_i being mutually Gaussian variates with $\mathbb{E}(x_i) = \mathbb{E}(y_i) = 0$, $\mathbb{E}(x_i^2) = \mathbb{E}(y_i^2) = \sigma_h^2$, and $\eta = \mathbb{E}(x_i y_i)/\sigma_h^2$. The phase θ is then obtained as

$$\theta = \arctan\left(\frac{y}{x}\right). \tag{8}$$

The impact of varying the physical parameters η and μ on the $\eta-\mu$ channel is shown in Fig. 2. It is important to notice the similar behavior of the channel with either increasing μ or decreasing η. As such, the impact of these changes on the SM/SSK systems' performance should be similar. Increasing the value of μ to 1.5 instead of 0.5 clearly increases the peaks of the joint PDF, which increases the similarities between different channel paths associated with different transmit antennas. Thus, correlation between different channel paths increases, which should degrade the SM/SSK performance. Recall also that the $\eta-\mu$ channel includes the Nakagami-m channel as a special case where $m = \mu$. As such, increasing the value of μ increases the number of multipath clusters and creates a highly correlated channel. As $m \to \infty$, the Nakagami channel becomes Gaussian, and MIMO communication would not be possible since it would be impossible to resolve different channel paths. Similarly, decreasing η increases the peaks of the joint PDF and the spatial correlation. Note that η represents the correlation coefficient between the scattered in-phase and quadrature components of each cluster of multipath. Thereby, as η increases, the correlation between in-phase and quadrature components of each cluster increases, but the spatial correlation among different channel paths associated with different antennas decreases, which should enhance SM/SSK performance. Nonetheless, the impact of decreasing η on the performance of SM/SSK systems is shown later to be less severe compared with increasing μ. These observations will be highlighted in detail in the results section and will be further discussed then.
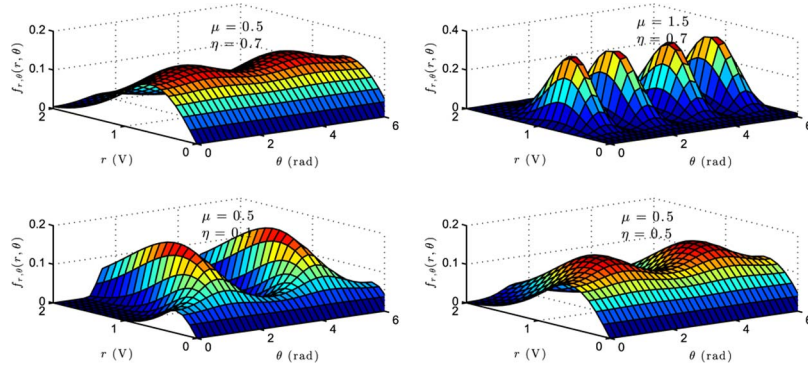
Fig. 2. Impact of the physical parameters η and μ on the $\eta-\mu$ channel for different values of η and fixed μ.

III. PERFORMANCE ANALYSIS

The average BER of an SM system can be calculated using the union-bound technique [3], [16], [31], [32] given by

$$P_{\text{ABER}} \leq \frac{1}{2^k k} \sum_{\jmath=1}^{N_t} \sum_{\imath=1}^{M} \sum_{\hat{\jmath}=\jmath+1}^{N_t} \sum_{\hat{\imath}=\imath+1}^{M} N_b \text{PEP}(\mathbf{x}_{\jmath,\imath} \rightarrow \mathbf{x}_{\hat{\jmath},\hat{\imath}}) \quad (9)$$

where $\text{PEP}(\mathbf{x}_{\jmath,\imath} \rightarrow \mathbf{x}_{\hat{\jmath},\hat{\imath}})$ represents the PEP of deciding on $\mathbf{x}_{\hat{\jmath},\hat{\imath}}$ given that $\mathbf{x}_{\jmath,\imath}$ is transmitted, with $\jmath, \hat{\jmath} = 1, 2, \ldots, N_t$ and $\imath, \hat{\imath} = 1, 2, \ldots, M$, and N_b is the number of bit errors associated with the corresponding PEP event. From (4), the general PEP of the SM system can be written as

$$\begin{aligned} \text{PEP}(\mathbf{x}_{\jmath,\imath} \rightarrow \mathbf{x}_{\hat{\jmath},\hat{\imath}}|\mathbf{H}) &= Q\left(\sqrt{\varphi\|\mathbf{H}\Psi\|_F^2}\right) \\ &= \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{\varphi\|\mathbf{H}\Psi\|_F^2}{2\sin^2\theta}\right) d\theta \quad (10) \end{aligned}$$

where $\varphi = (1/(2(\sigma_e^2\|x_{\jmath,\imath}\|^2 + N_0)))$, and Ψ for the SM-MIMO system is given by $\Psi = \mathbf{x}_{\jmath,\imath} - \mathbf{x}_{\hat{\jmath},\hat{\imath}}$, whereas for the SSK-MIMO system, $\Psi = \mathbf{x}_{\jmath} - \mathbf{x}_{\hat{\jmath}}$.

Taking the expectation of (10) yields

$$\text{PEP}(\mathbf{x}_{\jmath,\imath} \rightarrow \mathbf{x}_{\hat{\jmath},\hat{\imath}}) = \frac{1}{\pi} \int_0^{\pi/2} \mathcal{M}\left(-\frac{\varphi}{2\sin^2\theta}\right) d\theta \quad (11)$$

where $\mathcal{M}(\cdot)$ is the MGF of the random variable $\|\mathbf{H}\Psi\|_F^2$.

In the following, a novel approach to express the argument of the MGF in (11) is given, which is different than the existing analysis in literature for SM/SSK systems in the presence of channel estimation errors, and will be shown to lead to a general expression for the PEP. A similar approach was used to derive the performance of the STC system in correlated channels in [33] and to derive the PEP of a transmit diversity SM system over correlated Rayleigh fading channel in [19]. An alternative approach using Gil-Pelaez inversion theorem was considered in [20].

The MGF arguments in (11) can be written as [34]

$$\begin{aligned} \|\mathbf{H}\Psi\|^2 &= \text{tr}(\mathbf{H}\Psi\Psi^H\mathbf{H}^H) \\ &= \text{vec}(\mathbf{H}^H)^H \left(\mathbf{I}_{N_r} \otimes \Psi\Psi^H\right) \text{vec}(\mathbf{H}^H) \quad (12) \end{aligned}$$

where $\mathbf{I}_{N_r}$ is an $N_r \times N_r$ identity matrix, $\text{tr}(\cdot)$ is the trace operation, and $\text{vec}(\cdot)$ is the vectorization operation.
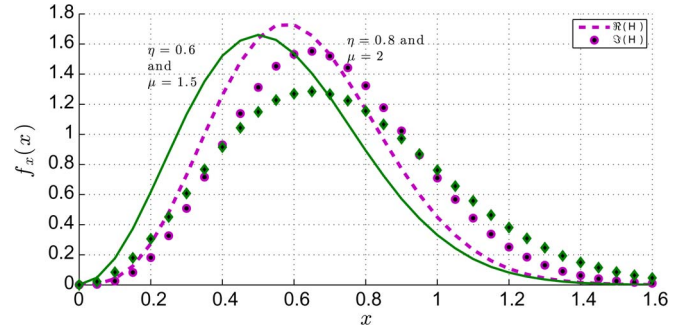

Fig. 3. Numerical estimation of the PDFs of the real and imaginary parts of the complex $\eta-\mu$ channel.

Let $\mathbf{Q}$ be a Hermitian matrix, and let $\mathbf{u}$ be a complex random variable with real and imaginary parts of its components being normally distributed and having equal mean and variance. It is assumed in this paper that the real and imaginary parts of the $\eta-\mu$ fading channel are normally distributed random variables. In the $\eta-\mu$ channel and as shown in (7), x_i and y_i are Gaussian random variables, whereas $x^2 = \sum_{i=1}^{2\mu} x_i^2$ and $y^2 = \sum_{i=1}^{2\mu} y_i^2$ are chi-squared distribution random variables with 2μ degrees of freedom. The squared envelope of the channel is the summation of these two random variables. To obtain the real and imaginary parts of the $\eta-\mu$ channel, (6) should be used. As clearly shown, a proof that the real and imaginary parts of the random channel follow a Gaussian distribution is very complicated and beyond the scope of this paper. However, a large number of complex $\eta-\mu$ channel entries are generated in MATLAB for different η and μ values, and the PDFs of the real and the imaginary parts of the complex channel are numerically computed and shown in Fig. 3. Reported results validate the assumption that the real and imaginary parts of the $\eta-\mu$ fading channel follow an approximate Gaussian distribution. In addition, it is shown in the next section that the analytical and simulation results demonstrate a close match for a wide range of SNR values, which corroborate the validity of this assumption.

Let the mean matrix of $\mathbf{u}$ be $\bar{\mathbf{u}}$, and let the covariance be $\mathbf{C}$. Then, from [35], for any Hermitian matrix $\mathbf{Q}$, the MGF of $\mathbf{u}^H\mathbf{Q}\mathbf{u}$ is

$$\mathcal{M}(s) = \frac{\exp\left(s\bar{\mathbf{u}}^H\mathbf{Q}(\mathbf{I} - s\mathbf{C}\mathbf{Q})^{-1}\bar{\mathbf{u}}\right)}{|\mathbf{I} - s\mathbf{C}\mathbf{Q}|} \quad (13)$$

where $\mathbf{I}$ denotes the identity matrix with proper dimensions.

Using (12) and (13), the MGF in (11) can be written as

$$\mathcal{M}(s) = \frac{1}{|\mathbf{I}_{N_t N_r} - s\mathbf{C}\Delta|}$$
$$\times \exp\left[s \times \text{vec}(\bar{\mathbf{H}}^H)^H \Delta \left(\mathbf{I}_{N_t N_r} - s\mathbf{C}\Delta\right)^{-1} \text{vec}(\bar{\mathbf{H}}^H)\right]$$
(14)

where $\Delta = \Psi\Psi^H$.

Now, plugging (14) into (11) yields

$$\text{PEP}(\mathbf{x}_\ell \to \mathbf{x}_{\hat{\ell}}) = \frac{1}{\pi} \int_0^{\pi/2} \frac{1}{\left|\mathbf{I}_{N_t N_r} + \frac{\varphi}{2\sin^2\theta}\mathbf{C}\Delta\right|} \exp\left[-\frac{\varphi}{2\sin^2\theta}\right.$$
$$\left. \text{vec}(\bar{\mathbf{H}}^H)^H \Delta \left(\mathbf{I}_{N_t N_r} + \frac{\varphi}{2\sin^2\theta}\mathbf{C}\Delta\right)^{-1} \text{vec}(\bar{\mathbf{H}}^H)\right] d\theta.$$
(15)

Accordingly, the PEP of SM/SSK-MIMO systems is given by

$$\text{PEP}(\mathbf{x}_\ell \to \mathbf{x}_{\hat{\ell}}) \leq \frac{1}{2} \frac{1}{\left|\mathbf{I}_{N_t N_r} + \frac{\varphi}{2}\mathbf{C}\Delta\right|}$$
$$\times \exp\left[-\frac{\varphi}{2}\text{vec}(\bar{\mathbf{H}}^H)^H \Delta \left(\mathbf{I}_{N_t N_r} + \frac{\varphi}{2}\mathbf{C}\Delta\right)^{-1} \text{vec}(\bar{\mathbf{H}}^H)\right].$$
(16)

Note that (16) is a general expression for the PEP of SM/SSK-MIMO systems in the presence of imperfect channel knowledge at the receiver.

Substituting (16) into (9) and using the Chernoff bound [31], an expression for the average BER in SM-MIMO systems operating over the $\eta-\mu$ fading channel is found as

$$P_{\text{ABER}}^{\text{SM}} \leq \frac{2^{-k}}{k} \sum_{j=1}^{N_t} \sum_{i=1}^{M} \sum_{\hat{j}=j+1}^{N_t} \sum_{\hat{i}=i+1}^{M} N_b \times \frac{1}{2} \frac{1}{\left|\mathbf{I}_{N_t N_r} + \frac{\varphi}{2}\mathbf{C}\Delta\right|}$$
$$\times \exp\left[-\frac{\varphi}{2}\text{vec}(\overline{\mathbf{H}}^H)^H \Delta \left(\mathbf{I}_{N_t N_r} + \frac{\varphi}{2}\mathbf{C}\Delta\right)^{-1} \text{vec}(\overline{\mathbf{H}}^H)\right]$$
(17)

which reduces to

$$P_{\text{ABER}}^{\text{SSK}} \leq \frac{2^{-k}}{k} \sum_{j=1}^{N_t} \sum_{\hat{j}=j+1}^{N_t} N_b \frac{1}{2} \frac{1}{\left|\mathbf{I}_{N_t N_r} + \frac{\varphi}{2}\mathbf{C}\Delta\right|}$$
$$\times \exp\left[-\frac{\varphi}{2}\text{vec}(\overline{\mathbf{H}}^H)^H \Delta \left(\mathbf{I}_{N_t N_r} + \frac{\varphi}{2}\mathbf{C}\Delta\right)^{-1} \text{vec}(\overline{\mathbf{H}}^H)\right]$$
(18)

for SSK systems.

However, the mean matrix $\bar{\mathbf{H}}$ and the covariance matrix $\mathbf{C}$ for the $\eta-\mu$ fading channel envelope must be given to evaluate the average BER in (17) and (18). From [26] and considering format 2 $\eta-\mu$ channel, the mean and variance can be obtained as

$$\mathbb{E}\{\mathbf{H}\} = \frac{(1-\eta^2)^{\mu+\frac{1}{2}}\Gamma\left(2\mu + \frac{1}{2}\right)}{\sqrt{2\mu}\Gamma(2\mu)}$$
$$\times {}_2F_1\left(\mu + \frac{3}{4}, \mu + \frac{1}{4}; \mu + \frac{1}{2}; \eta^2\right)$$
(19)

where ${}_2F_1(\cdot)$ is the Gauss hypergeometric function [36].

The variance is given by

$$\text{Var}\{\mathbf{H}\} = \mathbb{E}\{\mathbf{H}^2\} - (\mathbb{E}\{\mathbf{H}\})^2$$
(20)

where $\mathbb{E}\{\mathbf{H}^2\}$ is given by

$$\mathbb{E}\{\mathbf{H}^2\} = \frac{(1-\eta^2)^{\mu+1}\Gamma(2\mu+1)}{2\mu\Gamma(2\mu)}$$
$$\times {}_2F_1\left(\mu + 1, \mu + \frac{1}{2}; \mu + \frac{1}{2}; \eta^2\right).$$
(21)

Using (19) and (20), the mean matrix $\bar{\mathbf{H}}$ and the covariance matrix $\mathbf{C}$ can be given as

$$\bar{\mathbf{H}} = \mathbb{E}\{\mathbf{H}\} \times \mathbf{1}_{N_r \times N_t}$$
(22)

$$\mathbf{C} = \text{Var}\{\mathbf{H}\} \times \mathbf{I}_{N_r N_t}$$
(23)

where $\mathbf{1}_{N_r \times N_t}$ is an $N_r \times N_t$ all-ones matrix.

A. Asymptotic Analysis

In order to clearly show the system diversity gain and the effect of system parameters on the overall system performance, an asymptotic expression for the PEP is obtained in what follows. For high SNR (i.e., $\rho \gg 1$) and following similar steps as described in [33], (15) can be written as

$$\text{PEP}^{\text{Asym}}(\mathbf{x}_{j,\imath} \to \mathbf{x}_{\hat{j},\hat{\imath}}) \leq \frac{1}{2}\left(\frac{\varphi}{2}\Delta\text{Var}\{\mathbf{H}\}\right)^{-N_r}$$
$$\cdot \prod_{i=1}^{\delta} \frac{1}{\alpha_i}\left[\exp\left(-\frac{1}{\text{Var}\{\mathbf{H}\}}(\mathbb{E}\{\mathbf{H}\})^2\right)\right]^{N_r}$$
(24)

where $\delta = \text{rank}(\Psi\Psi^H)$, and α_i is the nonzero eigenvalue of $\Psi\Psi^H = (\mathbf{x}_{j,\imath} - \mathbf{x}_{\hat{j},\hat{\imath}})(\mathbf{x}_{j,\imath} - \mathbf{x}_{\hat{j},\hat{\imath}})^H$.

Note that $\delta = 1$ for the SM/SSK system and is zero when $\mathbf{x}_{\hat{j},\hat{\imath}} = \mathbf{x}_{j,\imath}$. In addition, the first eigenvalue of $\Psi\Psi^H$ is always zero for the SM/SSK system, i.e., $\alpha_1 = 0$, which eliminates the multiplication term. Hence, the asymptotic PEP in (24) reduces to

$$\text{PEP}^{\text{Asym}}(\mathbf{x}_{j,\imath} \to \mathbf{x}_{\hat{j},\hat{\imath}}) \leq \frac{1}{2}\left(\frac{\varphi}{2}\Delta\text{Var}\{\mathbf{H}\}\right)^{-N_r}.$$
(25)

It is evident from (25) that a diversity order of N_r is achieved for the SM/SSK system over general $\eta-\mu$ fading channel in the presence of imperfect channel knowledge.

IV. NUMERICAL AND SIMULATIONS RESULTS

In the analysis, a 4×4 MIMO system is considered, and 4-QAM modulation is used for SM systems. The channel is assumed to be static for each frame, and a frame length of 500 symbols is considered. Orthogonal Hadamard pilot symbols are inserted at the beginning of each frame for channel estimation purposes. The pilot sequences, each of length N_t symbols, are simultaneously transmitted from the multiple transmit antennas, and a least square algorithm is considered
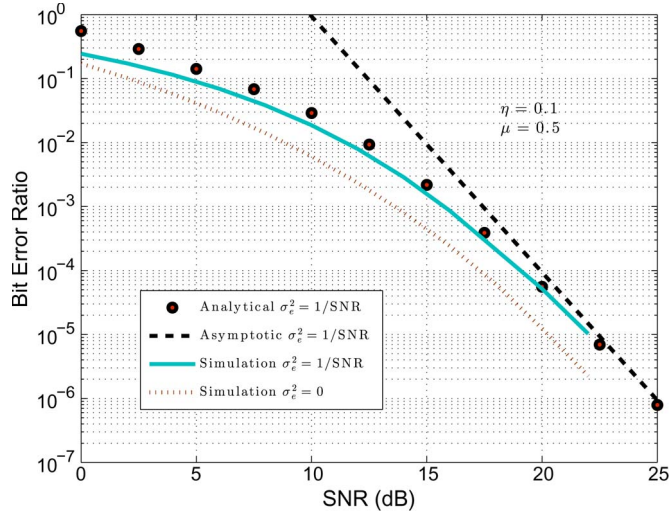
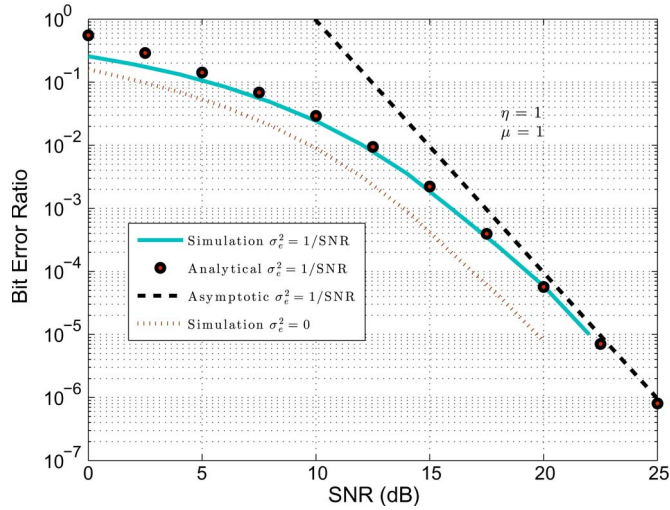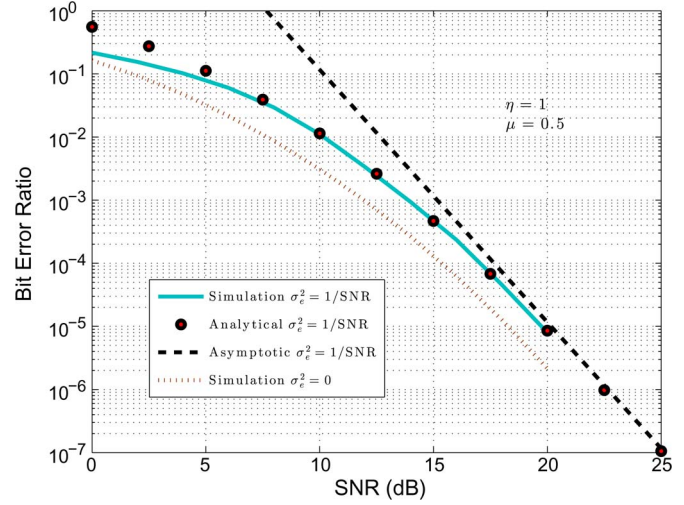Fig. 4.   Average BER performance of the SM system versus SNR for $\mu = 0.5$ and $\eta = 0.1$.

Fig. 6.   Average BER performance of an SM system versus SNR for $\mu = 0.5$ and $\eta = 1$.

Fig. 5.   Average BER performance of an SM system versus SNR for $\mu = 1$ and $\eta = 1$.

Fig. 7.   Average BER performance of an SM system versus SNR for $\mu = 0.5$ and $\eta = 0.8$.

at the receiver to estimate the channel. The variance of the Gaussian estimation error decreases as the SNR of the data symbols increases (the pilot symbols have the same energy as the data symbols), i.e., $\sigma_e^2 = (\text{SNR})^{-1}$ [18], [32]. For Monte Carlo simulation results, at least 10^6 bits are transmitted for each considered SNR value, and the number of transmitted bits increases up to 10^7 at high SNR values. Performance analyses for different values of η and μ are presented.

In Figs. 4–9, the performance of the SM system for different $\eta-\mu$ combination values is shown. In Fig. 4, analytical, simulation, and asymptotic results are shown for $\eta = 0.1$ and $\mu = 0.5$. Analytical results demonstrate close match to simulation results for a wide range of SNR values. Meanwhile, asymptotic results follow the slope of simulation results at high and pragmatic SNR values. Note that the analytical bound is a bit loose at low SNR but tightens fast as the SNR increases. SM performance, assuming perfect channel knowledge $(\sigma_e^2 = 0)$, is also depicted to show the impact of channel estimation error on the perfor-

mance of the SM system. As noticed in the figure, channel estimation error causes about 2-dB degradation in SNR. Similar behavior can be noticed in Figs. 5–7 for different combinations of $\eta-\mu$ values. The results validate the derived analysis and highlight the impact of channel estimation error on the SM system performance.
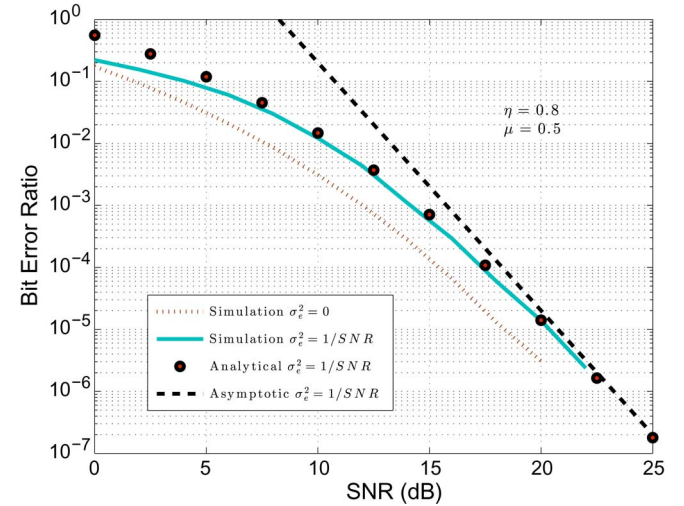
In Fig. 8, the impact of changing the values of the μ parameter for fixed $\eta = 0.4$ on the performance of the SM system is studied. Recall that the Nakagami-m channel is realized when $\eta = 1$ and $\mu = m$. As such, increasing the value of μ should result in similar performance behavior as increasing the value of m for the Nakagami-m fading channel. As previously discussed, the Nakagami-m distribution becomes Gaussian as $m \to \infty$, and the channel paths will be irresolvable unless power imbalance is considered among the transmit antennas, which makes MIMO communication impossible. Thus, SM performance degradation is expected for higher μ values, as clearly noticed in Fig. 8. Increasing μ from 0.5 to 1 degrades
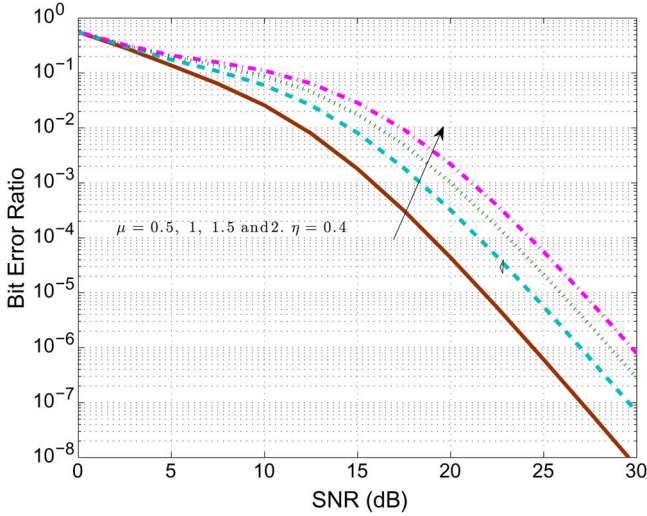
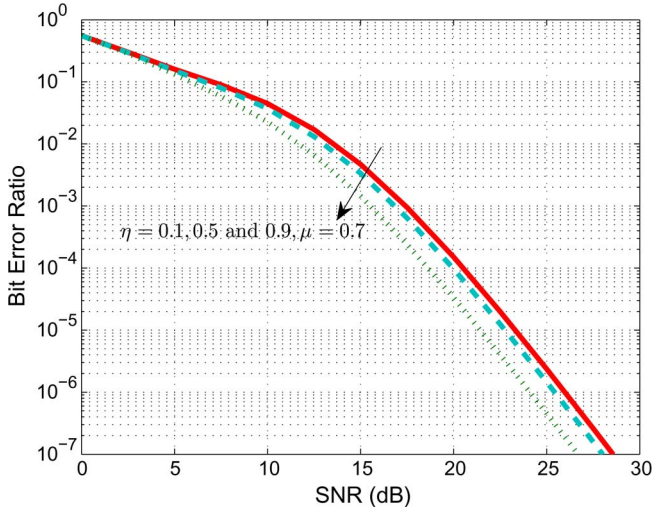Fig. 8. Impact of changing the μ parameter on the performance of the SM system for a fixed value of $\eta = 0.4$.

Fig. 9. Impact of changing the η parameter on the performance of the SM system for a fixed value of $\mu = 0.7$.

Fig. 10. Average BER performance of the 4×4 MIMO SSK system versus SNR for different values of μ and $\eta = 1$.

Fig. 11. Average BER performance of the SSK 4×4 MIMO system versus SNR for $\eta = 0.6$ and $\mu = 0.5$.

SM performance by about 2 dB, whereas 5-dB degradation can be noticed for $\mu = 2$.

Fig. 9 shows the impact of varying η for a fixed value of $\mu = 0.7$. Increasing η slightly enhances the performance of an SM system as anticipated from Fig. 2 previously. SM performance enhances by about 1 dB when increasing η from 0.1 to 0.9. Lower values of η increase the spatial correlation between different channel paths and degrade the performance of the SM system.

Finally, SSK results are given in Figs. 10 and 11. Again, analytical, simulation, and asymptotic results match for a wide range of SNR values. The impact of changing μ for $\eta = 1$ is shown in Fig. 10. As noticed in SM, higher μ values degrade the performance of SSK system. In addition, the curve for $\sigma_e^2 = 0$ is shown in the figure as a reference. Channel estimation errors degrade SSK performance by about 1 dB. The performance of the SSK system for $\eta = 0.6$ and $\mu = 0.5$ is shown in Fig. 11. This figure highlights the exactness of the analysis for noninteger η values of the SSK system.
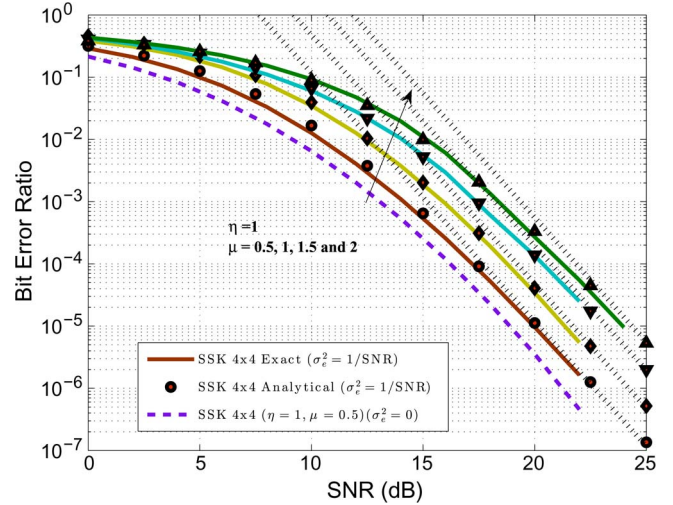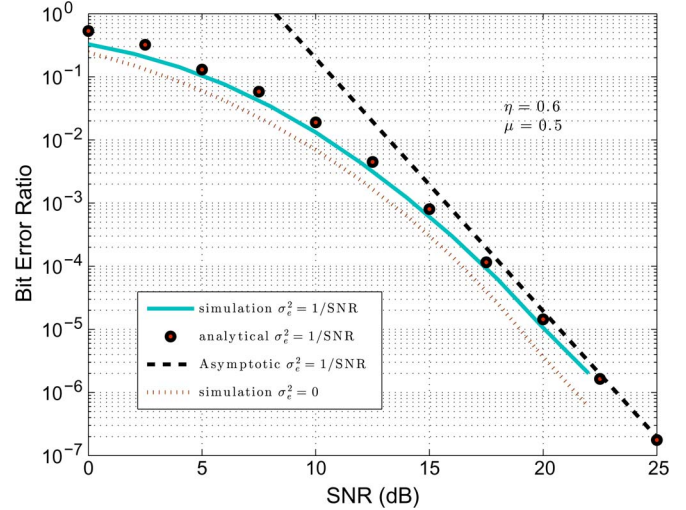
V. CONCLUSION

A novel performance analysis of SM/SSK MIMO systems over generalized $\eta - \mu$ fading channel in the presence of Gaussian imperfect channel estimation has been presented in this paper. The analyses consider the exact channel model and the fact that the phase is not uniform. A general closed-form expression for the PEP is derived, and an approximate expression for high SNR is also given. The derived expressions are general and can be readily used for various well-known fading channels such as Nakagami-m and Hoyt distributions. In addition, the derived analysis can be easily used to evaluate SM/SSK performance in the idealistic condition where full channel knowledge is assumed at the receiver by setting $\sigma_e^2 = 0$. In addition, the effect of varying η and μ fading parameters on the overall system performance is investigated. It is shown that, unlike the already reported results in literature, increasing μ value increases the spatial correlation among different channel paths associated with different transmit antennas, which significantly degrades system performance. However, system performance is slightly enhanced by increasing the value of η.

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