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What Drives Corporate Bond Market Betas?[*]

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Abstract

We study the cross-section of expected corporate bond returns using an intertemporal CAPM (ICAPM) with three factors: innovations in future excess bond returns, future real interest rates and future expected inflation. Our test assets are a broad range of bond market index portfolios of different default categories. We find that the ICAPM can explain the cross-section of expected bond returns and of the three factors, innovations about future inflation and future real interest rates are the most important. Our model provides an alternative to the *ad hoc* risk factor models used, for example, in evaluating the performance of bond mutual funds.

JEL classification: F31; F37.
Keywords: bond market, fixed income mutual funds, asset pricing model, variance decomposition, recursive utility, betas, factor pricing.

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[§]The first version of the paper was titled "News and the Cross-Section of Expected Bond Returns".

1 Introduction

We study the factors that explain the cross-section of expected corporate bond returns. Our model, adapts the Campbell (1993) ICAPM to the case of an investor who invests only in the bond market. The stochastic discount factor in our model does not, in contrast to the Campbell ICAPM for the equity market, contain any free parameters related to the risk aversion of the representative agent. We test our three factor ICAPM using bond index portfolios from seven different credit rating categories. Empirically, we find that the ICAPM can explain the cross-section of expected corporate bond returns over the 1993-2006 period. Of the three factors, innovations about future inflation and future real interest rates are more important than innovations in future excess bond returns in explaining the cross-sectional variation.

There is, surprisingly, little research on the cross-section of expected bond returns in comparison to that on the cross-section of stock returns. This is striking given that in 2005, according to the IMF (April 2007), the capitalization of the US bond markets was $24 trillion as compared to $17 trillion for the US stock markets. The relative sizes of the corporate and government bond markets were $18.1 trillion and $5.9 trillion respectively. More importantly from an investor's perspective, the most recent data (Investment Company Institute, 2007) shows that out of a total of $18 trillion under management in US mutual funds in 2006, as much as $ 2 trillion was invested in bond and money market funds compared to about $ 10 trillion in equity funds. In terms of number of funds, out of a total of about 8,100 mutual funds, 2,849 (35%) were classified as bond and money market funds, 4,770 (58%) as equity market funds and the remaining as hybrid funds.

Chang and Huang (1990) are, as far as we are aware, the first to investigate the relation between expected return and covariance risk measures in the case of corporate bonds. Using six portfolios of corporate bonds sorted according to credit rating categories, they find that two latent factors explain the cross-section of these portfolios over their 1963-1979 sample period. Fama and French (1993) use a five factor model to explain the cross-section of *both* stock and bond returns. They observe that when their two term structure factors are included in the bond regressions, the explanatory power of the stock market factors

disappears for all but low-grade corporate bonds.

More recently, Gebhardt, Hvidkjaer, Swaminathan (2005) evaluate the factor loadings versus characteristics debate in the context of the cross-section of expected bond returns. Their results imply that firm-specific information implicit in ratings and duration is not related to the cross-section of expected bond returns. Importantly, they find that a two factor model, using term and default factors (as in Fama and French, 1993) does a good job of explaining expected bond returns. Ferson, Kisgen and Henry (2006) take a first step in linking the stochastic discount factor to specific term structure variables to evaluate the performance of government bond funds.

Our main results are as follows. Using a return decomposition for a consol bond and the recursive preferences proposed by Epstein and Zin (Epstein and Zin, 1989; 1991), we obtain a three factor ICAPM in the spirit of Campbell (1993). We test this model using seven index portfolios of different default categories between 1993 and 2006 with a Fama-MacBeth chi-squared test. We find that our model cannot be rejected. Of the three factors within our ICAPM, innovations in future inflation rates (i.e. news about expected inflation) and future real rates were more important than innovations in expected excess bond returns in determining the cross-section of corporate bond returns. Our robustness tests show that our ICAPM results also hold for an expanded set of test assets which includes seven additional corporate industry bond portfolios.

The rest of the paper is organized as follows. Section 2, provides a brief outline of related research on the cross-section of expected bond returns while in Section 3 we describe the set up of our model and the test methodology. Next, in Section 4, we provide details of the data that we use and discuss our empirical results in Section 5. Section 6 presents some robustness checks and Section 7 concludes the paper.

2   Related Literature

We organize this brief review in two parts; the first focuses on related empirical research and the second on the ICAPM model used in this paper.

2.1 Empirical Background

As mentioned earlier, despite the relative large size of the US government and corporate bond markets relative to the equity markets and the substantial proportion of funds invested in bond-only mutual funds there has been surprisingly little research on the factors that drive bond market betas. As Chang and Huang (1990) point out, the perceived risks of bond are commonly identified as operating risk, default risk, interest rate risk, purchasing power risk and duration risk. However, while all but the last two are present in stocks there is an emphasis on systematic versus unsystematic risk in the case of stocks and on unsystematic risk for bonds. They suggest that perhaps "the lack of convincing empirical evidence to show that covariance risks are priced in bond markets contributes to ..(this) differential treatment". Chang and Huang (1990) construct six portfolios as test assets[1] based on Moody's rating quality as a criteria- Aaa, Aa, A, Baa, Ba and B. They point out that while there are a number of criteria according to which these test assets could be created they opt for credit ratings relying on the evidence in Weinstein (1981) that bond ratings may be significantly related to bond betas. They find using a latent variable approach, as in Gibbons and Ferson (1985), that excess returns on corporate bonds are driven by two unobservable factors. However, when observations for January are excluded, the data are consistent with a single latent variable specification.

Fama and French (1993) find that a five factor model that includes a term structure and a default premium factor in addition to the now familiar Market, SMB and HML factors explains well the cross-section of both stock and bond returns. Specifically, in the context of our paper, they observe that when the two term structure factors are included in the bond regressions, the explanatory power of the stock market factors disappears for all but the low-grade corporate bonds.

More recently, Gebhardt, Hvidkjaer, Swaminathan (2005) evaluate the factor loadings versus characteristics debate in the context of the cross-section of expected bond returns. Their innovation, in this horse race, is to use bond market data that, unlike in the case of the stock market, allows for both factor loadings and firm characteristics to have a clear

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[1]Prior work on the cross-section of expected corporate bond returns, for example Friend, Westerfield and Granato (1978), uses individual corporate bond data to test the CAPM.

risk-based interpretation. They find that default betas and term betas are able to explain the cross-section of bond returns after controlling for characteristics such as duration and ratings. Their results imply that firm-specific information implicit in ratings and duration is not related to the cross-section of expected bond returns. Importantly, they find that a two factor model, using term and default factors (as in Fama and French, 1993) does a good job of explaining expected bond returns. They conclude however, that "while the search for more complete factor models to explain average bond returns is far from over", their results do unambiguously favour a risk-based factor model over a characteristics-based model. Viceira (2007), in a recent contribution, examines the role of covariance risk for bonds with stocks and consumption growth. He finds that movements in both the short-term nominal interest rate and the yield spread are positively related to changes in subsequent realized bond risk and bond return volatility.

Ferson, Kisgen and Henry (2006) take the first step in linking the stochastic discount factor to specific term structure variables in the macroeconomy. For example, their single factor model depends on two "factors"; changes in the long and short term rates and on their averages. Their three factor model includes a discrete change in convexity and an average convexity factor. They estimate the conditional performance of the fund and the parameters of the SDF model simultaneously in a GMM framework that allows for differential "states" of the term structure.

As pointed out earlier there is a significant amount of investment in bond market mutual funds. The measurement of the performance of these funds using asset pricing models relies largely on *ad hoc* factor models. A recent example is Huij and Derwall (2005) who study the persistence in bond mutual fund performance using a sample of 3,500 US bond market funds. They build on a model derived from Blake, Elton and Gruber (1993) that uses proxies for the overall bond market, returns on low-grade debt and returns on a mortgage-backed securities index. This model is then augmented first with an aggregate stock market index return factor and then with three factors obtained by a principal components analysis of yield changes in certain ranges of the maturity spectrum.

We also note here that the literature on the predictability of holding period returns

on corporate bonds (in contrast to government bonds) is rather sparse. This is relevant in our context, since we need to identify state variables that have predictive power for excess corporate bond returns. Chang and Huang (1990) find that the one-month T-Bill yield, the six month T-Bill yield minus the one month T-Bill yield, the Baa-rate less the one month T-Bill rate and a January dummy have significant predictive power. Also surprisingly, these regressions have high R-squares between 22-36% compared to the usually low R-squares in stock return predictive regressions. Baker, Greenwood and Wurgler (2003) find that the real short rate and the term spread have significant predictive power on the excess returns of corporate bonds over commercial paper. The R-square's in their predictive regressions, using annual data from 1954-2000, range from 14% to 40%. Relative to the literature on the predictability of excess returns on corporate bonds, there is a larger literature on the variables that predict yields on Government bonds. In recent work, Cochrane and Piazessi (2005) find that linear combinations of forward rates add significant explanatory power to the variables identified by Fama and Bliss (1987) and Ludvigson and Ng (2005) identify principal components of a set of macroeconomic factors that also contribute to predictability over and above the Cochrane-Piazessi factors.

2.2   Model Background

Our model closely follows the ICAPM derived in Campbell (1993, 1996). Campbell uses a log-linear approximation to an investor's budget constraint to express unanticipated consumption as a function of current and future returns on wealth. This expression is then combined with the Euler equation resulting from the investor's utility maximization to substitute consumption out of the model. Campbell derives a cross-sectional asset pricing formula, using Epstein Zin preferences, where an asset's return is determined by its covariance with the market return and news about future market returns making no reference to consumption data. Using this framework, Campbell and Vuolteenaho (2004) derive a two factor ICAPM for the stock market; the covariance with the discount rate news and the covariance with the cash flow which they term as the good beta and the bad beta respectively. In order to obtain these news factors they rely on the methodology in Campbell and Ammer (1993) and Campbell (1991). This approach uses a log-linear

approximation to the present value formula for stocks to decompose unexpected excess stock returns into two components; news about future cash flows (dividend growth) and news about future discount rates. These factors are then extracted from the data using a VAR framework where the components of the VAR are chosen from variables that are known to have predictive power for stock returns. We also use the present value decomposition based on the price of a consol bond or perpetuity, as in Shiller and Beltratti (1992) and Engsted and Tanggaard (2001), that corresponds to the long term investment horizon of our investor.

Our version of the Campbell (1993,1996) ICAPM assumes that the investor can only invest in the bond market. This may seem a restrictive assumption but there are a large number of market participants like pension funds and insurance companies among others that are restricted in the application of their funds to fixed income securities. As much as $3 trillion is invested, out of a total of $12 trillion, in mutual funds that invest only in the bond markets. As Ferson, Kisgen and Henry (2006) put it "Ideally, one would like an SDF model or a set of factors to price both stocks and bonds. Empirically, however, this is challenging. Roll (1970) found that the capital asset pricing model does not work well for bonds. Mehra and Prescott (1985) observe that simple consumption models can not price both Treasury bills and stocks. Multiple-factor models with both bond and stock-related factors appear to fare better (Ferson and Harvey 1991, Campbell 1996). However, it is more common to find bond factors used for pricing bonds and stock factors for pricing stocks. We stick with this tradition, using term structure models to price government bond funds."

The estimation of the Campbell-Vuolteenaho model requires the specification of the VAR whose components are not dictated by theory but are essentially an empirical issue. This issue has been discussed in detail in Campbell, Lo and Mackinlay (1997) and Campbell and Ammer (1993). Recently, Chen and Zhao (2006) also show that the estimations of the innovations is sensitive to the specification of the VAR system particularly when some of the factors are estimated as a residual. We will discuss this in the empirical part of our paper. In general however, misspecification of the state variables will be an issue wherever theory does not dictate what the choice of the state variables ought to be.

Reasonable choices of state variables motivated by their predictive ability for the system and robustness tests on the specification can, as Chen and Zhao (2006) point out, help mitigate this problem.

3   Model Setup and Test Methodology

We now provide brief details of our ICAPM model and of the econometric methodology used in the paper. Full details are provided in the Appendix.

3.1   Bond Decomposition

In this paper we decompose bond returns using the present value for a consol bond (Shiller and Beltratti, 1992 and Engsted and Tanggaard, 2001) rather than that for zero coupon bonds used in Campbell and Ammer (1993) since our investor has a long-horizon.

3.1.1   Consol Bond

We denote the coupon by C and the price $P_{b,t}$, then the log one period gross return from t to $t+1$ is given by:

$$r_{b,t+1} = log\left(\frac{C + P_{b,t+1}}{P_{b,t}}\right) = log\left(C + \exp\left(p_{b,t+1}\right)\right) - p_{b,t} \tag{1}$$

We now take a first order Taylor expansion around the mean of $log\left(C + \exp\left(p_{b,t+1}\right)\right)$ to get

$$r_{b,t+1} = \kappa_b + \rho_b - p_{b,t+1} - p_{b,t} \tag{2}$$

where κ_b is a constant arising from the linearization and $\rho_b \equiv \frac{\exp\left(E_t p_{b,t+1}\right)}{C + \exp\left(E_t p_{b,t+1}\right)} \approx \frac{E\left(P_{b,t+1}\right)}{C + E\left(P_{b,t+1}\right)} = \frac{1}{E\left(r_{b,t+1}\right)}$, this is approximately equal to $R_{b,t+1} \equiv \frac{C + P_{bt+1}}{P_{bt}}$. We can solve this forward, imposing the usual transversality condition and take conditional expectations at time t to get:

$$p_{b,t} \equiv -E_t \sum_{j=0}^{\infty} \rho_b^j r_{b,t+1+j}. \tag{3}$$

We, can substitute this into Equation (2) and if we assume that $\rho_b = \rho$, (or that the linearization constant for bonds is approximately equal to the linearization coefficient for the intertemporal budget constraint) we can write:

$$(E_{t+1} - E_t) \, r_{b,t+1} = -(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{b,t+1+j}. \tag{4}$$

To obtain excess returns, we add and subtract the risk free rate, $r_{f,t}$, and use the fact that $(E_{t+1} - E_t) \, r_{f,t} = 0$; we get the decomposition for innovations in the excess bond returns:

$$
\begin{aligned}
(E_{t+1} - E_t) \, (r_{b,t+1} - r_{f,t+1}) \;=\; & -(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \, (r_{b,t+1} - r_{f,t+1+j}) \\
& -(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{f,t+1+j}.
\end{aligned} \tag{5}
$$

Next, we can substitute for the nominal risk free rate

$$r_{f,t+1} = r_{r,t+1} + \pi_{t+1},$$

where $r_{r,t+1}$ and π_{t+1} are respectively the real interest rate and inflation rate, and decompose excess bond returns as

$$
\begin{aligned}
(E_{t+1} - E_t) \, (r_{b,t+1} - r_{f,t+1}) \;=\; & -(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \, (r_{b,t+1} - r_{f,t+1+j}) \qquad (6) \\
& -(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{f,t+1+j}, \\
\;=\; & -(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \, (r_{b,t+1} - r_{f,t+1+j}) \\
& -(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{r,t+1+j} \\
& -(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \pi_{t+1+j}.
\end{aligned}
$$

For ease of exposition, we use the notation for "innovations" used by Campbell and Ammer (1993). Specifically, $\widetilde{x}_{b,t+1} = (E_{t+1} - E_t) \, (r_{b,t+1} - r_{f,t+1})$ is the innovation in the log

9

excess one-period return on a consol bond from t to $t+1$, $\widetilde{x}_{x,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j (r_{b,t+1} - r_{f,t+1+j})$ is the innovation in the *future* log excess one-period return on a consol bond held from t to $t+1$, $\widetilde{x}_{r,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{r,t+1+j}$ is the innovation in the log excess one-period real return and $\widetilde{x}_{\pi,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \pi_{t+1+j}$ is the innovation in the log excess one-period inflation. Substituting this into Equation (6) above we get

$$\widetilde{x}_{b,t+1} = -\widetilde{x}_{\pi,t+1} - \widetilde{x}_{r,t+1} - \widetilde{x}_{x,t+1}. \tag{7}$$

This expression implies that unexpected excess bond returns must be due to "news", i.e. changes in expectations about either future excess bond returns, or future inflation or future real interest rates or combinations of these three. For example, news that either inflation, real interest rates or excess returns will be higher (lower) in the future, will lead to a fall (increase) in excess bond returns. This expression is a dynamic accounting identity and holds by construction having been obtained from the definition of the return on a consol bond. However, it is important to note that if both the Fisher Hypothesis and the Expectations Theory hold then inflation news would be the only source of variation in excess bond return innovations. Specifically, if the Fisher Hypothesis holds then nominal bond yields move one-for-one with expected inflation so that the ex ante real interest rate is constant. This implies that "news about future real rates" is constant or the component $(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{r,t+1+j}$ is zero. If for example, the Expectations Hypothesis holds then we know that the long-term bond yield is given as the expected future short rates plus a time varying term premium. This implies that expected excess bond returns are constant so that the "news about future excess returns" component $(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j (r_{b,t+1} - r_{f,t+1+j})$ is zero.

3.2 Bond ICAPM

We follow Campbell (1993, 1996) and use the Epstein-Zin utility function, defined recursively, for an infinitely lived representative agent as

$$U_t = \left[(1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(E_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \tag{8}$$

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, ψ is the elasticity of intertemporal substitution, γ is the coefficient of relative risk aversion, δ is a time discount factor and C_t is consumption. The Euler equation for asset i , following Epstein and Zin (1989,1991), has an associated pricing equation in simple returns given by

$$1 = E_t \left[\left\{ \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\varphi}} \right\}^{\theta} \left\{ \left(\frac{1}{R_{B,t+1}} \right) \right\}^{1-\theta} R_{i,t+1} \right], \qquad (9)$$

with the corresponding SDF where $R_{B,t+1}$ is the return on the aggregate bond market and $R_{i,t+1}$ is the return on the asset in the bond market. We now define the SDF

$$M_{t+1} = \delta^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} \left(\frac{1}{R_{B,t+1}} \right)^{1-\theta}. \qquad (10)$$

The log of the SDF is

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} - (1-\theta) r_{B,t+1}. \qquad (11)$$

With some algebra, we can write the log SDF as

$$m_{t+1} = E_t (m_{t+1}) - \frac{\theta}{\psi} (c_{t+1} - E_t (c_{t+1})) - (1-\theta) (r_{B,t+1} - E_t (r_{B,t+1})). \qquad (12)$$

We next use the following result from Campbell (1993) (Equation 21, page 494) reproduced below:

$$c_{t+1} - E_t c_{t+1} = r_{b,t+1} - E_t (r_{b,t+1}) + (1-\psi)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{B,t+1+j} \qquad (13)$$

to substitute out consumption to in the expression for the SDF above to get

$$
\begin{aligned}
m_{t+1} &= E_t (m_{t+1}) - \gamma (E_{t+1} - E_t)(r_{B,t+1} - r_{f,t+1}) \\
&\quad + (1-\gamma)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j (r_{B,t+1+j} - r_{f,t+1+j}) \\
&\quad + (1-\gamma)(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{f,t+1+j}.
\end{aligned}
$$

Now we substitute using the following relation:

11

$$(E_{t+1} - E_t)(r_{B,t+1} - r_f) = -\tilde{x}_{\pi,t+1} - \tilde{x}_{r,t+1} - \tilde{x}_{x,t+1}$$

to get

$$m_{t+1} = E_t(m_{t+1}) + \tilde{x}_{\pi,t+1} + \tilde{x}_{r,t+1} + \tilde{x}_{x,t+1}.$$

Next we define

$$
\begin{aligned}
\mathbf{f}_{t+1} &= \left(\tilde{x}_{\pi,t+1}, \tilde{x}_{r,t+1}, \tilde{x}_{x,t+1}\right)', \\
\mathbf{b} &= (1,1,1).
\end{aligned}
$$

We use the standard result that the log of the SDF m_{t+1} is a linear function of the K risk factors $\mathbf{f}_{t+1}$

$$m_{t+1} = a + b'\mathbf{f}_{t+1}, \tag{14}$$

then the unconditional model in expected return form for returns in logs is

$$E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = b'cov(r_{t+1}, f_{t+1}), \tag{15}$$

which is a form of the expected return-beta form:

$$E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = \lambda'\beta_i \tag{16}$$

where

$\beta_i = [Var(f_{t+1})]^{-1} Cov(r_{t+1}, f_{t+1})$ is a vector with the K betas for asset i and

$\lambda = -Var(f_{t+1})b$ is a vector of factor risk prices.

We can also write

$$E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = -\sigma_{i,EBR} - \sigma_{i,RR} - \sigma_{i,INFL}$$

where $\sigma_{i,EBR} = Cov\left(r_{i,t}, \tilde{x}_{x,t+1}\right)$ is the covariance of the asset return with bond excess return news,

$\sigma_{i,RR} = Cov\left(r_{i,t}, \tilde{x}_{r,t+1}\right)$ is the covariance of the asset return with real interest rate news and

$\sigma_{i,INFL} = Cov\left(r_{i,t}, \widetilde{x}_{\pi,t+1}\right)$ is the covariance of the asset return with inflation news.

We can write equation above in terms of factor betas' risk prices as

$$E\left(r_{i,t+1} - r_{f,t+1}\right) + \frac{\sigma_i^2}{2} = -\sigma_x^2 \beta_{i,EBR} - \sigma_r^2 \beta_{i,RR} - \sigma_\pi^2 \beta_{i,INFL}, \tag{17}$$

where σ_{EBR}^2, σ_{RR}^2, and σ_{INFL}^2 are respectively the variances of $\widetilde{x}_{x,t+1}, \widetilde{x}_{r,t+1}$, and $\widetilde{x}_{\pi,t+1}$. The risk prices for betas can be derived by defining $\lambda = (\lambda_{EBR}, \lambda_{RR}, \lambda_{INFL})^T = \sigma_f b$, where σ_f is a diagonal matrix with the factor variances along its main diagonal. In addition we can rewrite the model in an expected return-beta representation, *i.e.* :

$$E\left(r_{i,t+1} - r_{f,t+1}\right) + \frac{\sigma_i^2}{2} = \boldsymbol{\lambda}^T \boldsymbol{\beta}_i = \lambda_{EBR}\beta_{i,EBR} + \lambda_{RR}\beta_{i,RR} + \lambda_{INFL}\beta_{i,INFL} \tag{18}$$

where $\boldsymbol{\lambda} = (\lambda_x, \lambda_r, \lambda_\pi)^T = -Var\left(\mathbf{f}_{t+1}\right)\mathbf{b}$ denotes the vector of factor risk prices and $\boldsymbol{\beta}_i = Var\left(\mathbf{f}_{t+1}\right)^{-1} Cov\left(r_{i,t+1}, \mathbf{f}_{t+1}\right)$ represents the (3×1) vector of multiple regression betas for asset i. The $\lambda's$ represent the risk prices of multiple regression beta risk for each of the factors. Finally we rewrite the left hand side in simple expected return form, to obtain our three beta model for the bond market

$$E\left(r_{i,t+1} - r_{f,t+1}\right) = \lambda_{EBR}\beta_{i,EBR} + \lambda_{RR}\beta_{i,RR} + \lambda_{INFL}\beta_{i,INFL}. \tag{19}$$

Equation (19) implies that, in the case of the bond markets, the risk premium for an investor depends on the variance future long-term excess bond returns, real interest rates and inflation but is independent of the long-term investor's relative risk aversion. This seems to be due to the fact that there is no uncertainty associated with the nominal cash flows from a bond in contrast to the news about future cash flows in the case of stocks.

3.3   VAR Estimation and Extraction of News Components

We can now use the VAR approach of Campbell and Shiller to extract the components of Equation (19) from the data. We use the following VAR, where the vector of state variables z_t is specified as follows:

$$z_t = (x_{b,t}, r_t, sprd_t).$$

Here $x_{b,t}, r_t$, and $sprd_t$ are respectively the excess return on the bond market, the real interest rate and the Baa-Aaa credit yield spread. We use these variables because the VAR necessarily needs to include the excess bond return and the real rate to be able to compute their corresponding news components. We include credit yield spread because many previous studies have found that this variable has significant predictive power for bond returns (see among others Chen and Zhao, 2006)[2]. Inflation is not included as its news component will be calculated as a residual, as explained below.

We can write a first order VAR (in companion form for higher lags if required) as:

$$z_{t+1} = Az_t + w_{t+1} \qquad (20)$$

where A is the VAR parameter matrix and w_{t+1} is the vector of error terms. We know that from Equation (20) the VAR estimate of $z_{t+1} - E_t z_t$ is w_{t+1}. Further, the estimate of $(E_{t+1} - E_t) z_{t+1+j}$ is $A^j w_{t+1}$. We can then define suitable unit vectors g_1 and g_2 that can pick out the first and second elements of z_t. Specifically, these VAR estimates are given by:

$$\widetilde{x}_{b,t+1} = -\widetilde{x}_{\pi,t+1} - \widetilde{x}_{r,t+1} - \widetilde{x}_{x,t+1} \qquad (21)$$

where now using the VAR estimate of A and the VAR residuals w_{t+1}

$$
\begin{aligned}
\widetilde{x}_{b,t+1} &= g_1 w_{t+1} \\
\widetilde{x}_{x,t+1} &= \rho g_1 A \left(I - \rho A\right)^{-1} w_{t+1} \\
\widetilde{x}_{r,t+1} &= \rho g_2 A \left(I - \rho A\right)^{-1} w_{t+1} \qquad (22) \\
\widetilde{x}_{\pi,t+1} &= -\widetilde{x}_{b,t+1} - \widetilde{x}_{r,t+1} - \widetilde{x}_{x,t+1}
\end{aligned}
$$

Thus here, we get the inflation news component as a residual since we know the other components in the dynamic accounting identity. Just like in the case of stocks where the residual term is the cash flow, here we also can avoid the difficulties of estimating the inflation component directly.

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[2]In our robustness checks we will experiment with additional state variables.

4   Data

We use monthly data, over the 1993-2006 period, on bond indices for the aggregate bond market and for different rating categories obtained from Lehman Brothers. For example, The Lehman U.S. Aggregate Index, which we use as a proxy for the US bond market, covers the dollar denominated investment-grade fixed-rate taxable bond market, including Treasuries, government-related and corporate securities, MBS pass-through securities, asset-backed securities, and commercial mortgage-based securities. To qualify for inclusion in the U.S. Aggregate Index, a bond or security must meet certain criteria; for example, they must have at least one year-to-final-maturity, regardless of call features; have at least $250 million par amount outstanding; must be rated investment grade (Baa3/ BBB-/BBB-) or better; must be USD-denominated and non-convertible and all corporate and asset-backed securities must be registered with the SEC. There are a number of measures of returns available on the Lehman Brothers bond indices. In this paper, we use monthly data on the since-Inception Total Return, i.e. the cumulative total return of the index since its inception. This number is indexed to zero at inception (which will reflect different inception dates for different indices) and tracks cumulative index total return. We obtain holding period returns for each month that include both capital gains and coupon payments made during each month. For the test assets, we use (percentage) holding period returns on the following indices from the Lehman Brothers Fixed Income database: AAA, AA, A, BAA, BA, B, CA. The credit spread yield data (Moody's Baa-Aaa) and the CPI data is from the FRED database. We use the three-month T-Bill rate from the CRSP as a proxy for the risk-free rate and the real rate is obtained as the difference between the risk-free rate and the growth rate in the CPI.

We note two points regarding the data. First, we use holding period returns on the bond indices computed as follows. For each index, the return between t and $t-1$ is given by, the ratio of the value of a dollar invested in the index constituents between these two time periods. The excess returns for each bond index is then computed as the excess over the risk free rate. Many studies use other measures like yields that are not useful in our context. We note also that these Lehman Brothers corporate bond portfolios consist

of the most representative and liquid issues in each rating category that are followed by the traders who always post bid-ask prices. The monthly portfolio returns use either transactions prices for issues that were traded in the beginning and end of the month, bid-ask prices where these exist and in the remaining cases matrix implied prices are used in order. As Sangavinatsos (2005) points out- "as long as the matrix pricing is limited the computed monthly returns should accurately reflect the actual realized corporate bond market returns". He also points out that Lehman Brothers corporate bond indices are used and replicated as benchmarks[3] by a large proportion of bond portfolio managers and hence the computed returns represent returns that can actually be realized.

4.1   Test Methodology

We use the standard Fama and MacBeth (1973) cross-sectional method to estimate our model, as in equation (19). This methodology is appropriate in our case since the factors do not represent portfolio returns. Moreover, alternative methodologies such as GMM assume that the payoffs are typically returns or excess returns, including returns scaled by instruments; which clearly is not the case given our methodology for estimating the factors[4].

In the first step of the method, for each test asset, the betas are estimated with a time series regression of excess returns onto a constant and the three factors:

$$R_{it}^e = \alpha_i + \beta_{1i}\widetilde{x}_{x,t+1} + \beta_{2i}\widetilde{x}_{r,t+1} + \beta_{3i}\widetilde{x}_{\pi,t+1} + \varepsilon_{it}. \tag{23}$$

We use, following much of the recent literature, estimates of betas over the full sample period. In the second step, for each period t, the risk premiums λ_{1t}, λ_{2t}, λ_{3t} are estimated

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[3]The Morningstar website has a number of examples of this: SunAmerica High Yield Bond A- this fund normally invests at least 80% of its assets in below investment grade US and foreign junk bonds without regard to the maturities of such securities or the Fidelity U.S. Bond Index Fund has more than 70% in AAA US corporate bonds.

[4]Petkova (2006) examines a different estimation approach to Fama-MacBeth's when factors are not portfolio returns but innovations; specifically the GMM. Petkova estimates innovations and prices of risk simultaneously. This is innovations of the VAR system and the coefficients in the SDF are estimated in one step. However, the results based on GMM estimation are very similar to those derived from the Fama-MacBeth procedure.

from a series of cross-sectional regressions of the excess returns on the estimated betas, *i.e.*

$$R_t^{ei} = \widehat{\beta}_{1i}' \lambda_{1t} + \widehat{\beta}_{2i}' \lambda_{2t} + \widehat{\beta}_{3i}' \lambda_{3t} + \alpha_{it} \qquad i = 1, 2, ...7.$$

We estimate each of the $\lambda's$ and $\alpha_i's$ as $\widehat{\lambda}_{j,FM} = \frac{1}{T} \sum \widehat{\lambda}_{j,t}$ for $j = 1, 2, 3$ and $\widehat{\alpha}_{i,FM} = \frac{1}{T} \sum \widehat{\alpha}_{i,t}$. The sampling errors for these estimates are respectively $\sigma_j^2 \left(\widehat{\lambda}_{j,FM} \right) = \frac{1}{T^2} \left(\widehat{\lambda}_{j,t} - \widehat{\lambda}_{j,FM} \right)^2$ and $\sigma^2 \left(\widehat{\alpha}_{i,FM} \right) = \frac{1}{T^2} \left(\widehat{\alpha}_{i,t} - \widehat{\alpha}_{i,FM} \right)^2$. Although the standard errors derived from the Fama-MacBeth technique correct for cross-sectional correlation in a panel, this technique assumes that the time series is not autocorrelated. Moreover, Fama-MacBeth standard errors do not correct for the fact that the betas are generated regressors. In response to the first issue, we report Fama-MacBeth standard errors corrected for autocorrelation[5]. To account for the fact that betas are estimated regressors we also report Shanken (1992) standard errors. However, Shanken standard errors are to be preferred to Fama-MacBeth's only in the case that the returns are conditional homoskedastic since the latter may be more precise when the returns are conditional heteroskedastic (see for example Jagannathan and Wang, 1996). In general, these tests give an indication of the statistical significance of each of the news components as an explanation of the cross-sectional variation in expected returns on our bond portfolios. To get some insight into the economic importance of each of the news components, we report plots of actual and predicted mean returns. Finally, we also test if the Fama-MacBeth pricing errors are jointly zero using a χ^2 test statistic. We obtain the latter by dividing the expected value of the Fama-MacBeth cross section residuals $\widehat{\alpha} = \frac{1}{T} \sum_{t=1}^T \widehat{\alpha}_t$, by $\left(\frac{1}{T} \right)$ times their covariance matrix, *i.e.* $cov \left(\widehat{\alpha}, \widehat{\alpha}' \right) = \frac{1}{T} cov \left(\widehat{\alpha}_t, \widehat{\alpha}_t' \right)$. This ratio leads to the chi-squared statistic

$$T\widehat{\alpha}' cov \left(\widehat{\alpha}_t, \widehat{\alpha}_t' \right)^{-1} \widehat{\alpha} \sim \chi^2_{N-K}, \tag{24}$$

where N is the number of test assets and K the number of parameters.

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[5] We account for correlated $\widehat{\lambda}_t$'s by using a long-run variance matrix $\sigma^2 \left(\widehat{\lambda}_{FM} \right) = \frac{1}{T} \sum_{j=-\infty}^{\infty} Cov_T(\widehat{\lambda}_t, \widehat{\lambda}_{t-j}')$ where we downweight the higher order correlations through a Bartlett estimate, as in Newey and West (1987). In a GMM framework Newey and West estimate the spectral density function as $\widehat{S} = \sum_{j=-k}^{k} \frac{k-|j|}{k} Cov(u_t, u_{t-j})$. Here, we compute the Fama-MacBeth standard errors corrected for autocorrelation as $\sigma^2 \left(\widehat{\lambda}_{FM} \right) = \sum_{j=-12}^{12} \frac{13-|j|}{13} Cov(\widehat{\lambda}_t \widehat{\lambda}_{t-j}')$.

To assess if the single beta CAPM explains the cross-section of bond returns, we use the bond decomposition in (7). We apply the Fama -MacBeth procedure and run the time series regression

$$R_{it}^e = \alpha_i + \beta_{1i}\left(-\widetilde{x}_{x,t+1} - \widetilde{x}_{r,t+1} - \widetilde{x}_{\pi,t+1}\right) + \varepsilon_{it},$$

from which we obtain $\widehat{\beta}_{1i}$. In the second-step we run the cross-sectional regression

$$R_t^{ei} = \widehat{\beta}_{1i}' \lambda_{1t} + \alpha_{it} \qquad i = 1, 2, ...7. \tag{25}$$

We can assess the validity of the traditional CAPM by testing if the pricing errors are jointly zero using the chi-squared Fama-MacBeth statistic described in (24).

5   Empirical Results

Table (1) provides some interesting summary statistics on our set of test assets. For example, unlike equity size portfolios, the average returns on bond portfolios are not monotonically related to the rating category. While, for example the BA-rated portfolio (the riskiest in terms of credit rating) has the highest return of all portfolios, the average returns of the AAA-rated portfolio are very similar to those of the AA portfolio. The median returns also have a similar pattern. Further, B and the CA-rated portfolios returns are more than twice as volatile as compared to the AAA and other higher quality bond portfolios. The mean returns in Table (1) are percent per month. There is a percent of spread of expected returns to explain. The spread is from .58 to .81 percent per month. We also need to emphasize here that we are using holding period returns on our bond market indices. In many related papers it is not always clear whether the returns are yields (i.e. inversely related to price) or holding period returns. We compute these returns using the index levels which reflect both capital gains, accrued interest and coupons. The cross-correlations between the test assets are reported in Table (2). We note that the magnitude of the cross-correlations are related closely, as might be expected, to the rating categories; for example for the period 1993-2006 the correlation between the AAA and the A portfolio is 0.96 but is only -0.05 with the CA-rating category portfolio. On the other hand the

cross-correlation between the portfolios decreases in a monotonic way as we move from the AAA to the CA-rating category portfolio.

We report, in Table (3), some summary statistics on our three state variables: the excess return on the aggregate bond market index, the real rate and the credit spread over the sample period 1993-2006. Here we find that the excess bond return is more than five times as volatile as the real interest rate and a hundred times more volatile than the credit spread. However the real interest rate and the spread appear to be more persistent than the excess bond return. We also provide statistics on the cross-correlation between state variables, in Table (9). The cross-correlations between the excess bond market return, the real rate and the credit spread are, in general, quite low.

5.1 VAR Results

Next, in Table 5, we report parameter estimates over the full sample period, 1993-2006, for the VAR that we estimate. The state variables are: the excess bond market return, the real rate and the credit term spread. We report coefficients based on OLS estimates and OLS standard errors. We also obtained bootstrapped standard errors but we do not report these since they are qualitatively similar. Finally, we report the R^2 and the F-statistic for each regression. Our results indicate that the real rate r_t and the spread $sprd_t$ have some ability to predict excess bond returns. Compared to the low R^2 (typically 2-4%) seen in VARS with predictive variables for excess stock returns the R^2 for the excess bond return regression is 5%.

Finally Table (7) shows the covariance between factors; while news about real rates and news about future expected bond returns have low and negative covariance, there is significant negative covariation between expected excess bond returns and expected future inflation. In other words when investors learn that long-run inflation will be higher than expected, they also tend to learn that excess bond returns will be lower than expected.

5.2 Fama-MacBeth Cross-sectional Regressions

When we calculate the chi-squared Fama-MacBeth statistic in (24) to assess if the single beta CAPM can explain the variation across bond returns, we find that the model is

rejected. The chi-squared statistic is 16.28 which is larger than its 5% critical value, *i.e.* 12.59.

Table (9) reports results for the second stage of the Fama-MacBeth regression:

$$R_t^{ei} = \widehat{\beta}_{1i}'\lambda_{1t} + \widehat{\beta}_{2i}'\lambda_{2t} + \widehat{\beta}_{3i}'\lambda_{3t} + \alpha_{it}, \qquad i = 1, 2, ...7$$

for each t. We estimate each of the $\lambda's$ and α_i as $\widehat{\lambda}_{j,FM} = \frac{1}{T}\sum \widehat{\lambda}_{j,t}$ for $j = 1, 2, 3$ and $\widehat{\alpha}_{iFM} = \frac{1}{T}\sum \widehat{\alpha}_{it}$ with their corresponding standard errors. We also report Fama-MacBeth standard errors corrected for autocorrelation (refer to Section 4.1) and Shanken corrected standard errors. We find that the coefficients λ for the news betas for expected future inflation and expected future real rates are statistically significant. Here, the Fama-MacBeth χ^2 *test* statistic has a p-value of 0.59 in other words the Fama-MacBeth test does not reject the null that the pricing errors are zero at any reasonable significance level.

We also report, following the literature (see for example Cochrane, 2006) plots of the actual mean returns versus the model predictions. These graphs allow us to focus on the economically interesting pricing errors themselves and not just on whether a test statistic is large or small by statistical standards. Figure (1) shows that our model does reasonably well, in terms of the test portfolios lining up along the 45-degree line, in pricing the test assets.

6   Robustness Checks

6.1   Sensitivity to additional state variables

Our basic VAR includes two variables: excess bond return and real rate. In this Section we include the dividend yield on the CRSP VW index, since there is some evidence that returns on low-grade bond portfolios are predictable by the dividend yield.

Descriptive statistics of this variable are reported in the last column of Table (3). Our results are not materially altered when we include the dividend yield as an additional explanatory variable to the VAR (see Table 6). Moreover, the cross-sectional regression results reported in Table (11) are very similar to those corresponding to the analysis based on the original state variables, e.g. the inflation news component and the real rate

news component are significant, whereas the excess bond market news remains small and insignificant. For this new specification, the Fama-MacBeth χ^2 *test* statistic shows that there is not enough evidence to reject the null that the pricing errors are zero.

6.2 Augmented portfolio

It has become common practice to increase the number of test assets by adding different categories of portfolios into the analysis. Lewellen, Nagel and Shanken (2006) suggest that to improve empirical tests it is advisable to expand the set of test portfolios. For example, they suggest adding to the size B/M portfolios: industry, beta, volatility or factor loading-sorted portfolios. This paper puts forward the idea that all portfolios should be priced at the same time and not in separate cross-sectional regressions. In a recent paper, Lustig and Verdelhan (2007) analyse the cross-section of foreign currency premia and consumption growth simultaneously using 8 currency portfolios sorted on interest rates, 6 equity portfolios sorted on size and book to market and 5 bond portfolios.

In this section we add seven corporate bond industry portfolios to our original test assets[6]. These portfolios are obtained from Citigroup and include the following industrial sector classification: Manufacturing, Service, Transportation, Utility, Consumer, Energy and Other. We provide summary statistics of these portfolios in Table (12) and their cross-correlations are presented in Table (13).

The main results of the expanded portfolio of 14 test assets are summarized in Table (14). We find that the excess bond market news remains insignificant whereas the inflation news is significant using either ordinary Fama-MacBeth standard errors, standard errors corrected for autocorrelation or Shanken corrected standard errors. The real rate news component is marginally significant with Fama-MacBeth standard errors but loses its predictive ability when we use Shanken corrected standard errors. More importantly, our Fama-MacBeth chi-squared statistic, which tests whether all the pricing errors are zero, cannot reject the null hypothesis. Here, the statistic is 20.77 which is smaller than its 5% critical value, *i.e.* 24.72.

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[6]Results for the corporate industry portfolios alone show similar results to those for portfolios sorted on default categories.

Figure (7) presents plots of the actual mean returns versus the model predictions using several estimation techniques. We note that for this augmented portfolio our model does reasonably well, in terms of the test portfolios lining up along the 45-degree line, in pricing the test assets.

7   Conclusion

Though the bond market constitutes a separate asset class with a larger market value than the entire equity market, there has been less attention paid to the covariance risk of expected excess returns of bonds belonging to different risk classes. Some examples of this research are Chung and Huang (1990) and Gebhardt, Hvidkjaer and B. Swaminathan (2005). Previous research has either used stock market factor models augmented to include additional factors that affect bonds or used ad hoc models with factors that seem important in the context of bond markets. For example, Huij and Derwall (2005) measure bond fund performance relative to the return predicted by a variety of multi-index models used in the literature. The factors used in their models include proxies for the overall bond market, low-grade debt, and mortgage-backed securities and principal component based factors extracted from yield changes in certain ranges of the bond maturity spectrum. In contrast, in this paper, we provide a motivation for our news factors based on a simple present value decomposition for consol bonds. Further, we operationalize this using a VAR framework, as in Campbell and Vuolteenaho (2004), to extract factors from variables that forecast bond returns. Clearly, the limitations of this approach are that it assumes that the econometrician knows enough about the investor's information set through these variables and that the parameter of the VAR represent changes in the investor's environment. However, despite this our three factor model when taken to the data is able to give a reasonable account of the cross-sectional variation in expected bond returns.

Our main results are as follows. We use a return decomposition for a consol bond, which combined with Epstein-Zin preferences leads to a three factor ICAPM in the spirit of Campbell (1993,1996). An interesting feature of our three factor ICAPM for bonds is that it does not have the risk aversion coefficient as a free parameter and that the bond betas with the three factors are entirely data dependant. We test this model using

seven index portfolios of different default categories between 1993 and 2006 with a Fama-MacBeth chi-squared test. These results show that our model cannot be rejected. Of the three factors within our ICAPM, innovations in future inflation rates and future real rates were more important than news about future excess bond returns in determining the cross section of expected corporate bond returns. Our robustness tests show that our ICAPM results also hold for an expanded set of test assets which included seven additional industry bond portfolios.

There are a number of ways in which this study could be extended. First, one obvious concern is that our results are sample specific especially with respect to the choice of state variables. In ongoing work we are investigating techniques for estimation that may allow us to be more agnostic about this choice. Second, it would be useful to see how the model performs in the analysis of the performance of bond market mutual funds relative to models that use *ad hoc* factor representations.

	AAA	AA	A	BAA	BA	B	CA
Mean	.5828	.6020	.5987	.6102	.8100	.7586	.7893
Median	.5927	.6535	.6807	.6259	1.0398	.9704	1.1207
Maximum	4.0383	4.3893	4.4064	4.3988	4.9602	11.7125	16.3030
Minimum	-2.8876	-3.6452	-3.2060	-3.3286	-8.4681	-9.1155	-17.4148
Std. Dev	.0116	.0123	.0123	.0131	.01751	.0262	.0478
Skewness	-.1411	-.2619	-.1958	-.1190	-1.5477	-.3538	-.3003
Kurtosis	3.4208	3.7808	3.6266	3.3096	8.9113	7.2866	5.5342

Table 1: Desciptive Statistics. Lehman Corporate Bond Portfolios for different rating categories. Sample 01/1993-08/2006. Percentage holding period bond returns. Intermediate Maturity.

	AAA	AA	A	BAA	BA	B	CA
AAA	1	.9859	.9560	.8764	.3411	.0921	-.0508
AA	.9859	1	.9761	.9165	.4055	.1605	.0169
A	.9560	.9761	1	.9533	.4924	.2634	.1112
BAA	.8764	.9165	.9533	1	.6525	.4118	.2485
BA	.3411	.4055	.4924	.6525	1	.8364	.6920
B	.0921	.1605	.2634	.4118	.8364	1	.8249
CA	-.0508	.0169	.1112	.2485	.6920	.8249	1

Table 2: Pairwise Correlation Matrix. Lehman Corporate Bond Portfolios for different rating categories. Sample 01/1993-08/2006. Percentage holding period bond returns. Intermediate Maturity.

	bondmkt	real rate	credit spread	dividend
Mean	.2597	.0923	.0670	.1535
Median	.2554	.1241	.0633	.1426
Maximum	2.9888	1.1255	.1175	.3892
Minimum	-3.0169	-.9134	.0458	.0641
Std. Dev.	1.03913	.2980	.0172	.0561
Skewness	-.1941	-.3486	1.1911	1.2437
Kurtosis	3.2611	3.7986	3.7231	4.9077
ACF	.173	.396	.951	.294

Table 3: State Variables. Descriptive Statistics. Sample 10/1992-08/2006. bondmkt is the excess bond market return measured as the Lehman Brothers monthly US Aggregate bond return in excess of the 3 months treasury bill; real rate is the monthly real short-term interest rate; credit premium is the difference between Moody's Seasoned BAA and AAA Corporate Bond Yields; dividend yield is the difference between vwretd and vwretx from CRSP. The real rate is obtained as the difference between the risk-free rate and the growth rate in the CPI. The credit premium data and the CPI data is from the FRED database. ACF refers to the autocovariance function.

	bondmkt	real rate	credit spread	dividend
bondmkt	1	.0716	.0499	-.0009
real rate	.0716	1	-.1831	.0881
credit spread	.0499	-.1831	1	-.1997
dividend	-.0009	.0881	-.1997	1

Table 4: State Variables. Pairwise Correlations. Sample 10/1992-08/2006. bondmkt is the excess bond market return measured as the Lehman Brothers monthly US Aggregate bond return in excess of the 3 months treasury bill; real rate is the monthly real short-term interest rate; credit premium is the difference between Moody's Seasoned BAA and AAA Corporate Bond Yields; dividend yield is the difference between vwretd and vwretx from CRSP. The real rate is obtained as the difference between the risk-free rate and the growth rate in the CPI. The credit premium data and the CPI data is from the FRED database.

	bondmkt	real rate	credit spread
	.1598	.0006	−.0001
bondmkt (-1)	(.0764)	(.0204)	(.0004)
	[2.0918]	[.0331]	[−.4383]
	.5054	.3599	.0001
real rate (-1)	(.2705)	(.0723)	(.0014)
	[1.8679]	[4.9776]	[.1221]
	4.1747	−3.3726	.9531
credit spread (-1)	(4.6829)	(1.2513)	(.0247)
	[.8914]	[−2.6951]	[38.5747]
R-squared	.0528	.1931	.9055
F-statistic	2.9935	12.8484	514.4390

Table 5: VAR. Sample 10/1992-08/2006. All variables have been demeaned and a constant term has been included. bondmkt is the excess bond market return measured as the Lehman Brothers monthly US Aggregate bond return in excess of the 3 months treasury bill; real rate is the monthly real short-term interest rate; credit premium is the difference between Moody's Seasoned BAA and AAA Corporate Bond Yields. The real rate is obtained as the difference between the risk-free rate and the growth rate in the CPI. The credit premium data and the CPI data is from the FRED database. Figures correspond to OLS estimates, standard errors are inside parenthesis and t-statistics in brackets.

	bondmkt	real rate	credit spread	dividend
bondmkt (-1)	.1598	.0006	−.0001	.0033
	(.0766)	(.0204)	(.0004)	(.0038)
	[2.0853]	[.0341]	[−.4369]	[.8829]
real rate (-1)	.5028	.3630	.0002	−.0410
	(.2717)	(.0724)	(.0014)	(.0135)
	[1.8505]	[5.0104]	[.1642]	[−3.0225]
credit spread (-1)	4.3496	−3.5828	.9490	−.5603
	(4.7854)	(1.2759)	(.0252)	(.2392)
	[.9089]	[−2.8079]	[37.6672]	[−2.3426]
dividend (-1)	.2762	−.3318	−.0064	.2796
	(1.4451)	(.3853)	(.0076)	(.0722)
	[.1911]	[−.8611]	[−.8473]	[3.8713]
R-squared	.0530	.1968	.9059	.1594
F-statistic	2.2408	9.8063	385.3331	7.5854

Table 6: VAR. Sample 10/1992-08/2006. All variables have been demeaned and a constant term has been included. bondmkt is the excess bond market return measured as the Lehman Brothers monthly US Aggregate bond return in excess of the 3 months treasury bill; real rate is the monthly real short-term interest rate; credit premium is the difference between Moody's Seasoned BAA and AAA Corporate Bond Yields; and the dividend yield is the difference between vwretd and vwretx from CRSP. The real rate is obtained as the difference between the risk-free rate and the growth rate in the CPI. The credit premium data and the CPI data is from the FRED database. Figures correspond to OLS estimates, standard errors are inside parenthesis and t-statistics in brackets.

	Bondmkt News	Real Rate News	Inflation News
Bondmkt News	.139	-.073	-.241
Real Rate News	-.073	.258	-.201
Inflation News	-.241	-.201	1.64

Table 7: Variance-Covariance Matrix. The news components were obtained from the residuals and the companion matrix of a VAR with the following state variables (we include a constant and demeaned variables): bondmkt, real rate and credit premium. bondmkt is the excess bond market return measured as the Lehman Brothers monthly US Aggregate bond return in excess of the 3 months treasury bill; real rate is the monthly real short-term interest rate; credit premium is the difference between Moody's Seasoned BAA and AAA Corporate Bond Yields. The real rate is obtained as the difference between the risk-free rate and the growth rate in the CPI. The credit premium data and the CPI data is from the FRED database. Inflation news were obtained as a residual.

	AAA	AA	A	BAA	BA	B	CA
Bond Mkt News							
Estimate	-1.1062	-1.0953	-1.0335	-1.1025	-.4140	-.3695	-1.1277
OLS t-stat	-10.11	-9.69	-8.13	-5.84	-0.79	-0.45	-0.74
GMM t-stat	-9.92	-9.67	-7.85	-5.71	-0.83	-0.44	-0.77
Inflation News							
Estimate	-1.1067	-1.1625	-1.1420	-1.1505	-.5619	-.2893	-.1606
OLS t-stat	-35.81	-36.32	-31.81	-21.58	-3.83	-1.25	-0.37
GMM t-stat	-31.62	-33.02	-27.78	-21.70	-3.95	-1.30	-.38
Real Rate News							
Estimate	-1.1585	-1.1863	-1.0232	-.8798	.1588	1.0508	1.3004
OLS t-stat	-15.89	-15.73	-12.06	-6.99	0.45	1.93	1.28
GMM t-stat	-14.46	-15.40	-11.29	-6.35	0.44	1.86	1.38

Table 8: Time Series: The news components were obtained from the residuals and the companion matrix of a VAR with the following state variables (we include a constant and demeaned variables): bondmkt, real rate and credit premium. bondmkt is the excess bond market return measured as the Lehman Brothers monthly US Aggregate bond return in excess of the 3 months treasury bill; real rate is the monthly real short-term interest rate; credit premium is the difference between Moody's Seasoned BAA and AAA Corporate Bond Yields. The real rate is obtained as the difference between the risk-free rate and the growth rate in the CPI. The credit premium data and the CPI data is from the FRED database. Inflation news were obtained as a residual. The Corporate Bond Portfolios are bond market index portfolios of different default categories from Lehman Brothers. Sample 01/1993-08/2006. Excess bond returns. Intermediate Maturity.

	Excess Bond Market News	Inflation News	Real Rate News
Estimate	.0701	-.6336	.3328
Fama-MacBeth t-stat	.3118	-2.7416	2.1837
Fama-MacBeth t-stat corrected for autocorrelation	.2536	-3.0667	2.0317
Shanken corrected t-stat	.2434	-2.5703	1.7225
Fama-MacBeth chi-squared statistic	14.4723		
R^2	61.71%		

Table 9: Cross-Section: The news components were obtained from the residuals and the companion matrix of a VAR with the following state variables (we include a constant and demeaned variables): bondmkt, real rate and credit premium. bondmkt is the excess bond market return measured as the Lehman Brothers monthly US Aggregate bond return in excess of the 3 months treasury bill; real rate is the monthly real short-term interest rate; credit premium is the difference between Moody's Seasoned BAA and AAA Corporate Bond Yields. The real rate is obtained as the difference between the risk-free rate and the growth rate in the CPI. The credit premium data and the CPI data is from the FRED database. Inflation news were obtained as a residual.The Corporate Bond Portfolios are bond market index portfolios of different default categories from Lehman Brothers.

	AAA	AA	A	BAA	BA	B	CA
Bond Mkt News							
Estimate	-1.1170	-1.1203	-1.0618	-1.1375	-.4555	-.4676	-1.1460
OLS t-stat	-10.57	-10.25	-8.64	-6.24	-.90	-.59	-.78
GMM t-stat	-10.24	-10.04	-8.48	-6.13	-.97	-.59	-.82
Inflation News							
Estimate	-1.1082	-1.1664	-1.1467	-1.1563	-.5698	-.3060	-.1636
OLS t-stat	-36.33	-37.02	-32.39	-22.02	-3.91	-1.34	-.38
GMM t-stat	-31.84	-33.04	-28.24	-22.10	-4.06	-1.41	-.40
Real Rate News							
Estimate	-1.1621	-1.1976	-1.0392	-.9019	.1227	.9784	1.2612
OLS t-stat	-16.46	-16.40	-12.65	-7.41	.36	1.86	1.28
GMM t-stat	-14.99	-16.01	-12.05	-6.73	.35	1.79	1.39

Table 10: Time Series: The news components were obtained from the residuals and the companion matrix of a VAR with the following state variables (we include a constant and demeaned variables): bondmkt, real rate, credit premium and dividend yield. bondmkt is the excess bond market return measured as the Lehman Brothers monthly US Aggregate bond return in excess of the 3 months treasury bill; real rate is the monthly real short-term interest rate; credit premium is the difference between Moody's Seasoned BAA and AAA Corporate Bond Yields; and the dividend yield is the difference between vwretd and vwretx from CRSP. The real rate is obtained as the difference between the risk-free rate and the growth rate in the CPI. The credit premium data and the CPI data is from the FRED database. Inflation news were obtained as a residual. The Corporate Bond Portfolios are bond market index portfolios of different default categories from Lehman Brothers. Sample 01/1993-08/2006. Excess bond returns. Intermediate Maturity.

	Excess Bond Market News	Inflation News	Real Rate News
Estimate	.0865	-.6692	.3510
Fama-MacBeth t-stat	.3665	-2.6429	2.2718
Fama-MacBeth t-stat corrected for autocorrelation	.2983	-2.9197	2.1964
Shanken corrected t-stat	.2799	-2.0821	1.7541
Fama-MacBeth chi-squared statistic	14.2126		
R^2	62%		

Table 11: Cross-Section: The news components were obtained from the residuals and the companion matrix of a VAR with the following state variables (we include a constant and demeaned variables): bondmkt, real rate, credit premium and dividend yield. bondmkt is the excess bond market return measured as the Lehman Brothers monthly US Aggregate bond return in excess of the 3 months treasury bill; real rate is the monthly real short-term interest rate; credit premium is the difference between Moody's Seasoned BAA and AAA Corporate Bond Yields; and the dividend yield is the difference between vwretd and vwretx from CRSP. The real rate is obtained as the difference between the risk-free rate and the growth rate in the CPI. The credit premium data and the CPI data is from the FRED database. Inflation news were obtained as a residual. The Corporate Bond Portfolios are bond market index portfolios of different default categories from Lehman Brothers. Sample 01/1993-08/2006. Excess bond returns. Intermediate Maturity.

	Manufacturing	Transport	Consumer	Energy	Service	Other	Utility
Mean	.5742	.6374	.6021	.5770	.5989	.5863	.5486
Median	.5485	.6765	.6342	.6301	.6192	.5665	.5712
Maximum	5.1549	5.5990	5.1849	5.3689	5.3889	10.1852	5.7564
Minimum	-3.3077	-4.6320	-4.7813	-4.9907	-4.0849	-5.5047	-4.2018
Std. Dev	.0143	.0169	.0153	.0160	.0153	.0181	.0166
Skewness	.0001	-.2724	-.2810	-.3077	-.0679	.5231	-.1076
Kurtosis	.3904	.6617	.7518	.8052	.7090	5.2462	.6390

Table 12: Descriptive Statistics. Industry Bond Portfolios: Citigroup. Manufacturing includes: Aerospace/Defence, Automotive Manufacturers, Building Products, Chemicals, Conglomerate, Electronics, Information/Data Technology, Machinery, Metals/Mining, Paper/Forest Products, Textiles/Apparel/Shoes, Vehicle Parts, Maufacturing-Other. Service includes: Cable/Media, Gaming/Lodging/Leisure, Healthcare Supply, Pharmaceuticals, Publishing, Restaurants, Food/Drugs, Retails Stores-Other, Service-Other. Transportation includes: Airlines, Railroads, Transportation-Other. Consumer includes: Beverage/Bottling, Consumer Products, Food Processors, Tobacco. Utility includes: Electric, Power, Gas-Local Distribution, Telecommunications, Utility-Other. Energy includes: Gas-Pipelines, Oil and Gas, Oilfield Machinery and Services. Sample 01/1993-08/2006. Percentage bond returns.

	manufacturing	transport	consumer	energy	service	other	utility
manufacturing	1	.9196	.9165	.8920	.9460	.8110	.8485
transport	.9196	1	.9189	.8813	.9138	.7571	.7810
consumer	.9165	.9189	1	.9078	.9456	.8042	.8239
energy	.8920	.8813	.9078	1	.9433	.7141	.8779
service	.9460	.9138	.9456	.9433	1	.8159	.9158
other	.8110	.7571	.8042	.7141	.8159	1	.6696
utility	.8485	.7810	.8230	.8779	.9158	.6696	1

Table 13: Pairwise Correlation Matrix. Industry Bond Portfolios: Citigroup. Manufacturing includes: Aerospace/Defence, Automotive Manufacturers, Building Products, Chemicals, Conglomerate, Electronics, Information/Data Technology, Machinery, Metals/Mining, Paper/Forest Products, Textiles/Apparel/Shoes, Vehicle Parts, Maufacturing-Other. Service includes: Cable/Media, Gaming/Lodging/Leisure, Healthcare Supply, Pharmaceuticals, Publishing, Restaurants, Food/Drugs, Retails Stores-Other, Service-Other. Transportation includes: Airlines, Railroads, Transportation-Other. Consumer includes: Beverage/Bottling, Consumer Products, Food Processors, Tobacco. Utility includes: Electric, Power, Gas-Local Distribution, Telecommunications, Utility-Other. Energy includes: Gas-Pipelines, Oil and Gas, Oilfield Machinery and Services. Sample 01/1993-08/2006.

	Excess Bond Market News	Inflation News	Real Rate News
Estimate	-.0462	-.4238	.2873
Fama-MacBeth t-stat	-.3265	-2.7918	1.7945
Fama-MacBeth t-stat corrected for autocorrelation	-.2741	-3.1626	1.4740
Shanken corrected t-stat	-.2834	-2.559	1.5622
Fama-MacBeth chi-squared statistic	20.7717		
R^2	63%		

Table 14: Cross-Section: The news components were obtained from the residuals and the companion matrix of a VAR with the following state variables (we include a constant and demeaned variables): bondmkt, real rate and credit premium. bondmkt is the excess bond market return measured as the Lehman Brothers monthly US Aggregate bond return in excess of the 3 months treasury bill; real rate is the monthly real short-term interest rate; credit premium is the difference between Moody's Seasoned BAA and AAA Corporate Bond Yields. The real rate is obtained as the difference between the risk-free rate and the growth rate in the CPI. The credit premium data and the CPI data is from the FRED database. Inflation news were obtained as a residual. Our test assets are 7 industry corporate bond portfolios obtained from Citigroup and 7 Corporate bond market index portfolios of different default categories from Lehman Brothers.
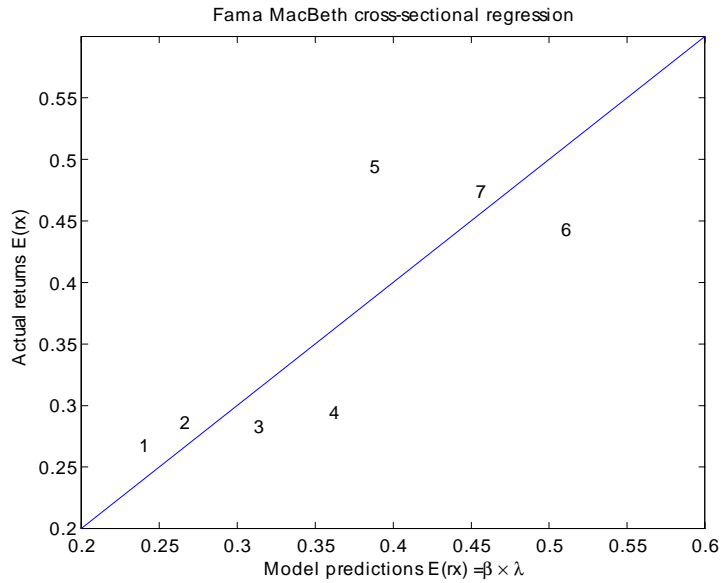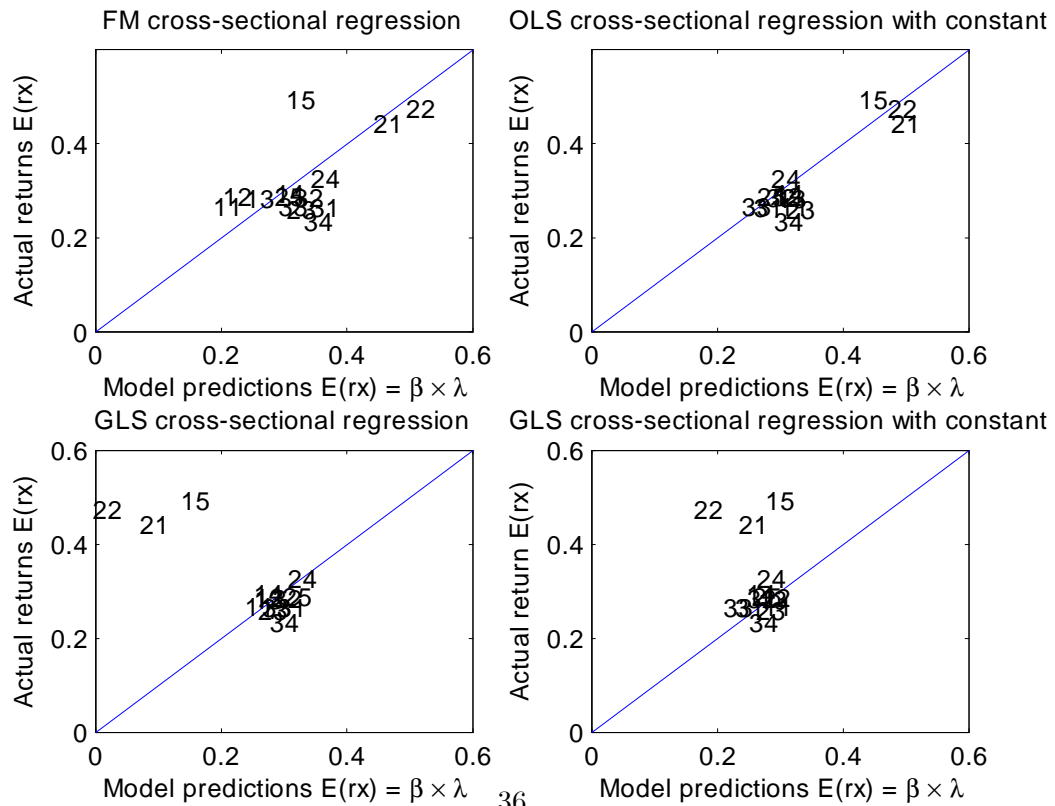
Figure 1: Sample 01/1993-08/2006. Index portfolios of different default categories from Lehman Brothers. Excess bond returns. Intermediate Maturity. The numbers correspond to 1: AAA, 2:AA, 3:A, 4:BAA, 5:BA, 6:B, 7:CA.

Sample 01/1993-08/2006. Seven intermediate maturity index corporate portfolios of different default categories from Lehman Brothers and seven corporate bond indices classified by Industry Sector from Citigroup. The numbers correspond to 11: AAA, 12: AA, 13: A, 14: BAA, 15: BA, 21: B, 22: CA, 23: manufacturing, 24: service, 25: transportation, 31: utility, 32: consumer, 33: energy, 34: other.

A P P E N D I C E S

This Appendix provides details of the bond return decomposition, the factor model and the VAR methodology used in the paper. It collects at one place and draws heavily on previous work by Campbell (1993, 1996), Campbell and Ammer (1993), Shiller and Beltratti (1992) and Campbell and Vuolteenaho (2004).

7.1 Bond Decomposition

There are two versions of the variance decomposition for bonds in the literature. The first uses a zero coupon bond and the second a consol bond.

7.1.1 Zero Coupon Bond

Following the notation in Campbell and Ammer (1993), we define:

$P_{n,t}$ = Price at time t of a discount bond maturing with n periods to maturity i.e. maturing and paying \$1 at $t + n$.

$P_{n,t} = \frac{1}{(1+Y_{n,t})^n}$, where $Y_{n,t}$ is the yield-to-maturity.

In logs, $p_{n,t} = \log(P_{n,t}) = \log\left(\frac{1}{(1+Y_{n,t})^n}\right) = -n y_{n,t}$ or $y_{n,t} = -\frac{1}{n} p_{n,t}$

The Holding Period Return, for one period from t to $t + 1$, is by definition $B_{n,t+1} = \frac{P_{n-1,t+1}}{P_{n,t}}$ since at time $t + 1$ the bond has $n - 1$ periods left to maturity. The log holding period return from t to $t + 1$, is therefore given by

$$b_{n,t+1} \equiv p_{n-1,t+1} - p_{n,t} \tag{A1}$$

where $log(B_{n,t+1}) = b_{n,t+1}$.

The above equation (A1) is a difference equation and we can write:

$$p_{n,t} = p_{n-1,t+1} - b_{n,t+1}$$

By recursive substitution

$$p_{n,t} = p_{n-1,t+1} - b_{n,t+1}$$

$$\therefore \quad p_{n-1,t+1} = p_{n-2,t+2} - b_{n-1,t+2}$$

$$\therefore \quad p_{n,t} = p_{n-2,t+2} - b_{n-1,t+2} - b_{n,t+1}$$

$$.....$$

$$\therefore \quad p_{n,t} = p_{n-n,t+n} - [b_{n,t+1} + b_{n-1,t+2} + \cdots + b_{1,t+n}]$$

But at maturity , $p_{n-n,t+n} = p_{0,t+n} = \ln(1) = 0$

$$\therefore \quad p_{n,t} = -[b_{n,t+1} + b_{n-1,t+2} + \cdots + b_{1,t+n}]$$

$$\therefore \quad p_{n,t} = -\sum_{i=0}^{n-1} [b_{n-i,t+i+1}]$$

we get:

$$p_{n,t} = -\sum_{i=0}^{n-1} [b_{n-i,t+i+1}] \tag{A2}$$

which holds both ex-post and ex-ante. Taking expectations at time t

$$E_t(p_{n,t}) = p_{n,t} = -E_t \sum_{i=0}^{n-1} [b_{n-i,t+i+1}] \tag{A3}$$

Next we substitute Equation (A3) in Equation (A1)

$$b_{n,t+1} \equiv p_{n-1,t+1} - p_{n,t}$$

$$b_{n,t+1} \equiv p_{n-1,t+1} + E_t \sum_{i=0}^{n-1} [b_{n-i,t+i+1}]$$

$$b_{n,t+1} \equiv p_{n-1,t+1} + E_t[b_{n,t+1}] + E_t \sum_{i=1}^{n-1} [b_{n-i,t+i+1}]$$

$$But, \quad p_{n-1,t+1} = -E_{t+1} \sum_{i=1}^{n-1} [b_{n-i,t+i+1}]$$

$$\therefore \quad b_{n,t+1} - E_t[b_{n,t+1}] \equiv -E_{t+1} \sum_{i=1}^{n-1} [b_{n-i,t+i+1}] + E_t \sum_{i=1}^{n-1} [b_{n-i,t+i+1}]$$

$$\therefore \quad b_{n,t+1} - E_t[b_{n,t+1}] \equiv -(E_{t+1} - E_t) \sum_{i=1}^{n-1} [b_{n-i,t+i+1}]$$

In economic terms the equation:

$$b_{n,t+1} - E_t\left[b_{n,t+1}\right] \equiv -\left(E_{t+1} - E_t\right)\sum_{i=1}^{n-1}\left[b_{n-i,t+i+1}\right] \tag{A4}$$

implies that since nominal bond returns are known over the life of the bond, unexpected positive nominal returns today are always offset by decreases in expected future nominal returns. Campbell and Ammer (1993) further write Equation. (A4) in terms of an excess bond return by defining the log one period excess bond return as :

$$x_{n,t+1} \equiv b_{n,t+1} - \pi_{t+1} - r_{t+1} \tag{A5}$$

where

$x_{n,t+1}$ =log excess one-period return on an n-period zero coupon bond held from t to $t + 1$.

π_{t+1} =log one-period inflation rate from t to $t + 1$ and

r_{t+1} = log one-period real rate from t to $t + 1$.

Substituting Equation (A5) into Equation (A4) gives:

$$\widetilde{x}_{n,t+1} = \left(E_{t+1} - E_t\right)\left\{-\sum_{i=1}^{n-1}\pi_{t+1+i} - \sum_{i=1}^{n-1}r_{t+1+i} - \sum_{i=1}^{n-1}x_{n-i,t+1+i}\right\} \tag{A6}$$

Using a more compact notation (again following Campbell and Ammer (1993) for convenience) where a tilde denotes an innovation, define:

$\widetilde{x}_{n,t+1}$ = the innovation in the log excess one-period return on a zero coupon bond held from t to $t + 1$

$\widetilde{x}_{x,t+1}$ = the innovation in the *future* log excess one-period return on a zero coupon bond held from t to $t + 1$

$\widetilde{x}_{r,t+1}$ = innovation in the *future* log excess one-period real return

$\widetilde{x}_{\pi,t+1}$ =innovation in the *future* log excess one-period inflation.

Substituting in Equation (A6) above we get

$$\widetilde{x}_{n,t+1} = -\widetilde{x}_{x,t+1} - \widetilde{x}_{r,t+1} - \widetilde{x}_{\pi,t+1} \tag{A7}$$

7.1.2 Consol Bond

Campbell (1993) uses a log-linear approximation to the return on a real consol bond that pays one unit of consumption good each period and with no maturity date. Here we follow Shiller and Beltratti (1992) and Engsted and Tanggaard (2001) and use a log-linear version of the present value of a nominal consol bond or a perpetuity. We denote the coupon by C and the price P_{bt}, then the log one period gross return from t to $t+1$ is given by:

$$r_{b,t+1} = \ln\left(\frac{C + P_{bt+1}}{P_{bt}}\right) = \ln\left(C + \exp\left(p_{b,t+1}\right)\right) - p_{b,t} \tag{A8}$$

We now take a first order Taylor expansion around the mean of $ln\left(C + \exp\left(p_{b,t+1}\right)\right)$

$$
\begin{aligned}
&\ln\left(C + \exp\left(p_{b,t+1}\right)\right) \\
&\approx \quad \ln\left(C + \exp\left(E_t\left(p_{b,t+1}\right)\right)\right) - E_t\left(p_{b,t+1}\right)\frac{\exp\left(E_t\left(p_{b,t+1}\right)\right)}{C + \exp\left(E_t\left(p_{b,t+1}\right)\right)}
\end{aligned}
$$

to get

$$
\begin{aligned}
r_{b,t+1} \; &= \; \underbrace{\ln\left(C + \exp\left(E_t\left(p_{b,t+1}\right)\right)\right) - E_t\left(p_{b,t+1}\right)\frac{\exp\left(E_t\left(p_{b,t+1}\right)\right)}{C + \exp\left(E_t\left(p_{b,t+1}\right)\right)}}_{\kappa_b} \\
&\quad + \underbrace{\frac{\exp\left(E_t\left(p_{b,t+1}\right)\right)}{C + \exp\left(E_t\left(p_{b,t+1}\right)\right)}}_{\rho_b}p_{b,t+1} - p_{b,t} \\
r_{b,t+1} \; &= \; \kappa_b + \rho_b p_{b,t+1} - p_{b,t}
\end{aligned}
$$

where $\kappa_b = \ln\left(C + \exp\left(E_t\left(p_{b,t+1}\right)\right)\right) - E_t\left(p_{b,t+1}\right)\frac{\exp\left(E_t\left(p_{b,t+1}\right)\right)}{C+\exp\left(E_t\left(p_{b,t+1}\right)\right)}$ is a constant arising from the linearization. The term ρ_b, given by

$\rho_b \equiv \frac{\exp\left(E_t\left(p_{b,t+1}\right)\right)}{C+\exp\left(E_t\left(p_{b,t+1}\right)\right)} \approx \frac{E\left(P_{b,t+1}\right)}{C+E\left(P_{b,t+1}\right)} \approx \frac{E\left(P_{b,t}\right)}{C+E\left(P_{b,t+1}\right)} = \frac{1}{E\left(R_{b,t+1}\right)}$, is approximately equal to $R_{b,t+1} \equiv \frac{C+P_{bt+1}}{P_{bt}}$.

Now,

$$r_{bt} = \kappa_b + \rho_b - p_{b,t+1} - p_{b,t} \tag{A9}$$

is a difference equation in the log bond price p_{bt}. We can solve Equation. (A9) forward, impose the usual transversality condition and take conditional expectations at time t, to get:

$$p_{b,t} \equiv -E_t \sum_{j=0}^{\infty} \rho_b^j r_{b,t+1+j} \qquad (A10)$$

We, can substitute this back into

$$r_{b,t+1} = \kappa_b + \rho_b p_{b,t+1} - p_{b,t} p_{b,t}$$

to get

$$r_{b,t+1} = \kappa_b + \rho_b \left(-E_{t+1} \sum_{j=0}^{\infty} \rho_b^j r_{b,t+2+j} \right) - \left(-E_t \sum_{j=0}^{\infty} \rho_b^j r_{b,t+1+j} \right)$$

$$r_{b,t+1} = \kappa_b - \rho_b \left(E_{t+1} \sum_{j=0}^{\infty} \rho_b^j r_{b,t+2+j} \right) + \left(E_t \sum_{j=0}^{\infty} \rho_b^j r_{b,t+1+j} \right)$$

$$r_{b,t+1} = \kappa_b - \rho_b \left(E_{t+1} \sum_{j=0}^{\infty} \rho_b^j r_{b,t+2+j} \right) + \left(E_t \sum_{j=0}^{\infty} \rho_b^j r_{b,t+1+j} \right)$$

$$\therefore \quad (E_{t+1} - E_t) \, r_{b,t+1} = - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho_b^j r_{b,t+1+j}$$

If we assume that $\rho_b = \rho$, in other words that the linearization constant for bonds is approximately equal to the linearization coefficient for the intertemporal budget constraint, then we get

$$(E_{t+1} - E_t) \, r_{b,t+1} = - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{b,t+1+j} \qquad (A11)$$

To obtain excess returns, we add and subtract the risk free rate and use the fact that $(E_{t+1} - E_t) \, r_{f,t} = 0$, we get the decomposition for innovations in the excess bond returns:

$$(E_{t+1} - E_t) \, (r_{b,t+1} - r_{f,t+1}) = - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \, (r_{b,t+1} - r_{f,t+1+j}) - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{f,t+1+j} \qquad (A12)$$

Next, we can write the nominal risk free rate as

$$r_{f,t+1} = r_{r,t+1} + \pi_{t+1}$$

where $r_{r,t+1}$ and $\pi_{t+1}$are the real interest rate and then the last term in the Equation (A12) can be written as:

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{f,t+1+j} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{rt+1+j} + (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \pi_{t+1+j} \quad (A13)$$

Thus we can write

$$
\begin{aligned}
(E_{t+1} - E_t)(r_{b,t+1} - r_{f,t+1}) &= -(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j (r_{b,t+1+j} - r_{f,t+1+j}) - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{f,t+1+j} \\
&= -(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j (r_{b,t+1} - r_{f,t+1+j}) - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{r,t+1+j} \\
&\quad -(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \pi_{t+1+j} \quad\quad (A14)
\end{aligned}
$$

Now, for ease of exposition, we use the notation in Campbell and Ammer (1993).for "innovations" and define

$\widetilde{x}_{b,t+1} = (E_{t+1} - E_t)(r_{b,t+1} - r_{f,t+1})$ is the innovation in the log excess one-period return on a consol bond from t to $t+1$,

$\widetilde{x}_{x,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j (r_{b,t+1} - r_{f,t+1+j})$ is the innovation in the *future* log excess one-period return on a consol bond held from t to $t+1$,

$\widetilde{x}_{r,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{r,t+1+j}$ is the innovation in the log excess one-period real return,

$\widetilde{x}_{\pi,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \pi_{t+1+j}$ is the innovation in the log excess one-period inflation.

Substituting in the above expression we get

$$\widetilde{x}_{b,t+1} = -\widetilde{x}_{\pi,t+1} - \widetilde{x}_{r,t+1} - \widetilde{x}_{x,t+1} \quad (A15)$$

This expression implies that unexpected excess bond returns must be due to "news" i.e. changes in expectations about either future excess bond returns, or future inflation or future real interest rates or combinations of these three. For example, news that either

43

inflation, real interest rates or excess returns will be higher (lower) in the future, will lead to a fall (increase) in excess bond returns. This expression is a dynamic accounting identity and holds by construction having been obtained from the definition of the return on a consol bond. Note that if both the Fisher Hypothesis and the Expectations Theory hold then inflation news would be the only source of variation in excess bond return innovations. For example, if the Fisher Hypothesis holds then i.e. nominal bond yields move one-for-one with expected inflation so that ex ante real interest rates are constants. This implies that "news about future real rates" is constant or the component $(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{r,t+1+j}$ is zero. If, for example, the Expectations Hypothesis holds then we know that the long-term bond yield is given as the expected future short rates plus a time varying term premium. This implies that expected excess bond returns are constant so that the "news about future excess returns" component $(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \left(r_{b,t+1} - r_{f,t+1+j} \right)$ is zero.

7.2 Expression for log SDF

We follow Campbell (1993, 1996) and use the Epstein-Zin utility function, defined recursively, for an infinitely lived representative agent as

$$U_t = \left[(1-\delta)^{\frac{1-\gamma}{\theta}} + \delta \left(E_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \tag{A16}$$

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, ψ is the elasticity of intertemporal substitution, γ is the coefficient of relative risk aversion, δ is a time discount factor and C_t is consumption. We assume that our investor can only invest in the bond market or in bond market mutual funds i.e. the aggregate bond market portfolio. The Euler equation for asset i, following Epstein and Zin (1989,1991), has an associated pricing equation in simple returns given by

$$1 = E_t \left[\left\{ \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\varphi}} \right\}^{\theta} \left\{ \left(\frac{1}{R_{B,t+1}} \right) \right\}^{1-\theta} R_{i,t+1} \right] \tag{A17}$$

with the corresponding SDF

$$M_{t+1} = \delta^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} \left(\frac{1}{R_{B,t+1}} \right)^{1-\theta} .; \tag{A18}$$

The log of the SDF is

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} - (1-\theta) r_{B,t+1}. \tag{A19}$$

Adding and subtracting both $\frac{\theta}{\psi} E_t\left(\Delta c_{t+1}\right) \Delta c_{t+1}$ and $(1-\theta) E_t\left(r_{b,t+1}\right)$ from the above equality leads to

$$
\begin{aligned}
m_{t+1} \;=\;& \theta \log \delta - \frac{\theta}{\psi}\Delta c_{t+1} \underbrace{-\frac{\theta}{\psi}E_t\left(\Delta c_{t+1}\right) + \frac{\theta}{\psi}E_t\left(\Delta c_{t+1}\right)} \\
& - (1-\theta)\, r_{B,t+1} \underbrace{- (1-\theta) E_t\left(r_{B,t+1}\right) + (1-\theta) E_t\left(r_{B,t+1}\right)}
\end{aligned}
$$

Regrouping terms

$$
\begin{aligned}
m_{t+1} \;=\;& \underbrace{\theta \log \delta - \frac{\theta}{\psi}E_t\left(\Delta c_{t+1}\right) - (1-\theta) E_t\left(r_{B,t+1}\right)}_{=E_t(m_{t+1})} \\
& - \frac{\theta}{\psi}\left(\Delta c_{t+1} - E_t\left(\Delta c_{t+1}\right)\right) - (1-\theta)\left(r_{B,t+1} - E_t\left(r_{B,t+1}\right)\right).
\end{aligned}
$$

The above expression can be written as

$$
m_{t+1} = E_t\left(m_{t+1}\right) - \frac{\theta}{\psi}\left(\Delta c_{t+1} - E_t\left(\Delta c_{t+1}\right)\right) - (1-\theta)\left(r_{B,t+1} - E_t\left(r_{B,t+1}\right)\right) \qquad \text{(A20)}
$$

We know that :

$$
\begin{aligned}
\Delta c_{t+1} - E_t\left(\Delta c_{t+1}\right) &= \log\left(\tfrac{C_{t+1}}{C_t}\right) - E_t\left(\log\left(\tfrac{C_{t+1}}{C_t}\right)\right) \\
&= c_{t+1} - c_t - E_t c_{t+1} - E_t c_t \\
&= c_{t+1} - E_t c_{t+1} \underbrace{-c_t - E_t c_t}_{=0} \\
\implies \Delta c_{t+1} - E_t\left(\Delta c_{t+1}\right) &= c_{t+1} - E_t c_{t+1}
\end{aligned}
$$

Hence we can write the log SDF as

$$
m_{t+1} = E_t\left(m_{t+1}\right) - \frac{\theta}{\psi}\left(c_{t+1} - E_t\left(c_{t+1}\right)\right) - (1-\theta)\left(r_{B,t+1} - E_t\left(r_{B,t+1}\right)\right) \qquad \text{(A21)}
$$

We next use the following result from Campbell (1993) (Equation 21, page 494) reproduced below:

$$
c_{t+1} - E_t c_{t+1} = r_{b,t+1} - E_t\left(r_{b,t+1}\right) + (1-\psi)\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty} \rho^j r_{B,t+1+j} \qquad \text{(A22)}
$$

to substitute out consumption in the expression (Equation. 21) for the SDF above to get

$$
\begin{aligned}
m_{t+1} &= E_t\left(m_{t+1}\right) - \frac{\theta}{\psi}\left(r_{B,t+1} - E_t\left(r_{B,t+1}\right) + (1-\psi)\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r_{B,t+1+j}\right) \\
&\quad - (1-\theta)\left(r_{B,t+1} - E_t\left(r_{B,t+1}\right)\right) \\
&\because \quad \theta = \frac{1-\gamma}{1-\frac{1}{\psi}}, \therefore \frac{\theta}{\psi} = \frac{1-\gamma}{\psi-1} \\
&\therefore \quad m_{t+1} = E_t\left(m_{t+1}\right) - \left(\frac{\theta}{\psi} + (1-\theta)\right)\left(r_{B,t+1} - E_t\left(r_{B,t+1}\right)\right) \\
&\quad - (1-\psi)\left(\frac{1-\gamma}{\psi-1}\right)\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r_{B,t+1+j} \\
&\because \quad \theta = \frac{1-\gamma}{1-\frac{1}{\psi}}, \therefore \frac{\theta}{\psi} = \frac{1-\gamma}{\psi-1} \\
&\therefore \quad m_{t+1} = E_t\left(m_{t+1}\right) - \gamma\left(r_{B,t+1} - E_t\left(r_{B,t+1}\right)\right) \\
&\quad + (1-\gamma)\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r_{B,t+1+j}.
\end{aligned}
$$

Adding and subtracting r_f from the right hand side gives

$$
\begin{aligned}
m_{t+1} &= E_t\left(m_{t+1}\right) - \gamma\left(r_{B,t+1} - E_t\left(r_{B,t+1}\right)\right) + (1-\gamma)\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r_{B,t+1+j} + r_f - r_f \\
&\therefore \quad m_{t+1} = E_t\left(m_{t+1}\right) - \gamma\left(E_{t+1} - E_t\right)\left(r_{B,t+1} - r_{f,t+1}\right) - \gamma\left(E_{t+1} - E_t\right)r_{f,t+1} \\
&\quad + (1-\gamma)\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j\left(r_{B,t+1+j} - r_{f,t+1+j}\right) + (1-\gamma)\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r_{f,t+1+j} \\
&\because \quad \left(E_{t+1} - E_t\right)r_{f,t+1} = 0 \\
m_{t+1} &= E_t\left(m_{t+1}\right) - \gamma\left(E_{t+1} - E_t\right)\left(r_{B,t+1} - r_{f,t+1}\right) \\
&\quad + (1-\gamma)\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j\left(r_{B,t+1+j} - r_{f,t+1+j}\right) + (1-\gamma)\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r_{f,t+1+j}.
\end{aligned}
$$

Now we substitute using the following relations:

$$
\left(E_{t+1} - E_t\right)\left(r_{B,t+1} - r_f\right) = -\tilde{x}_{\pi,t+1} - \tilde{x}_{r,t+1} - \tilde{x}_{x,t+1}
$$

to get

$$
\begin{aligned}
m_{t+1} =\ & E_t\left(m_{t+1}\right) - \gamma\left(E_{t+1} - E_t\right)\left(r_{B,t+1} - r_{f,t+1}\right) \\
& + (1-\gamma)\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j\left(r_{B,t+1+j} - r_{f,t+1+j}\right) + (1-\gamma)\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r_{f,t+1+j} \\
\therefore\quad m_{t+1} =\ & E_t\left(m_{t+1}\right) - \gamma\left(-\widetilde{x}_{\pi,t+1} - \widetilde{x}_{r,t+1} - \widetilde{x}_{x,t+1}\right) + (1-\gamma)\left(\widetilde{x}_{x,t+1}\right) + (1-\gamma)\left(\widetilde{x}_{\pi,t+1} + \widetilde{x}_{r,t+1}\right) \\
\therefore\quad m_{t+1} =\ & E_t\left(m_{t+1}\right) + \widetilde{x}_{\pi,t+1} + \widetilde{x}_{r,t+1} + \widetilde{x}_{x,t+1}
\end{aligned}
$$

Notice here, that unlike in the case of the stock market, the bond market decomposition does not have any free parameter i.e. γ. (see for example Campbell and Vuolteenaho, 2004).

7.3 The Expected Return Beta Model with Bond Market News Components

Next we use a standard result from Cochrane (2005)'s text which is as follow:

Given

$$
E_t\left(M_{t+1}R_{t+1}\right) = 1
$$

and assuming that the log of the SDF m_{t+1} is a linear function of the K risk factors $\mathbf{f}_{t+1}$

$$
m_{t+1} = a + b'\mathbf{f}_{t+1} \tag{A23}
$$

then the unconditional model in expected return form for returns in logs ($r_{i,t+1} = \ln\left(R_{t+1}\right)$) is

$$
E\left(r_{i,t+1} - r_{f,t+1}\right) + \frac{\sigma_i^2}{2} = b'cov\left(r_{t+1}, f_{t+1}\right) \tag{A24}
$$

which can be put in the expected return-beta form:

$$
E\left(r_{i,t+1} - r_{f,t+1}\right) + \frac{\sigma_i^2}{2} = \lambda'\beta_i \tag{A25}
$$

where

$\beta_i = [Var(f_{t+1})]^{-1} Cov(r_{t+1}, f_{t+1}) =$ vector with the K betas for asset i

$\lambda = -Var(f_{t+1})b =$ vector of factor risk prices

This can also be written in vector notation as follows

$$E(r_{i,t+1} - r_{f,t+1}\mathbf{1}_N) + \frac{1}{2}diag(Var(r_{i,t+1})) = \beta\lambda \qquad (A26)$$

where

$\beta = Cov(r_{t+1}, f_{t+1})[Var(f_{t+1})]^{-1} = N \times K$ factor beta matrix with row i of factor loadings for asset i and $\mathbf{1}$ is a N-dimension vector of ones.

Now, we define

$$\mathbf{f}_{t+1} = \left(\widetilde{x}_{\pi,t+1}, \widetilde{x}_{r,t+1}, \widetilde{x}_{x,t+1}\right)'$$
$$\mathbf{b} = (1,1,1)$$

Since

$$m_{t+1} = E_t(m_{t+1}) + \widetilde{x}_{\pi,t+1} + \widetilde{x}_{r,t+1} + \widetilde{x}_{x,t+1} \qquad (A27)$$

we get

$$E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = -\sigma_{i,EBR} - \sigma_{i,RR} - \sigma_{i,INFL} \qquad (A28)$$

where

$\sigma_{i,EBR} = Cov\left(r_{i,t}, \widetilde{x}_{x,t+1}\right) =$ covariance of asset return with bond excess return news

$\sigma_{i,RR} = Cov\left(r_{i,t}, \widetilde{x}_{r,t+1}\right) =$ covariance of asset return with real interest rate news

$\sigma_{i,INFL} = Cov\left(r_{i,t}, \widetilde{x}_{\pi,t+1}\right) =$ covariance of asset return with inflation news.

We can write equation above in terms of factor betas' risk prices as

$$E(r_{i,t+1} - r_{f,t+1}) + \frac{\sigma_i^2}{2} = -\sigma_x^2\beta_{i,EBR} - \sigma_r^2\beta_{i,RR} - \sigma_\pi^2\beta_{i,INFL}, \qquad (A29)$$

where σ_{RBR}^2, σ_{RR}^2, and σ_{INFL}^2 are the variances of $\widetilde{x}_{x,t+1}, \widetilde{x}_{r,t+1}$, and $\widetilde{x}_{\pi,t+1}$. The risk prices for betas can be derived by defining $\lambda = (\lambda_{EBR}, \lambda_{RR}, \lambda_{INFL})^T = \sigma_f b$, where σ_f is

a diagonal matrix with the factor variances along its main diagonal. In addition we can rewrite the model in an expected return-beta representation, *i.e.* :

$$E\left(r_{i,t+1} - r_{f,t+1}\right) + \frac{\sigma_i^2}{2} = \boldsymbol{\lambda}^T \boldsymbol{\beta}_i = \lambda_{EBR}\beta_{i,EBR} + \lambda_{RR}\beta_{i,RR} + \lambda_{INFL}\beta_{i,INFL} \quad \text{(A30)}$$

where $\boldsymbol{\lambda} = (\lambda_x, \lambda_r, \lambda_\pi)^T = -Var(\mathbf{f}_{t+1})\mathbf{b}$ denotes the vector of factor risk prices and $\boldsymbol{\beta}_i = Var(\mathbf{f}_{t+1})^{-1} Cov(r_{i,t+1}, \mathbf{f}_{t+1})$ represents the (3×1) vector of multiple regression betas for asset i. The $\lambda's$ represent the risk prices of multiple regression beta risk for each of the factors. Finally we take unconditional expectations and rewrite the left hand side in simple expected return form, to obtain our three beta model for the bond market

$$E\left(r_{i,t+1} - r_{f,t+1}\right) = \lambda_{EBR}\beta_{i,EBR} + \lambda_{RR}\beta_{i,RR} + \lambda_{INFL}\beta_{i,INFL} \quad \text{(A31)}$$

7.3.1   VAR Estimation and Extraction of News Components

We can now use the VAR approach of Campbell and Shiller to extract the components of Equation (A15) from the data. We now specify a VAR with excess bond returns, the real interest rate and other variables that help forecast returns and real rates. Suppose we use the following VAR where the vector z_t is specified as follows:

$$z_t = (x_{b,t}, r_t, cr\_sprd_t, dy_t) \quad \text{(A32)}$$

Here $x_{b,t}, r_t \ cr\_sprd_t$ and dy_t are the excess return on the bond market, the real interest rate, the BAA-AAA credit yield spread and the dividend yield on the CRSP VW index.

We need a few results before we can get compact expressions for the "news" components from the VAR. We know that we can write a first order VAR (in companion form for higher lags if required) as:

$$z_t = Az_{t-1} + w_t \quad \text{(A33)}$$

where A is the VAR parameter matrix and w_t is the vector of error terms.

We know, that this is a difference equation (see for example, Hamilton, 1994) and can be solved by recursive substitution as follows.

$$
\begin{aligned}
z_t &= Az_{t-1} + w_t \implies \\
z_{t+1} &= Az_t + w_{t+1} \\
z_{t+1+1} &= Az_{t+1} + w_{t+2} \\
&\quad\vdots \\
z_{t+1+1} &= A\left(Az_t + w_{t+1}\right) + w_{t+2} \\
z_{t+1+1} &= A^2 z_t + Aw_{t+1} + w_{t+2} \implies \\
\therefore\quad z_{t+1+j} &= A^{j+1} z_t + (\text{terms in } w) \\
\therefore\quad E_t\left(z_{t+1+j}\right) &= A^{j+1} E_t\left(z_t\right) \\
\therefore\quad E_t\left(z_{t+1+j}\right) &= A^{j+1} z_t.
\end{aligned}
$$

Now we need expressions for terms of the type

$$
\left(E_{t+1} - E_t\right)\left(z_{t+1+j}\right).
$$

So we can now expand the above expression

$$
\begin{aligned}
\left(E_{t+1} - E_t\right)\left(z_{t+1+j}\right) & \\
&= E_{t+1}\left(z_{t+1+j}\right) - E_t\left(z_{t+1+j}\right) \\
\text{using } E_t\left(z_{t+1+j}\right) &= A^{j+1} z_t, \text{ and} \\
E_{\underbrace{t+1}}\left(z_{\underbrace{t+1}+j}\right) &= A^j z_{\underbrace{t+1}} \\
\therefore\quad \left(E_{t+1} - E_t\right)\left(z_{t+1+j}\right) &= A^j z_{t+1} - A^{j+1} z_t \\
\therefore\quad \left(E_{t+1} - E_t\right)\left(z_{t+1+j}\right) &= A^j\left(z_{t+1} - Az_t\right) \\
\text{But, } z_{t+1} &= Az_t + w_{t+1} \\
\therefore\quad \left(E_{t+1} - E_t\right)\left(z_{t+1+j}\right) &= A^j w_{t+1}
\end{aligned}
$$

Now we need to extract from the VAR, expressions for the following news components:

$$(E_{t+1} - E_t)\left(r_{b,t+1} - r_{f,t+1}\right) \quad = \quad -\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j\left(r_{b,t+1} - r_{f,t+1+j}\right)$$

$$-\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r_{r,t+1+j}$$

$$-\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j \pi_{t+1+j}$$

We know that

$$z_t = \left(x_{b,t}, r_t, cr\_sprd_t, dy_t\right) \tag{A34}$$

We define row selection vectors $g_1{=}(1,0,0,0)$ and $g_2 = (0,1,0,0)$ so that we can pick out the first and second components we need from the VAR. For example,

$$\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j\left(r_{b,t+1} - r_{f,t+1+j}\right) \tag{A35}$$

$$= \quad \sum_{j=1}^{\infty}\rho^j g1' A^j w_{t+1}$$

$$= \quad g1' \sum_{j=1}^{\infty}\rho^j A^j w_{t+1}$$

$$= \quad g1' \left(\rho A + \rho^2 A^2 + \cdots + \cdots \infty\right) w_{t+1}$$

$$= \quad g1' \rho A \left(I + \rho A + \rho^2 A^2 + \cdots + \cdots \infty\right) w_{t+1}$$

$$= \quad g1' \rho A \left(I - \rho A\right)^{-1} w_{t+1}$$

Similarly,

$$\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j r_{r,t+1+j} = g_2' \rho A \left(I - \rho A\right)^{-1} w_{t+1} \tag{A36}$$

We can obtain the inflation news components, $\left(E_{t+1} - E_t\right)\sum_{j=1}^{\infty}\rho^j \pi_{t+1+j}$, as a residual since we know the other three components -note that $\left(E_{t+1} - E_t\right)\left(r_{b,t+1} - r_{f,t+1}\right) = g_1 w_{t+1}-$ in the dynamic accounting identity. This is:

$$(E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \pi_{t+1+j} = -g_1 w_{t+1} - g_1' \rho A \left(I - \rho A\right)^{-1} w_{t+1} - g_2' \rho A \left(I - \rho A\right)^{-1} w_{t+1}.$$

$$(\text{A37})$$

In the case of bonds we can avoid specifying the process for inflation as a state variable so long as we use the excess bond returns and the real rate in the VAR estimation. We can then obtain the inflation component of the decomposition as the residual term using the identity in Eq. (A15).

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