# Supplementary Information of “Modeling mutual feedback between users and recommender systems”

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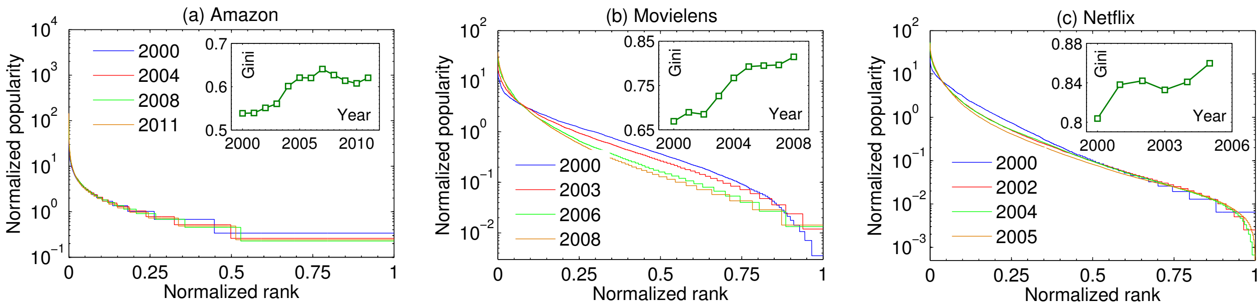
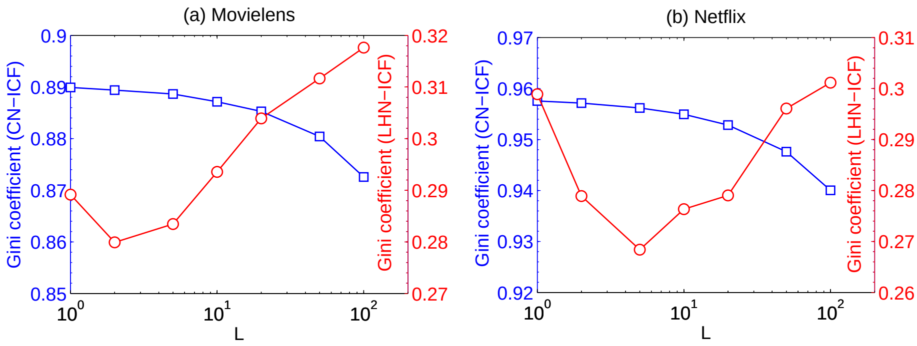
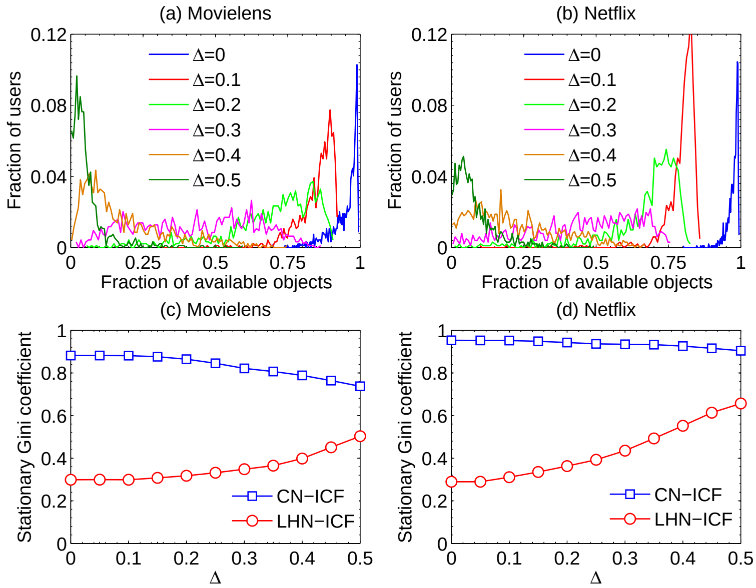
## Supplementary Figures



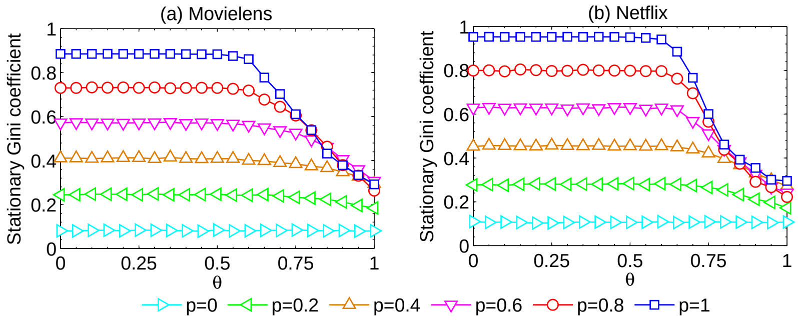

**Supplementary Figure** 1: **The evolution of heterogeneity of item popularity in real systems.** To show that the distribution of item popularity in online systems becomes more uneven with time, we show here plots of normalized item popularity versus normalized item rank in different years (main plots) and the evolution of the Gini coefficient [1] (insets) in three distinct real systems. The distribution of item popularity and the Gini coefficient for each year are based exclusively on links created by users in this given year. Normalized rank and popularity of item $\alpha$ are $R_\alpha/M$ and $k_\alpha/(\sum_\beta k_\beta)M$, respectively, where $R_\alpha$ is the degree rank of item $\alpha$, $M$ is the number of items, and $k_\alpha$ is the degree (popularity) of item $\alpha$. One can see here that both the normalized popularity of the most popular items and the Gini coefficient increase with time.



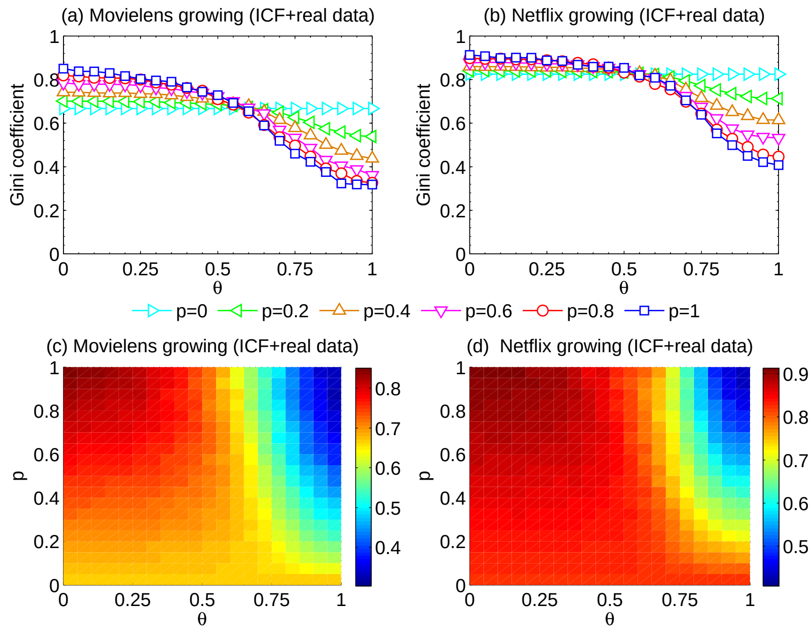

**Supplementary Figure** 2: **The effect of recommendation list length on the stationary Gini coefficient.** While recommendation list length of 20 is typically used in the information filtering literature, real systems and real users may behave differently. As one can see here, the stationary Gini coefficient varies little with list length $L$ for both popularity-favoring CN-ICF and diversity-favoring LHN-ICF.



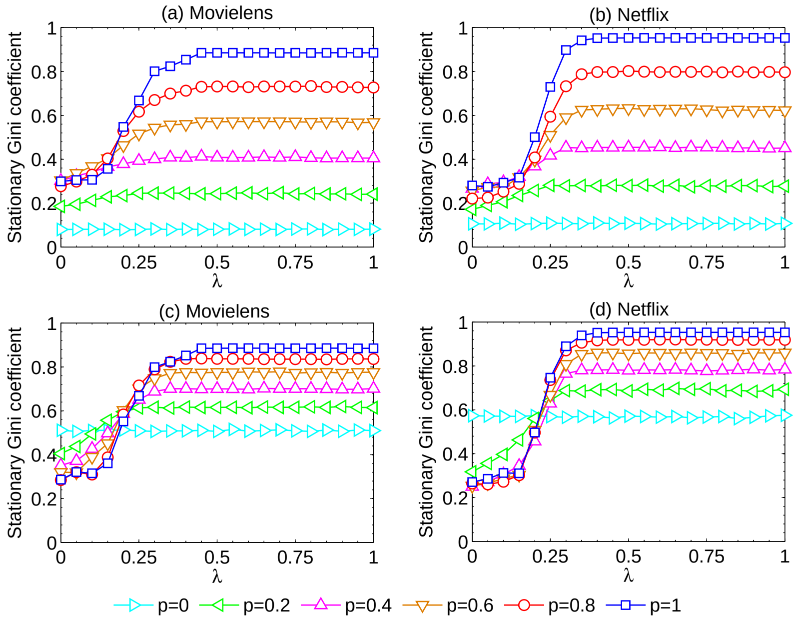

**Supplementary Figure 3: The effect of a similarity constraint on the stationary Gini coefficient.** Real links are quickly replaced with simulated ones in the rewiring process. To address a potential concern that the data becomes random by rewiring, we introduce a similarity constraint to the rewiring process in the following way. We first compute similarity between all user-item pairs in the input real data: for a given user $i$ and item $\alpha$, we compute what fraction $S_{i\alpha}$ of links between the neighbors of user $i$ and item $\alpha$ are actually present. This value lies between 0 and 1—the higher the value, the more similar the pair. We further set a similarity threshold $\Delta$: when presented with a recommendation list, user $i$ automatically discards all items $\alpha$ whose similarity $S_{i\alpha} \leq \Delta$. In this way, the rewiring process cannot drive the network to a state where users are linked with items that are entirely disconnected from them in the real data. In (a) and (b), we show how is the breadth of choice influenced by $\Delta$. When $\Delta$ is small, all users have a large fraction of available to be possible linked with. When $\Delta = 0.3$, a majority of users still have a wide choice—we consider this value a compromise between constraining the similarity and leaving freedom to the system. When $\Delta = 0.5$ and above, only a small fraction of all items are available to most users. In (c) and (d), we show the dependence of the stationary Gini coefficient on $\Delta$. One can see here that the difference between the cases $\Delta = 0$ (no constraint) and $\Delta = 0.3$ is small which means that our results are not particularly influenced by a lack of user-item similarity in rewired networks.



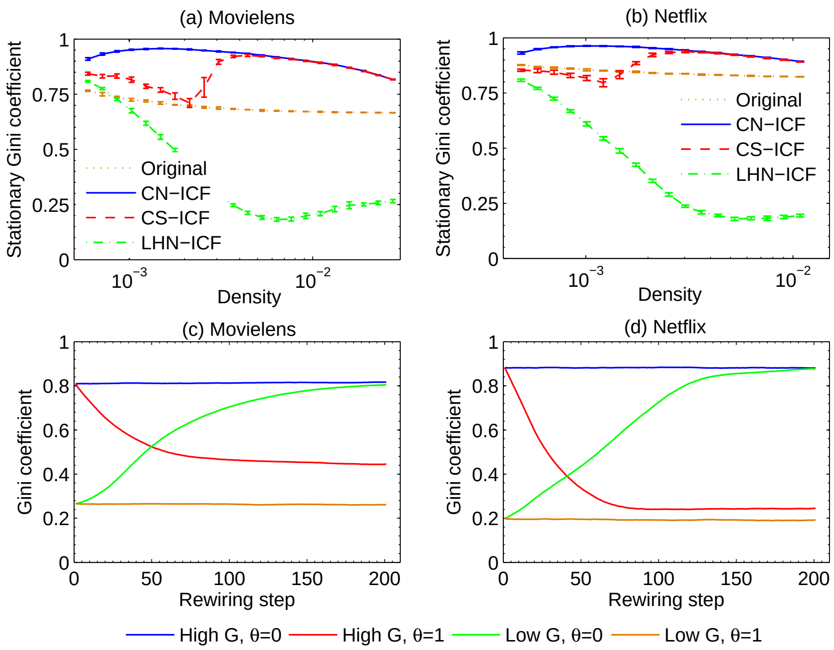

**Supplementary Figure** 4: **The stationary Gini coefficient obtained in the rewiring process.** $p$ is the fraction of rewired links whose target is chosen by recommendation; the target is chosen by random attachment otherwise. These results are similar to the case where users follow recommendation and preferential attachment (Figure 3 in the main text).



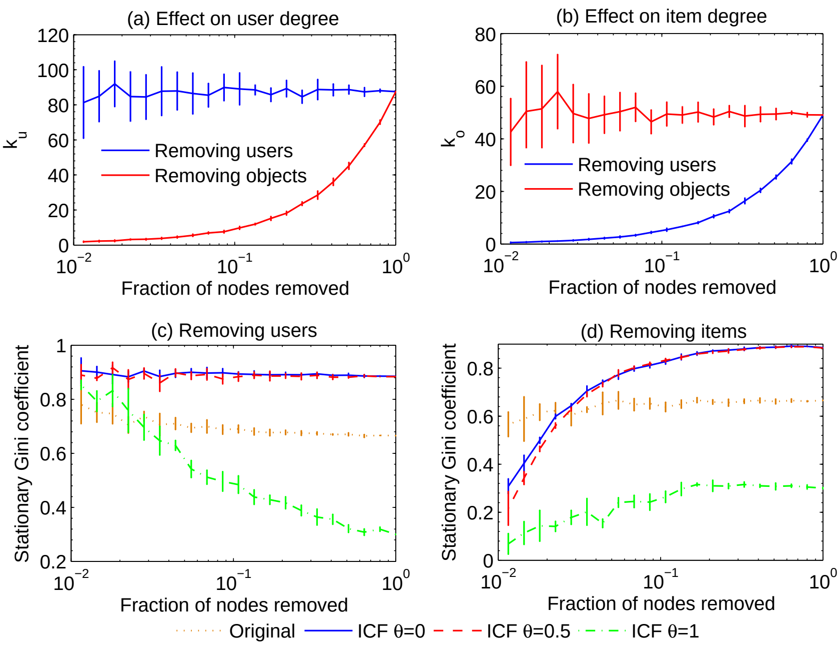

**Supplementary Figure** 5: **Rewiring with real time information.** Since we have the time information of the input real data, we can study the effect of recommendation on network evolution by replacing some of real future link with links in generated by recommendation. We start simulations at the moment when 20% of all links had been added in the system. In each step, the next added link is with probability $1-p$ the corresponding real link which had been added at this time and with probability $p$ it is a link drawn according to recommendation. The effect of $\theta$ on the Gini coefficient is shown for Movielens (a) and Netflix (b) data. Heatmaps of the Gini coefficient in the $[\theta, p]$ plane are shown in (c) and (d). Even though the system hasn't reached the stationary Gini coefficient yet, one can see that the results are consistent with other results presented in the manuscript.



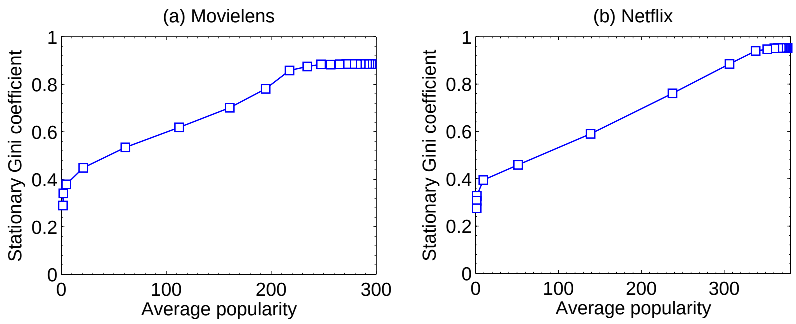

**Supplementary Figure** 6: **Stationary Gini coefficient obtained with a different recommendation algorithm.** In this figure, we further support the results presented in the main text by using a different well-known recommendation method which has a parameter to tune the algorithm's bias towards low or high degree items—the hybrid method combining mass diffusion and heat conduction processes [2]. We denote the components of the adjacency matrix as $a_{i\alpha}$ and the vector with initial resources as $\mathbf{f}^i$ where component $f^i_\alpha$ is the resource assigned to item $\alpha$. When computing recommendation for user $i$, the resource vector is initialized as $f^i_\alpha = a_{i\alpha}$, *i.e.*, one unit of resource is assigned to each item collected by user $i$. The recommendation scores $\widetilde{\mathbf{f}^i}$ are obtained as $\widetilde{\mathbf{f}^i} = \mathsf{W}\mathbf{f}^i$ where $W_{\alpha\beta} = \frac{1}{k_\alpha^{1-\lambda} k_\beta^{\lambda}} \sum_{j=1}^{N} \frac{a_{j\alpha} a_{j\beta}}{k_j}$ where $k_\beta$ is the degree of item $\beta$ and $k_j$ is the degree of user $j$. $\lambda \in [0, 1]$ is a parameter of the algorithm; as $\lambda$ increases from 0 to 1, the hybrid algorithm tends to recommend more and more popular items.




**Supplementary Figure** 7: **The effect of network density and the hysteresis phenomenon when** $p < 1$**.** Unlike Fig. 3(c)(d) and Fig. 4 in the main text where $p = 1$, we assume here $p = 0.9$ (90% of links are rewired according to recommendation, 10% according to preferential attachment). We consider this setting because it can mimic the case where new items constantly come to the system, i.e. the items with degree 0 still have some probability to receive links. However, these 0 degree items can never receive links if the network is rewired only based on ICF ($p = 1$). One can see that the results in the figure is qualitatively the same as that with $p = 1$ in Fig. 3 and Fig. 4.




**Supplementary Figure** 8: **Reducing network density by removing nodes.** In the paper, we show that the network density significantly influence the performance of recommendation algorithms. However, reducing network density by removing links from the network changes both user degree and item degree. Here, we use an alternative way to reduce network density by removing nodes. When only the user nodes are removed, the average degree of users is preserved while the average degree of items is reduced, see the blue curves in (a)(b). When only the item nodes are removed, the average degree of items is preserved while the average degree of users is reduced, see the red curves in (a)(b). We further study how these two scenario affects the performance of recommendation algorithms in (c)(d). When users are removed, the Gini coefficient increases with decreasing density, which is consistent with Fig. 3(c)(d). When items are removed, the Gini index decreases with decreasing density. In this figure, the curve labeled original is the Gini coefficient of the network before the rewiring process.




**Supplementary Figure** 9: **Comparison of short-term and long-term diversity.** Short-term recommendation diversity has been intensively studied recently [3]. One of the short-term diversity metric is recommendation novelty which is simply the average degree of items that appear in the recommendation lists [4]. Intuitively, the short-term diversity is connected with the long-term diversity: recommendation favoring large-degree items contributes to increasing the Gini coefficient in the long run. Here we change the $\theta$ parameter of the ICF recommendation method and report the resulting average popularity together with the corresponding stationary Gini coefficient. As expected, the obtained curves are monotonously increasing. At a certain level of average popularity, the stationary Gini saturates and does not increase further.

**Supplementary Note 1: Data description**

We now describe the data used for the empirical study in Fig. 1 and S1. The first dataset contains the Amazon movie review data (obtained from `snap.stanford.edu/data/web-Movies.html`) which after cleaning comprises $1,901,110$ reviews in the integer scale from 1 (worst) 5 (best) from $889,066$ users for $141,039$ items. The data spans $5,546$ days (August 1997-October 2012). To obtain an unweighted bipartite network, we represent all reviews with rating 4 or 5 as links between the corresponding user and item. After this operation, there are $960,374$ links whereas $497,308$ users and $88,858$ items have at least one link.

The second data set contains the Movielens movie rating data (obtained from `http://grouplens.org/`) which comprises $10,000,054$ ratings from $71,567$ users to $10,681$ movies in the online movie recommender service MovieLens. The ratings scale is the same as in the Amazon data—we thus apply the same procedure to build an unweighted bipartite network which then contains $8,242,124$ links between $69,878$ users and $10,677$ movies. The final data is from January 1995 to January 2009.

The third data set contains the Netflix movie rating data (download from the Netflix Prize web site `http://www.netflixprize.com/`) which comprises $100,481,826$ ratings from $480,189$ users to $17,770$ movies in the online DVD rental website Netflix. The ratings scale is the same as in the Amazon data—we thus apply the same procedure to build an unweighted bipartite network which then contains $85,730,791$ links between $479,760$ users and $17,770$ movies. The final data is from January 2000 to January 2006.


[1] Gini C., Variabilitá e mutabilitá (in English: Variability and Mutability) (C. Cuppini, Bologna) (1912).

[2] Zhou, T., Kuscsik, Z., Liu, J.-G., Medo, M., Wakeling, J. R. & Zhang, Y.-C. Solving the apparent diversity-accuracy dilemma of recommender systems. *Proc. Natl Acad. Sci.* **107**, 4511-4515 (2010).

[3] Lu, L., Matus, M., Yeung, C. H., Zhang, Y.-C., Zhang, Z.-K. and Zhou, T. Recommender system. *Phys. Rep.* **519**, 1-49 (2012).

[4] Zeng, A., Vidmer, A., Medo, M. and Zhang, Y.-C. Information filtering by similarity-

preferential diffusion processes. *Europhys. Lett.* **105**, 58002 (2014).