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Citation for published version:
Perry, MK & Sakovics, J 2004 'Auctions for Split-Award Contracts' ESE Discussion Papers, Edinburgh School of Economics, University of Edinburgh.

Link:
Link to publication record in Edinburgh Research Explorer

Document Version:
Peer reviewed version

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Auctions for Split-Award Contracts

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First Draft: June 1999
Revision: December 2001

Abstract

The buyer of a homogeneous input employs split-award contracting to divide his input requirements into two contracts that are awarded to different suppliers. The buyer uses a sequential second-price auction to award a larger primary contract and a smaller secondary contract. With a fixed number of suppliers participating in the auctions, we find that the buyer pays a higher expected price than with a sole-source auction. The premium paid to the winner of the secondary contract must also be paid to the winner of the primary contract as an opportunity cost of not winning the secondary contract. With fixed costs of participating in the auction, we identify the conditions under which a secondary contract can increase the number of suppliers and lower the expected price paid by the buyer. An optimal secondary contract can internalize the cost reductions from the new industry capacity and extract the rents of the suppliers. An optimal secondary contract can be particularly beneficial when the number of suppliers is limited by high fixed costs.

This paper examines split-award contracting in which a buyer divides the purchases of its input requirements into two contracts that are awarded to two different suppliers in two separate auctions. Split-award contracts are a common procurement practice for inputs and components used to produce final goods.[1] For example, automobile manufacturers employ multiple suppliers for various components of automobiles.[2] Similarly, firms in the beverage industries contract with multiple suppliers for cans and bottles. In addition, corporate or government buyers often employ auction mechanisms to award contracts for the inputs or components. Split-award auctions have also been employed by governments to privatize industries. In some cases, the governments have divided the markets and auctioned franchises to different suppliers.[3] Split-award contracting raises two interesting questions. First, what are the implications for competition among the suppliers? Second, what are the costs and potential benefits for the buyer? In order to address these questions, this paper proceeds in three parts.

In the first part of the analysis, we characterize the equilibrium expected price that a buyer would pay if he employed sequential second-price auctions to award the primary and secondary contracts to purchase his input requirements. In the second part of the analysis, we examine how the expected price is affected by the relative size of the primary and secondary contracts, the number of suppliers participating in the auctions, and the cost distribution of the suppliers. In the third part of the analysis, we examine how an optimal secondary contract can induce entry by a new supplier and result in a lower expected price paid by the buyer.

We consider a buyer with requirements for a homogeneous and divisible input, and three or more potential suppliers of this input. The buyer can hold a sole-source second-price auction for his total input requirements. Alternatively, the buyer can divide his input requirements into two contracts, a larger primary contract and a smaller secondary contract, and then hold a separate auction for each. Prior to the auctions, the suppliers obtain independent private realizations on their

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[1] Corey (1978) discusses some cases of split-award contracts in a variety of industries. See chapters 1-3. Woodside and Vyas (1987) examine the purchasing strategies of six firms for eighteen industrial products. The firms employed split-award contracting for eight of the products. Chapter 9 (pages 177-79) summarizes the reasons for the split awards.

[2] Smitka (1991) discusses subcontracting in the Japanese automobile industry. See also McMillan (1990) and Richardson (1993).

[3] See McAfee and McMillan (1994) and Cramton (1997) for discussions of the FCC spectrum auctions.

cost of producing the inputs from a known common distribution. Assuming that the buyer awards the contracts to two different suppliers, we derive the equilibrium bidding functions for second-price sequential auctions of the two contracts. The lowest cost supplier will win the larger primary contract, and the second lowest cost supplier will win the smaller secondary contract. However, the equilibrium bid at any cost realization is higher for both the primary and the secondary contracts than the bid for a corresponding sole-source contract. As a result, the buyer pays a premium using a split-award auction in that the expected price for the input requirements is higher than with a sole-source second-price auction.

The exact nature of this premium provides some insights into the buyer's costs of using a split-award auction. Obviously, the premium includes the expected profit of the supplier winning the secondary contract. However, the premium is twice this expected profit because the supplier of the primary contract must also receive the expected profit on the secondary contract. This ensures that the lowest cost supplier prefers to win the primary contract instead of the secondary contract. In other words, the expected profit on the secondary contract becomes an opportunity cost for any supplier with the lowest cost and is incorporated into the bids on the primary contract.

One of the common arguments for split-award contracting is that the buyer can induce a larger number of suppliers to compete for his input requirements.[4] This can occur in our model because split-award auctions can increase the expected profitability of suppliers. With an optimal secondary contract, the buyer may be able to simultaneously induce entry by an additional supplier and lower the expected price. Even with a premium over a sole-source auction, competition from the additional supplier can generate an expected price lower than a sole-source auction without the additional supplier. Two effects account for this finding. The first effect arises from the fact that an additional supplier reduces the expected industry costs of producing the input requirements. The second effect arises from the fact that a sole-source auction will not eliminate the expected rents of the existing suppliers. The additional supplier would allow the buyer to internalize the cost reduction and extract the rents of the existing suppliers, receiving both in the form of a lower expected price

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[4] This paper does not address several other arguments cited for the benefits of split-award contracting. First, split-award contracting may ensure the existence of an alternative supplier for the input requirements of the buyer in the event that one supplier is unable to fulfill his contract in a timely fashion. Second, split-award contracting may enable the buyer to discipline the suppliers in the non-price dimensions of performance by adjusting the

for the input requirements. We show that this benefit from a split-award auction is more pronounced when there are a small number of existing suppliers.

In Section 1, we briefly review the literature on second-sourcing, focusing primarily on the papers dealing with split-award contracting. In Section 2, we define the model. In Section 3, we characterize the equilibrium bidding functions for the auctions of the primary and secondary contracts. In Section 4, we show that expected price is higher with a split-award auction and examine the behavior of this premium for different parameters of the model. In Section 5, we examine the expected profits of the suppliers. We introduce fixed costs of participating in the auctions and demonstrate the circumstances under which a split-award auction can increase the number of suppliers. In Section 6, we define the optimal secondary contract and show how it can reduce the expected price for a wide range of participation costs. We then provide an illustration of the results in Section 7 and the intuition in Section 8. Finally, we offer some concluding remarks in Section 9.

1.   Related Literature

Split-award contracting is one form of second-sourcing. Second-sourcing has been used to describe procurement practices in which a buyer authorizes an increase in the number of suppliers in order to increase competition and reduce the price or cost.[5] Second-sourcing has also been used to describe procurement practices in which the buyer contracts with an initial supplier in one period but then switches to a second supplier in a subsequent period in order to limit the monopoly rents of the initial supplier.[6] Split-award contracting differs from these other two forms of second-sourcing

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purchases between them or by creating some form of yardstick competition. Third, split-award contracting may increase the number of suppliers by reducing the risk associated with participating in the auctions.

[5] In Rob (1986), a buyer finds it optimal to limit the size of an initial production contract to the developer of an input and auction the remaining input requirements among other suppliers. In Dasgupta and Spulber (1989/90) and Auriol and Laffont (1993), second-sourcing reduces the costs of purchasing the input because the suppliers have rising marginal costs of production. In Dana and Spier (1994) and McGuire and Riordan (1995), a regulator takes the place of the buyer and the benefit of second-sourcing is more competition and lower prices for consumers. See also Dick (1992).

[6] In Anton and Yao (1987) the option of switching to a second source allows the buyer to limit the informational rents of the incumbent on the initial production contract. In Laffont and Tirole (1988), second-sourcing of a production contract reduces the informational rents for the developer of the input, but undermines his investment incentives. In Riordan and Sappington (1989), the buyer's option to use a second source for production of the input can reduce the informational rents for the developer of the input and increase the probability that production will occur. See also Marshall, Meurer, and Richard (1994).

because the buyer divides his simultaneous input purchases and directly or indirectly limits competition among the potential suppliers.

The theoretical literature on split-award contracting originates with the work by Wilson (1979) and Bernheim and Whinston (1986) on "share" or "menu" auctions. Anton and Yao (1989 and 1992) build on these papers to examine split-award contracting when suppliers compete in a menu auction for all or part of the buyer's input requirements. Two suppliers submit bids for the whole contract and fractions of the contract. The buyer then selects the split that minimizes his costs of purchasing the input requirements. Sole-sourcing occurs when the buyer chooses one or the other supplier to produce the total input requirements.

In their 1989 paper, Anton and Yao consider the case in which the suppliers know each other's costs. The equilibrium price of a sole-source auction would be the cost of production for the higher cost supplier, and the lower cost supplier would win the auction earning rents equal to his cost advantage over the higher cost supplier. With diseconomies of scale in production (economies from splitting production), Anton and Yao then show that split awards reduce the total production costs, but increase the equilibrium menu bids and generate additional rents for the suppliers. In particular, both suppliers increase their sole-source bids to reflect the full cost reduction from split production. The buyer chooses the cost-minimizing split of production, but pays a total price equal to the cost of the higher cost supplier, plus the cost reduction from splitting production (relative to the cost of the lower cost sole-source supplier). Thus, the suppliers receive one rent from internalizing the cost reduction and a second rent from the higher price that is also equal to the cost reduction.

The buyer's use of a menu auction allows the suppliers to offer the efficient split of the production contracts. However, the menu auction also allows the suppliers to collude tacitly and raise the price to the buyer. The anomaly is that menu auction increases the price paid by the buyer precisely in the situation where splitting the production would lower costs. The buyer has effectively eliminated competition between the two suppliers by his inability to commit to a sole-source auction.[7]

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[7] Anton and Yao (1992) generalize their 1989 paper to incorporate incomplete information about costs. The suppliers have private signals about their costs and there is a parameter C which measures diseconomies of scale (C<1/2). They examine equilibria in which 50/50 split awards arise for all realizations of costs. Again, there must be diseconomies of scale, but the low cost realization provides an upper bound on the highest price that could

Our model differs substantially from the models of Anton and Yao. The buyer explicitly decides on the split of input production between the primary and secondary contract. The technology of production is constant returns so that the marginal cost is constant for each supplier. Each contract is auctioned separately and competition is preserved by assuming that there are more than two suppliers in the market. The expected price paid by the buyer increases with a split-award auction, but not because of any collusive effects or cost reductions from split production. Instead, the expected profit on the secondary contract becomes an opportunity cost for the suppliers in bidding on the primary contract. Thus, the suppliers bid less aggressively on both contracts and earn the additional expected profit from the secondary contract on the primary contract as well as the secondary contract.

The most closely related paper is Seshadri, Chatterjee, and Lilien (1991). In this paper, a buyer simultaneously auctions equal shares of his input requirements to a chosen number of suppliers. The buyer is not allowed to discriminate in the contract prices so the price for each contract is set at the highest accepted bid (as in a first-price auction). The authors make entry endogenous by assuming that each potential supplier draws an opportunity cost of participating in the auction from a common distribution. Since the expected price for each contract increases when the buyer increases the number of contracts awarded, the buyer can stimulate entry by awarding more contracts. The number of contracts that minimizes the expected price paid by the buyer depends on the manner in which the distribution of opportunity costs among the potential suppliers shifts as the number of entrants changes.[8]

Our model addresses the same tradeoff using a different specification for the split-award decision, an alternative model of the auctions, and a simpler version of the entry costs. In our model, the buyer chooses the relative size of two contracts, rather than the number of equal-size contracts. Indeed, we find that equal-size split-award contracts are not optimal. The two contracts are

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arise from an equilibrium split award. Depending on the cost distribution, this price may be higher or lower than the expected price from a sole-source auction. But again, the split award with two suppliers provides a mechanism for tacit collusion.

[8] Gilbert and Klemperer (2000) have a model of rationing with an interpretation for split awards. The buyer of an input wants to ensure that two suppliers (one high cost and the other low cost) each make an investment which will generate a positive probability of success, where success enables them to produce the input. For certain probabilities of success, the high cost supplier can be induced to make the investment at a lower expected price if the production is split when both suppliers are successful. In this model, the buyer retains the bargaining power over the suppliers and the split award weakens the participation constraint for the high cost supplier.

auctioned separately, and there is no constraint imposed by the buyer on the relationship between the awarded prices. In particular, we find that the expected price paid on the smaller secondary contract is greater than on the larger primary contract. Finally, we assume that the fixed costs of participating in the auctions are known to suppliers and to the buyer. In this way, we can fully illustrate the conditions under which a split-award auction would increase the number of suppliers and lower the expected price paid by the buyer.

2.   The Model of a Split-Award Auction

The model is a three-stage game. In the first stage, the buyer commits to a split between a larger primary contract and a smaller secondary contract. In addition, the buyer commits to a sequential second-price auction for these two contracts in which the primary contract is auctioned first and the winner of the primary contract cannot also win the subsequent auction for the secondary contract. In the second stage, potential suppliers decide simultaneously and independently whether to incur a known fixed cost of participating in the auctions. The suppliers only know the common distribution of production costs for themselves and the other suppliers, so their decision is based on the expected profits from participating in the auctions. This stage will be discussed in Sections 5 - 8. In the third stage, the suppliers that entered in the second stage decide what to bid for the contracts. In making their bids, they know their cost which is realized independently from the common cost distribution and they know the number of other suppliers participating in the auctions with costs drawn from the same common cost distribution. This stage will be examined in Sections 2 - 4 for a given number of suppliers. The distinguishing feature of the model is the buyer's choice of the size of the primary and secondary contracts in the first stage, taking into account how it will affect competition and profits in the third stage and thus entry in the second stage.

The buyer has a value v for the total input requirements. We assume that the value v is sufficiently larger than the highest possible cost realization of the suppliers, so that the buyer would not use a reserve price to reject bids within the range of feasible costs. The input is homogeneous and divisible so that it can be purchased from more than one supplier without any loss of productive efficiency for the buyer. The buyer awards two contracts, a primary contract for the majority of the input requirements and a secondary contract for the remainder. Let $a \geq 1/2$ be the fraction of the input requirements awarded to one supplier as the primary contract, and $b = 1 - a \leq 1/2$ be the

remaining fraction awarded to a second supplier as the secondary contract. We assume that the buyer can commit to award only one contract to a given supplier. Thus, the buyer cannot announce a split-award auction and then award both contracts to the lowest cost supplier. This commitment is the essence of split-award contracting because otherwise the model would be equivalent to a sole-source auction. In addition, we assume that the lowest cost supplier who wins the primary contract cannot re-contract with the winner of the secondary contract to produce the inputs for the secondary contract.[9]

When there is a secondary contract, we assume that there are $n > 2$, risk neutral suppliers competing to provide the input requirements of the buyer. If there were only two suppliers, the buyer would clearly use a sole-source auction to preserve competition to supply his input requirements. So when there is no secondary contract, we can allow $n = 2$. We also assume that there are constant returns in the production of the input.[10] Thus, if c is the cost of producing the entire input requirements for a supplier, then the cost of supplying the a-contract is $a \cdot c$, and the b-contract is $b \cdot c$.

The buyer awards the contracts to the potential suppliers using a competitive bidding procedure. Specifically, we assume that the buyer holds two sealed-bid second-price auctions for the two contracts in sequence, with the auction for the larger a-contract held first. For simplicity, we assume that the bids on the a-contract are not observed by the suppliers prior to submitting their bids for the b-contract. This sequential second-price auction need not be the optimal mechanism.[11] However, it is equivalent to other standard auction procedures, in terms of both the expected payments by the buyer and the award of the contracts to the suppliers. In particular, any mechanism yields the same expected outcome as this sequential second-price auction as long as all the suppliers are risk neutral, their costs are independent, the a and b contracts are won respectively by the

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[9] This assumption ensures that split-award contracting actually occurs as intended by the buyer. Presumably, the buyer would realize that one supplier was producing all the inputs, and would have some recourse. On a technical level, the expected outcome of a negotiation between the lowest cost supplier and the second lowest cost supplier over the price of supplying the inputs for the secondary contract would alter the bidding strategies for the auctions in relatively complex ways.

[10] If the marginal cost of production was increasing in the quantity produced, there would be an inherent technological reason for splitting the input requirements into smaller contracts. This explanation for split awards has already been explored by Dasgupta and Spulber (1989/90) and Auriol and Laffont (1993).

lowest and the second lowest cost suppliers, and a supplier with the highest possible cost earns no profit.[12] In particular, this sequential second-price auction is equivalent to a simultaneous, non-discriminatory unit price auction, where the potential suppliers bid unit prices, the lowest bidder obtains the larger *a*-contract, the second lowest bidder obtains the smaller *b*-contract, and both receive the unit price bid by the third lowest bidder. Similarly, our results would apply if we employed sequential first-price auctions.[13]

We assume that neither the buyer nor the other suppliers know the cost of any potential supplier prior to either auction. We also assume that the suppliers are symmetric in that each supplier draws his cost of producing the entire input requirements from the same distribution *G(c)* having a normalized support of [0,1]. Each supplier learns his cost prior to submitting a bid to the buyer, but need not incur this cost unless he wins one of the two auctions. Finally, we assume that the cost of each supplier is independently distributed from the costs of the other suppliers. Thus, the auctions are held in the setting of symmetric independent private values for the costs of the suppliers.

Our basic results on the equilibrium bidding functions will be valid for the general distribution function *G(c)* on the costs of suppliers. However, we also define a convenient one-parameter family of distribution functions *G(c;t)* in order to examine some comparative statics results over different distributions within this family. For $t > 0$, define $G(c;t) = 1 - [1 - H(c)]^t$, where *H(c)* is some given distribution function over the range [0,1]. The parameter *t* can be interpreted as the number of independent draws from the cost distribution *H(c)* that each supplier receives in order to obtain his lowest cost of producing the input requirements. For most of the comparative static results and examples, we will assume that *H(c)* is uniform so that *H(c) = c*.

The parameter *t* in *G(c;t)* shifts the probability of different cost realizations. If $t > 1$, then $G(c;t) > G(c;1)$ $^\circ$ *H(c)* and the probability of obtaining a cost lower than *c* is larger than under *H(c)*. Similarly, if $t < 1$, then $G(c;t) < G(c;1)$ and the probability of obtaining a cost lower than *c* is

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[11] Within the class of *ex post* efficient mechanisms, optimality is guaranteed if the informational rents of the buyers are extracted. This will happen if the buyer chooses the optimal secondary contract (Section 6).

[12] See Engelbrecht-Wiggans (1988) for the appropriate revenue equivalence theorem.

[13] On the other hand, the expected payments would be affected if the order of the two auctions were switched, i.e., *b* > *a* Since the lowest cost supplier may not win the primary contract, this change would increase the productive inefficiency, reduce the expected profits of the suppliers, and increase the expected price paid by the buyer.

smaller. Alternatively, consider the density function $g(c;t) = t \cdot [1 - H(c)]^{t-1} \cdot h(c)$, where $h(c)$ is the density function corresponding to $H(c)$. When t increases, the probability of lower cost realizations is higher, whereas the probability of higher cost realizations is lower. In other words, a distribution with a higher t stochastically dominates (in the first-order sense) a distribution with a lower t. For convenience, we will refer to t as the capacity of each supplier and refer to $T(n) = t \cdot n$ as the industry capacity. With the family $G(c;t)$, the distribution function for the lowest cost among all the suppliers is $G(c;T(n)) = 1 - [1 - H(c)]^{T(n)}$. [14] Thus, a larger capacity t or a larger number of suppliers n will lower the expected costs for both contracts. This will be important when we examine the impact of a split-award auction on entry and the expected price paid by the buyer.

3. The Bidding Strategies of Suppliers

The auction for the primary **a**-contract occurs before the auction for the secondary **b**-contract. For this reason, the bid of a supplier on the **a**-contract depends on the expected profit from winning the **b**-contract. The auction for the **b**-contract is a second-price auction among the $(n-1)$ suppliers who did not win the **a**-contract. Recall that, in a sole-source second-price auction, the dominant strategy for each supplier is to bid the true cost c. Thus, for a supplier with cost c, the probability of winning the auction for the **b**-contract is $[1 - G(c)]^{n-2}$. In equilibrium, the expected profit from the **b**-contract for a supplier with cost c, conditional on winning the **b**-contract, can be expressed as

$$(1) \qquad \boldsymbol{p_b}(c; \boldsymbol{a}, n) = (1 - \boldsymbol{a}) \cdot \int_c^1 (\boldsymbol{x} - c) \cdot (n - 2) \cdot \frac{[1 - G(\boldsymbol{x})]^{n-3}}{[1 - G(c)]^{n-2}} \cdot g(\boldsymbol{x}) \cdot d\boldsymbol{x} \ .$$

The normalized profit margin is $(\boldsymbol{x} - c)$, where $\boldsymbol{x}$ is the cost (and the equilibrium bid) of the second lowest cost supplier from among the $(n-1)$ suppliers bidding on the **b**-contract. As such, $\boldsymbol{x}$ is the equilibrium price in the second-price auction. The other terms in the integral are the density of $\boldsymbol{x}$,

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[14] The specification of $G(c;t)$ is not as special as it appears. Waehrer and Perry (2001) have shown that $G(c)$ must take the form $G(c;t)$ for some t and $H(c)$ under very reasonable assumptions. The key assumption is that the industry exhibits a form of constant returns to scale in that the distribution of the lowest cost draw among the suppliers (the first order statistic) depends on $T(n)$ only and not on n and t separately.

where x is equivalently defined as the lowest cost from among the other (n-2) suppliers bidding on the b-contract, conditional on c being the lowest cost of the supplier who wins the b-contract. The resulting expected profit must then be multiplied by $b = 1 - a$, which is the fraction of the input requirements purchased by the buyer under the b-contract.

Observe that p_b $(c;a,n)$ is the opportunity cost of winning the a-contract. This conditional expected profit from the b-contract is the profit that a supplier would forego by winning the a-contract. By the monotonicity of the equilibrium bidding strategy, the supplier who wins the a-contract would surely win the b-contract if it chose not to bid on the a-contract. In a sole-source auction (b=0), there would be no such opportunity cost, and thus no cost of bidding more aggressively on the a-contract. As such, a split-award auction should result in a higher equilibrium price on the a-contract.

Now consider the expected profit from the a-contract. Let $b(c)$ be the normalized equilibrium bidding function on the a-contract. In order to solve for $b(c)$, consider the optimal bid of one supplier when the other suppliers are using the equilibrium bidding function based on their realized costs. This supplier may bid $b(x)$ as if its cost x were above the true cost c. This reduces the probability of winning the a-contract, but ensures a higher profit margin after winning. The expected profit from the a-contract can be expressed as

(2)
$$p_a(x,c;a,n) \;=\; a \cdot \int_{x}^{1} \left(b(j) - c \right) \cdot (n-1) \cdot \left[1 - G(j) \right]^{n-2} \cdot g(j) \cdot dj \; .$$

The normalized profit margin is $b(j)$ - $c,$ where $b(j)$ is the equilibrium bid of the lowest cost supplier from among the other (n-1) suppliers bidding on the α-contract, and thus the equilibrium price in the second-price auction. The integral is evaluated from a lower bound of x because this supplier wins the a-contract whenever its bid $b(x)$ is below the bid $b(j)$ of the lowest cost supplier from among the other (n-1) suppliers. The remaining terms in the integral are the density of the lowest cost j from among the other (n-1) suppliers. Finally, the resulting expected profit must be multiplied by a, which is the fraction of the input requirements purchased by the buyer under the a-contract.

We can now express the total expected profit of a supplier in terms of its reported cost $x \geq c$:

(3) $$\boldsymbol{p}(x,c;\boldsymbol{a}n) = \boldsymbol{p_a}(x,c;\boldsymbol{a}n) + (n\text{-}1)\cdot G(x)\cdot[1-G(c)]^{n-2}\cdot\boldsymbol{p_b}(c;\boldsymbol{a}n).$$

The total expected profit is the expected profit from the $\boldsymbol{a}$-contract defined by (2), plus the expected profit from the $\boldsymbol{b}$-contract. The expected profit from the $\boldsymbol{b}$-contract is the conditional expected profit defined by (1), times the probability that the supplier will actually win the $\boldsymbol{b}$-contract. The supplier will win the $\boldsymbol{b}$-contract when (i) one of the other suppliers bids below $b(x)$ for the $\boldsymbol{a}$-contract and thus has a cost below x, and (ii) the remaining $(n\text{-}2)$ suppliers have costs above c (and x), thereby losing both contracts. The term $(n\text{-}1)$ is simply the number of combinations that (i) and (ii) can occur.

We can now state the following proposition about the equilibrium bidding function for the $\boldsymbol{a}$-contract.

Proposition 1: *The equilibrium bidding function for the $\boldsymbol{a}$-contract is*

(4a) $$b(c;\boldsymbol{a}n) \quad = \; c \; + \; [1/\boldsymbol{a}]\cdot\boldsymbol{p_b}(c;\boldsymbol{a}n)$$

(4b) $$= \; c \; + \; [(1\text{-}\boldsymbol{a})/\boldsymbol{a}]\cdot\int_c^1\left(\frac{1-G(\boldsymbol{q})}{1-G(c)}\right)^{n-2}\cdot d\boldsymbol{q}\;.$$

Proof: In a symmetric equilibrium with monotonic bidding strategies, it must be that a supplier with cost c does not benefit from reporting a cost different from c and thus making a bid different from $b(c)$, given that the rest of the suppliers are bidding according to the monotone bidding function $b(c)$. Consequently, the equilibrium condition is

$$\left. \frac{d\boldsymbol{p}(x,c;\boldsymbol{a},n)}{dx} \right|_{x=c} = 0 \ . \ [15]$$

Solving this equation for *b(c)*, we directly obtain (4a). The expression (4b) follows from integration by parts. QED

Proposition 1 states that the only symmetric and increasing equilibrium bidding strategy for a supplier in the first auction for the **a**-contract is to bid its true cost, where cost now includes both the actual production cost and the opportunity cost of losing the **b**-contract.[16] As such, the bidding strategy is a natural generalization of a sole-source second-price auction.

As assumed, the equilibrium bidding function *b(c)* is increasing with *c*. This is true despite the fact that the expected profit from the **b**-contract, conditional on winning, is decreasing with higher costs. The effect of a higher cost in supplying the **a**-contract dominates this effect from a lower opportunity cost of losing the **b**-contract.

The equilibrium bidding function shifts downward when the number of suppliers *n* is increased. With more competition for the **b**-contract, the expected profit from the **b**-contract is lower, even conditional on winning. In addition, the equilibrium bidding function shifts downward when the size of the **a**-contract is increased. Thus conversely, the equilibrium bids on the primary contract increase when the size of the secondary contract is increased.

The equilibrium bidding functions can be easily calculated for the one-parameter distribution function *G(c;t)* defined in the previous section. If we also assume that *H(c)* is uniform on the range [0,1], then the equilibrium bidding function simplifies to

(4c) $b(c;\boldsymbol{a},n,t) \ = \ c \ + \ [(1-\boldsymbol{a})/\boldsymbol{a}] \cdot \{(1-c) \, / \, [t(n-2)+1]\} \ .$

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[15] In principle, this is only a necessary condition. However, it can be shown that it is also sufficient. For example, see McAfee and Vincent (1993).

[16] Note that this strategy is not a dominant strategy. The term [1/**a**] is simply a normalization because the reported costs are the costs of producing the full input requirements. Thus, the bid on the primary contract can be expressed as $\boldsymbol{a}b(c;\boldsymbol{a},n) = \boldsymbol{a}c + \boldsymbol{p}_b$.

The bidding function clearly shifts downward as either the number of suppliers n or the capacity t of each supplier increases. When n is larger, more suppliers are drawing their costs from the same cost distribution. When t is larger, the same number of suppliers are drawing their costs from more favorable cost distributions. In either case, the probability of any one supplier winning either contract is lower for any given cost c. Thus, each supplier will bid more aggressively, and the equilibrium bids must decline.

4.   The Expected Price Paid by the Buyer

We can now define the expected prices (per unit of the input) that the buyer pays for the a-contract, the b-contract, and the total input requirements. Denote $c_1(n)$, $c_2(n)$, $c_3(n)$ as the expected values of the first, second, and third lowest cost from the n suppliers. Clearly, $c_1(n) < c_2(n) < c_3(n)$. The supplier with the lowest cost will win the a-contract, and the supplier with the second lowest cost will win the b-contract. The expected price paid by the buyer for the b-contract is simply the expected value of the third lowest cost $c_3(n)$. The expected price paid by the buyer for the a-contract is the expected value of the equilibrium bidding function in (4a) integrated over the distribution of the second lowest cost from among the n suppliers. The expected price paid by the buyer for his total input requirements is the weighted average of the expected prices on the two contracts. These expected prices can be expressed as follows:

(5a) $EP_b(n) \ = \ c_3(n)$,

(5b) $EP_a(a,n) \ = \ \int_0^1 b(c; a, n) \cdot n \cdot (n\text{-}1) \cdot G(c) \cdot [1 - G(c)]^{\,n\text{-}2} \cdot g(c) \times dc$,

(5c) $EP(a,n) \ = \ a \cdot EP_a(a,n) \ + \ (1\text{-}a) \cdot EP_b(n)$.

Substituting in the equilibrium bidding function and integrating by parts yields the following proposition.

Proposition 2: *The expected price for the total input requirements is given by*

(6a) $\qquad EP(\boldsymbol{a},n) \qquad = \boldsymbol{a} \cdot \{c_2(n) + [(1-\boldsymbol{a})/\boldsymbol{a}] \cdot [c_3(n) - c_2(n)]\} + (1-\boldsymbol{a}) \times c_3(n)$,

(6b) $\qquad\qquad\qquad\qquad = (2\boldsymbol{a} - 1) \cdot c_2(n) + 2 \cdot (1-\boldsymbol{a}) \cdot c_3(n)$,

(6c) $\qquad\qquad\qquad\qquad = c_2(n) + 2 \cdot (1-\boldsymbol{a}) \cdot [c_3(n) - c_2(n)]$.

Expression (6a) makes it clear that the expected price paid on the primary contract is less than the expected price paid on the secondary contract for $\boldsymbol{a} > 1/2$. The reason for the difference in expected prices is that the $\boldsymbol{a}$-contract is larger than the $\boldsymbol{b}$-contract. As a result, the opportunity cost $[c_3(n) - c_2(n)]$ decreases as it is re-normalized by $[(1-\boldsymbol{a})/\boldsymbol{a}] < 1$.[17] Expression (6b) makes it clear that the expected price is a linear combination of $c_2(n)$ and the expected price $c_3(n)$ on the secondary contract. Expression (6c) makes it clear that the expected price is linearly decreasing in $\boldsymbol{a}$ from $EP(1/2,n) = c_3(n)$ to $EP(1,n) = c_2(n)$.[18]

Consider now the difference between the expected price with a split-award auction and a sole-source auction defined as $\Delta(\boldsymbol{a},n) = EP(\boldsymbol{a},n) - EP(1,n)$. The function $\Delta(\boldsymbol{a},n)$ is the expected premium that the buyer must pay when he chooses to employ a split-award auction. This premium follows directly from expression (6c) in Proposition 2:

(7) $\qquad \Delta(\boldsymbol{a},n) = 2 \cdot (1-\boldsymbol{a}) \cdot [c_3(n) - c_2(n)]$.

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[17] The fact that the contracts are not awarded simultaneously at a uniform price is only a necessary condition for this difference in the expected prices. If the two contracts were identical, as in Seshadri, Chatterjee, and Lilien (1991), the expected prices would also be equal in our sequential auctions.

[18] Burguet and Sakovics (1997) examine a model of sequential auctions in which bidders are uncertain at the time of the first auction whether the second auction will occur. Under these circumstances, the selling price of identical goods auctioned sequentially decreases over time. These results are analogous to the increasing prices in this sequential procurement auction. Thus, the expected price for the total input requirements is lower when the secondary contract is smaller. If the buyer could induce the suppliers to undervalue the secondary contract as $\gamma < 1-\alpha$, then it is easy to show that the expected price becomes $EP(\boldsymbol{g}) = c_2(n) + (1-\boldsymbol{a}+\gamma) \cdot [c_3(n) - c_2(n)]$. As γ declines, the expected price also declines.

This premium has a natural interpretation. When the buyer creates the $\boldsymbol{b}$-contract, the second lowest cost supplier will win the contract at a price equal to the third lowest cost, with an expected value of $c_3(n)$. Since the expected price in a sole-source auction would have been the expected value $c_2(n)$ of the second lowest cost, the expected premium from the $\boldsymbol{b}$-contract covering $(1-\boldsymbol{a})$ of the buyer's input requirements is simply $(1-\boldsymbol{a}) \cdot [c_3(n) - c_2(n)]$. This half of the total premium is the expected profit for the supplier who is awarded the $\boldsymbol{b}$-contract. However, the total premium that the buyer must pay is twice this amount. The reason is that the expected profit on the $\boldsymbol{b}$-contract is an opportunity cost of winning the $\boldsymbol{a}$-contract. Thus, the bid of the second lowest cost supplier for the $\boldsymbol{a}$-contract must also be higher to reflect this expected profit on the $\boldsymbol{b}$-contract. In the second-price auction for the $\boldsymbol{a}$-contract, the bid of the second lowest cost supplier determines the price at which the contract is awarded. Thus, the premium that the buyer must pay in a split-award auction is twice the expected profit of the supplier winning the $\boldsymbol{b}$-contract.[19]

The behavior of the premium with respect to the number of suppliers n and the capacity t can be investigated using the distribution function $G(c;t)$ with $H(c)$ uniform on $[0,1]$. In this case, the bidding function $b(c)$ from (4c) is linear in c, so that the expected price paid by the buyer in the auction for the $\boldsymbol{a}$-contract is simply the equilibrium bid evaluated at the expected value of the second lowest cost from all n suppliers. Let $c_1(n,t)$, $c_2(n,t)$ and $c_3(n,t)$ denote the expected value of the first, second and third lowest costs from the distribution $G(c;t)$ with $H(c)$ uniform on $[0,1]$. The derivation of these expressions is contained in Appendix 1. All three expressions are decreasing in both the number of suppliers and the capacity of each supplier. When $t = 1$, $G(c;t)$ is uniform on

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[19] The intuition for the premium in our model has some similarities, as well as some differences, to the tacit collusion that arises in the model of Anton and Yao (1989). In the model of Anton and Yao, the diseconomies of scale make the split-award contracts more profitable for the suppliers. This generates an opportunity cost of winning a sole-source contract. The resulting higher bids on the sole-source contract then feedback as an opportunity cost on winning a split-award contract. In equilibrium, the price paid by the buyer incorporates the higher cost of sole-source production, plus the economies from split production relative to the lower cost of sole-source production. Thus, a similarity between the two models is that the bids are higher because the profit on a split-award contract becomes an opportunity cost. In the model of Anton and Yao, this profit is an opportunity cost for a sole-source contract; whereas, in our model, this profit is an opportunity cost for the primary contract. But there are also some other differences. In the model of Anton and Yao, a split-award contract does not create an opportunity cost unless there are diseconomies of scale. With constant returns or economies of scale, the equilibrium would result is a sole-source contract and a Bertrand price. In our model, the buyer selects the split of his input requirements between the primary and secondary contracts, and is not forced to choose among the menu bids of the suppliers. The opportunity cost then arises from the assumption that the buyer selects different suppliers for the two contracts. As a result, there are fewer suppliers competing for the secondary contract.

[0,1] and these expressions collapse to the familiar formulas: $c_i(n,1) = i/(n+1)$. Substituting these expressions into (7), the premium in a split-award auction can now be written as

$$(8) \qquad \Delta(\boldsymbol{a},n,t) = 2 \cdot (1-\boldsymbol{a}) \cdot \{n \cdot (n-1) \cdot t^2\} \, / \, \{[tn+1] \cdot [t(n-1)+1] \cdot [t(n-2)+1]\} \ .$$

Corollary 2.1 in Appendix 2 examines the behavior of this premium. Although there are no general results, the premium is declining in both n and t when $t \geq 1$. Thus, if the cost distribution is uniform or more favorable to low cost realizations, a split-award auction is less costly to the buyer when the number of suppliers is larger or when the capacity of each supplier is greater.

5.   The Expected Profit of Suppliers and Entry

In this section, we investigate how a split-award auction affects the *ex ante* expected profits of the suppliers. We then introduce a fixed cost of participating in the auctions and characterize the smallest secondary contract which can increase the number of suppliers participating in the auctions.

The expected profit of a supplier can be obtained by integrating expression (3) over c with $x = c$. But more intuitively, the expected profit is equal to the expected price minus the expected cost for the total input requirements of the buyer, divided by the number of suppliers. The expected cost of producing the total input requirements of the buyer is

$$(9) \qquad EC(\boldsymbol{a},n) = \boldsymbol{a} \cdot c_1(n) + (1-\boldsymbol{a}) \cdot c_2(n) = c_1(n) + (1-\boldsymbol{a}) \cdot [c_2(n) - c_1(n)] \, .$$

The second term of (9) is the expected inefficiency in production that arises from awarding a secondary contract to the second lowest cost supplier. Using (6c), we can now express the expected profit of a supplier as

$$(10) \qquad E\boldsymbol{p}(\boldsymbol{a},n) = n^{-1} \times \{\boldsymbol{a} \cdot [c_2(n) - c_1(n)] + 2 \cdot (1-\boldsymbol{a}) \cdot [c_3(n) - c_2(n)]\}.$$

The first term is the informational rent earned on the α-contract. The second term is the premium paid by the buyer equal to twice the expected profit from the $\boldsymbol{b}$-contract. Differentiating (10) with respect to $\boldsymbol{a}$, we obtain the following proposition.

Proposition 3: *The expected profit of each supplier is increasing with the size of the secondary contract if and only if*

$$(11) \qquad\qquad 2 \cdot [\, c_3(n) - c_2(n) \,] \; > \; c_2(n) - c_1(n) \; .$$

The left-hand side of (11) is the increase in the expected price with higher $\boldsymbol{b}$ (see (6c)), and the right-hand side of (11) is the increase in the expected cost with higher $\boldsymbol{b}$ (see (9)). In other words, the expected profits of the suppliers will increase with the size of the secondary contract when the expected premium is larger than the expected efficiency loss from awarding the secondary contract to the second lowest cost supplier. Whether condition (11) is satisfied or not depends on the cost distribution. If we restrict attention to the family of distributions $G(c,t)$ with $H(c)$ uniform on $[0,1]$, condition (11) can be greatly simplified. Thus, we can state the following corollary to Proposition 3.

Corollary 3.1: *Assume the costs are drawn from $G(c;t)$ with $H(c)$ uniform on $[0,1]$. The expected profit of each supplier is increasing with the size of the secondary contract if and only if the industry capacity is greater than unity, $T(n) = t{\times}n > 1$ (or $t > 1/n$).*

Proposition 3 and Corollary 3.1 demonstrate that a split-award auction need not increase the expected profits of the suppliers, even though it always increases the expected price (see Proposition 2). The reason is that a split-award auction also increases the expected cost of producing the input requirements. Instead of awarding the total input production to the lowest cost supplier, a split-award auction allows part of the production to be performed by the second lowest cost supplier. The expected price increases with a larger secondary contract, but a larger fraction of the input requirements is awarded to the second lowest cost supplier. For $T(n) < 1$, this increase in expected cost is greater than the increase in expected price, and thus the expected profit declines with higher $\boldsymbol{b}$. When $T(n) < 1$, the distribution of the lowest cost realization for the industry is skewed toward higher values of cost relative to the uniform distribution. Therefore, because of the smaller probability of low cost realizations, the difference between the first and second order statistics grows larger than the difference between the second and the third order statistics.

Having characterized the expected profit of suppliers, we now examine how a split-award auction can induce entry or prevent the exit of a supplier. Since there is always a positive expected

profit from having capacity t and participating in the auctions, we now assume that there is a fixed cost f of participating in the auctions with the capacity t. This fixed cost must be incurred by the suppliers prior to the auctions, as well as prior to their cost realizations for producing the input requirements. In addition, the fixed cost is independent of the actual input production that may or may not result from the contract awards. Thus, f is simply an investment in learning about the true cost of production.[20] Given this fixed cost, the size of the secondary contract can affect the number of suppliers that can profitably participate in the auction.[21]

By Corollary 3.1, if the industry capacity is less than unity, a split-award contract cannot induce entry and the buyer will always use a sole-source auction. On the other hand, if the industry capacity exceeds unity, the buyer may be able to use a split-award auction to induce the entry of a new supplier or prevent the exit of an existing supplier. Consider the case in which n suppliers would find it profitable to participate in a sole-source auction, but $(n+1)$ suppliers would not. Define $F_{n,t}$ as the set of fixed costs which generate exactly n suppliers with capacity t:

(12) $$F_{n,t} = (\ E\boldsymbol{p}(1,n+1,t)\ ,\ E\boldsymbol{p}(1,n,t)\ )\ .^{[22]}$$

Since $E\boldsymbol{p}(\boldsymbol{a},n+1,t)$ is decreasing in $\boldsymbol{a}$ when $T(n+1) > 1$, the buyer can offer the maximum possible expected profits to the suppliers when he awards two equal-size contracts ($\boldsymbol{a} = 1/2$). Thus, we can characterize the conditions for the existence of a secondary contract that can induce entry.

Corollary 3.2: *Assume the costs are drawn from G(c;t) with H(c) uniform on* [0,1] *and that* $T(n+1) = t\times(n+1) > 1$. *For f* $\widehat{\boldsymbol{I}}$ $F_{n,t}$ *such that f* $\boldsymbol{£}$ $E\boldsymbol{p}(1/2,n+1,t)$, *there exists a minimal secondary contract* $\boldsymbol{b}(f)$ *which can induce entry by the (n+1)st supplier.*

<u>Proof:</u> The condition $t\times(n+1) > 1$ ensures that the expected profit is increasing with $\boldsymbol{b}$ when there are $(n+1)$ suppliers. If $E\boldsymbol{p}(1,n,t) < E\boldsymbol{p}(1/2,n+1,t)$, there is a minimal secondary contract for all f $\widehat{\boldsymbol{I}}$

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[20] Alternatively, the fixed costs could also be interpreted as opportunity costs of reserving capacity to produce the inputs if the supplier wins one of the contracts.

[21] In this section, we assume again that the buyer can commit to the size of the secondary contract. The buyer would obviously benefit from switching to a sole-source auction after suppliers incurred their entry costs believing there would be a secondary contract. Such a commitment could be enforced in a repeated game with punishment strategies. Doing so in this model would be relatively cumbersome because the gain from a split-award auction depends on the fixed cost of the additional supplier who would be induced to enter.

[22] The expected profit of a supplier are strictly decreasing in n: $dE\boldsymbol{p}(\boldsymbol{a},n,t)/dn < 0$ for $n > 2$ and all $\boldsymbol{a}$ and t. Thus, the set $[E\boldsymbol{p}(1,n+1,t)\ ,\ E\boldsymbol{p}(1,n,t)]$ is clearly non-empty for any given $n > 2$.

$F_{n,t}$. However, if $E\boldsymbol{p}(1/2,n+1,t) < E\boldsymbol{p}(1,n,t)$ and $E\boldsymbol{p}(1/2,n+1,t) < f < E\boldsymbol{p}(1,n,t)$, there is no $\boldsymbol{b} \leq$ 1/2 which can increase the expected price sufficiently above $E\boldsymbol{P}(1,n+1,t)$ in order to make the $(n+1)$st supplier profitable. The buyer can induce entry by the $(n+1)$st supplier only if $f \leq E\boldsymbol{p}(1/2,n+1,t)$. The minimal secondary contract is $\boldsymbol{b}(f) = \boldsymbol{1} - \boldsymbol{a}(f)$ where $\boldsymbol{a}(f)$ is implicitly defined by $f = E\boldsymbol{p}(\boldsymbol{a},n+1,t)$. QED

Corollary 3.2 demonstrates that a split-award auction can induce entry for some fixed costs in $F_{n,t}$, but not necessarily all. We find that $E\boldsymbol{p}(1/2,n+1,t) < E\boldsymbol{p}(1,n,t)$ when $n < 3 + 1/t$. Thus, there exist interesting cases for which no secondary contract can induce the $(n+1)$st supplier to enter. For example, this can occur for $n \in \{2,3\}$ when $t \geq 1$; for $n \in \{2,3,4\}$ when $1/2 \leq t < 1$; and for $n \in \{2,3,4,5\}$ when $1/3 \leq t < 1/2$. However, for smaller $f \in F_{n,t}$ such that $f \leq E\boldsymbol{p}(1/2,n+1,t)$, a minimal secondary contract would induce entry. When a minimal secondary contract exists, the buyer would certainly not offer a larger secondary contract that would increase the expected price. But not all minimal secondary contracts will benefit the buyer. The next section examines the conditions under which the minimal secondary contract can lower the expected price.

6.   An Optimal Secondary Contract Can Reduce the Expected Price

For a fixed number of suppliers n and an industry capacity greater than unity, a split-award auction increases the expected profits of the suppliers and introduces an inefficiency from using the second lowest cost supplier for the secondary contract. However, if a split-award auction induces entry by the $(n+1)$st supplier, there is new capacity in the industry and new competition in the auction. The new capacity lowers the expected production costs of the input requirements, and the new competition can convert this cost reduction into a lower expected price paid by the buyer. In addition, the new competition can enable the buyer to extract rents from the existing suppliers, also in the form of a lower expected price. We can now state the key proposition characterizing when a secondary contract can lower the expected price.

Proposition 4: *Assume the costs are drawn from G(c;t) with H(c) uniform on* [0,1] *and that T(n+1) = t×(n+1) > 1. There exists a non-empty subset of $F_{n,t}$, denoted by (f_L,f_H), such that for all f $\in$ (f_L,f_H), an optimal secondary contract $\boldsymbol{b}^*(f) > 0$ induces entry by the $(n+1)$st supplier and lowers the expected price. The boundaries of this subset are defined by $f_L =$*

Ep(1,n+1,t) and f$_H$ = Ep(n/(n+1),n+1,t), and the optimal secondary contract is the minimal secondary contract: b*(f) = b¢(f). For f $\widehat{I}$ F$_{n,t}$ and f > f$_H$, b*(f) = 0.

<u>Proof:</u> We first define the smallest primary contract a(n) which would lower the expected price. The expected price EP(a,n,t) is decreasing in n and linearly decreasing in a (linearly increasing in b). One can show that EP(1,n+1,t) < EP(1,n,t) < EP(1/2,n+1,t). As a result, there exists an a(n)$\widehat{I}$ (1/2,1), implicitly defined by EP(a,n+1,t) = EP(1,n,t), such that for all a > a(n), EP(a,n+1,t) < EP(1,n,t). Interestingly, a(n) does not depend on t, and we can show that a(n) = n/(n+1) > 1/2.

We can now define the largest fixed cost f$_H$ which would allow entry of the (n+1)st supplier and reduce the expected price. Since a(n) is the smallest primary contract which lowers the expected price after entry, f$_H$ = Ep(a(n),n+1,t) < Ep(1/2,n+1,t). Recall that for f ³ Ep(1/2,n+1,t), a split-award auction cannot induce entry of the (n+1)st supplier. For f$_H$ < f < Ep(1/2,n+1,t), a split-award auction could induce entry by the (n+1)st supplier, but the expected price would be higher than a sole-source auction with n suppliers. However, for any f$_L$ < f < f$_H$, there exists a set of primary contracts (a(n),a¢(f)] which would induce entry of the (n+1)st supplier and lower the expected price.

We can now define the optimal primary and secondary contracts. Since the expected price decreases strictly with higher a for any given number of suppliers, the expected price is minimized at the largest a (smallest b) which induces the (n+1)st supplier to enter. Thus, the optimal primary contract a*(f) is implicitly defined by Ep(a*,n+1,t) = f and the optimal secondary contract is the corresponding minimal secondary contract b*(f) = b¢(f).

Finally, we need to show that a larger secondary contract cannot further reduce the expected price by inducing the (n+2)nd supplier to enter. This part of the proof is contained in Appendix 3, but the intuition is clear. The optimal secondary contract b*(f) extracts all the rents from the (n+1) suppliers. Thus, any further reduction in the expected price requires a reduction in the expected costs from (9). However, a larger secondary contract increases the productive inefficiency because a greater fraction of the input requirements are awarded to the supplier with the

second lowest cost. This greater inefficiency dominates the reduction in expected costs from having the capacity of an additional supplier. QED

Figures 2(a) and 2(b) illustrates Proposition 4. In Figure 2(a), we have drawn the expected prices $EP(a,n,t)$ and $EP(a,n+1,t)$. Both are decreasing functions of a, and the latter is uniformly lower with the additional supplier. Thus, $a(n)$ is the primary contract for which the expected price $EP(a(n),n+1,t)$ is equal to the expected price of a sole-source auction with n suppliers, $EP(1,n,t) = c_2(n)$. If $a > a(n)$, then the expected price would be lower with the $(n+1)$st supplier. Figure 2(b) illustrates the region of fixed costs (f_L,f_H) for which an optimal secondary contract can generate entry and a lower expected price. We have drawn the expected profit of the suppliers $Ep(a,n+1,t)$ which is a decreasing function of a. Thus, for an $f \; \hat{I} \; (f_L,f_H)$, the $a^*(f) = a(f)$ is illustrated in Figures 2(a) and 2(b) respectively as the largest a which would induce entry by the $(n+1)$st supplier. Thus, $b^*(f) = 1 - a^*(f)$ is the optimal secondary contract. Figure 2(a) illustrates that the resulting expected price $EP(a^*(f),n+1,t)$ is below the sole-source price $EP(1,n,t) = c_2(n)$ with n suppliers.

The set of fixed costs (f_L,f_H) with an optimal secondary contract generating $(n+1)$ suppliers clearly depends the number of suppliers n, but it also depends on the capacity t of each supplier. We can measure the magnitude of this set relative to $F_{n,t}$. In particular, $[(f_H - f_L) / (Ep(1,n,t) - f_L)] = 1/2 - 1/[2t(n+1)]$. When the industry capacity $t \times (n+1) = 1$, this set is empty because a secondary contract cannot increase expected profits. As $t \times (n+1) \rightarrow \infty$ with an increase in t (or n), the subset (f_L,f_H) approaches 50% of the set $F_{n,t}$, and does so rapidly.[23]

An equal split of the input requirements ($a = b = 1/2$) cannot be optimal. Even if an equal split can induce entry by the $(n+1)$st supplier (see Corollary 3.2), it is easy to show that the expected price will be higher: $c_3(n+1) > c_2(n)$.[24] The upper bound on the optimal secondary contract is $b(n) = 1 - a(n) = 1/(n+1) \leq 1/3$, and this occurs at $f = f_H$.

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[23] When $(n+1) = 3$, (f_L,f_H) is one-third of the range of $F_{n,t}$ for $t=1$. But for $(n+1) = 5$, (f_L,f_H) is 40% of $F_{n,t}$ for $t=1$. Thus, the set (f_L,f_H) is small only when t is substantially less that unity.

[24] Seshadri, Chatterjee, and Lilien (1991) assume that the contracts have an equal share of the input requirements. In our model, two equal-size contracts would have to generate two or more entrants in order to lower the expected price. Even if the expected price were lower, this would not be optimal for the reasons discussed in Appendix 3. Equal-size contracts generate the highest expected price, $c_3(n)$, and the largest

7.   An Illustration of the Optimal Secondary Contract

The benefit of an optimal secondary contract can be illustrated using the uniform cost distribution $G(c,1)$ where $t = 1$. For this case, $f_L = [(n+1)\cdot(n+2)]^{-1}$. With $a(n) = n/(n+1)$, $f_H = (n+1)^{-2}$. Thus, for $f_L < f < f_H$, the optimal primary contract is $a^*(f) = 2 - (n+1)\times(n+2)\cdot f$, and the expected price is $EP(a^*(f),n+1,t) = 2\cdot(n+1)\cdot f$. For $f_H < f < Ep(1,n,1) = [n\cdot(n+1)]^{-1}$, the optimal secondary contract is $b^*(f) = 0$, resulting in a sole-source auction with n suppliers.

For this case, Figure 3(a) illustrates the size of the optimal primary and secondary contracts which the buyer would employ for any given fixed entry cost. The fixed cost determines the number of suppliers as illustrated. The lines starting at 2 on the a-axis are the functions $a^*(f)$. For example, when $1/20 < f < 1/16$, the number of suppliers with a sole-source auction would have been $n = 3$. However, by creating a secondary contract $b^*(f)$, and reducing the size of the primary contract to $a^*(f)$, the buyer maintains $n+1 = 4$ suppliers participating in the auctions. For $1/16 < f < 1/12$, the buyer could continue to reduce the size of the primary contract and maintain 4 suppliers, but the expected price is lower by having a sole-source auction with 3 suppliers. Then, for $1/12 < f < 1/9$, the buyer again creates a secondary contract to maintain 3 suppliers. Finally, for $f > 1/9$, buyer uses a sole-source auction with two suppliers. The optimal secondary contract $b^*(f) = 1 - a^*(f)$ could be as large as 20% of the input requirements in order to maintain 5 suppliers, as large as 25% to maintain 4 suppliers, and as large as 33% to maintain 3 suppliers.

Figure 3(b) illustrates the expected price paid by the buyer using an optimal secondary contract. The expected price increases as the higher fixed cost reduces the number of suppliers participating in the auctions. With a sole-source auction, the expected price would be a step function, rising to $EP = 2/5$ at $f = 1/30$, to $EP = 1/2$ at $f = 1/20$, and to $EP = 2/3$ at $f = 1/12$. However, an optimal secondary contract lowers the expected price in the regions of fixed costs where $b^*(f) > 0$. In these regions, the expected price increases with f as the size of the optimal secondary contract increases. Figure 3(b) illustrates that a split-award auction is potentially

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efficiency loss $(1/2)\cdot[c_2(n) - c_1(n)]$. Thus, it is no surprise that optimal secondary contract is significantly smaller than the primary contract.

beneficial to the buyer for any number of suppliers, but can be very beneficial when the number of suppliers is small.[25]

8.   The Intuition for the Optimal Secondary Contract

Even though a split-award auction results in a higher expected price with a fixed number of suppliers, the entire schedule of expected prices as a function of the size of the secondary contract shifts downward with an increase in the number of suppliers (see Figure 2(a)). If the fixed entry cost is relatively low (close to f_L), the buyer can induce the $(n+1)$st supplier to enter with a relatively small secondary contract. The increase in expected price which would otherwise occur from introducing the secondary contract is more than offset by the reduction in expected price caused by the additional supplier.

The additional supplier causes two effects which result in a lower expected price. The first effect arises from the fact that the new supplier adds additional capacity to the industry. This new capacity lowers the expected costs of production for both the primary and secondary contracts because it lowers the expected value of the first and second lowest costs of the suppliers. An optimal secondary contract allows the buyer to internalize this reduction in expected costs in the form of a lower expect price. The second effect arises from the fact that the new supplier lowers the expected profits of the existing suppliers. In particular, $Ep(1,n,t) > Ep(a^*,n+1,t) = f$. Thus, the optimal secondary contract allows the buyer to extract all of the expected rents of the n existing suppliers defined as their expected profits minus their fixed costs.[26]

We can identify the relative magnitude of these two effects. Note that the expected profits of all the suppliers, defined by $EP(a,n) = n \times Ep(a,n)$ using (10), is equal to the expected price $EP(a,n)$ from (6), minus the expected costs $EC(a,n)$ from (9). Thus, the reduction in the expected

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[25] For $t=1$, the ratio of fixed costs to production costs for $f \in F_{n,t}$ ranges from $1/(n+2)$ to $1/n$.

[26] Anton and Yao (1989) have an example in which investment by the higher cost supplier can result in a lower expected price from split-award contracts. The higher cost supplier has no incentive to invest if the buyer is only awarding a sole-source contract because the investment cannot lower the costs sufficiently. However, if the buyer awards two contracts, the investment is profitable because it increases the profits from a split award. The investment lowers the price because it lowers the sole-source costs faster than it increases the economies from splitting the production. Thus, a split award can induce investment that would lower the price. In this example, the price reduction is driven by the decline in the sole-source production costs of the higher cost supplier which are an important determinant of the price. This differs from the two effects causing the price reduction in our model.

price from an optimal secondary contract is simply the reduction in expected costs, $EC(1,n)$ – $EC(\boldsymbol{a}^*,n+1)$, plus the reduction in expected industry profits, $E\boldsymbol{P}(1,n)$ – $E\boldsymbol{P}(\boldsymbol{a}^*,n+1)$. These expressions are easy to calculate for $G(c;t)$ with $H(c)$ uniform on [0,1]. We find that the share of the reduction in the expected price attributable to these two effects is independent of the size of the primary contract. As a result, the share of the reduction attributable to extracting rents from the existing suppliers has the following simple form:

(13) $\qquad S_P(n,t) = [t{\times}(n+1) - 1] / [2tn]$.

Since industry capacity $T(n+1) = t{\times}(n+1) > 1$, the share attributable to rent extraction increases with t, and approaches $(n+1)/(2n)$ in the limit as $t\rightarrow¥$. Thus, as the cost distribution of the suppliers becomes more favorable to low cost realizations, the share attributable rent extraction is larger and the share attributable to cost reduction is smaller. When $t = 1$, the reduction in expected price is shared equally between a reduction in the expected industry rents and a reduction in the expected industry costs.[27]

9.   Conclusions

The buyer's control over the size of the primary and secondary contracts is crucial for his ability to induce entry and lower the expected price. In particular, the benefits to the buyer depend on its ability to choose a primary contract which is significantly larger than the secondary contract. Split-award contracts of equal size are never optimal because they generate the largest inefficiency in production and the least aggressively bidding on the primary contract. The higher expected profits on the equally large secondary contract provide a large opportunity cost that is incorporated into the bidding on the primary contract. Thus, split-award contracts of equal size generate the highest expected price for a given number of suppliers. With a smaller secondary contract, the buyer can limit the productive inefficiency from the secondary contract and maintain aggressive bidding for the primary contract. When smaller secondary contracts can induce entry, then may also reduce the expected price.

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[27] When $t > 1$, the share attributable to rent extraction is decreasing with n. For this case, the share attributable to rent extraction declines from .75 (when $t\rightarrow\infty$) to .5 as $n\rightarrow\infty$. In other words, rent extraction becomes somewhat less important when there is a larger number of existing suppliers. The opposite occurs when $t < 1$, and the share attributable to rent extraction is increasing with n.

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Appendix 1

The expected value of the lowest cost from n independent draws:

$$c_1(n,t) = n \cdot \int_0^1 xg(x)\left[1 - G(x)\right]^{n-1} dx$$

$$= tn \cdot \int_0^1 x(1-x)^{tn-1} dx \quad = \frac{tn}{tn[tn+1]} \quad = \frac{1}{tn+1} \quad .$$

The expected value of the second lowest cost from n independent draws:

$$c_2(n,t) = n(n-1) \cdot \int_0^1 xg(x)G(x)\left[1 - G(x)\right]^{n-2} dx$$

$$= n(n-1) \cdot \int_0^1 xt(1-x)^{t-1}\left[1 - (1-x)^t\right](1-x)^{t(n-2)} dx$$

$$= n(n-1) \cdot t \cdot \int_0^1 x(1-x)^{t(n-1)-1} - x(1-x)^{tn-1} dx$$

$$= n(n-1) \cdot t \cdot \left[\frac{1}{t(n-1)[t(n-1)+1]} - \frac{1}{tn[tn+1]}\right]$$

$$= \frac{2tn - (t-1)}{[t(n-1)+1] \cdot [tn+1]} \quad .$$

The expected value of the third lowest cost from n independent draws:

$$c_3(n,t) = n\binom{n-1}{2} \cdot \int_0^1 xg(x)[G(x)]^2 \left[1 - G(x)\right]^{n-3} dx$$

$$= n\binom{n-1}{2} \cdot \int_0^1 xt(1-x)^{t-1}\left[1 - (1-x)^t\right]^2 (1-x)^{t(n-3)} dx$$

$$= n\binom{n-1}{2} \cdot t \cdot \int_0^1 x(1-x)^{t(n-2)-1}\left[1 - 2(1-x)^t + (1-x)^{2t}\right] dx$$

$$= n\binom{n-1}{2} \cdot t \cdot \left[\frac{1}{t(n-2)[t(n-2)+1]} - \frac{2}{t(n-1)[t(n-1)+1]} + \frac{1}{tn[tn+1]}\right]$$

$$= \frac{3t^2 n(n-1) - (t-1)(3tn - 2t + 1)}{[t(n-2)+1] \cdot [t(n-1)+1] \cdot [tn+1]} \quad .$$

Appendix 2

Corollary 2.1: *Assume the costs are drawn from G(c;t) with H(c) uniform on* [0,1]. *The premium* $\Delta(\boldsymbol{a},n,t)$ *with a split-award auction is*

(i) *increasing with t for t < t*(n), and decreasing with t for t > t*(n),*

   where $1/n < t^*(n)$, *and* $t^*(n) < 3/n < .75$ *for n* ≥ 4 .

(ii) *increasing with n for t < t**(n), and decreasing with n for t > t**(n),*

   where $t^{**}(n) < t^*(n)$ *and* $t^{**}(n) < .6$ *for n* ≥ 3 .

Proof of (i): The sign of the derivative of the premium with respect to t depends on a cubic equation in t: $2 + 3\cdot(n\text{-}1)\cdot t - n\cdot(n\text{-}1)\cdot(n\text{-}2)\cdot t^3 = 0$. The left-hand side of this equation is clearly positive for small t, but becomes negative for large t. There are two imaginary roots and one real positive root defined as $t^*(n)$. Thus, it is clear that the premium is increasing (decreasing) with t when $t > (<)$ $t^*(n)$. Figure 1 depicts $t^*(n)$.

For a given number of suppliers, the premium is maximized at $t = t^*(n)$. The explanation for this finding follows from the behavior of the premium as $t \rightarrow 0$ and $t \rightarrow \infty$. When $t \rightarrow 0$, the expected price approaches the upper bound on the cost distribution, normalized to 1, irrespective of the number of suppliers or the size of the $\boldsymbol{a}$-contract. Thus, there can be no premium. Similarly, when $t \rightarrow \infty$, the expected price approaches the lower bound on the cost distribution, normalized to 0, irrespective of the number of suppliers and the size of the $\boldsymbol{a}$-contract. Thus again, there can be no premium. Once capacity exceeds $t^*(n)$, the premium decreases with further increases in the capacity of each supplier. In particular, for $n \geq 4$, the premium is declining with t for all $t > 3/n$. Note that $\Delta(\boldsymbol{a},3,1) > \Delta(\boldsymbol{a},4,1)$. Since we are only interested in integer numbers of suppliers, this implies that the premium is declining in integer n for all $t > 1$.

Proof of (ii): The sign of the derivative of the premium with respect to n depends on another cubic equation in t: $1 - 2 \times n - 3 \times (n\text{-}1)^2 \times t + (2\text{-}4n+3n^2) \times t^2 + n^2 \times (n\text{-}1)^2 \times t^3 = 0$. This equation is clearly positive for large t, but becomes negative for small t. There are two imaginary roots and one real

positive root defined as $t^{**}(n)$. Thus, it is clear that the premium is increasing (decreasing) with n when $t < (>) t^{**}(n)$. Figure 1 depicts $t^{**}(n)$.

For a given $t < t^{**}(n)$, the premium declines when the number of suppliers in reduced below n. This surprising finding is limited to a small range of the parameter space. Thus, when $t > t^{**}(n)$, and the premium declines when the number of suppliers is increased from n. Since $t^{**}(n) \gg$ .56, this case occurs for all distributions skewed in favor of low costs, the uniform distribution, and many distributions skewed in favor of high costs.

Appendix 3

There is no gain to the buyer from inducing entry by more than one additional supplier. This is easy to demonstrate when $t = 1$. Let $e > 0$ be the number of new entrants, and define $\mathbf{a}'(e;n)$ as the largest $\mathbf{a}$ such that a split-award auction would induce e new entrants. Thus, $1 - \mathbf{a}'(e;n)$ is the minimal secondary contract for e entrants. If there is no entry, a sole-source auction minimizes the expected price so define $\mathbf{a}'(0;n) = 1$. We can solve for $\mathbf{a}'(e;n)$ by setting the expected profits equal to the fixed cost:

$$\frac{\mathbf{a}'(e;n)}{(n+e)(n+e+1)} + \frac{2\cdot(1-\mathbf{a}'(e;n))}{(n+e)(n+e+1)} = f , \text{ which yields } \mathbf{a}'(e;n) = 2 - f\cdot(n+e)\cdot(n+e+1) .$$

When $t = 1$, the expected price from (9) becomes a simple linear expression in terms of α:

$$EP(\mathbf{a}'(e;n),n+e,1) = \frac{4 - 2\cdot\mathbf{a}'(e;n)}{n+e+1} .$$

Substituting $\mathbf{a}'(e;n)$, we obtain $EP(\mathbf{a}'(e;n),n+e,1) = 2\times(n+e)\times f$. Since the expected price is increasing in e, the optimal secondary contract would induce only one new entrant: $\mathbf{a}^*(f) = \mathbf{a}'(1,n)$. One new supplier will lower the expected price if $f < f_H = (n+1)^{-2}$, but two new suppliers will provide no further reduction in the expected price. Note that the expected industry costs are $EC(\mathbf{a}'(e;n),n+e,1) = (n+e)\times f$. Thus, the minimal secondary contract for each new entrant generates progressively higher expected costs.

We can also show that two additional suppliers cannot further reduce the price for all $t > 1/n$ $(T(n) > 1)$. As in the case of $t = 1$, the proof begins by expressing the expected price as a function of the number of entrants using $\boldsymbol{a}¢(e;n)$, the largest primary contract which induces e entrants. One can then show that the expected price with two entrants must be larger than the expected price with one entrant for all $t > 1/n$ and all f such that $f_L < f < f_H$. The proof is very awkward, but the intuition is clear. The optimal secondary contract defined by $\boldsymbol{b}^*(f) = 1 - \boldsymbol{a}¢(1;n)$ exactly eliminates the expected rents of all the suppliers. Moreover, it has been shown (see McAfee and McMillan (1987)) that the optimal number of entrants can be induced by a sole-source auction. Therefore, the decline in the expected price with a split-award auction can arise only if it is accompanied by the extraction of some significant supplier rents. Consequently, no further gains are possible for the buyer beyond the optimal secondary contract $\boldsymbol{b}^*(f)$ which induces one additional supplier.