

THE IMPACT OF MONEY ILLUSION ON CONSUMPTION

An Empirical Investigation

THESIS

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Chapter 1

Introduction

Does money illusion have an impact on aggregate consumption? Every well-educated economist should answer this question in the negative, with the argument that the representative consumer is rational and, therefore, perfectly knows the difference between nominal and real values. In consequence, consumption should remain unaffected by variations in the general price level, such that the neutrality of money prevails. By theoretically and empirically unveiling a short- to medium-term impact of inflation on consumption, the present dissertation shows that the answer to this question is – contrary to the typical assumption – far from evident.

1.1 Motivation

The idea of analyzing the effect of money illusion on aggregate consumption is motivated by two observations: First, people typically present some degree of money illusion, and, second, aggregate consumption models completely leave out inflation from their analysis.

Money Illusion Exists at the Individual Level

With the success of behavioral economics in the 1990s, there has been a renewed interest in money illusion as a departure from the pure rationality assumption. Money illusion argues that consumers rely not only on real values, as assumed by the traditional models, but also partly on nominal values. As a result, money illusion reveals that inflation is a potentially significant determinant of consumption. In fact, money illusion need not be the reflection of an intrinsic irrationality or a foible on the part of the individuals; rather, it is equally consistent with the widely assumed rational-

ity assumption. Following the literature on rational inattention (see, for example, [Sims, 2003](#); [Mankiw and Reis, 2002](#)), this dissertation considers money illusion as pertaining to the fact that the consumer is constrained in her ability or willingness to collect and process information about inflation. Although surveys and experimental studies show that people are confused between nominal and real values (see, in particular, [Shafir et al., 1997](#); [Fehr and Tyran, 2001](#)), there is no certainty whether they really behave in an illusioned way and, consequently, whether this confusion has a real impact on aggregate values. The present thesis addresses this question by investigating whether inflation has a significant impact on real consumption and whether this impact can be attributed to money illusion.

Traditional Consumption Models Exclude any Role for Inflation

Inflation is typically defined as a purely monetary phenomenon, influencing nominal values but leaving real values unchanged, upon which most macroeconomic models rely. Based on the original literature on signal extraction, which predicted a real impact of unanticipated changes in nominal variables ([Lucas, 1972](#); [Barro, 1977](#)), many empirical works have focused on the possible effects of inflation on output and unemployment. While some find inflation to have an overall negative impact on the economy (e.g., [Barro, 1995](#); [Chari et al., 1995](#); [Bruno and Easterly, 1998](#)), others refute this nonneutrality of money (e.g., [Bullard and Keating, 1995](#); [McCandless and Weber, 1995](#); [Lucas, 1996](#)). However, with the notable exception of [Deaton \(1977\)](#), there are hardly any studies analyzing a possible dependency of consumption on fluctuations in the general price level. This thesis aims at filling this gap and provides an alternative justification, in terms of money illusion, for any significant short-term impact of inflation on consumption. An important insight to be gained from the different analyses is that inflation should not be excluded from any aggregate consumption model.

1.2 Outline

Since the central question of this dissertation remains largely uninvestigated, I explore different ways to apprehend and model the relationship between money illusion and consumption. All reported empirical results have been obtained with quarterly U.S. data for two reasons: first to facilitate comparison with other studies on consumption, and, second, because the U.S. inflation series can be neatly divided into

subperiods with different average inflation rates.

As a formal introduction, Chapter 2 presents the rational expectations–permanent income hypothesis model, which is used throughout the dissertation as a benchmark model representing the optimal behavior of the fully rational and illusion-free consumer. According to this standard model, consumption growth is totally independent of inflation because the consumer perfectly anticipates every fluctuation in the general price level. Next, Chapter 2 presents an example of how money illusion can be modeled directly in the consumer’s utility function as an extension of the money in the utility function model.

Chapter 3 derives stylized facts regarding inflation and consumption growth and reveals that inflation is, together with lagged consumption growth, the only significant short- to medium-term determinant of consumption growth. This result is obtained by extending the random walk model derived in Chapter 2 with current and lagged inflation and by controlling for other determinants of consumption typically evoked in the consumption literature. The resulting *money illusion consumption function* is estimated with ordinary least squares (OLS) and two stage least squares (TSLS) over different subperiods and using different inflation measures. The results reveal two opposing effects of inflation on consumption growth, each corresponding to a different effect of money illusion on consumption. First, contemporaneous inflation has a negative effect on consumption growth. This effect reflects money illusion as a signal extraction problem: that is, the consumer does not fully anticipate inflation and temporarily decreases her consumption when she observes an increase in the price level. Second, lagged inflation has a positive cumulative effect on current consumption growth. This effect reflects money illusion as a cost-saving rule of thumb, in which the consumer uses nominal values as a proxy for real values. Following this behavioral approach, the consumer’s subjective wealth increases when her nominal income is adjusted upward to meet inflation, stimulating her real consumption.

Chapter 4 focuses on the interpretation of money illusion as a signal extraction problem, in which the consumer decides not to be constantly and perfectly informed about inflation. The adopted modeling approach proposes that the consumer observes only her nominal income and wealth and must rely on assumptions about the unobserved evolution of her real income and inflation. This inflation extraction problem is estimated with a Kalman filter, thus yielding potentially more efficient estimates than those in Chapter 3. The estimation results confirm a strong and significant negative impact of current inflation on consumption growth in periods during

which inflation is particularly high, but find no robust evidence of any positive or negative inflation effect in periods with low inflation rates. This result suggests that consumers behave differently depending on the inflationary environment and that money illusion can have a potentially important impact on aggregate consumption in times of high inflation.

Based on the insights gained from the Chapter 4, Chapter 5 suggests modeling money illusion as a specification error within the benchmark rational expectations–permanent income hypothesis model. In this indirect measure of money illusion, the consumer is assumed to be aware that she might suffer from money illusion during times of high inflation and to be able to efficiently control for it. Using the robust control method suggested by Hansen and Sargent (2008), the estimation results show that the consumer seeks a higher robustness in high-inflation subperiods than in illusion-free, low-inflation subperiods. Moreover, the greater degree of robustness induces a high degree of precautionary savings on the part of the consumer. These results suggest a high degree of inflation uncertainty in times of high inflation, attributable to the higher degree of money illusion during such periods. Moreover, they suggest that money illusion has an important negative impact on consumption.

The intuitive and straightforward models presented in this dissertation clearly show that inflation is an important determinant of consumption growth, particularly in times of high inflation. One should be very cautious, however, when interpreting the uncovered inflation effects as stemming from money illusion only. In fact, it is not possible to perfectly disentangle money illusion from other inflation effects when using only aggregate data. On one hand, this drawback calls for further investigations into the exact relationship between inflation and consumption. On the other hand, the different models and estimation results presented in this dissertation show that money illusion cannot be excluded *a priori* at the aggregate level, which supports the idea that modern macroeconomics should pay greater attention to behavioral phenomena.

Chapter 2

Two Potential Benchmark Models

As a first step, this chapter introduces the rational expectations–permanent income hypothesis model, which is used throughout the dissertation as a benchmark empirical model for the analysis of the impact of money illusion on consumption. The main advantage of this model lies in its relative simplicity and in its straightforward and testable implication: that no variable other than current consumption helps predict future consumption.

The second possible benchmark model suggested in this chapter is the money in the utility function model. This monetary model is appealing because it provides a potential explanation for why people suffer from money illusion: They want to hold money balances because such balances facilitate transactions. As an example, the last section of this chapter shows how money illusion can be modeled directly in the consumer’s utility function.

The main purpose of this chapter is to present the assumptions underlying the models and to introduce the notation and derivation techniques that will accompany this thesis from beginning to end. Readers familiar with these models and with standard dynamic programming optimization techniques can safely skip the introductory sections and start with the example in Section 2.3.2 or with the actual empirical analyses in Chapters 3, 4 and 5.

Keywords: Rational expectations-permanent income hypothesis, random walk hypothesis, money in the utility function, money illusion.

JEL classification: D11, D91, E21.

2.1 Introduction

Money illusion can be interpreted as the consumers' confusion between nominal and real values (see Section 3.1). The theoretical consequence of money illusion is that consumption is affected by purely nominal variations in its determinants, i.e., inflation becomes a further determinant of consumption. However, since people are typically assumed to be rational, money illusion can only have a short-term effect – if any – on consumption. To measure this temporary effect of money illusion on consumption, it is useful to have a benchmark model representing the average consumer's behavior over the long run.

This chapter presents two suitable benchmark models that can be used for an empirical analysis of the relationship between inflation and consumption growth. The first model is the rational expectations–permanent income hypothesis model (REPIH) developed by Hall (1978). Although this model does not explicitly deal with inflation or monetary issues, its simplicity and intuitive random-walk implications for consumption growth render it the ideal empirical benchmark model. For this reason, the REPIH is used in Chapter 3 to analyze the relationship between inflation and consumption and extended in Chapter 5 to control for money illusion as a misspecification.

As a matter of completeness, this chapter further presents the money in the utility function (MIU) model developed by Sidrauski (1967a,b). Contrary to Hall's model, Sidrauski's model allows for a non-neutrality of money in the short run by attributing a direct utility to the holdings of money balances. In Section 2.3.2, I present an example of how money illusion can be controlled for directly within the utility function, using a method suggested by Miao and Xie (2013).

2.2 The Rational Expectations–Permanent Income Hypothesis

The permanent income hypothesis (PIH) dates back to the work of Friedman (1956), who developed a consumption theory aimed at explaining the fact that aggregate consumption is much smoother than aggregate income (in that it presents fewer and smaller fluctuations over time). To explain this phenomenon, he introduces the dual concepts of permanent income y^p and transitory income y^t , such that both add up to total income y_t in every period (i.e., $y_t = y_t^p + y_t^t$). The idea behind permanent income is that the individuals base their consumption plan for each period on their

total wealth, which corresponds to the amount of resources they expect to have at their disposable for all remaining periods of their lives. The PIH then states that only very short-term fluctuations in consumption are induced by transitory income fluctuations, while long-term fluctuations in consumption are generated by fluctuations in permanent income. Compared to the absolute consumption model developed by Keynes (1936), $c_t = a + by_t$, in which b is the marginal propensity to consume (MPC) out of contemporaneous income, Friedman (1956) suggests that the MPC out of transitory consumption is much smaller than the MPC out of permanent income. In consequence, in order to understand consumption, one must be able to precisely capture permanent income. However, the main problem arising from the original PIH model is that y^p is an unobserved subjective concept and, thus, cannot directly be measured or estimated.

Friedman’s PIH, being foremost a statistical concept, only forms a complete consumption behavior theory when combined with the life cycle hypothesis, which was developed by Modigliani and Brumberg (1954) and later extended by Ando and Modigliani (1963). Indeed, Modigliani and Brumberg (1954) provided the utility analysis that set the foundations for what is sometimes referred to as the life cycle–permanent income hypothesis (LCPIH). This model, despite its sound theoretical assumptions and plausible implications, still suffers from the measurement problem of permanent income and from the fact that the consumers are neither fully forward-looking nor able to account for uncertainty in their decisions.

Hall (1978) was the first to directly address the uncertainty problem in deriving the PIH implications for a representative and fully rational consumer by modelling aggregate consumption as obeying the first order conditions for the optimal choice of the representative consumer. His approach was revolutionary in the sense that his micro-founded REPIH model is immune to the so-called “Lucas (1976) critique” and provides a solution for many of the drawbacks of the original PIH. First, the direct measure of permanent income becomes secondary because it is completely captured by contemporaneous consumption. Second, by taking into account uncertainty about the future, the model implies that future consumption is determined by no other variable than current consumption. Consequently, consumption growth is unpredictable and follows a random walk.

The following section derives the REPIH and shows which assumptions underlie the random walk hypothesis, which is used as a benchmark model and extended in Chapter 3.

2.2.1 The Consumer's Problem Under Uncertainty

Starting from the general maximization problem for a single rational consumer, this section presents how we can derive, in a stepwise manner, under minimal assumptions and with dynamic programming optimization techniques, a simple and intuitive consumption function that can be empirically tested.

Under uncertainty, the consumer's dynamic optimization problem can be specified as follows.¹ Consider a representative consumer who maximizes her lifetime utility $U_t = U(c_t, c_{t+1}, \dots)$ over an infinite horizon:

$$\max_{\{c_t\}} U_t = \mathbb{E}_t \left[\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t u(c_t) \right] \quad (2.1)$$

subject to

$$A_{t+1} = (1+r)(A_t + y_t - c_t) \quad (2.2)$$

where A_0 is given and

$$\lim_{j \rightarrow \infty} \left(\frac{1}{1+r} \right)^j A_{t+j} \geq 0. \quad (2.3)$$

The variables c_t , y_t and A_t denote, respectively, the beginning-of-period consumption, labor income and assets in period t . The uncertainty in the model is induced by the income process $\{y_t\}$, which is stochastic. A_t can be negative, meaning that the consumer is allowed to freely borrow or lend. The constants r and $\rho > 0$ are the certain, risk-free interest rate and the subjective discount rate, or the consumer's intertemporal rate of time preference. The within-period utility function $u(\cdot)$ is strictly concave; that is, it has diminishing marginal utility with $u'(\cdot) > 0$ and $u''(\cdot) < 0$. A constant ρ in this discrete setup implies that utility is additive, which means that the total utility over all periods is simply the sum of the discounted utilities of each period. For now, the utility function is also assumed to be time-separable, meaning that each period's utility depends only on consumption within the given period. Later in this chapter, two types of utilities fulfilling the presented requirements will be suggested: namely, the quadratic and the constant elasticity of substitution utility functions. In Chapter 3, however, the model will also allow

¹In this section, I broadly follow [Bagliano and Bertola \(2007, ch. 1\)](#).

for non time-separability of the lifetime utility. Individuals characterized by this particular type of utility function, in which current period utility also depends on past consumption, display some degree of habit formation (see Section 3.3.1). Yet, another type of utility function – namely the money in the utility function – will be considered in Section 2.3, when we turn to modeling money illusion by directly including a role for money in the individuals’ preferences.

Equation (2.2) is the accounting identity representing the evolution of assets: The gross savings ($A_t + y_t - c_t$) are carried from period t to period $t + 1$ and remunerated with r . In the present case, I use the original notation of Hall (1978), in which consumption and labor income take place at the beginning of period t . This notation follows the suggestion of Samuelson (1969) and is widely used in the literature (see, for example, Blanchard and Fischer, 1989; Deaton, 1992; Carroll, 1997). However, it is also very common to assume that consumption occurs at the end of the period, in which case the asset evolution equation is equal to $A_{t+1} = (1 + r)A_t + y_t - c_t$ (as in Flavin (1981) or Muellbauer and Lattimore (1999)). Whereas the choice of the specification for equation (2.2) can slightly alter the appearance of the equilibrium conditions, it does not change their substance. Moreover, it can be easily shown that the solution to the simple consumer problem presented in this chapter is independent from the specification of the accounting identity (Azar, 2012).

The second constraint, equation (2.3), guarantees that the consumer does not run a *Ponzi game* in which the consumer infinitely borrows in order to attain infinite utility at the end of her life through infinite consumption. Note that this constraint is equal to the non-negativity constraint $A_{T+1} \geq 0$ in a finite-horizon setup, in which T denotes the end-of-life period. More importantly, the inequality becomes a strict equality if the marginal utility is always positive (which is the case) because the agent will be able to attain a higher total utility by increasing her consumption until she has consumed all her resources. In this case, the no-Ponzi-scheme constraint is called the *transversality condition*, which states that the individual consumes exactly all of her lifetime resources over her life cycle.

Under uncertainty, the maximization problem depends on future payoffs that are not certain and that depend on a particular probability distribution. This uncertainty is reflected by the operator \mathbb{E}_t , which represents the consumer’s expectations, conditional on all information available up to period t . Typical uncertain variables include income and the interest rate, for which the individual forms rational expectations based on the optimal use of all information available at the time the expectations are built. As a matter of simplicity – and in order to better stress

the main implication of the REPIH – the interest rate is considered certain and constant, which leaves future income as the only uncertain variable. Due to its uncertain nature, $\{y_t\}$ is to be interpreted as an exogenous endowment process.

There are several ways to solve this optimization problem. The standard method, presented by [Hall \(1978\)](#) in his original paper, is to set up the Lagrangian for the dynamic optimization problem and to derive the first order conditions (FOC) with respect to c_t and c_{t+1} .² The idea here is to analyze the utility loss of moving from consumption in period t to consumption in a period infinitesimally forward in time, which is equivalent to $t + 1$ in this discrete setup. It can then be shown that it is optimal for the consumer to set $u'(c_t)$ equal to $\mathbb{E}[u'(c_{t+1})]$, which is the main result of the REPIH. The following section shows how to obtain exactly the same result using another derivation technique: namely, dynamic programming.

Dynamic Programming: Derivation of the Bellman Equation

The reason for using dynamic programming is that, not only does it provide a more general framework, which offers additional flexibility, but it is also particularly powerful in dealing with problems under uncertainty.³ In addition, dynamic programming is also used to derive the solution to the money in the utility function model in [Section 2.3](#) and to the robust consumption model in [Section 5.3](#).

Two preliminary conditions (which are met in the above-presented consumer's problem) are necessary for the use of dynamic programming: First, the objective function must be time-separable. Second, there must exist a dynamic accumulation constraint in each period (in our case, [equation \(2.2\)](#)). The general idea behind dynamic programming is to decompose, through backward recursion, the infinite-period problem into a sequence of two-period optimization problems. To do this, assume that the consumer faces a finite maximization problem, which ends at time T , and that final wealth is non-negative (i.e., $A_{T+1} \geq 0$). At the beginning of the last period, the consumer faces the following problem:

²See [Romer \(2006, ch. 7\)](#) for a detailed textbook presentation.

³The approach presented in this section follows the notation and intuition of [Bagliano and Bertola \(2007, pp. 36-41\)](#). For a detailed specification of the assumptions underlying dynamic programming and a complete description of the alternative derivation techniques, refer to [Ljungqvist and Sargent \(2004, ch. 3\)](#), who enthusiastically define dynamic programming as “the imperialism of recursive methods”.

$$\begin{aligned} & \max_{\{c_T\}} \left\{ \mathbb{E}_T [u(c_T)] \right\} \\ \text{subject to} & \\ & A_{T+1} = (1+r)(A_T + y_T - c_T) \\ \text{and} & \\ & A_{T+1} \geq 0, \end{aligned}$$

where A_T is given. Solving for the optimal solution (while assuming that the non-negativity constraint is binding) we get the optimal level of consumption in period T as a function of wealth and income at time T ; that is, $c_T = c_T(A_T, y_T)$. To see this, we can first insert $A_{T+1} = 0$ into the accounting identity and then substitute for c_T in the objective function.

By definition, the solution to the maximization problem yields the consumer's maximum utility that the consumer can obtain in the last period. The value of this maximum utility is summarized by a so-called value function, V_T , which can be thought of as the value of the consumer's objective function in the last period, depending on the then-available resources. Just as the solution to the consumer's problem, the value function depends only on resources available in the last period, i.e., $V_T(A_T, y_T)$.

The same line of thought can be applied to the penultimate period for a given value of wealth A_{T-1} . The problem in period $T-1$ can then be written as the following two-period problem:

$$\begin{aligned} & \max_{\{c_{T-1}\}} \left\{ u(c_{T-1}) + \left(\frac{1}{1+\rho} \right) \mathbb{E}_T [V_T(A_T, y_T)] \right\} \\ \text{subject to} & \\ & A_T = (1+r)(A_{T-1} + y_{T-1} - c_{T-1}). \end{aligned}$$

Analogously to the final period, we get the optimal solution $c_{T-1} = c_{T-1}(A_{T-1}, y_{T-1})$ and further define the value function $V_{T-1}(A_{T-1}, y_{T-1})$, which summarizes the maximum value of utility that the consumer can obtain over the two periods ($T-1$) and T .

As we continue the backward recursion, we find ourselves at the initial period, where the infinite-period optimization problem condenses into a maximization prob-

lem over only two periods. The consumer now optimizes only over her first-period utility function and her (forward-looking) value function at $t + 1$, representing the intertemporal utility after maximization in all future periods. This two-period representation of the consumer's discrete-time problem is called the *Bellman equation*, which has the following general form:

$$V(A_t, y_t) = \max_{\{c_t\}} \left\{ u(c_t) + \left(\frac{1}{1 + \rho} \right) \mathbb{E}_t [V(A_{t+1}, y_{t+1})] \right\} \quad (2.4)$$

subject to

$$A_{t+1} = (1 + r)(A_t + y_t - c_t). \quad (2.5)$$

Unlike the original maximization problem described by equations (2.1) through (2.3), the Bellman equation breaks down the dynamic problem by forming a sequence of maximization problems, in which the consumer faces a trade-off between opting for immediate utility and utility tomorrow. As such, the right-hand side (RHS) of equation (2.4) comprises the current objective u , plus the consequences V , of the discounted objective of behaving optimally in the future. Note that, in the Bellman equation, V no longer depends on time anymore because it converges to a constant as $T \rightarrow \infty$.

Another simple way of understanding the Bellman equation and expressing the consumer problem as a two-period maximization problem is suggested by [Chen and Funke \(2007, pp. 6-8\)](#). Beginning with the objective function (equation (2.1)), we can separate the current period t from all following periods and re-index the sum from $t = 1$ to $t = 0$:

$$\begin{aligned} \max_{\{c_t\}} U_t &= \mathbb{E}_t \left[\sum_{t=0}^{\infty} \left(\frac{1}{1 + \rho} \right)^t u(c_t) \right] \\ &= u(c_t) + \mathbb{E}_t \left[\sum_{t=1}^{\infty} \left(\frac{1}{1 + \rho} \right)^t u(c_t) \right] \\ &= u(c_t) + \left(\frac{1}{1 + \rho} \right) \mathbb{E}_t \left[\sum_{t=0}^{\infty} \left(\frac{1}{1 + \rho} \right)^t u(c_{t+1}) \right] \\ &= u(c_t) + \left(\frac{1}{1 + \rho} \right) \mathbb{E}_t [U_{t+1}(\cdot)]. \end{aligned}$$

We can restate the previous equation in terms of the value function $V(A_t, y_t)$

which measures the total lifetime utility U that the consumer can obtain by maximizing all one-period utilities. We find ourselves at the same Bellman equation, (2.4), subject to the usual accumulation constraint and transversality condition.

A particularity of dynamic programming is that the Bellman equation is expressed in terms of *control* and *state* variables. The idea underlying dynamic programming is to find the optimal control and state “today”, taking as a given that later behavior will be optimal. In the present example, c_t is the control variable, whose level can be chosen by the consumer in each period so as to maximize her overall utility. The choice of the control variable in t affects the amount of wealth that the consumer will have at her disposal for consumption in the next period $t + 1$. This amount is given by the endogenous state variable A_{t+1} , which is also referred to as the controlled state. The labor income variable y_t is an exogenous state variable following a certain stochastic process. This exogenous state cannot be influenced by the consumer and, hence, does not enter the consumption-borrowing plan $\{c_t, A_{t+1}\}$ chosen as optimal by the consumer over her lifetime.

There are two possibilities for solving the dynamic programming optimization problem of equation (2.4) and equation (2.5).⁴ The first is to take the *Euler route* by directly inserting c_t from the constraint into the value function and then differentiating with respect to A_{t+1} . The second possibility, presented hereafter, is to set up a Lagrangian by adding the constraint with an appropriate multiplier λ_t . In either case, the solution is a sequence of optimal control $\{c_t\}$, optimal state $\{A_{t+1}\}$ and optimal shadow price $\{\lambda_t\}$ that satisfies the first order and transversality conditions, given the exogenous path $\{y_t\}$. Note that, since the interest rate r is constant over time, the shadow price of consumption is also constant over time; that is, $\lambda_t = \lambda$. The Lagrangian for the Bellman equation is given by:

$$\mathcal{L} = u(c_t) + \left(\frac{1}{1+\rho}\right) \mathbb{E}_t [V(A_{t+1}, y_{t+1})] + \lambda_t \left[A_t - \left(\frac{1}{1+r}\right) A_{t+1} + y_t - c_t \right],$$

⁴In this chapter, I only illustrate how to find the Euler equation representing the solution for the consumer’s general maximization problem. A complete solution for the dynamic programming problem also, however, implies a derivation of the exact form of the value function of the Bellman equation. This is done in Section 5.3.2 for the robust consumption model.

with the corresponding FOC:

$$c_t : \frac{\partial u(c_t)}{\partial c_t} - \lambda_t = 0 \quad (2.6)$$

$$A_{t+1} : \left(\frac{1}{1+\rho}\right) \mathbb{E}_t \left[\frac{\partial V(A_{t+1}, y_{t+1})}{\partial A_{t+1}} \right] - \left(\frac{1}{1+r}\right) \lambda_t = 0 \quad (2.7)$$

$$\lambda_t : A_t - \left(\frac{1}{1+r}\right) A_{t+1} + y_t - c_t = 0.$$

The FOC for c_t states that the marginal utility of consumption is constant in every period. Since marginal utility is uniquely determined by the consumption level c_t , consumption must also be constant over time; that is, it becomes already clear that the optimal behavior will be $c_0 = c_1 = c_2 = \dots$ (Romer, 2006, p. 347). Moreover, since marginal utility is equal to the multiplier λ , it corresponds to the marginal utility of an infinitesimal increase in the consumer's available resources. Substituting (2.6) into (2.7), we get the following FOC:

$$u'(c_t) = \left(\frac{1+r}{1+\rho}\right) \mathbb{E}_t \left[\frac{\partial V(A_{t+1}, y_{t+1})}{\partial A_{t+1}} \right], \quad (2.8)$$

which must hold in every period. To solve for $\frac{\partial V(A_{t+1}, y_{t+1})}{\partial A_{t+1}}$, we make use of the envelope theorem, which gives the effect of an infinitesimal change in A_t on $V(A_t, y_t)$; that is:

$$\frac{\partial V(A_t, y_t)}{\partial A_t} = \lambda_t = \frac{\partial u(c_t)}{\partial c_t}.$$

Since the first order conditions are valid for all periods, we can update the previous equation from period t to period $t+1$:

$$\frac{\partial V(A_{t+1}, y_{t+1})}{\partial A_{t+1}} = \frac{\partial u(c_{t+1})}{\partial c_{t+1}}.$$

Reinserting this into equation (2.8), we finally get the optimal consumption rule, or the so-called *Euler equation*:

$$u'(c_t) = \left(\frac{1+r}{1+\rho}\right) \mathbb{E}_t [u'(c_{t+1})]. \quad (2.9)$$

The Euler equation (2.9) shows that the optimal consumption behavior rests upon the indifference between the current marginal utility and the future discounted marginal utility of consumption. It states that, given a consumer's subjective discount rate, she is indifferent between consuming one unit of a particular good today and consuming $(1 + r)$ units of the same good in the following period. Consequently, equation (2.9) reflects the stability of the marginal utility relationship between any two successive periods.

An important implication of uncertainty is that future realizations of utility may well deviate from its expectations. Using the definition of expectation, we can write $u'(c_{t+1}) = \mathbb{E}_t u'(c_{t+1}) + \nu_{t+1}$, where, for rational consumers, the ν_t s have a zero mean and are serially uncorrelated. Thus, under rational expectations, equation (2.9) becomes the stochastic equation:

$$u'(c_{t+1}) = \left(\frac{1 + \rho}{1 + r} \right) u'(c_t) + \nu_{t+1}, \quad (2.10)$$

where ν_{t+1} can be interpreted as new and unpredicted information about wealth in $t + 1$. More precisely, since y_t is the only random variable in the model, ν_{t+1} measures the unpredictable news about labor income accruing between the two successive periods t and $t + 1$. Note that, formally, the sequence ν_t is a *martingale difference sequence*, defined as a process for which $\mathbb{E}_t(\nu_t) = 0$ for $t = 1, 2, \dots$ and $\mathbb{E}_t(\nu_t | \nu_{t-1}, \dots, \nu_1) = 0$ for $t = 2, 3, \dots$ (Lütkepohl, 2005, p. 689).

Depending on the nature of the consumer's utility function, different optimal consumption behaviors can be derived from this Euler equation. I briefly present two strictly concave utility functions scrutinized by Hall (1978): the quadratic utility function and the isoelastic utility function.

Case One: Quadratic Utility

Recall that $u(c)$ is an increasing and concave function of consumption. One type of utility function presenting these characteristics is the quadratic utility function, which has the convenient feature of yielding linear marginal utility.⁵ Consider, for example, the following intra-period quadratic utility (see, for example, Romer, 2006, p. 353):

⁵The advantage of linear marginal utility is that, in this case, $\mathbb{E}_t[u'(c_{t+1})]$ is equal to $u'(\mathbb{E}_t[c_{t+1}])$ (Romer, 2006, p. 353).

$$u(c) = c - \frac{a}{2}c^2,$$

where $a > 0$. Inserting the differentiated utility function into equation (2.9) yields⁶

$$\mathbb{E}_t(c_{t+1}) = \frac{(1 + \rho)ac_t + (r - \rho)}{(1 + r)a}. \quad (2.11)$$

If we further assume that the interest rate is equal to the subjective discount rate (i.e., $r = \rho$), we get Hall's famous result:

$$c_t = \mathbb{E}_t(c_{t+1}), \quad (2.12)$$

which states that the optimal consumption path for the rational consumer is to keep her level of consumption constant over time.⁷ In probabilistic terms, the stochastic process c_t is then said to be a *martingale* (Ljungqvist and Sargent, 2004, p. 36). In other words, the *ex ante* current consumption is the best forecast of the next period's consumption. Thus, equation (2.12) implies that no variable other than c_t (as, for example, current wealth, income, interest rates or inflation) can help predict the evolution of consumption. Consequently, the REPIH not only excludes alternative consumption behaviors as habits or liquidity constraints, but also fails to leave room for money illusion to impact consumption growth via inflation, which is the main assumption of the present thesis.

This central theoretical result becomes even more striking when we use of the above-mentioned definition of expectations and insert the marginal quadratic utility into equation (2.10). This gives:

$$\Delta c_{t+1} = \varepsilon_{t+1}, \quad (2.13)$$

where $\Delta c_{t+1} = c_{t+1} - c_t$ and $\varepsilon_{t+1} = \left(\frac{-1}{a}\right)\nu_{t+1}$ is a martingale difference sequence, with $\mathbb{E}_t(\varepsilon_{t+1}) = 0$ by construction under rational expectations, the exact inter-

⁶Note that, if we define $\beta = \frac{1}{1+\rho}$ and $R = 1 + r$, as in Sections 2.3.1 and 5.3, equation (2.11) becomes $\mathbb{E}_t(c_{t+1}) = \frac{ac_t + (\beta R - 1)}{a\beta R}$, while equation (2.12) remains unchanged under $\beta R = 1$.

⁷As we will see in Section 3.3.2, the common assumption that $r = \rho$ is everything but slight, since it excludes the possibility for the expected inflation to have an impact on current consumption through the real interest rate.

pretation of which will be derived in Section 2.2.2. Equation (2.13) states that, under quadratic preferences and with an interest rate being equal to the discount rate, consumption growth follows a random walk.⁸ This result, uncovered by Hall (1978), represents the stochastic implication of the PIH under rational expectations and has led to much debate across empirical studies. However, this model possesses the tremendous advantage of simplicity, and it yields both microeconomically and theoretically founded implications that can be directly tested with aggregate macroeconomic data.

I undertake such a test in Chapter 3, where I use the random walk hypothesis as a benchmark model and extend it with variables controlling for money illusion, habit formation, liquidity constraints and wealth.

Case Two: CES Utility

Consider now the constant elasticity of substitution (CES) utility function, which is commonly used in macroeconomic models. This function also presents the convenient properties of being increasing, concave and additively separable. We have:

$$u(c) = \begin{cases} \frac{c^{1-\gamma}-1}{1-\gamma} & \text{for } \gamma \neq 1 \\ \log(c) & \text{for } \gamma = 1, \end{cases}$$

where the constant $\gamma > 0$ represents the consumer's degree of risk aversion. Note that, in the special case of $\gamma = 1$, the limit $u(c) = \log(c)$ results from *l'Hôpital's rule*.⁹ In this particular case, again under the assumption that $r = \rho$, the Euler equation (2.9) becomes $c_t = \mathbb{E}_t(c_{t+1})$. As a consequence, the logarithmic utilities yield the same optimal consumption path as the quadratic preferences analyzed above. The picture is, however, slightly different if the degree of risk aversion is not equal to one ($\gamma \neq 1$). In this case, the Euler equation is equal to:

$$c_t^{-\gamma} = \mathbb{E}_t(c_{t+1}^{-\gamma}).$$

⁸Although the random walk hypothesis rests upon the assumption of quadratic utility, the result that departures of consumption growth from its average value are not predictable arises under more general assumptions (Romer, 2006, p. 356).

⁹According to l'Hôpital's rule,

$$\lim_{\gamma \rightarrow 1} \frac{c^{1-\gamma} - 1}{1 - \gamma} = \log(c).$$

Taking the logarithm on both sides of the equation, we can apply *Jensen's inequality*¹⁰ by virtue of which $\log[\mathbb{E}_t(c_{t+1}^{-\gamma})] \leq \mathbb{E}_t[\log(c_{t+1}^{-\gamma})]$, and derive the following result:

$$\log(c_t) \leq \mathbb{E}_t[\log(c_{t+1})].$$

We see that, without additional constraints, CES utility consumers have similar, but not congruent optimal consumption paths as quadratic utility consumers. Consequently, the REPIH implication that consumption follows a random walk is generally restricted to quadratic utility functions. This result can, however, be extended to other types of utilities, provided that more restrictive assumptions are imposed.¹¹

2.2.2 Definition of Permanent Income

In order to deduce the optimal decision rule, reconsider the initial accounting identity equation (2.2), which describes the dynamic evolution of assets. Rearranging this equation for current assets, we get:

$$A_t = \left(\frac{1}{1+r}\right) A_{t+1} + c_t - y_t. \quad (2.14)$$

Since the initial endowment A_0 is known, we can solve the previous equation forward by substituting for $A_{t+1} = \left(\frac{1}{1+r}\right) A_{t+2} + c_{t+1} - y_{t+1}$, A_{t+2} , ..., A_{t+j} in equation (2.14). This gives the following solution:

$$A_t = \left(\frac{1}{1+r}\right)^j A_{t+j} + \sum_{i=0}^{j-1} \left(\frac{1}{1+r}\right)^i c_{t+i} - \sum_{i=0}^{j-1} \left(\frac{1}{1+r}\right)^i y_{t+i}.$$

Making use of the transversality condition equation (2.3), we find that the first term on the RHS of the previous equation tends towards zero as $j \rightarrow \infty$. We obtain:

$$\sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^i c_{t+i} = A_t + \sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^i y_{t+i}. \quad (2.15)$$

¹⁰If the utility function is concave, Jensen's inequality states that the expected utility is smaller than the utility of the expected value.

¹¹It can be shown, for example, that if the logarithm of income (and consumption) follows a normal distribution, then consumption follows a random walk with drift. See, for instance, [Hansen and Singleton \(1983\)](#) or [Romer \(2006, p. 381\)](#) for details.

Equation (2.15) is the forward recursive solution of the evolution of assets, which becomes the infinite horizon budget constraint of the representative consumer once the transversality condition is taken into account. Starting in any period t , the consumer will finance the present value of current and future consumption with her current assets, as well as the present values of the flows of current and future labor income. Allowing for uncertainty about the future, equation (2.15) can be rewritten as:¹²

$$\sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_t(c_{t+i}) = A_t + \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_t(y_{t+i}).$$

We can now substitute the general definition of $\mathbb{E}_t(c_{t+i})$ under quadratic utility (2.11) into the previous equation:

$$\begin{aligned} \frac{(1+\rho)ac_t + (r-\rho)}{(1+r)a} \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i &= A_t + \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_t(y_{t+i}) \\ \frac{(1+\rho)ac_t + (r-\rho)}{(1+r)a} \left(\frac{1+r}{r} \right) &= A_t + \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_t(y_{t+i}) \\ c_t &= \left(\frac{r}{1+\rho} \right) \left[A_t + \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_t(y_{t+i}) \right] - \frac{r-\rho}{(1+\rho)a}. \end{aligned} \quad (2.16)$$

This equation is the general expression relating consumption to wealth and the constant parameters ρ and r . Though the complete formulation of equation (2.16) is being deepened and discussed in Section 5.3,¹³ for now, it is important to analyze its implications in the case where $r = \rho$, as suggested by Hall (1978). In this case, we have $c_t = \mathbb{E}_t(c_{t+i})$, and the consumption function (2.16) melts down to:

$$c_t = \left(\frac{r}{1+r} \right) \underbrace{\left[A_t + \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_t(y_{t+i}) \right]}_{y_t^p}. \quad (2.17)$$

Equation (2.17) represents the consumption function resulting from the PIH;

¹²Taking the expectation of both sides of equation (2.15) is a necessary step for the derivation of the solution to this problem. This convention is questionable from a strict mathematical point of view, but it is nonetheless widely used in the economic literature.

¹³In Chapter 5, under the definitions of y_t^p , β and R , the consumption function (2.16) is rewritten as $c_t = \beta(R-1)y_t^p - \frac{1-\beta R}{a}$.

it reflects exactly how consumption c_t depends on its determinants: namely, assets A_t and labor income y_t . It states that it is optimal for an individual to consume in every period t a constant fraction (i.e., the annuity value $\frac{r}{1+r}$) of her discounted total lifetime resources. These lifetime resources comprise the value of current financial wealth A_t and the present value of all expected future labor incomes, $\sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^i \mathbb{E}_t(y_{t+i})$. The latter term can be thought of as the consumer's human capital, denoted by H_t , as used, for instance, in Hall (1978, p. 975) and Bagliano and Bertola (2007, p. 8). On the RHS of equation (2.17), the term in square brackets corresponds to Friedman's definition of *permanent income*, denoted by y_t^p . For Friedman (1956), permanent income is, thus, closely related to total wealth, which comprises both nonhuman and human wealth. Consequently, equation (2.17) gives the amount that an individual can consume in every period, while leaving wealth unaltered. This straightforward interpretation of the consumption function – corresponding to the main assumption of the LCPIH – implies that people want to smooth their consumption, keeping consumption levels constant across their lifetimes.

With his derivation of the consumption function under rational expectations, Hall (1978) first showed that the problem of identifying and measuring permanent income becomes redundant because it is entirely captured by current consumption. While the marginal effect of y_t^p on c_t , $\left(\frac{r}{1+r}\right)$, is invariant, all short-term fluctuations in the consumption level are induced by transitory changes in the income level. Second, equation (2.17) shows that an individual's consumption in t is not determined by her wealth or income in the current period only, but rather by all current and future incomes she can expect to earn, given her information set in t . This is the forward-looking feature of the REPIH, which fully accounts for the uncertainty the individual faces regarding the exact evolution of her income in the future.

By relating consumption to its determinants (i.e., financial and human wealth) it is possible to give an exact interpretation of the random walk hypothesis presented above.

Interpretation of the Consumption Function

Making use of the insight that current consumption c_t is equal to current permanent income y_t^p and of the random walk hypothesis summarized in equation (2.13), it is possible to give a precise definition and interpretation of the error term ε_t .

From equation (2.17) the permanent income is given by:

$$y_t^p = A_t + \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_t(y_{t+i}). \quad (2.18)$$

Considering permanent income in period $t+1$ and adding conditional expectations on both sides, we get:

$$\mathbb{E}_t(y_{t+1}^p) = \mathbb{E}_t(A_{t+1}) + \mathbb{E}_t \left[\sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_{t+1}(y_{t+1+i}) \right]. \quad (2.19)$$

From the definition of the evolution of assets in equation (2.2), we know that $\mathbb{E}_t(A_{t+1}) = A_{t+1}$, since next period's wealth depends only on current period values. Using the definition of permanent income, $c_t = \left(\frac{r}{1+r} \right) y_t^p$, we can further deduce from the Euler equation (2.12) that $\mathbb{E}_t(y_{t+1}^p) = y_t^p$. This means that, analogously to consumption, the best and only predictor of permanent income in $t+1$ is the permanent income in t . If we now wish to measure the difference between the actual permanent income in period $t+1$ and the permanent income for period $t+1$ that was expected in the period t , we can subtract equation (2.19) from equation (2.18), updated one period ahead. This yields:

$$\begin{aligned} y_{t+1}^p - \mathbb{E}_t(y_{t+1}^p) &= A_{t+1} + \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_{t+1}(y_{t+1+i}) - \\ &\quad A_{t+1} + \mathbb{E}_t \left[\sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_{t+1}(y_{t+1+i}) \right] \\ y_{t+1}^p - y_t^p &= \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_{t+1}(y_{t+1+i}) - \mathbb{E}_t \left[\sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i \mathbb{E}_{t+1}(y_{t+1+i}) \right] \\ y_{t+1}^p &= y_t^p + \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i [\mathbb{E}_{t+1} - \mathbb{E}_t](y_{t+1+i}). \end{aligned} \quad (2.20)$$

Note that, for the last step, we can make use of the fact that $\mathbb{E}_t[\mathbb{E}_{t+1}(y)] = \mathbb{E}_t(y)$ by the law of iterated expectations. Equation (2.20) states that the evolution of permanent income from one period to another is governed only by the difference in expectations regarding labor income between these two periods. If the information set in t is identical to the information set in $t+1$, then the expectations about income do not change ($\mathbb{E}_t = \mathbb{E}_{t+1}$) and the permanent income will not change between t and $t+1$ (i.e., $y_{t+1}^p = y_t^p$).

Again, under the equality $c_t = \left(\frac{r}{1+r}\right) y_t^p$, we can rewrite equation (2.20) in terms of the deviation of consumption from one period to another:

$$\Delta c_{t+1} = \left(\frac{r}{1+r}\right) \left\{ \sum_{i=0}^{\infty} \left(\frac{1}{1+r}\right)^i [\mathbb{E}_{t+1} - \mathbb{E}_t] (y_{t+1+i}) \right\} \quad (2.21)$$

$$\Delta c_{t+1} = \varepsilon_{t+1}. \quad (2.22)$$

Equation (2.22) is equal to the above-defined equation (2.13), which states that, under quadratic utility and an interest rate identical to the discount rate, the optimal consumption path follows a random walk. Equation (2.21) now gives us a precise definition of the innovation ε . It becomes evident that consumption growth is dictated only by revisions of the expectations regarding future labor incomes. These revisions in expectations occur only if new and previously unpredictable information about income emerges between two periods. Since such news is unpredictable, it means that the error terms are orthogonal to each other, which implies that $\mathbb{E}_t(\varepsilon_{t+i}) = 0$ for all $i = \{1, 2, \dots\}$. This is another way of stating that, if the news about future income were predictable at t , it would already be accounted for in the current consumption level c_t . The fact that the error terms have a zero mean that is conditional on all previously observed information is convenient in that the lagged values of income and consumption become valid instruments for the estimation of the consumption function. This fact will be used in Chapter 3 for the two-stage least squares estimation of the money illusion consumption function. Note, finally, that wealth completely disappeared from equation (2.21), leaving current consumption as the only measurable and relevant predictor for future consumption.

2.3 The Money in the Utility Function Model

The present thesis is devoted to the analysis of the impact of money illusion on consumption. The central question in performing such an analysis remains how to best model money illusion. In the later chapters, I suggest three approaches to modeling money illusion – namely, as a rule of thumb (Chapter 3), as a signal extraction problem (Chapter 4) and as a source of model ambiguity (Chapter 5) – all of which directly or indirectly rely on the assumptions of the REPIH. Since money illusion is, *per se*, a monetary phenomenon, another potential way to address this problem is to depart from the previously derived REPIH, which depicts a nonmonetary economy (in which no money is used as a medium of exchange).

One way to introduce money into the consumer's problem is to assume that the representative consumer wants to constantly hold a positive amount of real money balances because holding such a balance eases transactions and is, hence, more time-efficient. Since the consumer attributes an intrinsic utility to money as a transaction facilitator, this model is called the MIU model. Even though the original formulation of this model excludes the presence of money illusion by focusing only on real values, money illusion is related to the MIU model in two ways. First, money illusion implies a non-neutrality of money in the short run, which is a feature of the standard MIU model. As discussed in more detail in Chapter 3, money illusion induces individuals to seek increases in their nominal income and wealth, regardless of the inflation level (Shafir et al., 1997). In consequence, people's utility and consumption depend partially on inflation and, thus, are not neutral to monetary values.

Second, the fact that consumers retain positive utility from the mere holding of money balances can explain why people suffer from money illusion. Since having more money is always preferred to having less money, the money-illusioned consumer will be reluctant to undergo cuts in her nominal income or nominal wealth – a typical feature of money illusion. This approach is advocated by Miao and Xie (2013), who incorporate money illusion into the MIU model by assuming that consumers retain a direct utility from a mixture of real and nominal money balances. The standard (illusion-free) MIU model is derived in the next section, followed, in Section 2.3.2, by a brief presentation of the extension, incorporating money illusion, suggested by Miao and Xie (2013).

2.3.1 The Original Sidrauski Model

The MIU model goes back to Sidrauski (1967a,b) who was the first to link money, inflation and growth by attributing an intrinsic role to money.¹⁴ Since then, the MIU model has been widely used in the literature on monetary economics to analyze the impact of monetary policies and the welfare cost of inflation (see, for example, Poterba and Rotemberg, 1986; Blanchard and Kiyotaki, 1987; Farmer, 1997).

In the standard MIU model, the representative consumer's problem can be written as follows. The consumer wants to maximize her lifetime utility:

$$U_t = \mathbb{E}_t \left[\sum_{t=0}^{\infty} \beta^t u(c_t, m_t) \right],$$

¹⁴In this section, I loosely follow Walsh (2010, Chapter 2) regarding notation and derivations.

where $\beta = \frac{1}{1+\rho}$ is the intertemporal discount factor, subject to the following wealth governing constraints:

$$W_t = f(k_{t-1}) + \tau_t + (1 - \delta)k_{t-1} + \frac{(1 + i_{t-1})b_{t-1} + m_{t-1}}{1 + \pi_t} \quad (2.23)$$

$$W_t = c_t + k_t + b_t + m_t. \quad (2.24)$$

The within-period utility is assumed to be strictly concave, continuously differentiable and increasing in both real consumption c_t and the end-of-period real money balances $m_t = \frac{M_t}{P_t}$. While the inclusion of m_t in the utility function corresponds to Sidrauski's intuition, it is also the main criticism of the MIU model: Even if money holdings are never used for the purchase of consumption goods and services, they yield utility to the consumer. On one hand, this seems to contradict the full rationality assumption.¹⁵ On the other hand, if one assumes that money illusion can prevail in the short-term, then leaving the money balances in $u(c_t, m_t)$ is, again, a reasonable representation of the consumer's preferences.

The first constraint, equation (2.23), represents the consumer's sources of wealth, which consist of a production function $f(k_{t-1})$ with constant returns to scale, real lump-sum transfers per capita τ_t and capital stock per capita k_{t-1} (adjusted by the rate of depreciation δ), as well as the previous period's money balances m_{t-1} and bonds b_{t-1} holdings (where i_{t-1} is the nominal interest rate on bond holdings, and π_t is the inflation rate, such that $1 + \pi_t = \frac{P_t}{P_{t-1}}$). Equation 2.24 states that the consumer can spend her income on consumption, invest it in capital, save it in bonds or keep it as money balances.

This dynamic optimization problem can be formulated in terms of a value function with the following Bellman equation:

$$V(W_t) = \max_{\{c_t, m_t, b_t\}} \left\{ u(c_t, m_t) + \beta \mathbb{E}_t [V(W_{t+1})] \right\} \quad (2.25)$$

subject to

$$\begin{aligned} W_{t+1} = & f(W_t - c_t - b_t - m_t) + \tau_{t+1} + \\ & + (1 - \delta)(W_t - c_t - b_t - m_t) + \frac{(1 + i_t)b_t + m_t}{1 + \pi_{t+1}}. \end{aligned} \quad (2.26)$$

¹⁵An alternative model is the cash-in-advance (CIA) model, which does not attribute an intrinsic utility to money, but, instead, imposes the additional constraint $P_t c_t \leq M_t$, which states that every good must be paid with cash in advance (see Clower, 1967; Lucas, 1980). Interestingly, the CIA model yields very similar conclusions to those of the MIU model, be they in cases without or with consideration for money illusion (for the latter case, see Appendix B of Miao and Xie, 2013).

Note that equation (2.23) has been inserted into equation (2.24) to eliminate k_t , such that the maximization needs to be done over the control variables c_t , m_t and b_t , as well as over the controlled state W_{t+1} .

In the same fashion as in the previous section, we can set up the corresponding Lagrangian for equation (2.25) and derive the corresponding FOC:

$$c_t : u_c(c_t, m_t) = \lambda_t [f_k(k_t) + (1 - \delta)] \quad (2.27)$$

$$m_t : u_m(c_t, m_t) = \lambda_t \left[f_k(k_t) + (1 - \delta) - \frac{1}{1 + \pi_{t+1}} \right] \quad (2.28)$$

$$b_t : f_k(k_t) + (1 - \delta) = \frac{1 + i_t}{1 + \pi_{t+1}} \quad (2.29)$$

$$W_{t+1} : \beta \mathbb{E}_t \left[\frac{\partial V(W_{t+1})}{\partial W_{t+1}} \right] = \lambda_t, \quad (2.30)$$

accompanied by the following transversality conditions, which ensure that the consumer does not over-accumulate debts in terms of capital, money or bonds to attain a higher lifetime utility:

$$\lim_{t \rightarrow \infty} \beta^t \lambda_t k_t = 0$$

$$\lim_{t \rightarrow \infty} \beta^t \lambda_t m_t = 0$$

$$\lim_{t \rightarrow \infty} \beta^t \lambda_t b_t = 0.$$

Making use of the envelope theorem for equation (2.30), we get

$$\frac{\partial V(W_t)}{\partial W_t} = \lambda_t [f_k(k_t) + (1 - \delta)] = u_c(c_t, m_t),$$

updated one period:

$$\frac{\partial V(W_{t+1})}{\partial W_{t+1}} = \lambda_{t+1} [f_k(k_t) + (1 - \delta)] = u_c(c_{t+1}, m_{t+1}). \quad (2.31)$$

Substituting (2.31) back into (2.30) yields the definition of λ_t :

$$\lambda_t = \beta \mathbb{E}_t [u_c(c_{t+1}, m_{t+1})].$$

Combining equations (2.27), (2.29) and (2.30) gives us the following Euler equation for consumption:

$$u_c(c_t, m_t) = \beta \mathbb{E}_t[u_c(c_{t+1}, m_{t+1})] \left(\frac{1 + i_t}{1 + \pi_{t+1}} \right). \quad (2.32)$$

It states that the utility loss from consumption today equals the utility from consumption tomorrow, adjusted for the real gain from keeping bonds. When we define the real interest rate as $1 + r_t = \frac{1+i_t}{1+\pi_{t+1}}$, the Euler equation for consumption is similar to the one obtained in the REPIH (i.e., equation (2.9)). The crucial difference is that money balances enter the utility function and, thus, potentially have a direct impact on the consumer's optimal consumption path.¹⁶ Note, further, that the just-used definition of the real interest rate is called the *Fisher relationship* and corresponds to the real return on capital, as shown by the FOC for b_t in equation (2.29).

Making use again of the definition of λ_t and of equation (2.29), equation (2.28) gives the Euler equation for money. This can be interpreted as the marginal cost, in terms of future consumption, of retaining real money balances today:

$$u_m(c_t, m_t) = \beta \mathbb{E}_t[u_c(c_{t+1}, m_{t+1})] \left(\frac{i_t}{1 + \pi_{t+1}} \right). \quad (2.33)$$

Finally, we can combine equation (2.32) with equation (2.33) to get the marginal rate of substitution (MRS) between money and consumption:

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \frac{i_t}{1 + i_t}. \quad (2.34)$$

This is a central result of Sidrauski's MIU model, which characterizes the money demand. It appears that the intertemporal ratio of marginal utilities is equal to the relative price, $\frac{i_t}{1+i_t}$, which can thus be considered as the opportunity cost (in terms of consumption goods) of holding money balances. We see from the Fisher relationship that inflation increases the nominal interest rate and the RHS of equation (2.34). In consequence, inflation is said to be equivalent to a distortionary *inflation tax* (Walsh, 2010, p. 53). The counterpart to the demand for money is the growth of real money balances, which is set by the government and is governed by the following equation:

$$\frac{m_{t+1}}{m_t} = \frac{1 + \vartheta_t}{1 + \pi_{t+1}},$$

where $(1 + \vartheta_t)$ is the growth rate of the nominal money balances. Again, we see

¹⁶This is, however, only the case if the utility function is non-separable, as in the example presented further below, on page 27.

that inflation has a negative impact on the growth of real money balances.

The Non-Separable MIU Utility Function

Of crucial importance for this section is the shape of the MIU utility function, which typically takes the following CES form:

$$u(c_t, m_t) = \frac{1}{1-\gamma} \left[\alpha(c_t)^{1-\varphi} + (1-\alpha)(m_t)^{1-\varphi} \right]^{\frac{1-\gamma}{1-\varphi}}, \quad (2.35)$$

where $\gamma > 0$ is the risk aversion parameter, $\frac{1}{\varphi} > 0$ is the elasticity of substitution between consumption goods and money, and $\alpha \in (0, 1)$ represents the relative weights of consumption and money. This general specification comprises several specific cases. For instance, $\gamma = 1$ implies a logarithmic utility function (similar to the example on page 17). Further, the case in which $\gamma = \varphi$ condenses down to a separable utility function, since the marginal utilities of consumption and money are then independent from one another. Note, however, that the case of non-separability, in which $\gamma \neq \varphi$, seems more plausible because the mere fact that holdings of real money balances facilitate transactions signifies that the consumer's utility is not independent of the amount of goods she is willing to purchase.

The partial derivatives of u with respect to c_t and m_t are:

$$\begin{aligned} u_c(c_t, m_t) &= \alpha c_t^{-\varphi} \left[\alpha(c_t)^{1-\varphi} + (1-\alpha)(m_t)^{1-\varphi} \right]^{\frac{\varphi-\gamma}{1-\varphi}} \\ u_m(c_t, m_t) &= (1-\alpha) m_t^{-\varphi} \left[\alpha(c_t)^{1-\varphi} + (1-\alpha)(m_t)^{1-\varphi} \right]^{\frac{\varphi-\gamma}{1-\varphi}}. \end{aligned}$$

Looking at the first derivative, we see that the marginal utility of consumption depends on the real money balances. In consequence, inflation has, potentially, two different effects on consumption. First, the inflation tax weighs on the consumer's budget and, thus, is expected to have a negative (wealth) effect on consumption. Second, inflation will have an intertemporal substitution effect because the optimal consumption path (described by the consumption Euler equation (2.32)) will also depend on money.

Inserting these equations back into equation (2.34) yields the following optimal money demand for the MIU consumer:

$$\begin{aligned} \frac{1-\alpha}{\alpha} \left(\frac{m_t}{c_t}\right)^{-\varphi} &= \frac{i_t}{1+i_t} \\ \frac{m_t}{c_t} &= \left[\left(\frac{\alpha}{1-\alpha}\right) \left(\frac{i_t}{1+i_t}\right) \right]^{-\frac{1}{\varphi}}. \end{aligned} \quad (2.36)$$

The money demand in terms of consumption goods, $\frac{m_t}{c_t}$, depends on the time-invariant parameters φ and α , as well as on the nominal interest rate i_t . The simple and reduced form of this equation presents an easily testable empirical model of the real money balances as a function of real consumption and the nominal interest rate. Again, an increase in inflation decreases the RHS of equation (2.36) and implies, for a constant c_t , a decrease in m_t . While this is an interesting feature, equation (2.36) merely describes the ratio of real money to consumption, which is expressed in their *levels* and for a given period. With regard to money illusion and to the effective impact of inflation on the real economy, the crucial question is to what extent consumption *growth* is affected by this strict monetary phenomenon. As is presented in the next section, the answer to this question is not unambiguous; rather, it relies heavily on the governing parameters and the assumptions about the underlying processes.

2.3.2 Incorporating Money Illusion

Miao and Xie (2013) extended the standard MIU model developed by Sidrauski (1967a,b) by assuming that the representative consumer suffers from money illusion, which is defined as her (partial) reliance on both nominal and real values. This definition is similar to the one used in Chapters 3 and 4, and it implies that the consumer's demand function is not anymore homogeneous of degree zero with respect to its determinants (see Section 3.1.2). To account for this apparent irrationality, Miao and Xie (2013) suggested including the price level in the MIU utility function, as follows:

$$U(c_t, M_t, P_t) = \frac{1}{1-\gamma} \left\{ \alpha \left[c_t^{1-\theta} (P_t c_t)^\theta \right]^{1-\varphi} + (1-\alpha) \left[\left(\frac{M_t}{P_t} \right)^{1-\theta} M_t^\theta \right]^{1-\varphi} \right\}^{\frac{1-\gamma}{1-\varphi}}, \quad (2.37)$$

where the variables and parameters are the same as defined in the previous section, except for the new money illusion parameter $\theta \in (0,1)$. According to this

money illusion MIU utility function, people again retain utility from consumption and money. Money illusion comes into play through the θ parameter, which attributes utility not only to real consumption c_t and real money balances $\frac{M_t}{P_t} = m_t$, but also to nominal consumption $P_t c_t$ and nominal money balances M_t . If there is no money illusion and the consumer is fully rational (i.e., if $\theta = 0$), then the utility function is identical to the one analyzed in the previous section (equation (2.35)). The other extreme case is $\theta = 1$, in which case the consumer suffers from complete money illusion. In this case, she seeks to maximize her nominal consumption and nominal money balances only. The fact that the utility is now a function of the general price level in $U(c_t, M_t, P_t)$ does not mean that the consumer prefers higher prices to lower prices; rather, it reflects the fact that the consumer suffering from money illusion derives her total utility from the “misperceived mixture of real and nominal consumption, as well as real and nominal money balances” (Miao and Xie, 2013, p. 87).

Similarly to the standard MIU model analyzed in the previous section, $\gamma = \varphi = 1$ implies that the utility function is log-linear and completely separable in c_t , M_t and P_t . In this case, the consumption decision is affected neither by money illusion nor by the money balances, such that neutrality of money prevails even with a positive θ . To allow for a certain effect of money illusion, Miao and Xie (2013) restricted their analysis to the special case where $\gamma = \varphi$, corresponding to an additively separable utility function over money and consumption and under money illusion. This assumption is fairly restrictive, and it needs to be kept in mind when interpreting their main results. To illustrate the difficulty of drawing conclusive and incontestable results regarding the real impact of money illusion on the consumer’s optimal behavior, consider the partial derivatives of the (non-separable) MIU utility function with respect to consumption (c_t) and real money balances ($\frac{M_t}{P_t}$):

$$u_c(c_t, m_t) = \alpha P_t^\theta (c_t P_t^\theta)^{-\varphi} \left\{ \alpha \left[c_t^{1-\theta} (P_t c_t)^\theta \right]^{1-\varphi} + (1-\alpha) \left[\left(\frac{M_t}{P_t} \right)^{1-\theta} M_t^\theta \right]^{1-\varphi} \right\}^{\frac{\varphi-\gamma}{1-\varphi}}$$

$$u_m(c_t, m_t) = (1-\alpha) P_t^\theta \left(\frac{M_t}{P_t} P_t^\theta \right)^{-\varphi} \left\{ \alpha \left[c_t^{1-\theta} (P_t c_t)^\theta \right]^{1-\varphi} + (1-\alpha) \left[\left(\frac{M_t}{P_t} \right)^{1-\theta} M_t^\theta \right]^{1-\varphi} \right\}^{\frac{\varphi-\gamma}{1-\varphi}}.$$

In line with the previous section, the marginal utility of consumption depends, not only on consumption, but also on the (nominal and real) money balances and the general price level. The impacts of these variables on the consumption path depend entirely on the choices of the parameters θ , α , γ and φ , as well as on the

assumptions about the processes governing the evolution of the price and money levels in the economy. Furthermore, recall that the case in which $\gamma \neq \varphi$ probably describes the MIU consumer's preferences more accurately than the computationally easier case of $\gamma = \varphi$. Indeed, it seems more plausible that the demands for money holdings and consumption are not independent from one another because at least some current money balances are being held as a transaction facilitators for future purchases of consumption goods. This argument clearly favors the general case with non-separable preferences.

Assuming $\gamma = \varphi > 1$ and introducing some specific monetary and productivity shocks, [Miao and Xie \(2013\)](#) found a negative relationship between inflation and long term economic growth. Intuitively, an increase in the expected inflation rate encourages the consumer to consume more and save less, which impairs future growth. In their setting, the money-illusioned agent's consumption and saving decisions were also influenced by her distorted perception of the riskiness of real wealth. Note that, while money illusion might well impact the real economy via this wealth channel, the following chapters of this thesis assume that, for psychological reasons (which will be exposed in [Section 3.1.2](#)), money illusion's main impact on real consumption growth works via the income channel.

In order to define the optimal money demand for the money-illusioned MIU consumer, we can insert the marginal utilities of consumption and money into the Euler equation [\(2.34\)](#) and get

$$\begin{aligned} \frac{(1-\alpha) P_t^\theta \left(\frac{M_t}{P_t} P_t^\theta\right)^{-\varphi}}{\alpha \frac{P_t^\theta}{P_t^\theta} (c_t P_t^\theta)^{-\varphi}} &= \frac{i_t}{1+i_t} \\ \frac{\left(\frac{M_t}{P_t}\right)}{c_t} &= \left[\left(\frac{\alpha}{1-\alpha}\right) \left(\frac{i_t}{1+i_t}\right) \right]^{-\frac{1}{\varphi}}, \end{aligned} \quad (2.38)$$

which is equal to equation [\(2.36\)](#) in cases with only real money balances (m_t) and without money illusion ($\theta = 0$). Interestingly, even a fully money-illusioned consumer (i.e., a consumer who retains utility solely from nominal values) will have the same optimal money demand. To see this, plug $\theta = 1$ into equation [\(2.37\)](#) and maximize over $c_t P_t$ (instead of c_t) and M_t (instead of $\frac{M_t}{P_t}$). The result will be the same money demand as in equations [\(2.36\)](#) and [\(2.38\)](#). The fact that a money-illusioned consumer's optimal money demand is the same as the rational consumer's (i.e., the two consumers have the same optimal money-consumption ratio) can be

interpreted as the fact that money illusion, just like any irrational behavior, is not optimal or viable in the long run and, in consequence, can only reflect short-term deviations from the optimal money-to-consumption path. Beware, however, that, as alluded in the previous section, equation (2.38) is solely a static observation of the factors influencing the consumer's optimal demand for money and consumption, since this equation is valid in every period. Of course, any deeper and more thorough analysis of the putative impact of money illusion on consumption should focus on its impact on consumption growth. Due to its heavy reliance on the *ad hoc* assumptions regarding the governing parameters and the underlying processes, I depart from this extended MIU model in the following chapters in order to use empirically more suitable models to estimate the relationship between inflation and consumption growth.

2.4 Conclusion

This chapter has presented two widely used macroeconomic models that are relevant to the main topic of the present dissertation: money illusion, which assumes the existence of a direct and causal relationship between inflation and consumption growth. The first model, the REPIH model, shows that a fully rational and forward-looking agent seeks to keep her consumption constant over time and that deviations from this optimal path are only driven by random shocks to income. In this nonmonetary economy, money illusion is rejected *a priori* because inflation has no impact on the real economy or consumption growth. Since the REPIH yields a very convenient and intuitive dynamic framework for analyzing consumption growth and testing for its determinants, it is used as a long-term benchmark model in the next chapters, which perform empirical investigations on the putative link between money illusion and consumption growth.

The second model analyzed in this chapter was the MIU model, which attributes an intrinsic value to the holding of real money balances. In this model, inflation has – through these money holdings – a potentially important impact on the real economy. [Miao and Xie \(2013\)](#) extend this model to allow for the representative consumer to suffer from money illusion (i.e., to retain utility from both real and nominal consumption and money balances). They show that, under some specific assumptions, money illusion stimulates immediate consumption and impedes long-term economic growth through unexpected inflation. Chapter 3 further investigates this potentially positive impact of inflation on short-term consumption growth and

provides an alternative justification for it: namely, the nominal income channel (see Section 3.1.2). Note, however, that the empirical investigations in chapters 4 and 5 do not confirm this positive impact of inflation on consumption; rather, they advocate in favor of a short-term depressing effect of money illusion on consumption growth.

Chapter 3

Estimating a Money Illusion Consumption Function

Money illusion, defined as the misperception of nominal values as real values, implies that consumption is not homogeneous of degree zero with respect to its determinants and, thus, reacts to variations in the price level. This chapter tests for the presence of money illusion in aggregate U.S. quarterly consumption data by adding inflation as an explanatory variable to the random walk hypothesis model. It appears that inflation is, in fact, the most important determinant of consumption growth and that money illusion has two distinct effects on consumption. First, contemporaneous inflation negatively impacts on current consumption growth, which reflects that consumers misinterpret inflation as individual price increases and, thus, reduce their immediate consumption. This negative inflation effect reflects money illusion as a signal extraction problem. Second, lagged inflation has a positive cumulative effect on current consumption growth. This positive inflation effect reflects the presence of money illusion as a cost-saving rule of thumb, such that consumers use nominal values as proxies for real values and increase their consumption when nominal incomes are adjusted upward towards inflation. The results are robust to the inclusion of other determinants of consumption growth, do not depend on the choice of the inflation measure and are valid across periods experiencing different average inflation rates.

Keywords: Money illusion, aggregate consumption function, random walk hypothesis.

JEL classification: D11, D12, D91, E21, E31.

3.1 Introduction

Money illusion is a well-known concept among economists that is widely used to describe a variety of apparently irrational behaviors that are, in principle, all related to a latent confusion between nominal and real values. In order to better apprehend this still vague phenomenon, Section 3.1.1 presents, in a first step, a brief literature overview of the concept and applications of money illusion. A major problem related to money illusion is that, even though survey studies show that individuals are confused between nominal and real values, there is not yet certitude regarding whether they really behave in an illusioned way and, consequently, whether money illusion has a real impact on aggregate values. In a second step, in Section 3.1.2, I derive a refined definition of money illusion that allows for a short- to medium-term impact of inflation on aggregate real consumption. It is important to emphasize that this assumption does not necessarily contradict the traditional macroeconomic models, which are based on the full rationality assumption and the long-run non-neutrality of money.¹

In the present chapter, it is not only assumed that inflation is a major short-term determinant of consumption growth, but also that its impact differs depending on the time horizon considered. In the very short term, inflation has a negative impact on consumption because consumers misinterpret (unanticipated) increases in the aggregate price level as rises in the prices of individual goods they typically purchase and, consequently, substitute consumption with saving. In the short to medium term, money illusion has a positive impact on consumption because money-illusioned consumers “feel richer” as their nominal incomes increase and, thus, increase their consumption. Note that this dual impact of inflation on consumption growth assumed in this chapter is quite different from the sole positive wealth effect of inflation on consumption growth suggested by the extended MIU model of [Miao and Xie \(2013\)](#) (see Section 2.3.2).

The subsequent sections derive and estimate empirical models that allow to test for the presence of money illusion in aggregate consumption. First, in Sec-

¹In particular, the present assumption is totally compatible with the early signal extraction literature of [Lucas \(1972, 1973\)](#) and [Barro \(1977, 1978\)](#), who allowed for inflation to have a short-term positive impact on output and, similarly, a negative impact on unemployment. Their idea was that incomplete information prevents people from perfectly distinguishing nominal money shocks from (demand-induced) real shocks. Consequently, some nominal shocks are unanticipated and induce short-term non-neutrality. Using the theoretical implications of money illusion for consumption, the present study extends this intuition to consumption by allowing unanticipated nominal shocks to have a real impact on consumption. As in the original models, we will see in Chapter 4 that money illusion can be interpreted and modeled as a signal extraction problem.

tion 3.2, inflation is tested as a determinant of consumption growth within the rational expectations–permanent income hypothesis (REPIH) model. The resulting *inflation-augmented random walk hypothesis model* is then extended in Section 3.3 to form the *money illusion consumption function*, which controls for other possible determinants of consumption growth.

The estimation of these models with quarterly U.S. data (Section 3.4) shows interesting results and presents the following new insights on the relationship between inflation and consumption. First, inflation is the most important short-term determinant of consumption growth and should, as a consequence, not be excluded from any study modeling and forecasting aggregate consumption.² Second, the inflation effect on consumption growth is, as expected, negative for contemporaneous inflation and (mostly) positive for lagged inflation rates, confirming the presence of different kinds of money illusion depending on the considered time horizon. Last, inflation has a more long-lasting effect on consumption during periods with low average inflation rates than during periods with high inflation rates. This indicates that money illusion through the nominal income channel decreases as inflation increases, reflecting a greater awareness of inflation when inflation accelerates or lies above a certain threshold. Note that, while the positive inflation effect is not confirmed by the estimations of the alternative consumption models under money illusion in Chapters 4 and 5, these estimations also show a more pronounced effect of inflation on consumption in periods with high average inflation.

3.1.1 Money Illusion in the Economic Literature

As early as in the beginning of the twentieth century, money illusion was considered to be a cognitive bias, through which, as defined by Camerer et al. (2004, p. 32) people “make decisions based on nominal quantities rather than converting those figures into real terms by adjusting for inflation”. For Fisher (1928, p. 4), who coined the term and devoted an entire book to it, money illusion is “the failure to perceive that the dollar, or any other unit of money, expands or shrinks in value”.

In particular, money illusion becomes crucial for Keynesian economists because it violates the long-run neutrality proposition of the quantitative theory of money and provides an explanation for the observed downward nominal wage and price rigidities. Describing the observed tendency for workers to reject nominal wage cuts

²From a long-term and growth perspective, consumption and income (output) grow at the same rate. In consequence, the main long-term determinant of consumption growth is, indeed, income, as is evident from the consumption function represented by equation (2.17) in the previous chapter.

but to accept real wage cutbacks, Keynes (1936, p. 9) wrote: “whether logical or illogical, experience shows that this is how labour in fact behaves”. As Akerlof and Shiller (2009, Chapter 4) state it, money illusion is, in Keynes’ view, one of five *animal spirits* (or apparent irrationalities) that govern much of the economic activity and cause the economic fluctuations. Concerning the nominal wage rigidities, modern field studies by Baker et al. (1994) and Bewley (1999) show, for example, that firms are reluctant to cut nominal wages due to such a move’s negative impact on employees’ satisfaction. This could be explained by the fact that workers suffering from money illusion focus on their nominal instead of their real wage fluctuations. For example, as Shafir et al. (1997, p. 364) learned through questionnaires, “many people [...] who would strongly object to a 1 percent cut in salary in times of no inflation, are less likely to complain when they get a 5 percent raise in times of 6 percent inflation”.³ Since wages and prices are generally closely related, there might also be a potential and indirect link between downward price rigidities and money illusion, in that people will dislike price decreases because of their negative impact on nominal wages and, thus, their satisfaction. Since price rigidities can typically be explained by more straightforward illustrations such as menu costs, I deliberately ignore the possibility that money illusion may have an impact on the real economy via the downward rigidity of wages and prices.

During the 1970s, with the breakthrough of the rational expectations revolution, money illusion was considered irrational and costly to decision makers and, hence, completely lost its interest for researchers. This change in paradigm is perfectly illustrated by Tobin (1972, p. 3) who warned that “an economic theorist can, of course, commit no greater crime than to assume money illusion”. The REPIH model developed by Hall (1978) and presented in Chapter 2 perfectly mirrors this paradigm, since the agent is fully rational and, consequently, does not attribute any role to money or to nominal values.

It is not until the 1990s and the emergence of behavioral economics that money illusion regained some popularity, especially as an example of irrational behavior. Unfortunately, the very broad definition of the concept of money illusion, which was thought to arise with “any change in relative prices” (Shafir et al., 1997, p. 347), resulted in the use of money illusion to describe a plethora of inscrutable situations.

³As suggested by (Fehr et al., 2009), it is probable that other mechanisms, such as fairness or morale, rather than money illusion, play a crucial role in these downward rigidities. However, one cannot exclude that these fairness concerns do not, in fact, stem from money illusion. Part of the answer was potentially provided by Boes et al. (2007) who analyze money illusion through people’s wage satisfaction and did not find evidence in favor of money illusion.

For example, not only was money illusion revived as an explanation for nominal downward wage and price rigidity (Fehr and Tyran, 2001), but it was also newly used to describe confusion after a currency changeover.⁴ Moreover, money illusion was also transposed onto different markets (in which the distinction between nominal and real values plays an important role) to describe, for example, an investors' reliance on nominal values in the financial and housing markets.^{5,6}

Despite the numerous studies focusing on money illusion, evidence in favor of it can only be found at the individual level. In particular, such evidence has been obtained through survey studies (Shafir et al., 1997; Shiller, 1997), laboratory experiments (Fehr and Tyran, 2001, 2005, 2008; Weber et al., 2009) and in the fact that the fraction of inflation-indexed contracts is low and does not increase when inflation accelerates (Howitt, 1987, p. 518). Many behavioral studies focusing on irrationalities suggest that even a small amount of irrationality at the individual level can potentially have a huge and long-lasting aggregate impact.⁷ Consequently, a deeper understanding of the mechanisms underlying money illusion and of its aggregate impact is of greatest importance. There exist, however, hardly any studies trying to investigate the presence of money illusion at the aggregate level.⁸ The following sections aim to fill this gap by deepening the understanding of the general relationship between consumption and inflation. In light of the overuse of the term of “money illusion”, Section 3.1.2 first suggests a narrower and more testable definition of money illusion.

⁴Consumers' “illusion” with respect to newly introduced currencies is analyzed by Kooreman et al. (2004), Cannon and Cipriani (2006), Wertenbroch et al. (2007) and Dzokoto et al. (2010).

⁵In financial markets, money illusion is also referred to as the Modigliani-Cohn hypothesis, in which investors discount future real payoffs at nominal rather than at real rates (Modigliani and Cohn, 1979). More recent studies also support this finding, for example Ritter and Warr (2002), Sharpe (2002), Campbell and Vuolteenaho (2004), Cohen et al. (2005) and Basak and Yan (2010).

⁶Examples of studies supporting the presence of money illusion on the housing market are Genesove and Mayer (2001), Brunnermeier and Julliard (2008), Piazzesi and Schneider (2007) and Stephens and Tyran (2012).

⁷See, in particular, works from George Akerlof (Akerlof and Yellen, 1985a,b; Akerlof, 2002). A few studies directly address money illusion by supposing its potentially important aggregate impact, but none directly test for it using aggregate data. See, for instance, Shafir et al. (1997), Fehr and Tyran (2001) and Basak and Yan (2010).

⁸Exceptions are, for example, Akerlof et al. (1996) and Akerlof et al. (2000), who construct a long-term *money illusion Phillips curve* and analyze its consequences for the real economy. It seems unrealistic, however, that money illusion can subsist in the long run, as suggested by assumption (A2) below. Another example is the MIU model of Miao and Xie (2013), which is presented in Section 2.3.2.

3.1.2 Refining the Concept of Money Illusion

In this thesis, money illusion is defined as the consumer's confusion between nominal and real values, causing real consumption to react to purely nominal variations of its determinants. Intuitively, money illusion can be interpreted as the *underperception* of inflation on the part of the consumer. In spite of this rather straightforward definition of money illusion, several factors may explain why a consumer might present some degree of money illusion in her consumption behavior.

Rather than using the original definition of money illusion as a pure irrationality, I focus on money illusion as a cost- and time-saving rule that is fully consistent with rationality.⁹ Since the continuous collecting and processing of information about inflation is costly for the consumer, she updates her information set only occasionally or when the inflation rate changes significantly. In this respect, the present understanding of money illusion is closely related to the rational inattention model, which assumes that people are constrained in their ability to acquire and process information. The rational inattention model, also referred to as the sticky information model, is advocated by [Sims \(2003\)](#), [Mankiw and Reis \(2002\)](#) and [Reis \(2006\)](#).

The informational constraint implies two different types of money illusion: namely, money illusion as a signal extraction problem and money illusion as a rule of thumb. Whereas the former predicts a direct and negative impact of inflation on consumption, the latter implies an indirect but positive and more long-lasting effect of inflation on consumption:

- (i) The negative *unanticipated inflation effect*: This channel, described by [Deaton \(1977\)](#), stems from the fact that consumers purchase their goods in a sequential manner. When inflation is unanticipated, or when consumers are not perfectly informed about inflation, then (at least some) consumers temporarily misinterpret the price rises of goods they typically purchase as relative price increases rather than as inflation, (i.e., rather than as increases in the general price level). The result of this money illusion is that, in the case of unexpected inflation, each consumer attempts to adjust her purchases (i.e., favoring saving, or future consumption, to immediate consumption), such that contemporaneous real consumption automatically falls. In this case, money

⁹In this chapter, I do not consider money illusion to reflect the consumer's preference for positive holdings of money balances in order to facilitate transactions. This particular case is discussed in [Section 2.3](#).

illusion as a bounded rationality is observationally equivalent to a pure cognitive bias: For some arbitrary reason, the consumer makes a mistake in her calculations, either because she suffers from incomplete information or because she is unable (or unwilling) to continuously collect and process the available inflation. This is a typical example of a signal extraction problem, which can also be apprehended with specific modeling and estimation techniques (see Chapter 4). Of course, this contemporaneous and negative effect of inflation on consumption disappears very rapidly as more and more consumers realize that the price increases of the individual goods are due to inflation. Similarly, the unanticipated inflation effect disappears as the rational consumer updates her information set after a nominal price shock.

- (ii) The positive *nominal income effect*: According to this channel, consumers base their (real) consumption decisions, not only on their real income, but also on their nominal income. In this case, inflation indirectly stimulates real consumption because money-illusioned consumers misinterpret nominal income increases as real income increases in the case of a positive price shock. The idea of this argument, first advocated by [Branson and Klevorick \(1969\)](#) and supported by [Shafir et al. \(1997\)](#), is that, if money-illusioned consumers see their nominal income increase, they “feel richer” and increase their current consumption. This type of money illusion can be interpreted as an efficient cost- and time-saving rule of thumb, in which consumers use nominal values as proxies for real values. Consumers could, indeed, rationally decide to not fully account for inflation because nominal values are more salient, easier to gauge, more available, and good proxies for real values in periods of stable inflation ([Shafir et al., 1997](#); [Fehr and Tyran, 2001](#)).¹⁰ Note that this positive nominal income–money illusion effect does not take place immediately after an unanticipated positive price shock, but only once nominal incomes are adjusted to account for the new price level. On the other hand, the effect could easily last for several periods, depending on the efficiency of the rule of thumb.

At present, the distinction between the negative unanticipated inflation effect

¹⁰There is extensive literature on the heuristics employed in making judgments under uncertainty, which dates back to [Tversky and Kahneman \(1974\)](#). In particular, [Cochrane \(1989\)](#) demonstrated that simple rules of thumb for consumption (so-called “near rational” alternatives) incur only very small utility losses. Furthermore, [Uhlig and Lettau \(1999\)](#) considered how such rules of thumb can arise endogenously in a learning context. Finally, [Akerlof et al. \(2000\)](#) provided a good overview of different psychological justifications for the use of money illusion as a rule of thumb, in particular during periods of low inflation.

and the positive nominal income effect is not crucial for the mere identification of money illusion in the aggregate consumption. Formally, both of these money illusion mechanisms can be described as violations of *homogeneity postulate* developed by Patinkin (1965), which asserts that demand and supply functions must be homogeneous of degree zero in all nominal prices (see also Leontief, 1936; Dusansky and Kalman, 1974). Applied to consumption, this definition implies that real consumption should be a function of real income and real wealth, but not of the price level. Building on the definition of money illusion and of the homogeneity postulate, three empirically testable assumptions emerge, all of which are reconcilable with the evidence of money illusion at the individual level and with the standard rationality assumption:

(A1) Real consumption is not neutral to nominal shocks to its determinants.

This assumption describes the violation of the homogeneity postulate due to money illusion as either a signal extraction problem or a cost- and time-saving rule of thumb. It can be tested in an extended REPIH model (presented in Section 3.2) by simply adding inflation as an additional determinant of consumption growth. In particular, breaking down nominal income into real income and the price level makes it possible to distinguish the change in real consumption which is due to the change in real income from that which is due to the change in the price level. The idea is that, if all consumers are fully rational, they react only to changes in their real income and are unaffected by changes in the price level. Money-illusioned consumers, however, do not have homogeneity of degree zero demand functions and, thus, also react to changes in the price level.

(A2) Money illusion is a short-term phenomenon.

Money illusion cannot persist in the long term. As a signal extraction problem, money illusion is supposed to fade out rapidly as the consumer updates her information set about inflation. Money illusion as a rule of thumb is also limited in time, since the consumer is not able to hold her real consumption level constant if inflation exceeds her real income increase over a long period of time. In this case, the persistence of the rule of thumb depends on its efficiency, which is inversely proportional to the inflation level. It further depends on the number of people adopting this rule, on the depth and persistence of the

nominal (income or price) shock and on the after-shock inflation level.¹¹ In the empirical analysis, this assumption is tested by allowing lagged inflation to have an effect on contemporaneous consumption.

(A3) The degree of money illusion depends on the inflation level.

In the absence of inflation, money illusion is insignificant and has no impact on real consumption. Intuitively, the aggregate impact of money illusion increases in the inflation level and, potentially, is very important in periods or countries with high and volatile inflation rates. On the other hand, it is also likely that peoples' awareness of inflation increases as the inflation level increases, as the quality of the rule of thumb worsens (see, for example, [Brainard and Perry, 2000](#)) and as the media reporting increases ([Badarinza and Buchmann, 2009](#); [Lamla and Lein, 2010](#)). If money illusion stems from consumers' reliance on nominal values as proxies for real values, it should disappear as the cost of using this rule of thumb increases (i.e., with an increasing, volatile or high inflation level). This suggests the existence of a maximum money illusion level, where money illusion's impact is potentially most important.¹² Another reason for the decrease in money illusion is that, at high inflation rates, durables are substituted for money, since the latter becomes more expensive to hold ([Deaton, 1977](#)). This decreases consumers' sensitivity to inflation and, hence, lowers the impact of money illusion. The dependence of money illusion on the inflation level is tested in two ways. First, every model is tested over two periods with different inflation rates in order to test whether consumers adapt their behavior to different inflationary environments. Second, every model is tested for a nonlinear relationship between inflation and consumption, which would indicate the presence of a threshold above which money illusion loses its impact.

There are, of course, many additional testable hypotheses that could be for-

¹¹For this reason, the strong evidence in favor of money illusion reported by [Shafir et al. \(1997\)](#) needs to be dampened. They based their conclusions solely on one-time survey experiments and exclude any learning effect. On the other hand, [Fehr and Tyran \(2001\)](#) suggest that already a very small degree of money illusion at the individual level can have a very large aggregate effect through strategic interaction and that it does both increase and extend the impact of a price shock. In other words, it suffices for (some) individuals to believe that (some) others suffer from a certain degree of money illusion to spur a snowball effect. The authors' prediction that money illusion may subsist in the equilibrium is, however, directly contrary to assumption (A2).

¹²[Akerlof et al. \(2000\)](#) estimated this threshold to occur at a 6% inflation level. In the following sections, however, high inflation periods roughly correspond to inflation rates exceeding 3%, which seems to be a more realistic assumption.

mulated in the context of money illusion.¹³ However, the focus of this chapter is primarily to estimate whether there is *any* presence of money illusion in aggregate consumption.

3.2 A Simple Empirical Model for the U.S.

This section shows, in a first step, how money illusion can be measured within the REPIH framework by including inflation as a single explanatory variable within the simple random walk (RW) hypothesis. In a second step, the inflation-augmented RW model is tested with U.S. data. The estimation results suggest that inflation is a potentially important determinant of consumption growth, but do not allow to draw robust conclusions with respect to the presence and the nature of money illusion in aggregate consumption data.

3.2.1 The Inflation-Augmented Random Walk Hypothesis

The first attempt to directly estimate the impact of money illusion on aggregate consumption dates back to [Branson and Klevorick \(1969\)](#) who simply added the price level as an explanatory variable to the life cycle–permanent income hypothesis. The original idea is that a coefficient on the price level significantly different from zero corresponds to a violation of the [Patinkin \(1965\)](#) homogeneity postulate and, hence, indicates the presence of money illusion. [Branson and Klevorick \(1969\)](#) found evidence in favor of the presence of money illusion, but their model was subject to Lucas’ critique and their results were obtained with out-of-date estimation techniques. This section can be interpreted as an attempt to revive their intuition by transposing it within a more elaborate framework that is better able to account for the effects of money illusion on consumption and that can be estimated more efficiently.

The most appropriate benchmark consumption model is the widely used REPIH developed by [Hall \(1978\)](#) and presented in Chapter 2. As derived in Section 2.2.1, this model rests upon the postulate of rational expectations, according to which aggregate consumption obeys the first order conditions for the optimal consumption

¹³A few examples are: (i) Due to loss aversion, the effect and persistence of money illusion on consumption depends on the positive or negative nature of the shocks, as suggested by [Fehr and Tyran \(2001\)](#); (ii) The magnitude of money illusion differs depending on the nature of the goods and depending on the income and age distribution of the population; and (iii) The cost of money illusion is higher in terms of growth than in terms of welfare loss ([Miao and Xie, 2013](#)).

choice of a fully rational, forward-looking representative consumer. The main implication of the REPIH model is equation (2.13), which states that, under quadratic utility and a constant real interest rate equal to the subjective discount rate, consumption growth is unpredictable. That is, real consumption approximately follows the following RW without drift:

$$\Delta c_t = \varepsilon_t, \quad (3.1)$$

where c_t is the logarithm of aggregate real consumption and $\varepsilon_t \sim i.i.d.(0, \sigma^2)$ is the error term summarizing all new and unpredictable information about future income available in period t . According to the RW hypothesis derived by Hall (1978), no variable other than past consumption can help predict current consumption because all other relevant information has already been taken into account by the rational agent in her past consumption behavior. However, if Branson and Klevorik's intuition is correct and money illusion is present at the aggregate level, then adding inflation to equation (3.1) improves the REPIH model's predictive power of consumption growth and violates the homogeneity of degree zero postulate. The resulting inflation-augmented RW models are designed to fully capture money illusion:

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \varepsilon_t \quad (3.2)$$

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \sum_{j=0}^J \lambda_j^* L^j (\Delta p_t)^2 + \varepsilon_t, \quad (3.3)$$

where α is the constant; p_t is an appropriate price level index, such that Δp_t is the one-period inflation; and λ_i denotes the coefficient measuring the impact of a particular price change on current consumption growth. L^i is the lag operator, representing the i^{th} lag of the variable Δp_t . The innovations are, again, $\varepsilon_t \sim i.i.d.(0, \sigma^2)$ and represent unexpected news about real income. Equation (3.2) reflects the linear relationship between inflation and real consumption. In the strict sense, if any λ_i is significantly different from zero, then the homogeneity postulate is violated, validating assumption (A1). In this case, current and past prices contain useful information that has not yet been included in current consumption. This reflects an imperfect adjustment to past information and, hence, contradicts fully rational behavior.

If $0 < \sum_{i=1}^I \lambda_i \leq 1$ then the positive nominal income effect, advocated by Branson and Klevorick (1969) prevails because the consumer reacts positively to a positive

nominal shocks captured by the Δp_t terms. On the contrary, $-1 \leq \sum_{i=1}^I \lambda_i < 0$ indicates the presence of the negative unanticipated inflation effect on consumption suggested by Deaton (1977). As discussed in Section 3.1.2, these opposite inflation effects are not mutually exclusive, since the latter is immediate and disappears rapidly, while the former operates with a certain time lag and can potentially last for several periods. The distributed lags in inflation, represented by $\sum_{i=1}^I$, capture any effect of lagged inflation on current consumption growth. They allow the model to not only show the depth of money illusion, but also to test how much inertia (if any) money illusion contains, thus confirming or invalidating assumption (A2).

The linear model of equation (3.2) can be extended to the more general model described by equation (3.3), which also allows for a nonlinear relationship between inflation and consumption, as suggested by assumption (A3). The variables and intuitions are the same, except for the second summation, which captures the possible effect of squared current and past inflation on current real consumption growth. If (A3) is true, then the λ_j^* s are expected to be negative, so that the impact of inflation on consumption decreases after it exceeds a certain threshold.

Henceforth, equations (3.2) and (3.3) will be referred to as the “linear” model and the “nonlinear” model, respectively.

3.2.2 Estimation Results Over the Whole Sample

In order to obtain a broad picture of the relationship between inflation and consumption, this section presents the estimations of the linear and nonlinear inflation-augmented RW hypothesis models (3.2) and (3.3) for the U.S. over the whole-sample period (1959Q1 to 2012Q1). In a second step, in Section 3.2.3, the analysis focuses on separately estimating the linear and nonlinear models within two distinct high- and low-inflation subperiods to determine whether the representative consumer behaves differently in different inflationary environments.

The main estimation results for the linear and nonlinear models are summarized in Tables 3.1 and 3.2, respectively. They are obtained using detrended time series based on quarterly data. For a detailed description of the data and their properties, see Appendix A. As a matter of completeness, and to add robustness to the results, two alternative price indexes are used for the price variable p_t and to deflate all nominal time series. The first index is the implicit price deflator (IPD) for the consumption of nondurables and services, which is the most widely used measure of inflation in the consumption literature. The second index is the consumer price index

(CPI), which captures the price evolution of a representative basket of consumer goods and services. Another reason for also reporting CPI-based estimations is that this index makes particular sense in the context of money illusion because it reflects the price increase the consumers face when making their purchases over time.

The reported estimates in Tables 3.1 and 3.2 have been obtained with OLS. Although a simple Hausman test could not exclude the endogeneity of inflation with respect to current consumption growth over the whole sample at the 5% significance level, the endogeneity of inflation is clearly rejected within the two subperiods.¹⁴ As a matter of simplicity and since, as we shall see below, the estimations over the whole sample are subject to structural issues and cannot be considered accurate, only the OLS estimates are reported in this section.

Estimation of the Linear Model, Equation (3.2)

The first regressions of interest are models (i) and (ii) in Table 3.1. They represent the “preferred” IPD and CPI models in Tables B.1 and B.2 in Appendix B, where preferred refers to the retained lag length chosen for inflation, which corresponds to the upper limit I of the summation in the inflation-augmented RW equation (3.3).

All preferred models reported in the text have been chosen according to the adjusted R^2 , the Akaike information criterion (AIC), the Schwartz information criterion (BIC) and the last significant coefficients for lagged inflation (λ_i), when starting at $I = 10$ and consecutively reducing the number of lags. Regression (i) of Table 3.1, for example, corresponds to model (ix) in Table B.1. This is retained as the preferred model because it performs much better than the RW model (regression (x)), and because adding more lags does not significantly improve the quality of the model. For clear statistical and theoretical reasons, fewer lags in inflation are always preferred to longer distributed lags, especially if the coefficients and their significances are stable and close when reducing the number of lags in the model. Applying the same procedure to the CPI linear model in Table B.2, the retained model (model (viii)) contains three lags for inflation and is reported as regression (ii) in Table 3.1.

Looking at models (i) and (ii) in Table 3.1, the first striking result is that adding inflation to the RW model greatly improves the predictability of current consumption

¹⁴The Hausman (1978) test, also called the Wu-Hausman test, confirms the endogeneity of a variable if its OLS estimate β_{OLS} is significantly different from the consistent β_{IV} estimate. If the null hypothesis of $\beta_{OLS} = \beta_{IV}$ is rejected, the OLS estimate is not consistent, so the instrumental variable (IV) method should be preferred. For the IV estimate, I have chosen as instruments for inflation the quarterly growth of the M3 U.S. money stock in the periods t to $t - 5$ and lagged inflation in the periods $t - 3$ to $t - 5$.

growth, with coefficients of determination in the IPD and CPI models of $\bar{R}^2 = 0.72$ and 0.34, respectively. According to this statistic, the simple IPD model with only two lags in inflation explains over 70% of the variability of consumption growth, which is a surprisingly high value. The large difference in the goodness-of-fit between the IPD and the CPI models can be explained by two main factors that bring the IPD closer to the consumption series, measured through the personal consumption expenditures (PCE):¹⁵ First, the IPD includes a broader category of goods and services and, thus, better reflects the households' consumption, as measured by the PCE. Second, the IPD is a Fisher-type index that accounts for the evolution of the reference basket of consumption goods and services in its weights and, thus, follows the PCE closer than the CPI, which is based on the Laspeyres index and measures the price evolution of a fixed reference basket.

Apart from this difference, the IPD and the CPI models yield similar results with respect to the impact of inflation on current consumption growth. First, both models confirm that inflation has a strong predictive power for consumption growth and imply a rejection of the RW hypothesis. This may indicate that consumers do not – or are not able to – fully incorporate information about inflation in their past and current consumption. Since both current and lagged inflation (up to, respectively, two and three lags for IPD and CPI) have significant impacts on current consumption growth, the homogeneity postulate is violated, which possibly indicates the presence of money illusion in aggregate consumption. Furthermore, the high F -statistics in Table 3.1 imply that the null hypothesis of all coefficients (except the constant) being equal to zero is strongly rejected, thus validating assumption (A1).

The estimates of models (i) and (ii) in Table 3.1 are also in line with assumption (A2). The inflation effect is important for current inflation, but all coefficients for lagged inflation are rather small and become insignificant above the second lag in (i) and the third lag in (ii). This could indicate that money illusion is only a short-term phenomenon and disappears rapidly as consumers update their information sets. The small coefficients on lagged inflation further suggest that money illusion is not as deep or widespread as assumed by Shafir et al. (1997) and Fehr and Tyran (2001).

A more surprising result, which is visible in Tables B.1 and B.2, is that the

¹⁵Details on the differences between IPD and CPI can be found on the website of the U.S. Bureau of Economic Analysis at http://bea.gov/faq/index.cfm?faq_id=555. For more information on the problems pertaining to the measurement of inflation and its implications, see, for instance, the publications of the Boskin Commission focusing on the accurate measure of the cost of living (Boskin et al., 1998).

coefficients for lagged inflation do not present homogeneous or consistent signs across the distributed lags. For example, looking at the first regression with 10 lags in B.2, we see that $\hat{\lambda}_0$, $\hat{\lambda}_3$, $\hat{\lambda}_5$, $\hat{\lambda}_6$ and $\hat{\lambda}_7$ have negative signs, whereas $\hat{\lambda}_1$, $\hat{\lambda}_2$, $\hat{\lambda}_4$, $\hat{\lambda}_8$, $\hat{\lambda}_9$, $\hat{\lambda}_{10}$ have positive signs. This irregularity in the inflation impact is incompatible with the intuition of money illusion. It is likely that these variations in the signs reflect the presence of deeper, model-specific biases, such as, for example, omitted variables, structural breaks or the presence of nonlinearities. These potential biases are addressed individually in later sections.

In fact, the only coefficient that presents a constant sign and remains significant across the different models and periods analyzed in this chapter is the coefficient for the strong negative impact of current inflation on consumption growth. For example, in model (i) of Table 3.1, the coefficient of -0.69 on current IPD inflation means that a 1% increase in the IPD growth implies a *ceteris paribus* decrease in current consumption growth of 0.69%. This result clearly advocates in favor of the negative effect of unanticipated inflation on consumption. Further note that Davidson et al. (1978) argued that a large negative effect of current inflation on consumption growth reflects the erosion of the value of liquid assets from inflation. Note that this hypothesis is rejected in Section 3.3 when financial assets are directly controlled for.

The last important insight to be gained from regressions (i) and (ii) of Table 3.1 pertains to the coefficient for ε_{t-1} , which represents the estimated first-order moving-average (MA(1)) coefficient of the error term in the equation. This has been added to the estimated model whenever needed to control for serial correlation in the residuals. Note that, if it lies inside the unit circle, the MA(1) process is invertible. Since the coefficients are highly significant at the 1% level, there is a large degree of autocorrelation in the residuals of models (i) and (ii). Since the U.S. data are seasonally adjusted, it is more likely that the serial correlation stems from the fact that current consumption also depends on lagged consumption, reflecting the presence of habit persistence. The estimations in Section 3.3 indeed show that consumption habits are a further important predictor of consumption growth.

Alternatively, the presence of serial correlation can be explained by the *time aggregation* problem pointed out by Working (1960): There is a time inadequacy between the model and the data because the consumer's purchases are made at a relative high frequency, whereas the consumption and income data used for the analysis are measured as three-months averages (and not at points in time). This discrepancy mechanically induces first-order serial correlation in the model and possibly

induce incorrect inferences. Similarly, Granger (1980) showed that the aggregation of simple dynamic models can induce long memory in aggregate consumption even if the consumption of every individual household follows a RW. This aggregation problem can be attributable to incomplete information (Thornton, 2014).

Estimation of the Nonlinear Model, Equation (3.3)

The question remains to know as to whether the relationship between inflation and consumption is better explained by a nonlinear model (in our case, a quadratic model, as described by equation (3.3)). Following assumption (A3), the effect of inflation on consumption growth is supposed to decline when inflation exceeds a certain threshold, at which the general awareness about inflation increases.

The preferred estimated nonlinear models for the IPD- and CPI-based inflation-augmented RW models are summarized, respectively, in regressions (i*) and (ii*) of Table 3.2. The preferred models were chosen from the different estimations summarized in Tables B.7 and B.8 for the IPD and the CPI models, respectively. The selection procedure for the preferred models was the same as for the linear model, in that I constantly reduced the number of lags for inflation (the summation limit I in equation (3.3)) and for squared inflation (the upper limit J) until the coefficients λ_i and λ_j^* became significant. According to the significance and to the usual summary statistics, the retained models (i*) and (ii*) for the IPD and CPI models correspond to the models (vi*) and (vii*) in their respective tables.

Moving from the preferred linear models (i) and (ii) to the preferred nonlinear models (i*) and (ii*), four noteworthy features emerge. First, the RW hypothesis is again strongly rejected by the U.S. data for both the IPD and CPI models, since the nonlinear models have even better fits than the linear ones. Second, both inflation measures reveal the existence of a nonlinear relationship between inflation and consumption growth: In both preferred equations, squared inflation has a strong negative and significant impact on consumption growth. The estimated λ_j^* s all have the expected negative sign, which indicates a decline in the inflation effect above a certain inflation level. This negative impact is stronger for the CPI model (reflected by higher estimated $\hat{\lambda}_j^*$ s), but it disappears rapidly in both models. Third, the inclusion of squared inflation in the regression alters the linear effects captured by the λ_i s. In the IPD model, the coefficients on inflation from zero to two lags are roughly the same as those in Table 3.1. For the CPI model, the picture is slightly different: Adding squared inflation cuts the impact of current inflation approximately

Table 3.1: Impact of Inflation on Consumption – Linear Model

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \varepsilon_t$$

	Whole Sample		High Inflation		Low Inflation	
	IPD (i)	CPI (ii)	IPD (iii)	CPI (iv)	IPD (v)	CPI (vi)
α	0.01*** (0.00)	0.01*** (0.00)	0.02*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)
λ_0	-0.69*** (0.03)	-0.38*** (0.09)	-0.72*** (0.07)	-0.66*** (0.09)	-0.67*** (0.08)	-0.33 (0.24)
λ_1	0.08*** (0.02)	0.04 (0.07)	–	–	0.21*** (0.07)	0.24 (0.23)
λ_2	0.10** (0.03)	0.15** (0.06)	–	–	-0.11* (0.05)	-0.42** (0.17)
λ_3	–	-0.15** (0.07)	–	–	0.08* (0.04)	0.14 (0.20)
λ_4	–	–	–	–	-0.30*** (0.05)	-0.58*** (0.16)
λ_5	–	–	–	–	0.17*** (0.06)	-0.01 (0.17)
λ_6	–	–	–	–	0.06* (0.03)	0.34* (0.17)
λ_7	–	–	–	–	–	-0.31 (0.17)
λ_8	–	–	–	–	–	0.41 (0.24)
λ_9	–	–	–	–	–	-0.37** (0.17)
ε_{t-1}	0.46*** (0.07)	0.43*** (0.08)	0.46*** (0.10)	0.40*** (0.09)	0.05 (0.18)	0.14 (0.14)
\bar{R}^2	0.72	0.34	0.75	0.56	0.66	0.45
AIC	-7.81	-7.90	-8.04	-7.95	-8.94	-8.74
BIC	-7.73	-7.80	-7.94	-7.85	-8.57	-8.24
DW	1.72	1.84	1.99	1.97	1.98	1.93
F_P	133.07 [0.00]	22.18 [0.00]	93.57 [0.00]	43.28 [0.00]	10.68 [0.00]	3.99 [0.00]

This table summarizes the preferred models of Tables (B.1) to (B.6) resulting from the regression of consumption growth on current and lagged inflation for quarterly U.S. data. The IPD and CPI report, respectively, the inflation measure and the deflator used for the corresponding regression. The “Whole Sample” period is from 1959Q1 to 2012Q1, whereas the “High Inflation” period is from 1966Q1 to 1981Q3 for IPD and from 1966Q1 to 1982Q3 for CPI. The “Low Inflation” period is from 1991Q1 to 2001Q1. The coefficient for ε_{t-1} is the estimated first-order moving-average (MA) coefficient of the error term in the equation. In parenthesis are the Newey-West heteroscedasticity and autocorrelation consistent (HAC) standard errors. The reported statistics are standard and are described in detail in Table B.1.

Table 3.2: Impact of Inflation on Consumption – Nonlinear Model

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \sum_{j=0}^J \lambda_j^* L^j (\Delta p_t)^2 + \varepsilon_t$$

	Whole Sample		High Inflation		Low Inflation	
	IPD (i*)	CPI (ii*)	IPD (iii*)	CPI (iv*)	IPD (v*)	CPI (vi*)
α	0.01*** (0.00)	0.00*** (0.00)	0.02*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)
λ_0	-0.69*** (0.03)	-0.18*** (0.05)	-0.72*** (0.07)		-0.76*** (0.11)	-0.33 (0.24)
λ_1	0.10*** (0.02)	0.23*** (0.06)	-	-	0.14** (0.06)	0.24 (0.23)
λ_2	0.11*** (0.02)	0.22*** (0.06)	-	-	-0.10* (0.06)	-0.42** (0.17)
λ_3	0.01 (0.03)	-0.10 (0.07)	-	-	0.06 (0.06)	0.14 (0.20)
λ_4	0.09*** (0.03)	0.14** (0.07)	-	-	-0.19*** (0.06)	-0.58*** (0.16)
λ_5	0.06*** (0.02)	-	-	-	0.05 (0.04)	-0.01 (0.17)
λ_6	0.06** (0.03)	-	-	-	0.16*** (0.04)	0.34* (0.17)
λ_7	-	-	-	-	-0.10* (0.05)	-0.31* (0.17)
λ_8	-	-	-	-	0.15** (0.06)	0.41 (0.24)
λ_9	-	-	-	-	-0.13** (0.05)	-0.37** (0.17)
λ_0^*	-2.33*** (0.49)	-11.37*** (2.47)	-	-12.66*** (3.32)	14.88* (8.08)	-
λ_1^*	-2.17*** (0.72)	-10.00*** (3.39)	-	-7.98 (6.07)	-	-
λ_2^*	-1.28*** (0.48)	-	-	8.15* (4.46)	-	-
λ_3^*	-2.07*** (0.67)	-	-	-7.74** (3.65)	-	-
λ_4^*	-	-	-	5.32** (2.33)	-	-
ε_{t-1}	0.44*** (0.08)	0.40*** (0.07)	0.46*** (0.10)	0.39*** (0.13)	-	0.14 (0.14)
\bar{R}^2	0.76	0.42	0.75	0.61	0.72	0.45
AIC	-7.92	-8.01	-8.04	-8.00	-9.10	-8.74
BIC	-7.72	-7.87	-7.94	-7.77	-8.59	-8.24
DW	1.81	1.85	1.99	1.92	1.93	1.93
F_P	54.45 [0.00]	19.67 [0.00]	93.57 [0.00]	17.92 [0.00]	10.42 [0.00]	3.99 [0.00]

This table summarizes the preferred models of Tables (B.7) to (B.12) resulting from the regression of consumption growth on current inflation, lagged inflation and lagged squared inflation for quarterly U.S. data. The variables, periods and statistics are identical to those described in Table 3.1.

in half (moving from $\hat{\lambda}_0 = -0.38$ to $\hat{\lambda}_0 = -0.18$), while the estimated values and significances for λ_1 , λ_2 and λ_4 are substantially higher. Finally, we observe that all significant coefficients on lagged inflation have the same sign in (i*) and (ii*), which could resolve the previous puzzling result of undetermined signs in the linear models of Table 3.1. This suggests the presence of the unanticipated inflation effect on contemporary inflation suggested by Deaton (1977), compensated by the nominal income money illusion effect on lagged inflation suggested by Branson and Klevorick (1969) after nominal incomes have been adjusted for inflation.

Overall, the nonlinear models (i*) and (ii*) in Table 3.2 yield superior results (when compared to the linear models (i) and (ii) in Table 3.1) for U.S. data over the period from 1959Q1 to 2012Q1. This result is confirmed when we perform a *Regression Equation Specification Error Test* (RESET test) suggested by Ramsey (1969) to verify whether model (3.2) is misspecified and whether a nonlinear model is more accurate with regard to describing the data.¹⁶ The result of the RESET test on the (preferred) linear inflation-augmented RW models is that, over the whole sample and using the second- and third-degree polynomials in predicted values, the null hypothesis of no misspecification is rejected at the 1% level.

Despite this result, however, it would be misleading to conclude that the nonlinear preferred models (i*) and (ii*) do not suffer from any misspecification and best describe the relationship between inflation and consumption growth. The next sections show that the analyzed models indeed suffer from several specification errors. In particular, the next section reveals the presence of numerous structural breaks in the data, which call for separate analyses of the model in distinct subsamples. Moreover, it is plausible that many other determinants of consumption growth are omitted from the model (see Section 3.3). Finally, note that the same RESET test does not reject linearity when different subsamples or further exogenous variables are considered. Consequently, the nonlinear model cannot definitively be preferred over the linear model at this stage.

¹⁶The RESET test determines whether nonlinear combinations of the fitted values have an explanatory power for the dependent variable. In particular, for our regression $\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i(\Delta p_t) + \varepsilon_t$, the RESET test consists of estimating the extended model $\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i(\Delta p_t) + \delta_1 \Delta \hat{c}_t^2 + \delta_2 \Delta \hat{c}_t^3 + u_t$, with the null hypothesis of $\delta_1 = \delta_2 = 0$. If the null hypothesis is rejected, the original linear model is wrongly specified, favoring a nonlinear reformulation of the relationship between inflation and consumption growth and, thus, indicating that assumption (A3) may well be correct.

3.2.3 Estimations for High- and Low-Inflation Subperiods

The estimations over the whole sample suggest that inflation is a potentially important determinant of consumption growth. This result is contested by assumption (A3), which suggests that consumers behave differently depending on the inflation level. In order to better isolate the assumption that money illusion vanishes in periods of accelerating inflation, it is judicious to re-estimate models (3.2) and (3.3) for two subperiods with similar average inflation rates: one experiencing relatively high average inflation and the other one experiencing low average inflation.

In order to determine the two subperiods, first consider Figure 3.1, which shows the annualized quarter-to-quarter percentage growth rates of the IPD (upper graph) and the CPI (lower graph). Both inflation rates present similar evolutions since 1959, though the IPD inflation appears to be more volatile than the CPI inflation, in particular since the end of the 1990s. We see in both graphs that the level and volatility of inflation has been subject to many significant changes since the 1960s. Inflation rose sharply in the first quarter of 1973 and was fairly volatile until the first quarter of 1991, where it remained stable for roughly one decade before again becoming more volatile.

Another reason for seeking similar inflation levels within the selected subperiods is that, in light of the very irregular evolution pattern of the U.S.'s quarter-to-quarter inflation levels, the inflation-augmented RW model might be subject to several structural breaks over the whole-sample period. To test for different structural breaks, I make use of a Quandt-Andrews breakpoint test (Andrews, 1993) instead of the simple Chow (1960) test for a single structure break. The latter test requires the structural break date to be known, whereas the former allows for more flexibility because it consists of performing a Chow test at every time observation within a given interval.

By performing the Quandt-Andrews test on both IPD- and CPI-based linear and nonlinear models (i.e., equations (3.2) and (3.3)), in which the first and last 7.5% of the observations have been excluded, I have retrieved the likelihood ratio (LR) F-statistic for each period. These LR F-statistics are drawn in Figure 3.2, where the upper graph presents the LR F-statistic series for the IPD-based models and the lower graph presents the same statistics for the CPI inflation. Note that the greater the LR F-statistic, the higher the probability of a rejection of the null hypothesis of no structural break. The graphs clearly indicate the presence of structural breaks in all the models, with three peaks at the beginning of each decade from the 1980s to

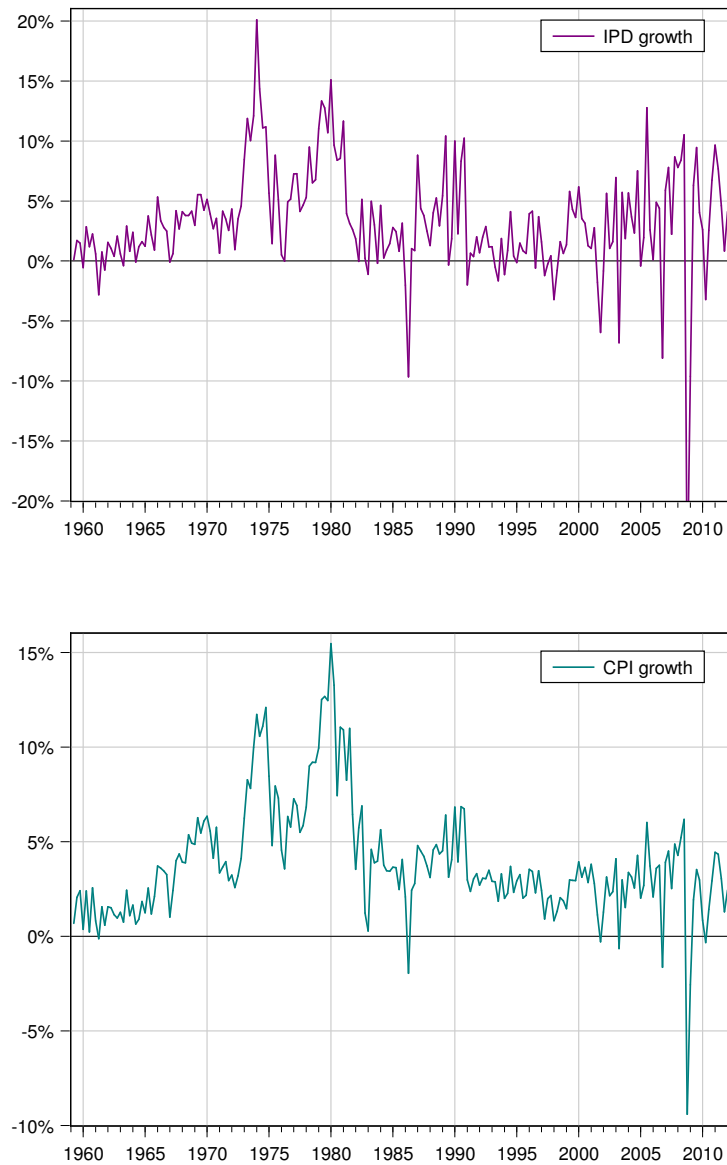


Figure 3.1: Evolution of U.S. IPD and CPI Quarterly Inflation

The two graphs represent the IPD (top) and CPI (bottom) annualized quarter-to-quarter growth rates in percentages for the U.S. over the sample period from 1959Q1 to 2012Q1. Data source: Federal Reserve Economic Data.

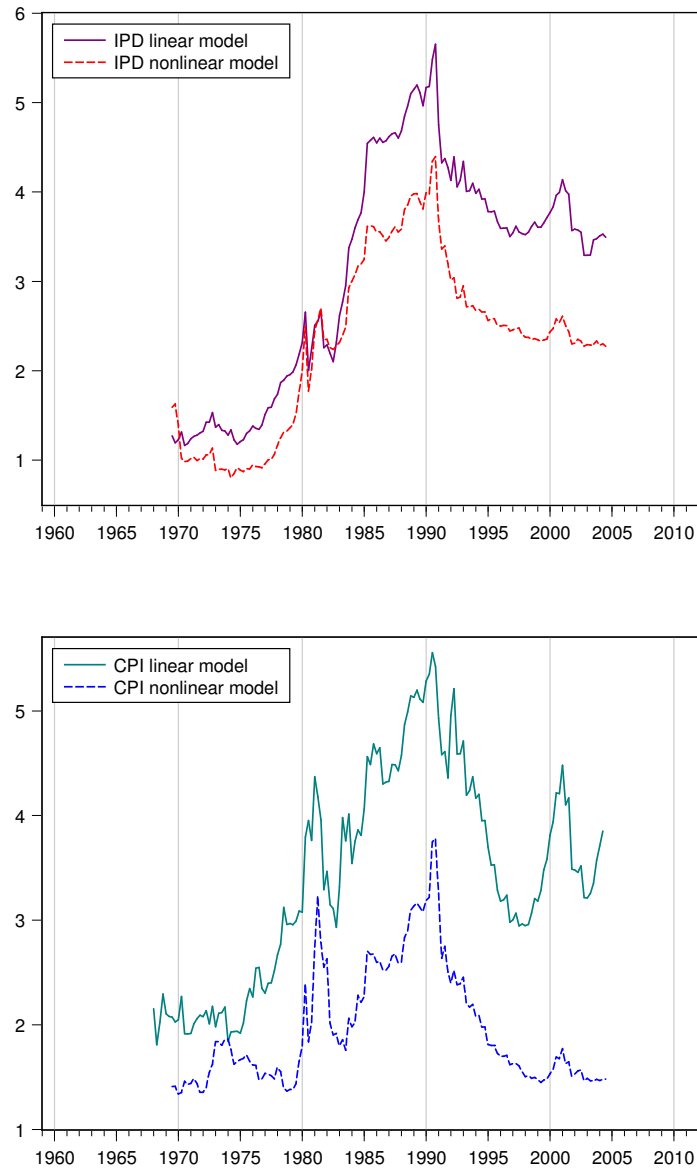


Figure 3.2: LR F-Statistic Series

The two graphs represent the LR F-statistic series for the presented IPD (top) and CPI (bottom) linear and nonlinear models resulting from a Quandt-Andrews unknown breakpoint test with 15% trimmed data. Data source: Federal Reserve Economic Data.

the 2000s. A direct consequence of this finding is that the estimations previously obtained over the whole-sample period might not depict the true relationship between inflation and consumption growth.

To avoid structural breaks within the two chosen subperiods and to better differentiate the high-inflation period from the low-inflation period, I ignore most of the 1980s and the 2000s because they are characterized by inflation rates that are both relatively low and particularly volatile in historical comparison. Based on the average inflation rates and the highest LR F-statistics, the chosen high-inflation periods are, respectively, 1966Q1 to 1981Q3 and 1966Q1 to 1982Q3 for IPD and CPI, respectively, whereas the chosen low-inflation period is 1991Q1 to 2001Q1 for both inflation measures. Within these selected subperiods, “high” inflation corresponds to a period during which inflation is mostly above 3% and particularly volatile, whereas “low” inflation period refers to a stable inflation rate that does not exceed 3%. Note that the end of the high inflation subperiod corresponds roughly to the beginning of the period that [Stock and Watson \(2002\)](#) have called the *Great Moderation*: namely, an important decline in the volatility of the U.S. Gross Domestic Product that has taken place in the first half of the 1980s.¹⁷

The estimation results for models (3.2) and (3.3) over the selected high- and low-inflation subperiods yield the preferred linear models (iii) to (vi) in Table 3.1 and the preferred nonlinear models (iii*) to (vi*) in Table 3.2. These are derived from Tables B.3 to B.6 for the linear models and B.9 to B.12 in Appendix B using the same lag-reducing technique as for the estimations over the entire sample period.

Looking at the differences between the high-inflation and the low-inflation subperiods in Tables 3.1 and 3.2, the most striking result is the fact that the response of consumption growth to current and lagged inflation differs greatly from one subperiod to another. While the inflation-augmented RW models still outperform the standard RW hypothesis model in both subperiods (thus confirming the importance of inflation as a determinant of consumption growth, and corroborating (A1)) the duration of the inflation effect diverges across the subperiods. For the low-inflation period, we see in Table 3.1 that lagged inflation is significant for up to six lags for the IPD model (v), or up to nine lags for the preferred CPI model (vi). On the contrary, there is absolutely no significant impact of lagged inflation on current consumption growth in the high-inflation preferred models (iii) and (iv). This is in line with assumptions (A2) and (A3), which suggest that money illusion as a cost-saving rule of

¹⁷For more details about the origins and characteristics of the Great Moderation, see [Blanchard and Simon \(2001\)](#), [Summers \(2005\)](#), [Fang and Miller \(2008\)](#) and [Burren and Neusser \(2010\)](#).

thumb is efficient when the inflation level is low and can last for several periods. In times of high and volatile inflation, using nominal values as proxies for real values is too costly for the consumer, which causes the positive nominal income effect of inflation on consumption disappear in the high-inflation subperiod.

In line with this observation, the estimates in Table 3.2 indicate that the nonlinear model (3.3) does not perform particularly better than the linear model in the subperiods, as opposed to the nonlinear models over the whole sample, which do perform better. In fact, squared inflation is never significant for the IPD models in the high-inflation subperiod. This explains why the preferred model (iii*) is the same as the linear preferred model (iii) in Table 3.1. The same insight is valid for the CPI model in the low-inflation period, where (vi*) corresponds to the linear preferred model (vi) in Table 3.1. A surprising result occurs in model (iv*), in which the linear effect of inflation totally disappears (including λ_0), but where squared inflation has a rather strong and long-lasting nonlinear effect on consumption growth. In this regard, model (iv*) seems to be a special case that needs to be further investigated. Concerning the low-inflation subperiod, the squared inflation IPD model (v*) performs slightly better than the preferred linear IPD model over the same period (v) with a more long-lasting effect of inflation (up to nine lags) on current consumption growth. However, since the low-inflation subperiod is relatively short (41 observations), it is particularly important to choose models with fewer lags, provided that the loss in the fit of the model is reasonable. In order to enhance the quality of the estimation results, particular attention is devoted to this issue throughout the rest of the analysis.

Even though the estimation of the inflation-augmented RW model over the selected subperiods shows that inflation is an important determinant of consumption growth and that consumers adjust their behavior to the changing inflationary environments, several open questions remain. First, the coefficients of lagged inflation have changing signs in all preferred models for the low-inflation period, as well as in the preferred, nonlinear, high-inflation CPI model (iv*). Since there is no sound economic theory explaining this phenomenon, it may pertain to statistical problems or reflect some deeper misspecifications. Second, the unanticipated inflation effect, captured by λ_0 , is not significant anymore in the CPI preferred model for the low-inflation period. In consequence, money illusion stemming from the sequential purchase of goods might not be as important as previously thought. Third, the preferred inflation-augmented RW models have surprisingly high coefficients of determination. For example, the mere adding of contemporaneous (IPD-based) in-

flation to the RW in model (iii) allows to explain as much as 75% of the variance in consumption growth. Even if money illusion is an important and widespread phenomenon, it seems unrealistic that inflation causes most variations in aggregate consumption. In fact, it is possible that the significant and high coefficients on current and lagged inflation do not exclusively reflect money illusion and that the λ_i s work as proxies for other consumption-driving variables that are not controlled for in the baseline models (3.2) and (3.3).

These remaining problems call for further investigations in order to better isolate the impact of inflation on consumption growth and to determine whether this effect is due to money illusion. The following section considers an extension of the inflation-augmented RW model, labelled as the money illusion aggregate consumption function, which controls for additional variables and incorporates other consumption behaviors that could potentially determine consumption growth.

3.3 The Money Illusion Aggregate Consumption Function

The inflation-augmented RW model estimated in Section 3.2 is a rather simplistic extension of the REPIH because it assumes that only inflation can help predict consumption growth and *a priori* excludes the idea that the consumer can be influenced by other phenomena. In order to obtain a more robust and general consumption model, this section further extends the benchmark RW model to control for other possible determinants of consumption growth: namely, liquidity constraints (LC), habit formation (HF), wealth, the interest rate and an error-correction term. I refer to the resulting model as the *money illusion consumption function* because the additional variables ensure that the inflation effect does not work as a proxy for other determinants and that it captures predominantly the impact of money illusion on consumption.

In the style of the inflation-augmented linear and nonlinear RW models, the linear and nonlinear money illusion consumption function (MICF) can be written as follows:

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i(\Delta p_t) + \sum_{k=1}^K \delta_k L^k(\Delta c_t) + \eta \Delta y_t + \omega a_{t-1} + \varphi [c_{t-1} - y_{t-1}] + \varepsilon_t \quad (3.4)$$

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i(\Delta p_t) + \sum_{j=0}^J \lambda_j^* L^j(\Delta p_t)^2 + \sum_{k=1}^K \delta_k L^k(\Delta c_t) + \eta \Delta y_t + \omega a_{t-1} + \varphi [c_{t-1} - y_{t-1}] + \varepsilon_t \quad (3.5)$$

where $L^k(\Delta c_t)$ is consumption growth lagged k times capturing HF, Δy_t is current income capturing LC, a_{t-1} is the lagged wealth-to-income ratio capturing both wealth and the interest rate and $[c_{t-1} - y_{t-1}]$ is an ECT capturing the short-term deviation from the long-term relationship between consumption and income. These variables are briefly presented in the next sections. In a second step, equations (3.4) and (3.5) are estimated in order to refine the relationship between inflation and consumption and to find the best predictive model for consumption growth.

3.3.1 Liquidity Constraints and Habit Formation

In the consumption literature, consumers who do not behave exactly as predicted by the standard REPIH model are typically said either to face LC or to present a certain degree of HF.

First, it is plausible to assume that a constant fraction of consumers face LC, which prevents them from borrowing in order to smooth consumption.¹⁸ Campbell and Mankiw (1989, 1990, 1991) were first to empirically show that there is a large fraction of “hand-to-mouth” consumers in the populations of many developed countries. This behavior is later justified with the existence of “buffer-stock-savers” (Carroll, 1997): either individuals who hold small buffer stocks as a precaution against very bad income shocks or near-zero-net-worth people who simply do not have the financial wherewithal to smooth their consumption (Mankiw, 2000). Formally, the distinctive feature of LC consumers is that they consume all of their current real income Y_t in every period, as presented in equation (3.6):

$$c_t = y_t. \quad (3.6)$$

¹⁸LC consumers are often referred to as rule-of-thumb consumers. To avoid confusion and to stay in line with the above-presented characteristics of money illusion, rule-of-thumb behavior in this chapter is only used to describe the adoption of nominal values as proxies for real values.

The inability of these current income consumers to smooth their consumption over time leads to an aggregate excess sensitivity of consumption to income in comparison to the predictions of the REPIH model.¹⁹

The second type of consumers who do not behave as predicted by the REPIH are those who, without regard to their real income evolution, are willing to keep their consumption level constant in each period. This HF has garnered a great deal of attention because it generates persistence in consumption growth.²⁰ In particular, HF is often used to explain the hump-shaped response of U.S. consumption to income shocks (Fuhrer, 2000) or the equity premium puzzle (Constantinides, 1990; Campbell and Cochrane, 1999). In its simplest form, HF can be modeled as in equation (3.7), such that the consumer consumes, in every period, exactly what she had consumed in the previous period:

$$c_t = c_{t-1}. \quad (3.7)$$

The implication of HF is (for positive income shocks) contrary to that of the LC in that it leads to an excessively smooth consumption in comparison to the REPIH's predictions. It is important to note that equation (3.7) does not necessarily only capture consumption habits; it can also reflect any phenomenon encouraging the consumer to save more than suggested by the long-term REPIH benchmark. For example, both the HF and the rational inattention model, though different in their microeconomic foundations, have identical implications with regard to aggregate data and can be formulated the same way (Reis, 2006, p. 1788). Consequently, the general formulation of equation (3.7) can also be understood as capturing different sources of uncertainty that stimulate precautionary savings on the part of the consumer. This relationship is analyzed in more detail in Chapters 4 and 5.

There is still much debate regarding whether LC or HF contributes more to consumption growth,²¹ but it appears evident that neither of them should be left out of a model attempting to capture the determinants of consumption growth.

¹⁹Many studies have contributed to describing this excess sensitivity of consumption. See, in particular, Flavin (1981), Blinder and Deaton (1985), Campbell and Mankiw (1989, 1990, 1991), Attanasio and Weber (1995) and Carroll (2001). Interestingly, hyperbolic discounting behavior, suggested by Laibson (1997), yields a similar outcome.

²⁰Consumption habits became successful starting with Muellbauer (1988). His original formulation has often been reused without major modifications (e.g., Campbell and Deaton, 1989; Sommer, 2004; Carroll et al., 2011). Micro-evidence for habits can be found, for example, in Ferson and Constantinides (1991), Tallarini and Zhang (2005) and Ravn et al. (2006).

²¹See, for example, Malley and Molana (2006), Sommer (2007), Kiley (2010), Carroll et al. (2011) and Di Bartolomeo et al. (2011).

Note that the MICF described in equation (3.4) allows current consumption to also depend on consumption lagged more than one period, which is, of course, a sound assumption in the context of persistent consumption habits.

A crucial difference between money illusion, LC and HF is that the last two behaviors rely solely on real values. Depending on its nature, however, money illusion can induce an aggregate effect that is similar to either LC or HF. In particular, money as a rule of thumb induces excessive sensitivity of aggregate consumption in the case of a nominal income increase, and money illusion as a signal extraction problem implies excessive smoothness of aggregate consumption via the unexpected inflation channel.

3.3.2 Real Interest Rate and Wealth

In order to further improve the REPIH model and to better understand the factors explaining consumption growth, the real interest rate and wealth are good candidates for inclusion as additional control variables. Furthermore, these are potentially important in the present context of money illusion because the general confusion between nominal and real values concerns, not only income, but also, potentially, the interest rate (which is directly available to individuals only in nominal terms) and wealth (which is a determinant of consumption in the REPIH, as can be seen in equation (2.17)).

To understand the exact relationship between the real interest rate and consumption growth, consider again the Euler equation (2.9) representing the optimal consumption path of the representative consumer in the REPIH model. If we drop the standard assumptions of the real interest rate being constant and equal to the discount rate (i.e., giving us $r_t \neq \rho$), then an increase in the real interest rate has a positive impact on consumption growth.²²

Furthermore, loosening the just-mentioned original assumptions also allows for an indirect effect of inflation on consumption growth. Indeed, as we can see from the standard (loglinearized) Fisher equation, denoted $r_t = i_t - \pi_{t+1}^e$, a *ceteris paribus* increase in the expected inflation decreases the contemporaneous real interest rate, which again decreases the current growth rate of consumption. Consequently, it is possible that the statistically significant negative impact of current inflation on contemporaneous consumption growth found in Section 3.2 captures, not only the

²²It becomes particularly intelligible if we consider logarithmic utility. In this case, the Euler equation is equal to $\frac{c_{t+1}}{c_t} = \frac{1+r_t}{1+\rho}$. An increase in r_t has to be compensated by either an increase in c_{t+1} or a decrease in c_t , fostering consumption growth.

unexpected inflation effect, but also, to a certain extent, expected inflation. For this reason, including (a proxy for) the real interest rate in the MICF helps to distinguish between the two effects. Note, however, that most empirical studies on consumption – which also omit inflation from their estimated equations – find no statistically significant evidence for this theoretical link between real interest and consumption.²³

Wealth is not much better in predicting consumption growth, as was theoretically and empirically shown by [Hall \(1978\)](#). The reason is that the consumer with rational expectations already takes into account her wealth in her current consumption. Nevertheless, in order to match the above-mentioned intuition and as a means of robustness, the MICF includes a measure for both wealth and the real interest rate to verify whether these have an influence on the estimates for inflation. An interesting variable is the ratio of liquid household assets (i.e., financial wealth) to income, defined as $a = \frac{A_{t-1}}{y_t}$, as suggested by [Muellbauer and Lattimore \(1999\)](#). The advantage of this weighted ratio over a direct wealth measure is that this ratio gives illiquid assets a lower weight than liquid assets ([Muellbauer and Lattimore, 1999](#), Section 11) and, more importantly, captures time variations in the interest rate ([Carroll et al., 2011](#), p. 1137).

3.3.3 An Error Correction Model Reformulation

The last control variable included in the MICF is an error-correction term capturing both the short-run and the long-run relationships between consumption and income, as presented in the seminal paper of [Davidson et al. \(1978\)](#) and the subsequent literature. [Davidson et al. \(1978\)](#) described the UK consumption growth using an error correction model (ECM) of the following type: $\Delta c_t = \beta_0 + \beta_1 \mathbb{E}_{t-1} \Delta y_t + \beta_2 [c_{t-1} - y_{t-1}] + \varepsilon_t$. This model is, in fact, an ADL(1,1) *autoregressive distributed lag model*, in which $\Delta c_t = \beta_0 + \beta_1 \Delta y_t + \varepsilon_t$ represents the long-run equilibrium relationship between income and consumption. The short-term deviation from that equilibrium is then captured by $[c_{t-1} - y_{t-1}]$, corresponding to lagged savings, and $\beta_2 [c_{t-1} - y_{t-1}]$ is the corresponding error-correction term.²⁴

In this setup, β_2 is expected to be negative and represents the speed of return to the equilibrium relationship between income and consumption after a shock. Fur-

²³See, for example, [Hall \(1988\)](#), [Campbell and Mankiw \(1989, 1990, 1991\)](#), [Parker and Preston \(2005\)](#) and [Kiley \(2010\)](#).

²⁴Note that the ECM proposed here can be extended to a VECM if all the variables of interest (in particular, consumption, income and prices) are cointegrated time series.

ther note that, even though Davidson et al. (1978) used a different approach than the Euler equation model of Hall (1978) to characterize consumption growth, the ECM can be used to describe, to a certain extent, the behavior of a LC consumer within the REPIH (Davidson and Hendry, 1981; Muellbauer and Lattimore, 1999): Consider an LC consumer who is willing to keep constant her marginal propensity to consume at a high level. The ECM allows her to deviate from this equilibrium in the short run in response to a certain shock. For example, she may temporarily decide to “save for a rainy day” – in the terms of Campbell (1987) – and, consequently, increase her consumption to return to her long-run objective. Conversely, if this consumer’s previous-period consumption was higher than her previous-period income, her consumption in the current period will tend to be lower, *ceteris paribus*.

3.4 Estimation of the Money Illusion Consumption Function

In Section 3.4.1, the preferred models resulting from the estimation of the inflation-augmented RW model over the selected subperiods are re-estimated with the MICF in order to test whether the inflation effects are robust to the inclusion of additional control variables. The results allow the exclusion of the nonlinear models and suggest that inflation and HF are the main determinants of consumption growth. This result is confirmed in Section 3.4.2, which derives the optimal IPD- and CPI-based MICFs for the high- and low-inflation subperiods. In particular, it is demonstrated that the MICF yields substantially better results than aggregate consumption models that do not control for inflation.

3.4.1 Extended Preferred Inflation-Augmented RW Models

Table 3.3 describes the estimation results of the linear and nonlinear MICFs (equations (3.4) and (3.5)), in which the number of distributed lags on inflation, I , and squared inflation, J , are chosen according to the preferred models of the inflation-augmented RW hypothesis presented in Tables 3.1 and 3.2. For simplicity, only the first lag of consumption growth is reported in Table 3.3 (i.e., K has been arbitrarily set to 1). Because of the endogeneity of income, the results have been calculated with a two-stage least-squares (TSLS) model. In line with the literature and the model assumptions, the natural candidate instrumental variables are the lagged values of the exogenous variables because these are correlated with themselves, but not

with the error terms. Note that all instruments have to be lagged by at least two periods to avoid the time aggregation problem pointed out in Section 3.2.2.

Comparing the MICF estimates in Table 3.3 with the previous results in Tables 3.1 and 3.2, the most striking difference is that squared inflation is no longer a significant determinant of consumption growth (cf. models (iii) and (v)). Consequently, the nonlinear inflation-augmented RW model (equation (3.3)) and the nonlinear MICF (equation (3.5)) can probably be ignored in further analyses. It is important to stress, however, that this does not contradict assumption (A3) of an inflation effect that decreases with the inflation level, since the nonlinear effect is already controlled for by analyzing subperiods with differing average inflation rates.

Concerning the impact of current inflation on consumption growth, we see that λ_0 remains highly significant in all models (except for model (vi)), with an important negative impact of current inflation on consumption growth. This means that current inflation cannot be considered as simply a proxy for either financial wealth, as suggested by Davidson et al. (1978), nor as solely capturing other consumption behaviors, such as HF or LC. As a result, The unanticipated inflation effect appears to be important. Inflation should, therefore, not be excluded from any analysis focusing on the determinants of consumption growth. On the other hand, the effect of lagged inflation on consumption growth is much less important in Table 3.3, and even almost completely disappears in model (vi). This could indicate that there is no money illusion as a rule-of-thumb behavior and that consumers do not systematically use nominal values as proxies for real values. Note, further, that adding the other possible determinants of consumption growth to the model could not resolve the puzzle of the changing signs in lagged consumption, a result that again calls for further investigation.

Contrary to many empirical studies on this subject, this study finds that the coefficients on the other variables in the MICF do not yield clear or homogeneous results - a finding that calls for additional analyses. Although the coefficient δ_1 on consumption growth lagged one period is significant in models (i), (iii) and (vi), its sign and magnitude differ depending on the period, the inflation measure and the number of lags on inflation. Since these results for δ_1 are somewhat counterintuitive, their robustness can be tested by including more lags on consumption growth, using, for example, the same lag-reducing technique as used for inflation in Section 3.2.2. Having a $K > 1$ also makes economical sense because it is very possible that consumption habits overlap across several quarters. Moreover, we see that there is still some degree of serial correlation, as reflected by the significant MA(1) term in (i)

Table 3.3: Re-Estimation of the Preferred Inflation-Augmented RW Models

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \sum_{j=0}^J \lambda_j^* L^j (\Delta p_t)^2 + \delta_1 \Delta c_{t-1} + \eta \Delta y_t + \omega a_{t-1} + \varphi [c_{t-1} - y_{t-1}] + \varepsilon_t$$

	IPD			CPI		
	High (i)	Low linear (ii)	Low nonlinear (iii)	High linear (iv)	High nonlinear (v)	Low (vi)
α	0.00 (0.03)	0.00 (0.01)	-0.01 (0.02)	-0.02 (0.02)	-0.01 (0.03)	-0.01 (0.03)
λ_0	-0.54*** (0.13)	-0.50** (0.22)	-0.78*** (0.22)	-0.58*** (0.17)	-	-0.27 (0.56)
λ_1	-	0.30 (0.32)	0.58*** (0.12)	-	-	0.51 (0.45)
λ_2	-	-0.11 (0.07)	-0.11* (0.06)	-	-	-0.50 (0.40)
λ_3	-	0.11 (0.09)	0.13 (0.11)	-	-	0.51 (0.38)
λ_4	-	-0.27*** (0.09)	-0.18* (0.09)	-	-	-0.54 (0.34)
λ_5	-	0.19** (0.07)	0.09 (0.06)	-	-	0.26 (0.26)
λ_6	-	0.00 (0.06)	0.10 (0.09)	-	-	0.07 (0.25)
λ_7	-	-	-0.09 (0.08)	-	-	-0.26 (0.23)
λ_8	-	-	0.16 (0.12)	-	-	0.59 (0.38)
λ_9	-	-	-0.18** (0.08)	-	-	-0.54* (0.30)
λ_0^*	-	-	13.34 (19.65)	-	-20.59 (23.53)	-
λ_1^*	-	-	-	-	8.58 (17.76)	-
λ_2^*	-	-	-	-	10.95 (17.84)	-
λ_3^*	-	-	-	-	-9.63 (10.08)	-
λ_4^*	-	-	-	-	3.55 (8.48)	-
δ_1	-0.11** (0.05)	0.19 (0.28)	0.47*** (0.10)	-0.14 (0.14)	0.68 (0.51)	0.59*** (0.19)
η	0.10 (0.10)	-0.01 (0.03)	0.08 (0.13)	0.08 (0.09)	-0.02 (0.53)	0.12 (0.15)
ω	0.07** (0.03)	0.16 (0.11)	0.03 (0.02)	0.03 (0.02)	-0.10 (0.14)	0.06* (0.03)
φ	-0.05 (60.10)	-0.02 (0.03)	-0.07 (0.07)	-0.12* (0.07)	-0.05 (0.11)	-0.05 (0.12)
ε_{t-1}	0.48*** (0.13)	-0.39 (0.30)	-0.52* (0.27)	0.58** (0.22)	-	-
$AR(1)$	-	-	-	-	-	-0.51*** (0.17)
\bar{R}^2	0.79	0.57	0.71	0.62	0.27	0.46
DW	1.83	1.74	2.01	2.02	1.98	2.25
F_P	27.25 [0.00]	3.14 [0.01]	5.93 [0.00]	13.98 [0.00]	3.49 [0.00]	3.39 [0.00]

This table presents the TSLS estimates of the preferred models represented in Tables (3.1) and (3.2), controlling for HF, LC, wealth and an error correction term. The instruments used are the considered inflation variables lagged two periods, as well as: Δc_{t-3} to Δc_{t-5} , Δy_{t-2} to Δy_{t-4} , a_{t-3} to a_{t-5} and $[c_{t-3} - y_{t-3}]$ to $[c_{t-5} - y_{t-5}]$. The inflation measures, periods and statistics are identical to those described in Table 3.1. $AR(1)$ is the estimated coefficient of the first-order autoregressive (AR) term of the MICP. Note that, in contrast to the previously reported OLS estimations, the \bar{R}^2 has no direct statistical meaning in the context of IV estimations.

and (ii) and the significant AR(1) term in (vi).²⁵ If, as previously assumed, serial correlation comes from the presence of HF in aggregate consumption, adding further lags on consumption growth could remove the remaining autocorrelation in the residuals and add consistency to the results.

The remaining variables in equation (3.4) do not seem to play an important role in predicting consumption growth. Their coefficients are mostly insignificant and remain very small in comparison to those for inflation or lagged consumption growth. This casts some doubt, for example, on the LC model hypothesizing that a constant and relatively large fraction of the population lives from hand to mouth. Starting from the general model (equation (3.4)), the next section looks for the models that best explain consumption growth in both high-inflation and low-inflation periods.

3.4.2 Finding the Best Aggregate Consumption Function

The estimations in the previous sections uncover new insights about the nature of the relationship between inflation and consumption growth. In particular, current and past inflation seem to be important determinants of consumption growth, and consumers seem to behave differently in different inflationary environments. However, neither the negative nor the positive inflation effects of the inflation-augmented RW models are totally robust to the inclusion of additional variables. In order to better identify the effect of money illusion on consumption growth and to find the most reliable aggregate consumption models for the selected subperiods and inflation measures, this section focuses on the estimation of the linear MICF (equation (3.4)) without significant initial restrictions. The nonlinear model (3.5) can be excluded because there is no nonlinear relationship between inflation and consumption growth within the two analyzed subperiods.

The different estimation results of equation (3.4) for the high- and low-inflation subperiods are summarized in Tables 3.4 and 3.5, respectively. In order to compare the performance of the MICF with aggregate consumption functions from the literature, I also report the estimations of (3.4) excluding inflation (i.e., setting $\lambda_i = 0$ for $i = 0, \dots, I$).²⁶ Contrary to these “traditional” aggregate consumption functions, however, I also allow for distributed lags in consumption growth in order to control

²⁵In model (vi), the MA(1) term has been replaced by the first-order autoregressive term AR(1) because this term better explains the data. This choice does not, however, change the interpretation of autocorrelation in the residuals.

²⁶Examples of inflation-excluding aggregate consumption models can be found in [Carroll et al. \(2011\)](#), [Kiley \(2010\)](#), [Fuhrer \(2000\)](#), [Malley and Molana \(2006\)](#) and [Sommer \(2007\)](#).

for consumption habits that overlap across several quarters. For the estimation, the maximum numbers of lags in inflation and consumption growth have been set to $I, K = 5$ for three reasons: first, because the coefficients on further lags are almost never significant in the estimation of the inflation-augmented RW model; second, to remain in an economically realistic range; and third, to avoid, as much as possible, the small sample problem for the low-inflation subperiod (see Section 3.2.3).

The reported models in Table 3.4 (high-inflation subperiod) and Table 3.5 (low-inflation subperiod) are as follows. The first and fifth benchmark regressions (e.g., (i)^H or (v)^H for the IPD and CPI models in Table 3.4) report the TSLS estimations of equation (3.4), in which inflation is excluded from the list of regressors. The second and the sixth regressions correspond to the same inflation-excluding model after sequentially removing each least-significant variable, until all remaining variables (or the last lag of the variables) are statistically significant. The resulting models can be interpreted as the preferred standard aggregate consumption functions, which do not attribute any predictive role to inflation. The third and seventh regressions report the estimation of the MICF that correspond to the benchmark model augmented by current and past inflation. The last presented models are the preferred MICFs for each subperiod and inflation measure, obtained through the same variable-reducing technique as before. They are to be compared to the preferred benchmark models without inflation. The preferred MICF models are highlighted in bold because they unambiguously outperform the benchmark models that exclude inflation, as well as the previously preferred inflation-augmented RW models and, thus, represent the final estimation results of this chapter.

Looking at Tables 3.4 and 3.5, we see that the traditional aggregate consumption function models, which include all variables but exclude inflation, do not perform well in describing consumption growth. In fact, no variable has a significant impact on consumption growth that is statistically significant and constant across the subperiods and inflation measures. Reducing these benchmark models to keep only those variables that have a significant impact on consumption growth slightly improves the fit, but yields estimates that depend entirely on the subperiod and the inflation measure considered. In the preferred IPD model (ii)^H, for example, the significant variables are current income and consumption growth lagged two periods, which advocate in favor of LC with some presence of HF. In the CPI model (vi)^H, however, the LC effect is cut in half and is only significant at the 10% level, HF is more long-lasting and wealth has the most significant coefficient at the 5% level. The preferred models for the low-inflation subperiod are completely different. In the

Table 3.4: Money Illusion Consumption Function – High-Inflation Period

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \sum_{k=1}^K \delta_k L^k (\Delta c_t) + \eta \Delta y_t + \omega a_{t-1} + \varphi [c_{t-1} - y_{t-1}] + \varepsilon_t$$

	IPD				CPI			
	Excluding Inflation		Including Inflation		Excluding Inflation		Including Inflation	
	(i) ^H	(ii) ^H	(iii) ^H	(iv) ^H	(v) ^H	(vi) ^H	(vii) ^H	(viii) ^H
α	0.00 (0.04)	0.00 (0.00)	-0.03 (0.08)	0.01*** (0.00)	-0.03 (0.03)	0.00** (0.00)	0.14 (2.04)	0.01*** (0.00)
λ_0	–	–	-1.48 (2.14)	-0.76*** (0.07)	–	–	5.53 (82.27)	-0.75*** (0.12)
λ_1	–	–	1.06 (2.04)	0.27* (0.15)	–	–	-6.04 (91.56)	0.34** (0.16)
λ_2	–	–	-0.27 (1.00)	0.14 (0.10)	–	–	10.75 (143.01)	–
λ_3	–	–	-0.59 (0.83)	-0.20* (0.11)	–	–	-5.60 (61.71)	–
λ_4	–	–	0.84 (0.92)	0.17** (0.08)	–	–	6.99 (85.21)	–
λ_5	–	–	0.07 (0.68)	–	–	–	-4.64 (61.96)	–
δ_1	-0.03 (0.32)	-0.15 (0.15)	1.26 (2.20)	0.38*** (0.11)	-0.01 (0.24)	-0.03 (0.29)	4.41 (49.94)	0.36*** (0.12)
δ_2	0.11 (0.24)	0.27** (0.14)	-0.51 (1.35)	–	0.15 (0.18)	0.17 (0.23)	7.02 (98.72)	–
δ_3	0.05 (0.20)	–	0.03 (0.68)	–	0.03 (0.12)	0.01 (0.15)	-2.70 (32.88)	–
δ_4	-0.11 (0.14)	–	0.48 (0.86)	–	-0.22* (0.13)	-0.23* (0.12)	-0.62 (11.85)	–
δ_5	-0.01 (0.17)	–	0.11 (0.75)	–	0.24 (0.16)	0.30* (0.17)	0.22 (4.74)	–
η	0.39 (0.29)	0.66*** (0.14)	-0.06 (0.89)	–	0.41** (0.19)	0.38* (0.21)	5.65 (72.46)	–
ω	0.15 (0.36)	–	-0.36 (0.59)	–	0.13* (0.07)	0.16** (0.07)	-0.45 (4.50)	–
φ	-0.04 (0.15)	–	-0.10 (0.27)	–	-0.12 (0.11)	–	1.13 (15.74)	–
$\sum_{i=1}^I \lambda_i$	–	–	–	0.38*** (0.11)	–	–	–	0.34** (0.16)
\bar{R}^2	0.85	0.52	-0.91	0.75	0.49	0.38	-121.86	0.55
DW	1.73	2.17	2.09	1.78	1.99	2.02	1.75	1.84
F_P	8.23 [0.00]	8.36 [0.00]	1.14 [0.35]	32.28 [0.00]	5.98 [0.00]	5.74 [0.00]	0.02 [1.00]	27.37 [0.00]

This table presents the estimation results of the MICF (3.4) for the U.S. over the subperiods 1966Q1 to 1981Q3 and 1966Q1 to 1982Q3 for the IPD and CPI models, respectively. The superscript *H* stands for “high-inflation period”. The regressions containing income have been estimated with TSLS, using as instruments each utilized variable, lagged by two periods. In the regressions that exclude inflation, lagged inflation has been added to the instrument set with two lags up to the highest lags of the instruments for lagged consumption. Models that do not contain income have been estimated with OLS.

Table 3.5: Money Illusion Consumption Function – Low-Inflation Period

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \sum_{k=1}^K \delta_k L^k (\Delta c_t) + \eta \Delta y_t + \omega a_{t-1} + \varphi [c_{t-1} - y_{t-1}] + \varepsilon_t$$

	IPD				CPI			
	Excluding Inflation		Including Inflation		Excluding Inflation		Including Inflation	
	(i) ^L	(ii) ^L	(iii) ^L	(iv) ^L	(v) ^L	(vi) ^L	(vii) ^L	(viii) ^L
α	-0.02 (0.02)	0.00*** (0.00)	0.02 (0.11)	0.01*** (0.00)	0.00 (0.04)	0.00** (0.00)	-0.15 (2.85)	0.01** (0.00)
λ_0	-	-	-0.83 (1.56)	-0.72*** (0.07)	-	-	-3.52 (60.36)	-0.53*** (0.18)
λ_1	-	-	-0.44 (2.54)	0.38*** (0.06)	-	-	-5.82 (97.93)	0.25 (0.28)
λ_2	-	-	-0.06 (1.44)	-0.19** (0.09)	-	-	-3.47 (51.45)	-0.71*** (0.22)
λ_3	-	-	0.16 (1.74)	0.53*** (0.08)	-	-	1.36 (19.44)	0.59*** (0.16)
λ_4	-	-	0.45 (4.62)	-0.53*** (0.09)	-	-	5.97 (110.93)	-0.65*** (0.20)
λ_5	-	-	-0.69 (4.62)	0.22*** (0.04)	-	-	-6.23 (113.25)	0.51** (0.21)
δ_1	0.04 (0.16)	-	-0.63 (3.43)	0.26*** (0.07)	0.22 (0.25)	0.22 (0.17)	-9.08 (157.76)	0.10 (0.13)
δ_2	-0.01 (0.10)	-	0.02 (1.31)	0.00 (0.08)	0.27 (0.36)	0.09 (0.14)	1.12 (22.23)	0.05 (0.13)
δ_3	0.09 (0.16)	-	-0.01 (2.40)	0.41*** (0.09)	0.18 (0.22)	0.28** (0.12)	-0.97 (24.32)	0.42*** (0.10)
δ_4	0.05 (0.19)	-	0.57 (4.36)	-0.31*** (0.10)	0.00 (0.29)	-	6.85 (118.93)	-
δ_5	-0.02 (0.13)	-	-0.45 (2.70)	-	-0.16 (0.21)	-	-2.10 (36.12)	-
η	0.44 (0.27)	0.42** (0.04)	0.79 (4.23)	-	0.08 (0.30)	-	5.36 (92.61)	-
ω	0.05 (0.06)	-	0.12 (0.70)	-	0.13 (0.06)	0.03* (0.02)	0.16 (2.51)	-
φ	-0.09 (0.08)	-	0.05 (0.41)	-	0.01 (0.18)	-	-1.03 (18.29)	-
$\sum_{i=1}^I \lambda_i$	-	-	-	0.40** (0.16)	-	-	-	-0.02 (0.32)
$\sum_{k=1}^K \delta_k$	-	-	-	0.37* (0.19)	-	-	-	0.57** (0.23)
\bar{R}^2	0.08	0.29	-3.67	0.81	-0.57	0.24	-150.15	0.51
DW	2.94	2.77	1.73	1.88	2.70	2.35	1.63	1.81
F_P	1.54 [0.18]	5.36 [0.03]	0.30 [0.99]	17.61 [0.00]	1.58 [0.17]	4.13 [0.01]	0.01 [1.00]	5.55 [0.00]

This table shows the estimation results of the MICF (3.4) for the U.S. over the subperiod 1991Q1 to 2001Q1. The superscript L stands for “low-inflation period”. The regressions containing income have been estimated with TSLS, using as instruments each utilized variable, lagged by two periods. In the regression that excludes inflation, lagged inflation has been added to the instrument set with two lags up to the highest lags of the instruments for lagged consumption, except for the case of regression (ii), for which the instruments are income lagged by two periods and inflation lagged by two to five periods. Models not containing income have been estimated with OLS.

IPD model, the only potentially relevant variable is current income, since all other effects vanish as the insignificant variables are being removed. However, this LC effect is rather weak because it is only present when more than four lags of inflation are added to the instruments list. In the last retained benchmark model (vi)^L, the LC effect completely disappears, replaced by a weakly significant wealth effect and some degree of HF.

The fact that the preferred benchmark models differ greatly across the selected subperiods and inflation measures indicates, not only that the standard aggregate consumption functions underestimate the changes in the consumer behavior depending on the inflationary environment, but also that they are inappropriate for drawing robust conclusions about any consumption behavior due to their extreme sensitivity to the model specifications.

Compared to this benchmark case, simply adding current and lagged inflation in the third and sixth regressions does not increase the fit of the model. In contrast, in this case, no coefficient is significant, and some have implausible signs and values. The estimations of these full MICFs perform particularly badly in the CPI case, as reflected by very large estimated standard errors. In fact, the models that include all additional variables are not able to beat the RW model suggested by Hall (1978), since the F-tests cannot reject the null hypothesis of all coefficients being equal to zero (as shown by the low F-statistics in (v)^H, (i)^L and (v)^L).

The picture changes dramatically when the full MICF is reduced to include only those variables that have significant coefficients, as reported by the fourth and seventh models in Tables 3.4 and 3.5. These preferred MICF models clearly outperform the preferred inflation-ignoring models in terms of the general fit of the model and of the robustness of the estimated coefficients. In fact, both high-inflation and low-inflation models confirm the importance of current and past IPD- and CPI-based inflation, as well as of HF for consumption growth. This corroborates assumption (A1) by showing that consumption growth is not neutral with respect to inflation.

Interestingly, the preferred MICF models yield very similar results across the subperiods. The main difference lies in the number of significant lags on inflation and consumption growth. In the high-inflation period, only one lag on consumption growth is (highly) statistically significant, presenting similar values in the IPD and CPI models. In models (iv)^H and (viii)^H, the negative unanticipated inflation effect (λ_0) is highly significant and very large, at around -0.75 . There is a slight difference with regard to lagged inflation, however, in that the IPD model suggests a longer

lasting nominal income effect than the CPI model. That said, the overall effect of lagged inflation is very similar in both model. Table 3.5, in contrast, suggests more long-lasting effects for inflation and HF. Both the IPD model (iv)^L and the CPI model (viii)^L indicate that inflation with up to five lags and that consumption with, respectively, four- and three-period lags have significant effects on current consumption growth. The fact that the inflation effect lasts longer in the low-inflation subperiod corresponds to assumption (A3), which suggests that money illusion as a rule of thumb is more costly when inflation is high and that consumption behaviors should be analyzed separately in different inflationary environments.

The superiority of the preferred MICFs represents a novel finding because it shows that the only stable model capable of successfully predicting consumption growth across different subperiods and using different inflation measures is the simple RW model augmented by current and lagged inflation, as well as lagged consumption growth. In fact, the preferred MICF models outperform, not only the benchmark models that exclude inflation, but also outperform the linear inflation-augmented RW models presented in Table 3.1. If we compare, for example, models (iv)^H and (viii)^H in Table 3.4 with models (iii) and (iv) of Table 3.1, we see that allowing past consumption to be correlated with current consumption not only substantially reduces the serial correlation in the model, but also validates lagged inflation rates as significant predictors for current consumption growth. Table 3.5 presents a similar picture for the low-inflation period: Both preferred MICF models (iv)^L and (viii)^L appear to be simple extensions of models (v) and (vi) of Table 3.1. The MICF models clearly outperform the inflation-augmented RW models by using fewer lags on inflation but allowing for the presence of HF.

There are two further noteworthy features of the preferred MICF. First, note that the IPD-based MICF models have a substantially better fit than the CPI models, which is reflected in much higher coefficients of determination in Tables 3.4 and 3.5. This feature illustrates the fact that the IPD is closer to the PCE time series than the CPI (cf. Section 3.2.2). Second, even though equation (3.4) contains many different explanatory variables and does not impose any particular structural form, all preferred models present the particular form of autoregressive distributed lag models, or ADL(I, K) models, with I lags of the exogenous variable Δp_t and K lags of the endogenous variable Δc_t . Since one of the major issues generally associated with ADL models is the presence of multicollinearity, the preferred MICF models might suffer from some degree of multicollinearity as well. On one hand, this weakness could be particularly important in the low-inflation models of Table 3.5,

which contain a relatively high number of lags.²⁷ On the other hand, the major issue with multicollinearity is the prevalence of particularly large standard errors, which can lead to erroneously declaring coefficients statistically insignificant. In fact, this is not the case for the coefficients on lagged inflation in the low-inflation subperiod.

The encouraging results with respect to the importance of inflation for consumption growth should not eclipse the puzzle mentioned earlier (namely, the fact that some coefficients on lagged inflation are negative). If all coefficients were positive, this would unequivocally indicate the presence of a widespread money illusion affecting consumption through the nominal income channel and corroborate the existing literature on money illusion, as exposed in Section 3.1.1. Regardless, this puzzle does not refute the presence of this type of money illusion because the negative coefficients could also capture some remaining statistical issues, such as remaining seasonality patterns in the data or other omitted variables that are not controlled for in the MICF.

Two facts support the possible presence of remaining statistical issues. First, the signs on lagged inflation appear to be negative over regular intervals, as can be seen in the negative signs of λ_0 , λ_2 and λ_4 in the preferred MICF models (iv)^L and (viii)^L. Second, lagged consumption growth also seems to suffer from the same bias, in the manner of δ_4 in the preferred MICF model (iv)^L. The simplest and most intuitively appealing way of addressing this issue is to analyze the cumulative effect of the distributed lags on inflation and consumption growth. In fact, the total nominal income effect of lagged inflation on consumption growth is entirely captured by the sum $\sum_{i=1}^I \lambda_i$, as reported in Tables 3.4 and 3.5. The cumulative effect of past consumption growth, $\sum_{k=1}^K \delta_k$, is also reported in Table 3.5 and can be interpreted as the total HF effect influencing current consumption growth.

Concerning the total effect of lagged inflation in Table 3.4, we see that $\sum_{i=1}^4 \lambda_i$ is clearly positive and highly significant, despite the negativity of λ_3 . Note that $\sum_{i=1}^4 \lambda_i$ is very close in value to the only significant coefficient λ_1 in the CPI model (viii)^H, which indicates some robustness of the money illusion effect within the high-inflation subperiod. Consequently, both preferred MICFs advocate for the presence of both money illusion mechanisms presented in Section 3.1.2: namely, the negative unanticipated inflation effect of current inflation and the positive nominal income effect of lagged inflation on current consumption growth. Interestingly, it appears

²⁷In order to avoid the negative effects associated with multicollinearity, it is possible to impose restrictions on the parameters to be estimated, as suggested by [Koyck \(1954\)](#), or to use the [Almon \(1965\)](#) technique by imposing a rapid speed of decay on the coefficients of the exogenous variable.

that the negative effect is approximately twice as big as the positive effect and the HF effect, which both yield similar impacts on consumption growth.

In Table 3.5, the IPD and the CPI models present somewhat divergent results for the low-inflation subperiod. On one hand, the preferred IPD MICF model presents cumulative effects for lagged inflation and lagged consumption growth that are very similar to those in the high-inflation subperiod in Table 3.4. In particular, model (iv)^L confirms the presence of both money illusion effects. On the other hand, the preferred CPI model for the low-inflation subperiod, (viii)^L, mitigates these results. Even though the unanticipated inflation effect and the HF are still important and significant (despite the former being slightly lower and the latter slightly higher than in the other preferred models), the cumulative effect of lagged inflation becomes insignificant and almost null ($\sum_{i=1}^5 \lambda_i = -0.02$). As we can see in the table, this is due to the fact that the negative coefficients are much larger than those in the other models and that, in consequence, the different coefficients on lagged inflation cancel out their respective effects.

As a concluding remark, there is little doubt that both current and past inflation play a central role in current consumption growth, indicating that money illusion might be far more important than has been thus far admitted. The estimation results for the MICFs show a particularly high and significant negative impact of current inflation on consumption growth, validating money illusion as a signal extraction problem, as suggested by Deaton (1977). However, the estimation results are not as conclusive for the presence of money illusion as a rule of thumb, thus calling for further investigation in order to remove the remaining statistical biases and to be able to determine the exact depth and duration of the nominal income channel suggested by Branson and Klevorick (1969).

3.5 Conclusion

Contrary to most empirical macroeconomic studies on aggregate consumption, which leave inflation completely out from their analysis, this chapter shows that both contemporaneous and lagged inflation play an important role in real consumption growth. This novel result is robust to the choice of the inflation measure (i.e., IPD or CPI) and valid for periods with both high and low average inflation rates.

When extending the model to control for liquidity constraints, habit formation, wealth, the interest rate and an error-correction term, all estimated models reduce to an autoregressive distributed lag model, in which current consumption

growth depends only on past consumption growth, current inflation and past inflation. Whereas the significance of past consumption growth is typically attributed to consumption habits, this chapter assumes that the impacts of current inflation and past inflation are due to money illusion, which influences consumption growth via two distinct channels.

Contemporaneous inflation has a negative effect on consumption, which is attributable to the fact that consumers misinterpret inflation as a price increase in individual goods and, thus, postpone their current consumption. This negative effect of current inflation on consumption growth is found to be highly significant and particularly important, independent of the chosen inflation measure and sample period. To further investigate whether this negative unanticipated inflation effect is as important as suggested by the money illusion consumption function, Chapter 4 directly addresses money illusion as a signal extraction problem, which allows for the use of more sophisticated estimation techniques.

The second identified impact of money illusion is reflected by a positive effect of lagged inflation on consumption growth. This effect captures money illusion as a rule-of-thumb behavior, in which the consumer uses nominal income as a proxy for real income. In this case, a positive nominal income shock increases the consumer's subjective wealth and stimulates her consumption. The estimation results show that this positive effect of money illusion on consumption growth is significant and lasts for only one period when inflation is high, reflecting the fact that the efficiency of this rule of thumb worsens rapidly when nominal and real income decouple. In the examined low-inflation period, the effect of lagged inflation on consumption growth is found to be more long-lasting, although it is only significant and positive in the IPD inflation models.

The contrasting results of the CPI models call for a deeper analysis of the relationship between inflation and consumption growth. Moreover, the assumed link between money illusion and the observed inflation effect should be interpreted with some caution. Showing that inflation has a significant effect on consumption growth violates the homogeneity postulate, but is it not a sufficient condition to prove the existence of money illusion at the aggregate level. Even though the money illusion consumption function excludes the possibility that the inflation effects works as a proxy for LC, HF, the interest rate or wealth, there may still be other inflation-to-consumption mechanisms at work that are totally independent of the money illusion phenomenon.

For example, the inflation effect could also reflect an uncertainty effect ([Koskela](#)

and Viren, 1985; Barro, 1995) or a substitution effect between durable and non-durable goods (Cukierman, 1972; Fortune, 1982). Concerning such a substitution effect, recall that the IPD accounts for substitution effects between different non-durable goods and between non-durables and services, but not between durables and non-durables. To directly address this possible substitution effect in future studies, a solution could be to re-estimate the money illusion consumption function with disaggregated price indexes.

The argument that inflation solely captures an uncertainty effect can be questioned. The presented results show a highly significant inflation effect, even after controlling for income, wealth and the interest rate. This suggests that, if the inflation effect works as a proxy for some remaining uncertainty, this uncertainty is not directly related to the consumer's lifetime wealth. Furthermore, the positive and significant effect of lagged consumption growth on current consumption growth, which is attributed solely to HF in this chapter, predominantly reflects a prudent behavior on the part of the consumer. The resulting excessive consumption smoothing certainly captures a high degree of uncertainty and not only HF. Note, finally, that this argument can only address the negative effect of current inflation on consumption growth, which works as an additional consumption smoother, and cannot address the uncovered positive effect of lagged inflation.

Chapter 4

Money Illusion as a Signal Extraction Problem

Money illusion can be interpreted as a signal extraction problem, in which the consumer observes only her nominal income and “overlooks” the relevant inflation at the time that she makes her consumption decisions. In this chapter, I develop two state-space models for this signal extraction problem. The first one takes an unobserved components approach to model a fully money-illusioned consumer, while the second one treats inflation as an observed variable. Given the same U.S. quarterly data and sample periods as in Chapter 3, the Kalman filter estimation results are as follows. Due to its heavy reliance on the assumed processes governing inflation and real income growth, the first model yields rather poor predictions for actual consumption growth, such that full money illusion can be rejected. The second state-space model performs much better in explaining real consumption growth, but yields different results than the model in Chapter 3. While the negative inflation effect suggested by [Deaton \(1977\)](#) is significant and large in the high-inflation period, its presence remains undetermined in the low-inflation subperiod. Furthermore, the models do not confirm any presence of the positive inflation effect on consumption growth advocated by [Branson and Klevorick \(1969\)](#). Finally, the estimates uncover a high degree of consumption smoothness in the high-inflation subperiod, attributable to increased precautionary savings when inflation uncertainty is high.

Keywords: Money illusion, signal extraction, state-space model, Kalman filter.

JEL classification: E21, C61, E31, C32.

4.1 Introduction

A consumer is considered to be money-illusioned if she does not fully anticipate and account for inflation in her consumption decisions, thus violating the homogeneity of degree zero postulate (see Section 3.1.2). The resulting confusion between nominal and real income can be modeled as a signal extraction problem, which can be understood as a measurement problem in which the consumer would like to anchor her consumption to her real income, but only observes her nominal income.

The economic literature dealing with signal extraction problems goes back to the early 1970s, when incomplete information was first introduced to justify the nonneutrality of money in the short run. The idea is that people are not able to disentangle changes in monetary policy from changes in demand or relative prices, thus causing unexpected monetary shocks to have an impact on real economic variables (typically output or labor supply).¹ Of course, all changes are anticipated in the long run, such that the neutrality of money prevails. Deaton (1977), by making a similar distinction between anticipated and unanticipated inflation, was the first to establish a direct and positive link between (unexpected) inflation and savings. Following more recent studies on rational inattention (Sims, 2003; Mankiw and Reis, 2002; Reis, 2006), the signal extraction problem analyzed in this chapter is completely in line with the full rationality assumption (i.e., the consumer deliberately chooses not to be constantly and perfectly informed about inflation).

The analysis in Chapter 3 of the inflation-augmented random walk (RW) model and the money illusion consumption function (MICF) suggests that money illusion, via inflation, has two distinct impacts on contemporaneous consumption growth. On one hand, unexpected inflation implies a negative impact of current inflation (Deaton, 1977), while, on the other hand, the nominal income effect suggested by Branson and Klevorick (1969) implies a positive impact of lagged inflation on current consumption growth. The purpose of this chapter is to provide, as a robustness check for the previous OLS and TSLS estimation results, an alternative and potentially more efficient way to model and estimate money illusion. In particular, money illusion as a signal extraction problem can be estimated by means of a Kalman filter, which allows us to optimally extract the relevant information from unobserved (state) variables.

In this chapter, I develop two alternative state-space models that characterize the signal extraction problem faced by two consumers suffering from different de-

¹See Lucas (1972, 1973), Sims (1972) or Barro (1977, 1978).

degrees of money illusion. The first model, developed in Section 4.2 and estimated in Section 4.2.1, considers a consumer who is completely money-illusioned. The particularity of this consumer is that she only observes two variables – namely, her real consumption and her nominal income growth – while she entirely relies on assumptions about the evolution of the unobserved inflation and real income growth.

One advantage of this “unobserved components” approach is that, despite its simplicity and the minimal amount of information available to the consumer, it captures, through the correlations between the states, different consumption behaviors that are relevant for the present analysis: namely, money illusion, habit formation (HF) and liquidity constraints (LC). The reverse side of this coin, however, is that the two conflicting inflation effects, as well as the opposing HF and LC effects, are mutually exclusive. Interestingly, the estimation results show that the completely money-illusioned consumer, having access to no information about inflation, behaves according to the RW hypothesis of Hall (1978) (i.e., exactly the same way as the fully rational consumer).

The second state-space model, analyzed in Section 4.3 and estimated in Section 4.3.1, is less restrictive than the first one, in that it considers that the consumer observes inflation, but might not be able – or willing – to completely account for it in her evaluation of her real income growth. Moreover, it allows opposite inflation effects and consumption behaviors to have simultaneous effects on current consumption growth. The resulting model is similar to the MICF of Section 3.3, in which money illusion, HF and LC are directly controlled for in the estimated equation, except for the fact that, in this model, real income growth is considered as unobserved and follows a specific process.

Thanks to the greater amount of information about inflation at her disposal, the second consumer is able to better anticipate the evolution of her real income. With regard to money illusion, the estimation results show that, while the negative inflation effect is still present, the supposed positive effect of lagged inflation on current consumption growth does not survive the robustness checks, which suggests that the significant effect in Chapter 3 might capture other phenomena that are not explicitly controlled for.

4.2 Modeling Inflation as an Unobserved Variable

In a fashion similar to that of Sargent (1979, p. 209), and in keeping with the definition of money illusion as an underperception of inflation, we can state the signal ex-

traction problem faced by the money-illusioned consumer as follows: Consider a consumer who wants to estimate her real income growth, defined as $\Delta y_t^r = \Delta y_t^n - \Delta p_t$, but only observes her nominal income growth Δy_t^n . She sees the random variable Δy_t^n , but overlooks the pertinent inflation Δp_t at the time that she makes her consumption decisions. Recall that this underperception of inflation does not necessarily reflect an irrationality; it can also be attributed to some intentional rule-of-thumb behavior or to incomplete information about the state of the general price level (see Section 3.1.2).

This signal extraction problem is, conceptually, a measurement error problem that can be controlled for via a Kalman filter, which is a powerful recursive algorithm that provides potentially more efficient estimates than the standard OLS and TSLS methods used in Section 3.4.² In the consumption context, Kalman filters can be used to account for measurement errors in consumption growth (Sommer, 2007; Carroll et al., 2011), but also (and mainly) to distinguish between the different components of the income process, such as, particularly, permanent and transitory income (Malley and Molana, 2006; Morley, 2007; Pozzi, 2010). Compared to the latter case, this approach of modeling inflation (rather than permanent income) as a signal might be more accurate, since aggregate shocks to the log level of income are essentially permanent. As a result, the distinction between permanent and transitory income is obsolete, since, at the aggregate level, permanent income is equal to actual income (Carroll et al., 2011). This fact is of great importance in the following analysis because it implies that apparent transitory income shocks stem from other mechanisms, such as inflation shocks.

To model the signal (here: inflation) extraction problem in such a way that it can be estimated with a Kalman filter, it is useful to write the model in a state-space form. To do so, I adopt an approach similar to that of Morley (2007) and first decompose consumption and nominal income into two components: an idiosyncratic stationary component and a stochastic innovation component. Even though stationarity is not a prerequisite for an optimal estimation of state-space models, I use the same detrended variables as in Chapter 3 to facilitate a direct comparison. The second step is to develop a correlated unobserved components model for aggregate consumption and income growth.

Consider the following measurement (or observation) equations:

²There are, of course, other methods that depart from this linear quadratic control filtering, but can be used to model and estimate concerns about mismeasurement or model misspecification. One of these is robust control, which is presented in detail in Chapter 5.

$$\Delta c_t = \beta + \eta \Delta y_t^r + u_t \quad (4.1)$$

$$\Delta y_t^n = \Delta y_t^r + \Delta p_t, \quad (4.2)$$

where y_t^n is (the logarithm of) nominal income, y_t^r is (the logarithm of) real income, and u_t and Δp_t are the measurement errors of consumption and income growth, respectively. The constant β can be thought of as reflecting the precautionary motive and collateral nonlinearities (as in [Carroll et al., 2011](#)). The parameter η is the consumer's marginal propensity to consume out of her current real income.

An important assumption in this first model is that inflation (Δp_t) is entirely unobserved, in the sense that the consumer does not have (or seek) immediate access to information about the evolution of the general price levels of goods and services. In other words, this model depicts the signal extraction problem of a consumer suffering from complete money illusion. Since the consumer only observes the (control) variables Δc_t and Δy_t^n , she needs to make assumptions about the processes governing all unobserved (state) variables. First, assume that real income growth follows a first-order autoregressive AR(1) process:

$$\Delta y_t^r = \mu + \phi_y \Delta y_{t-1}^r + v_t, \quad (4.3)$$

where $v_t \sim iidN(0, \sigma_v^2)$ and μ is a drift parameter that can be interpreted as capturing a constant long-term growth rate in the economy. In addition, the measurement errors of real consumption growth and nominal income growth both follow AR(1) processes, as suggested by the data and the literature:

$$u_t = \phi_u u_{t-1} + \varepsilon_{ut} \quad (4.4)$$

$$\Delta p_t = \phi_p \Delta p_{t-1} + \varepsilon_{pt}, \quad (4.5)$$

where $\varepsilon_{ut,pt} \sim iidN(0, \sigma_{u,p}^2)$. Equations (4.1) to (4.5) can be rewritten in a state-space form. First, the measurement equation of the state-space model is the following:

$$\begin{bmatrix} \Delta c_t \\ \Delta y_t^n \end{bmatrix} = \begin{bmatrix} \beta \\ 0 \end{bmatrix} + \begin{bmatrix} \eta & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta y_t^r \\ u_t \\ \Delta p_t \end{bmatrix}. \quad (4.6)$$

This can be summarized as $\tilde{y}_t = Ax_t + H'\xi_t$, where \tilde{y}_t is the (2×1) vector of observed variables, x_t is a scalar one and ξ_t is a vector containing the unobserved state variable Δy_t^r and the measurement errors.³ The accompanying state (or transition) equation, written as $\xi_t = \tilde{\mu} + F\xi_{t-1} + \tilde{v}_t$ in its general formulation, is equal to:

$$\begin{bmatrix} \Delta y_t^r \\ u_t \\ \Delta p_t \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_y & 0 & 0 \\ 0 & \phi_u & 0 \\ 0 & 0 & \phi_p \end{bmatrix} \begin{bmatrix} \Delta y_{t-1}^r \\ u_{t-1} \\ p_{t-1} \end{bmatrix} + \begin{bmatrix} v_t \\ \varepsilon_{ut} \\ \varepsilon_{pt} \end{bmatrix}, \quad (4.7)$$

with the associated variance-covariance matrix $\mathbb{E}(\tilde{v}_t\tilde{v}_t') = Q$:

$$Q = \begin{bmatrix} \sigma_v^2 & \rho_{vu}\sigma_v\sigma_u & \rho_{vp}\sigma_v\sigma_p \\ \rho_{vu}\sigma_v\sigma_u & \sigma_u^2 & \rho_{up}\sigma_u\sigma_p \\ \rho_{vp}\sigma_v\sigma_p & \rho_{up}\sigma_u\sigma_p & \sigma_p^2 \end{bmatrix}, \quad (4.8)$$

where

$$\begin{aligned} \rho_{vu} &= \text{corr}(v_t, \varepsilon_{ut}) \\ \rho_{vp} &= \text{corr}(v_t, \varepsilon_{pt}) \\ \rho_{up} &= \text{corr}(\varepsilon_{ut}, \varepsilon_{pt}). \end{aligned}$$

Contrary to the basic state-space model, in which the elements of Q outside the diagonal are set to zero, this chosen representation, in which the unobserved components are cross-correlated, allows us to distinguish between different theories of aggregate consumption and inflation. Of particular interest for the present analysis are the correlations ρ_{up} and ρ_{vu} , which incorporate, respectively, the different money illusion and consumption theories presented in Chapter 3.⁴

The former correlation, ρ_{up} , directly relates the measurement error in consumption growth to the measurement error in income growth (i.e., inflation). Consequently, a positive correlation of $\rho_{up} > 0$ indicates that inflation and consumption growth move in the same direction. Such a positive relationship indicates the presence of money illusion in the [Branson and Klevorick \(1969\)](#) sense (i.e., con-

³The notation follows [Hamilton \(1994, Chapter 13\)](#), with the exception that the measurement errors are included in the state vector to allow for correlations between the unobserved components.

⁴Note that, in this chapter, the ρ s stand for the correlations and should not be mistaken for the subjective discount rate used in Chapter 2.

sumers “feel richer” and increase their consumption after a positive nominal income shock). This behavior implies that consumption growth is more sensitive to income growth than predicted by the REPIH. On the other hand, a negative correlation (i.e., $\rho_{up} < 0$), implies that consumption growth is less reactive to a shock to the nominal income growth, favoring additional consumption smoothing. This negative correlation between inflation and consumption growth corresponds to the unanticipated inflation effect suggested by Deaton (1977), predicting that consumers decrease their consumption after an unexpected positive price shock.

With regard to the second correlation of interest, $\rho_{vu} < 0$ implies a partial adjustment of consumption to income shocks in each period, resulting in consumption smoothness, as predicted by the HF model. On the other hand, LC consumption is characterized by a greater sensitivity of consumption to income shocks and suggests $\rho_{vu} > 0$. Note, finally, that the REPIH implies that consumption reacts to nothing apart from permanent income (i.e., $\sigma_u^2 = 0$ and $\rho_{vu} = \rho_{up} = 0$).

In this model, I assume the shocks to inflation and real income growth to be independent from one another, such that the third correlation is equal to zero ($\rho_{vp} = 0$). The reason for this choice is twofold. First, the literature does not provide any robust evidence for this relationship. Second, this assumption allows for a necessary degree of freedom. Given $\rho_{vp} \neq 0$, the model would face an underidentification problem because there would be only two known variables from which to estimate three correlations.⁵ To add further robustness, the following section presents the Kalman filter estimation results for when only one correlation is set to zero (i.e., $\rho_{vp} = 0$; $\rho_{vu}, \rho_{up} \neq 0$), as well as the results for the case in which two correlations are set to zero (i.e., $\rho_{vp}, \rho_{vu} = 0$; $\rho_{up} \neq 0$). The Kalman filter for the state-space model summarized by equations (4.6) to (4.8) is briefly presented in Appendix C.

4.2.1 Kalman Filter Estimation Results

The Kalman filter estimation results for the first state-space model, characterized by equations (4.6) to (4.8), are summarized in Tables 4.1 and 4.2. To allow for a better comparison of the results and to account for the previously gained insights, the estimations are done for the same U.S. quarterly data, sample periods and inflation measures as in Chapter 3 and as described in Appendix A. Recall, however, that since inflation is unobserved in the present model and follows an assumed and predetermined process, the different price indexes (i.e., IPD and CPI) enter

⁵I thank Professor James D. Hamilton (University of California) for pointing out this identification problem.

the model only as deflators for the consumption series. Furthermore, since the money-illusioned consumer observes her nominal income (and not her real income), measured as the disposable personal income per capita, nominal income is deflated by neither IPD nor CPI inflation.

Tables 4.1 and 4.2 report the estimates for the parameters β , η , μ , ρ_{up} (in both tables) and ρ_{vu} (Table 4.2 only), which maximize the log-likelihood and minimize the AIC. The only difference between the two tables is that Table 4.1 allows for only one correlation (namely, ρ_{up}) to be unequal to zero, whereas Table 4.2 estimates both correlations (ρ_{up} and ρ_{vu}). Consequently, Table 4.1 is more restrictive than the second table, but it releases one additional degree of freedom for estimating the model presented in the previous section.

In order for the Kalman filter to find a recursive solution to the signal extraction problem, initial values need to be provided for $\xi_{1|0}$ and $P_{1|0}$ (see Appendix C). Instead of arbitrarily fixing the initial values, I generated them randomly, iterated the model until convergence was obtained, and selected the model with the highest log-likelihood. These “preferred” models are reported in Tables 4.1 and 4.2. For plausibility reasons, the estimated coefficients for η and the correlations ρ_{up} and ρ_{vu} have been restricted to lie, respectively, within the $(0, 1)$ and $(-1, 1)$ intervals.

Before we turn to the interpretation of the estimation results, it would be judicious to recall what we can expect, in light of the previous results from Section 3.4, with respect to the coefficients reported in Tables 4.1 and 4.2. The supposedly money-illusion-capturing coefficient ρ_{up} should be negative, because the total positive Branson and Klevorick (1969) effect, denoted $\sum_{i=1}^I \lambda_i$ in Section 3.4, despite its significance and magnitude, was only half as important (in most cases) as the negative inflation effect advocated by Deaton (1977).

With regard to the excess smoothness or excess sensitivity of consumption with respect to income, HF was high and significant (with largely positive $\sum_{k=1}^K \delta_k$) in all preferred models, while LC was never significant once inflation was included in the regression equation. Consequently, we expect η to be insignificant in both tables, while ρ_{vu} should be negative in Table 4.2. Regardless of the fact that the present state-space model uses as little information as possible and seeks to be very general and unrestricted in its formulation, nesting the different consumption theories solely within only one or two correlations does not allow for the simultaneous presence of inflation and income effects that operate in opposite directions. Another concern of the present model in comparison to the MICEF of Chapter 3 is that it does not allow for distributed lags to have a contemporaneous impact, even though the MICEF

suggested a simple ADL model to be the most suitable.⁶

Looking at the estimates of the first state-space model in Tables 4.1 and 4.2, we see that they clearly do not match the results from Chapter 3 and that they lead to contrasting conclusions. Consider, first, the most important parameter in this model: the correlation ρ_{up} between inflation, as an unobserved shock to nominal income, and the unexpected component in real consumption growth. In both tables, this is positive for the whole-sample period and for the low-inflation period, but negative for the high-inflation subperiod. On one hand, this regularity indicates that consumers might behave differently in different inflationary environments (i.e., that the negative inflation effect prevails over the positive nominal income effect in high-inflation periods, with the opposite being true for the low-inflation period). The importance of analyzing the high-inflation and the low-inflation subperiods separately is suggested, not only by the antagonistic signs on the correlations, but also by the different performances of the models, as measured with the log-likelihood and the AIC. As can be seen in the tables, the AICs for the low-inflation subperiod models are conspicuously lower than the AICs for the high-inflation subperiods for the whole sample. Moreover, the different log-likelihoods reported in Tables 4.1 and 4.2 also suggest the presence of structural breaks when estimating the models over the whole-sample period: Summing up the log-likelihood values for the different subperiod models, we obtain a sensibly higher value than that for the whole-sample models.⁷ Consequently, it is important to primarily consider the estimation results of the models in the subperiods to avoid biased estimates in the whole-sample models. This is in line with the findings of the MICEF in Section 3.2.3. Interestingly, note that we obtain different results for the two subperiods, even though none of the price indexes are used as explanatory variables in the model.

On the other hand, the estimates of ρ_{up} are neither significant nor robust to the inclusion (in Table 4.2) of the second estimated correlation ρ_{vu} , or when considering alternative inflation measures. This advocates strongly against the presence

⁶The second state-space model, presented in Section 4.3, corrects for these drawbacks by directly controlling for HF in the measurement equation and is, thus, closer to the MICEF in its formulation and interpretation.

⁷Adding up the log-likelihood values of the three subperiods from 1966Q1 to 2001Q1 in Table 4.1, we have $435.23 + 253.38 + 316.19 = 1004.80 > 969.35$ (IPD models) and $474.67 + 240.27 + 328.03 = 1042.97 > 1016.48$ (CPI models). Note that the log-likelihoods for the “middle” periods of 1981Q4 to 1990Q4 (IPD models) and 1982Q4 to 1990Q4 (CPI models) are not reported in the tables and that the log-likelihoods for the whole-sample models have been calculated for the truncated sample of 1966Q1 to 2001Q1 in order to fit the subperiods. Table 4.1 also suggests the presence of structural breaks, since the corresponding log-likelihood values are $435.48 + 253.62 + 316.21 = 1005.31 > 969.86$ (IPD models) and $474.29 + 240.61 + 327.27 = 1042.17 > 1016.51$ (CPI models).

Table 4.1: Inflation Unobserved: Estimations for a Single Correlation

$$\Delta c_t = \beta + \eta \Delta y_t^r + u_t; \quad \Delta y_t^n = \Delta y_t^r + \Delta p_t; \quad \Delta y_t^r = \mu + \phi_y \Delta y_{t-1}^r + v_t;$$

$$u_t = \phi_u u_{t-1} + \varepsilon_{ut}; \quad \Delta p_t = \phi_p \Delta p_{t-1} + \varepsilon_{pt}; \quad \varepsilon_{ut,pt}, v_t \sim iidN(0, \sigma_{u,p,v}^2)$$

	Whole Sample		High Inflation		Low Inflation	
	(i) ^{IPD}	(ii) ^{CPI}	(iii) ^{IPD}	(iv) ^{CPI}	(v) ^{IPD}	(vi) ^{CPI}
β	0.00 (0.00)	0.00 ^{***} (0.00)	-0.01 (0.01)	0.00 (0.00)	0.01 [*] (0.00)	0.00 (0.00)
η	0.26 (0.16)	0.06 [*] (0.04)	0.69 ^{**} (0.34)	0.12 (0.08)	0.15 (0.32)	0.43 ^{**} (0.22)
μ	0.00 (0.00)	0.01 ^{**} (0.01)	0.02 ^{***} (0.00)	0.02 (0.00)	0.01 (0.01)	0.01 ^{**} (0.00)
ρ_{up}	0.02 (0.07)	0.15 (0.17)	-1.00 (0.79)	-1.00 ^{***} (0.22)	0.05 (0.71)	0.33 (0.77)
AIC	-13.18	-14.33	-13.50	-13.86	-14.95	-15.57
log-L	969.35	1016.48	435.23	474.66	316.19	328.02

This table reports the Kalman filter estimation results of the state-space model described by equations (4.6) to (4.8) for quarterly U.S. data over the sample period of 1959Q1 to 2012Q1. The IPD and CPI indicate the price index used to deflate the consumption series. The “High-Inflation” subperiods are 1966Q1 to 1981Q3 and 1966Q1 to 1982Q3 for IPD and CPI, respectively, whereas the “Low-Inflation” subperiod is 1991Q1 to 2001Q1 for both inflation measures. The variance-covariance matrix Q for the shocks is restricted to be positive definite, and the initial values for the parameters are generated randomly. Parameter restrictions have been set for η to take values between 0 and 1 and for the estimated correlations to lie within the $(-1, 1)$ interval. The corresponding estimated standard errors (in parentheses) have been calculated with the delta method. $\{*, **, ***\}$ denote the statistical significance of the coefficient at the $\{10, 5, 1\}$ percent levels. AIC is the Akaike information criterion. $log-L$ reports the (maximum) estimated log-likelihoods for each model, except for the values given for the “Whole-Sample” models, which have been calculated over the sample period from 1966Q1 to 2001Q1.

Table 4.2: Inflation Unobserved: Estimations for Two Correlations

$$\Delta c_t = \beta + \eta \Delta y_t^r + u_t; \quad \Delta y_t^n = \Delta y_t^r + \Delta p_t; \quad \Delta y_t^r = \mu + \phi_y \Delta y_{t-1}^r + v_t;$$

$$u_t = \phi_u u_{t-1} + \varepsilon_{ut}; \quad \Delta p_t = \phi_p \Delta p_{t-1} + \varepsilon_{pt}; \quad \varepsilon_{ut,pt}, v_t \sim iidN(0, \sigma_{u,p,v}^2)$$

	Whole Sample		High Inflation		Low Inflation	
	(i*) ^{IPD}	(ii*) ^{CPI}	(iii*) ^{IPD}	(iv*) ^{CPI}	(v*) ^{IPD}	(vi*) ^{CPI}
β	0.00 (0.00)	0.00*** (0.00)	-0.02 (0.02)	-0.02*** (0.01)	0.01 (0.02)	0.00 (0.01)
η	0.42* (0.25)	0.03 (0.10)	1.00 (0.76)	1.00*** (0.23)	0.02 (1.58)	0.66 (1.12)
μ	0.01** (0.01)	0.01* (0.01)	0.02*** (0.01)	0.02*** (0.00)	0.01 (0.00)	0.01*** (0.00)
ρ_{up}	0.22 (0.21)	0.61*** (0.17)	-0.38 (0.84)	-0.81 (0.84)	0.06 (0.72)	0.72 (9.17)
ρ_{vu}	0.40** (0.18)	-0.07 (0.18)	0.50 (0.64)	0.29 (0.66)	-0.17 (2.08)	1.00 (10.50)
AIC	-13.18	-14.32	-13.48	-13.83	-14.89	-15.43
log-L	969.86	1016.51	435.48	474.29	316.21	327.27

All notes for Table 4.1 apply to this table.

of money illusion or any related inflation effect. In addition, one should be very careful when considering restricted coefficients (in our case the η s and the correlations), which lie very close to their imposed boundaries. This is the case, for instance, in equations (iii)^{IPD} and (iv)^{CPI}, in which the estimated correlations are equal to negative one. In fact, the coefficients are restricted using a nonlinear logit transformation that yields “reconstructed” values that are non-symmetrical, especially near the boundaries. Furthermore, the standard errors are calculated using the delta method, which yields poorer approximations for parameters with nonlinear distributions, making inferences difficult for restricted coefficients that are equal or close to the boundary values. Consequently, the correlations in (iii)^{IPD} and (iv)^{CPI} that are equal to -1.00 are more likely to reflect a statistical feature related to the restrictions imposed on the parameters than any actual consumption behavior. The same reasoning and circumspection apply to the estimated correlation ρ_{vu} in (iv)^{CPI} and to the estimated η s in equations (iii*)^{IPD} and (iv*)^{CPI}.

Concerning the relationship between income and consumption (captured by the parameters η and ρ_{vu}), no clear picture appears. Even though a few η s are significantly positive at the 5% or 10% significance level, these results are not robust when considering different inflation measures or when estimating an additional correlation. This inconsistency shows up even more clearly in the estimates for ρ_{vu} in Table 4.2, which have contradictory signs and values across the different equations. Consequently, there is no clear sign of the presence of excess sensitivity or excess smoothness of consumption to income in the Kalman filter estimations of the first state-space model. Rather, it indicates quite the reverse: that consumption growth is free of behaviors like HF or LC.

Even though some estimates of the constants β and μ are significant in both tables, they are always very close to zero and, hence, can be left out of the interpretation. In light of all of these results, the estimations for the state-space model described by equations (4.6) to (4.8) indicate that consumption growth is independent from inflation, HF and LC. In fact, Tables 4.1 and 4.2 rather advocate in favor of the REPIH that consumption growth is essentially unpredictable and, thus, follows an RW without drift (see Hall (1978) and the derivation of equation (2.13) in Chapter 2). At first glance, this result seems surprising, since it lies in stark contrast to the estimation results of the MICF in Chapter 3, which strongly reject the REPIH irrespective of the chosen period or inflation measure. On the other hand, recall that the consumer in this signal extraction problem suffers from total money illusion, is assumed to have no access whatsoever to information about inflation, and

relies entirely on a guess about its governing process.

The consequence of the imposed informational constraint is perfectly illustrated by Figure 4.1. It depicts, for the CPI models in the high- (upper graph) and low-inflation (lower graph) subperiods, the consumer's prediction of CPI inflation for the case in which one correlation ("predicted(1)" series) or two correlations ("predicted(2)" series) are estimated.⁸ To evaluate the predictions' accuracy, the predicted series are being compared to the "true" inflation series (solid line), which has been demeaned so that the graphs only show deviations from the within-period average inflation rate. Recall that, since inflation is assumed to follow an AR(1) process without drift, as defined in equation (4.5), the Kalman filter-estimated state series for inflation represent predictions of deviations around a zero mean. An alternative modeling procedure would be to add a constant to equation (4.5) and to leave the true inflation series unchanged. However, though this would have no impact on the prediction's accuracy, it would imply the unnecessary loss of degrees of freedom that would negatively impinge on the statistical inference. As suggested in Section 3.2.3, this problem can be particularly pronounced in the low-inflation subperiod, since its sample period is relatively short. Note that, in this context, the estimation results reported in Tables 4.1 and 4.2 do not change when the inflation series is demeaned.

The upper graph suggests that the consumer's best predictions of inflation are rather poor in the high-inflation subperiod. The model estimating only one correlations (i.e., the predicted(1) series) appears to overestimate true inflation over most of the subsample, since it almost never anticipates negative inflation shocks. Furthermore, this model is not capable of predicting most of the peaks and troughs, as reflected by the series' relative smoothness. The predicted(2) series better follows the actual inflation's deviations from the mean, but provides fairly inaccurate predictions by either overestimating or underestimating true inflation.

The lower graph, which depicts the measured inflation rates in the low-inflation period, shows a sensibly more favorable picture. On one hand, the predictions are more accurate than in the high-inflation subperiod. Not only is the average prediction error smaller in this period (which is attributable to the lower average inflation rates), but the peaks and troughs in the true CPI inflation evolution also

⁸On account of the apparent inappropriateness of the whole-sample models, and since the CPI models perform better than the IPD models in terms of their AIC values, I show only the graphs for the CPI models in the subperiods. Note, however, that the IPD models yield a similar picture as Figure 4.1, except for the fact that the predicted(1) and predicted(2) series are sensibly closer from each other. In particular, the predicted(1) series for the high-inflation subperiod (upper graph) is not as smooth as in Figure 4.1 and follows a very similar path as the more volatile predicted(2) series.



Figure 4.1: Predicted and Actual CPI Inflation (demeaned)

The graphs compare the evolution of the filtered state estimates of the unobserved CPI inflation, Δp_t , with the “true” demeaned CPI inflation rates for the high-inflation subperiod (upper graph) and the low-inflation subperiod (lower graph). The measured series “predicted(1)” and “predicted(2)” correspond, respectively, to the models in which one or two correlations are estimated.

seem to be better anticipated. This provides a further explanation for the lower AIC values of the low-inflation models in comparison to the high-inflation models.

On the other hand, even though the predicted(1) and predicted(2) series broadly follow the evolution of the actual inflation, correctly anticipating the positive and negative directions of the inflation shocks most of the time, the predictions always over-shoot their real magnitude over this subperiod. This higher volatility in the predicted values certainly pertains to the fact that the true inflation rates are very stable and low (i.e., the deviations from the mean are very low, on average). The resulting relative inaccuracy of the inflation predictions implies that the consumer cannot objectively rely on her predictions about inflation for her consumption decisions in either the high- or in the low-inflation subperiod.

Overall, the poor predictions of actual inflation suggest that the assumption of full money illusion (i.e., that the consumer does not possess any information about inflation) is too restrictive and should probably be relaxed. The fact that the completely money-illusioned consumer ends up adopting the same optimal behavior as the fully rational consumer (i.e., her future consumption is determined only by current consumption and unexpected shocks) is, nonetheless, not surprising. Of course, the illusioned consumer knows from Figure 4.1 that her best predictions of the true inflation are rather unsatisfactory, regardless of the prevailing inflationary environment. Consequently, she is, on the average, better off not relying on these predictions if she does not want to wrongly diverge too much from her main objective, which is optimal intertemporal consumption smoothing. Recall, however, that even though this rule of thumb makes the completely money-illusioned consumer behave like the fully rational one, both consumers still base their consumption decisions on different information sets, with the former one relying on her nominal income and the latter one relying on her real income.

The heavy reliance of the model on the very restrictive and rather implausible assumption of full money illusion, as well as the fact that the model is incapable of predicting consumption growth, regardless of the considered sample periods or inflation measures, calls for an alternative state-space model capable of yielding more stable estimates. Furthermore, this first state-space model has the disadvantage of drawing conclusions based, to a large extent, on correlations. Yet, a nonzero correlation between two variables, even if it is statistically significant, does not necessarily imply a causal relationship. In any case, since the correlations in Tables 4.1 and 4.2 are not significant for the two subperiods, it seems judicious to search for an alternative specification, rather than deepening the analysis of this specific

state-space model. The second state-space model, which is presented and estimated in the following section, allows for different consumption behaviors to have simultaneous impacts on consumption growth. Moreover, its greater precision renders it theoretically and empirically more appealing.

4.3 Modeling Inflation as an Observed Variable

This section presents an alternative and more direct way to model money illusion as a signal extraction problem. In contrast to the above-derived model, the money-illusioned consumer now observes past and current inflation. She no longer necessarily suffers from complete money illusion and she can decide to what extent she wants to make use of information about (past and current) inflation in her consumption decisions. The most straightforward way to account for this new possibility is to include contemporaneous and lagged inflation directly within the signal equation of the state-space model. This less constrained informational setup seems more realistic than the one presented in Section 4.2 and is, at the same time, closer to the approach adopted in Chapter 3, allowing different inflation-to-consumption channels to simultaneously impact on contemporaneous consumption growth. Pursuing the same line of thought, we can also include lagged consumption growth as an observed variable in order to better control for HF and LC behaviors and to permit them to coexist.

Taking these theoretical modifications into account, the measurement equations (4.1) and (4.2) can be rewritten as:

$$\Delta c_t = \beta + \delta \Delta c_{t-1} + \lambda_0 \Delta p_t + \lambda_{-1} \Delta p_{t-1} + \eta \Delta y_t^r + u_t \quad (4.9)$$

$$\Delta y_t^n = \Delta y_t^r + \Delta p_t. \quad (4.10)$$

Since inflation is now observed, Δp_t is no longer an unknown inflation state; rather, it denotes inflation as an exogenous and deterministic variable. As a result, the only unknown (state) variable remaining in this model is that of real income growth, Δy_t^r . Apart from the central distinction between nominal and real income and the inclusion in the model of the latter as an explanatory variable, equation (4.9) appears to be similar to the ADL model of Section 3.4.2, which emerged as the most appropriate model to describe real consumption growth. For interpretation and computational reasons, lagged inflation and lagged consumption are restricted

to only one lag.⁹ With these new assumptions, the state-space model simplifies to two signal equations and two state equations (instead of two signal and three state equations in the previous setup). Moreover, only one correlation (ρ_{vu}) is now estimated. The other correlations are being set to zero: the first one, ρ_{vp} , for the same reason as before (by assumption), and the second one, ρ_{vp} , because the inflation effect is being already controlled for in the measurement equation.

Assuming real income growth (Δy_t^r) and the measurement error of real consumption growth (u_t) to follow the same AR(1) processes as in equations (4.3) and (4.4), the measurement equation $\tilde{y}_t = Ax_t + H'\xi_t$ and state equation $\xi_t = \tilde{\mu} + F\xi_{t-1} + \tilde{v}_t$ can be written, respectively, in the following state-space form:

$$\begin{bmatrix} \Delta c_t \\ \Delta y_t^n \end{bmatrix} = \begin{bmatrix} \beta \\ 0 \end{bmatrix} + \begin{bmatrix} \delta & \lambda_0 & \lambda_{-1} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta c_{t-1} \\ \Delta p_t \\ \Delta p_{t-1} \end{bmatrix} + \begin{bmatrix} \eta & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta y_t^r \\ u_t \end{bmatrix} \quad (4.11)$$

$$\begin{bmatrix} \Delta y_t^r \\ u_t \end{bmatrix} = \begin{bmatrix} \mu \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_y & 0 \\ 0 & \phi_u \end{bmatrix} \begin{bmatrix} \Delta y_{t-1}^r \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} v_t \\ \varepsilon_{ut} \end{bmatrix}. \quad (4.12)$$

The variance-covariance matrix melts down to the following (2×2)-matrix:

$$Q = \begin{bmatrix} \sigma_v^2 & \rho_{vu}\sigma_v\sigma_u \\ \rho_{vu}\sigma_v\sigma_u & \sigma_u^2 \end{bmatrix}, \quad (4.13)$$

where the only correlation left, $\rho_{vu} = \text{corr}(v_t, \varepsilon_{ut})$, captures the residual excessive sensitivity ($\rho_{vu} > 0$) or excessive smoothness ($\rho_{vu} < 0$) of consumption growth with respect to real income growth, after a one-period HF behavior has been accounted for. The benchmark REPIH model predicts that $\delta = \lambda_0 = \lambda_{-1} = \rho_{vu} = 0$, such that contemporaneous real consumption growth remains unpredictable.

4.3.1 Kalman Filter Estimation Results

The Kalman filter estimations of the state-space model treating inflation as an observed variable, characterized by equations (4.11) to (4.13), are summarized in Table 4.3. Contrary to the state-space model estimated in Section 4.2, which impugns

⁹Note, further, that in state-space models, lags of observed variables are typically treated as predetermined rather than as exogenous (as, for instance, in Chapter 3). In such an approach, the dependent variable and its lagged values are included in the state vector, with the advantage that the predicted values directly contribute to the prediction MSE, thus improving multi-step ahead forecasting. See Harvey (1990, pp. 367-373) for more details and further examples.

the importance of money illusion and other consumption smoothers (HF) or stimulators (LC) for consumption growth, the estimation results of the present model imply a rejection of the RW hypothesis in every period, for both the IPD- and CPI-based inflation models.

With regard to the importance of inflation for consumption growth, the parameters of interest are λ_0 and λ_{-1} . All of the estimates for λ_0 have a largely negative effect. In light of the results from Chapter 3 and the economic intuition behind the unexpected inflation effect, the negative sign is expected. Note, further, that the estimates are all highly significant (at the 1% level), except for those of the CPI model in the low-inflation period, (xii)^{CPI}. For the high-inflation period, the estimates of λ_0 are close to those of Table 3.4, which are obtained from the estimation of the MICF. However, the same observation does not hold for both low-inflation models. On one hand, the contrasted results for the second subperiod highlight, again, the importance of distinguishing between different inflationary environments when scrutinizing aggregate consumption. As was the case in the previous state-space model, the difference between the subperiods is also confirmed by the log-likelihood values reported in Table 4.3, which suggest the presence of a structural break.¹⁰

On the other hand, the contrasted results cast some doubt on the importance and validity of the unexpected inflation effect for the low-inflation period. This doubt is corroborated by the fact that the estimated models for the low-inflation subperiod ((xi)^{IPD} and (xii)^{CPI}) perform equally well in terms of their AIC values (-15.63 and -15.64, respectively), even though they find diametrically opposite predictors of consumption growth: In (xi)^{IPD}, the only significant effect is the negative inflation effect, whereas (xii)^{CPI} attributes a significant role solely to HF. While this confirms that both money illusion and HF play potentially important roles in explaining the evolution of real aggregate consumption, the present state-space model is not able to unambiguously conclude in favor of one effect or the other within the selected low-inflation subperiod.

The estimates for λ_{-1} call for even more wariness, this time with regard to the presence of a positive impact of (lagged) inflation on aggregate consumption growth. Even though the coefficients are largely positive and highly significant over the whole sample, in (vii)^{IPD} and (viii)^{CPI}, this effect, while remaining positive as expected, almost entirely loses its significance when different inflation levels are accounted

¹⁰Again, the sum of the log-likelihoods for the three subperiods is higher than the log-likelihood of the whole-sample models: We have $445.68 + 265.08 + 331.52 = 1042.28 > 1019.21$ for the IPD models and $482.51 + 244.85 + 331.53 = 1058.89 > 1040.98$ for the CPI models.

Table 4.3: Estimation Results When Inflation Is Observed

$$\Delta c_t = \beta + \delta \Delta c_{t-1} + \lambda_0 \Delta p_t + \lambda_{-1} \Delta p_{t-1} + \eta \Delta y_t^r + u_t; \quad \Delta y_t^n = \Delta y_t^r + \Delta p_t;$$

$$\Delta y_t^r = \mu + \phi_y \Delta y_{t-1}^r + v_t; \quad u_t = \phi_u u_{t-1} + \varepsilon_{ut}; \quad \varepsilon_{ut}, v_t \sim iidN(0, \sigma_{u,v}^2)$$

	Whole Sample		High Inflation		Low Inflation	
	(vii) ^{IPD}	(viii) ^{CPI}	(ix) ^{IPD}	(x) ^{CPI}	(xi) ^{IPD}	(xii) ^{CPI}
β	0.00** (0.00)	0.00 (0.00)	0.00 (0.01)	0.00 (0.00)	0.00** (0.00)	0.00 (0.00)
δ	0.59*** (0.06)	0.50*** (0.09)	0.17 (0.15)	0.26* (0.16)	0.24 (0.25)	0.39*** (0.15)
λ_0	-0.55*** (0.04)	-0.35*** (0.05)	-0.59*** (0.09)	-0.56*** (0.12)	-0.45*** (0.11)	-0.29 (0.30)
λ_{-1}	0.48*** (0.04)	0.27*** (0.06)	0.28* (0.16)	0.27 (0.20)	0.18 (0.18)	0.01 (0.29)
η	0.27*** (0.09)	0.43** (0.20)	1.00 (0.73)	1.00*** (0.03)	0.33 (0.28)	0.33 (0.33)
μ	0.01*** (0.00)	0.00 (0.00)	0.00** (0.00)	0.00** (0.00)	0.01*** (0.00)	0.00** (0.00)
ρ_{vu}	-0.15 (0.29)	-0.41 (0.37)	-0.94*** (0.13)	-0.91*** (0.03)	-0.37 (0.72)	-0.28 (0.71)
AIC	-14.14	-14.59	-13.80	-14.08	-15.63	-15.64
log-L	1019.21	1040.99	445.68	482.51	331.52	331.53

This table reports the Kalman filter estimation results of the state-space model described by equations (4.11) to (4.12). For more details, see the notes for Table 4.1.

for. The effect remains weakly significant at the 10% level only in the high-inflation subperiod and for the IPD inflation model. Interestingly, in this case, $\lambda_{-1} = 0.28$ is very close to the estimate for the preferred linear ADL model in the previous chapter, which yielded an estimate of $\lambda_{-1} = 0.27$ and was significant at the same confidence level (see Table 3.4). However, the positive effect of inflation lagged one period is no longer significant in the three remaining models for the two subperiods: namely, (x)^{CPI}, (xi)^{IPD} and (xii)^{CPI}. Even though the present state-space models are not necessarily directly comparable to the MICF estimated in Section 3.4, the estimates of λ_{-1} in Table 4.3 clearly call for more caution with regard to the presence of the nominal income effect of inflation on real consumption growth.

The estimates of the parameters characterizing the inflation-to-consumption relationship (namely δ , η and ρ_{vu}) also yield contrasting results. The HF parameter δ always has the expected positive sign, but differs in its significance and magnitude when different periods or inflation measures are considered. While it is very high (over 0.50) and significant at the 1% level over the whole-sample period, its magnitude and significance drop sharply across the two subperiods. In both the high- and low-inflation subperiods, δ is insignificant in the IPD model but remains significant in the CPI model at the 10% and 1% levels for the high and low subperiod, respectively. The fact that the highly significant HF effect in equations (vii)^{IPD} and (viii)^{CPI} is not robust to estimations over subperiods and alternative inflation measures highlights, again, not only that it is crucial to distinguish between different inflationary environments when analyzing the relationship between consumption growth and its potential determinants, but also that the choice of the inflation measure can engender antagonistic conclusions about consumption behaviors, even if the model does not directly address money illusion or other inflation-related phenomena.

The correlations between the innovations to consumption growth and real income growth (ρ_{vu}) also indicate a smooth adjustment of consumption to income shocks. Contrary to the previous state-space model, ρ_{vu} is now negative in all models and presents similar values within each period, independent of the inflation measure. The estimates for the whole sample and for the low-inflation subperiod are negative and moderate within the -0.15 to -0.41 range, but never significant. On the other hand, ρ_{vu} is highly significant for both high-inflation models and is very close to negative one (its lower boundary). This could indicate the presence of additional excessive smoothness in consumption growth, which could reflect either that consumption habits are even more persistent (i.e., present in further lags of consumption growth) or the existence of other mechanisms not accounted for in the model, like

precautionary savings or imperfect insurance on the credit markets.

The fact that the estimations of the MICF in Chapter 3 do not find consumption growth lagged more than one period to be significant in the high-inflation subperiod (see Table 3.4), speaks more in favor of alternative explanations than of additional HF. Furthermore, since ρ_{vu} is highly significant only in the subperiod experiencing particularly high and volatile average inflation, it could well reflect the higher uncertainty that the risk-averse consumer faces in this period regarding the future evolution of her real income path.¹¹ The higher uncertainty clearly supports the argument for additional precautionary savings. This observation represents an interesting complement to the previous estimations of the MICF, in which the HF coefficient is found to be similar in both subperiods, but where no other sources of consumption smoothness are controlled for. Moreover, recall that, in the MICF, the significant HF coefficient does not necessarily only reflect consumption habits, but also captures further sources of excessive consumption smoothing as uncertainty or sticky information (see Section 3.3.1). Finally, the assumption that HF and precautionary savings might prevail simultaneously in uncertain environments potentially extends the existing inflation-ignoring literature, which either omits the possibility of additional consumption “smoothers” (Carroll et al., 2011) or is unable to disentangle the former behavior from the latter (Morley, 2007).

Note, however, that since the estimates for ρ_{vu} in the high-inflation subperiod are close (but not equal) to their imposed lower boundary (-1), they could also reflect a statistical bias and should, therefore, be interpreted with prudence (see the explanation in Section 4.2.1 above). While whether ρ_{vu} is really affected by this statistical bias is questionable, it undoubtedly applies to the estimates for η in the high-inflation subperiod. Consequently, the second state-space model does not confirm the presence of LC in aggregate consumption growth, even though this effect is highly significant over the whole sample (as can be seen in the first two equations of Table 4.3). This rejection of LC behavior is in line with the results obtained from the previous state-space model and the MICF in Section 3.4.2.

Comparing the estimation results of the second state-space model in Table 4.3 with the results of the previous state-space model and treating inflation as an unobserved variable (see Tables 4.1 and 4.2), several further features are noteworthy. First, the constants, or drift parameters, β and μ , are, again, very close to 0 and do not play any particular role in the model, aside from their statistical significance.

¹¹This negative impact of uncertainty on consumption is closely related to the concept of ambiguity aversion, which is analyzed in detail in Chapter 5.

Second, the present state-space model performs better, in terms of the estimated AIC (and log-likelihood) values, than the first state-space model, irrespective of the chosen period, the inflation measure, or the number of estimated correlations. This is no surprise, since, in the second model, the consumer observes inflation and takes it as given, whereas, in the first model, she needs to make an assumption about its evolution. In consequence, the consumer in the second model has more information at her disposal to calculate her optimal behavior, which is reflected by the lower AIC values.

Third, the CPI models still outperform the IPD models in Table 4.3, but the difference is much less pronounced than in the previous state-space model and is even negligible for the low-inflation subperiod. Overall, the most accurate models are those for the low-inflation subperiods. This does not only reflect the fact that the considered time series are subject to fewer shocks and lower volatility during this period. In the second state-space model, it also pertains to the fact that the discrepancy between the evolution of the observed nominal income and the evolution of the unobserved, but relevant real income is much lower and precisely determined, since it exactly corresponds to inflation (which is also observed).

Figure 4.2 depicts the evolution of the predicted shocks to consumption growth, which is defined as the unobserved state variable u_t in equation (4.1), in comparison to the evolution of real income growth. Just as for the above-presented Figure 4.1, I deliberately show in Figure 4.2 the graphs for the CPI models in the subperiods only, since models (x)^{CPI} and (xii)^{CPI} outperform their IPD inflation-based counterparts and since the models over the whole sample lack robustness. Note, further, that the IPD models report very similar evolutions of the estimated state variables that are very similar to those of the CPI models, such that most conclusions drawn from Figure 4.2 are also valid for the IPD models.

The two graphs of Figure 4.2 compare the estimated state values of the two state variables in the present state-space model (i.e., the two elements of $\tilde{\xi}_t$ in equation (4.12)). Note that, since inflation is observed in this state-space model, the consumer will make no mistake in estimating the value of her real income growth (even though she only observes her nominal income growth), but still needs to make an assumption about its evolution. Consequently, the estimated $\Delta\hat{y}_t^r$ is equal to the true Δy_t^r in this model.

In the upper graph, which graphically represents the innovations to consumption growth and the predicted real income growth in the high-inflation subperiod, the two series seem to be highly negatively correlated. As we can see, both series experience

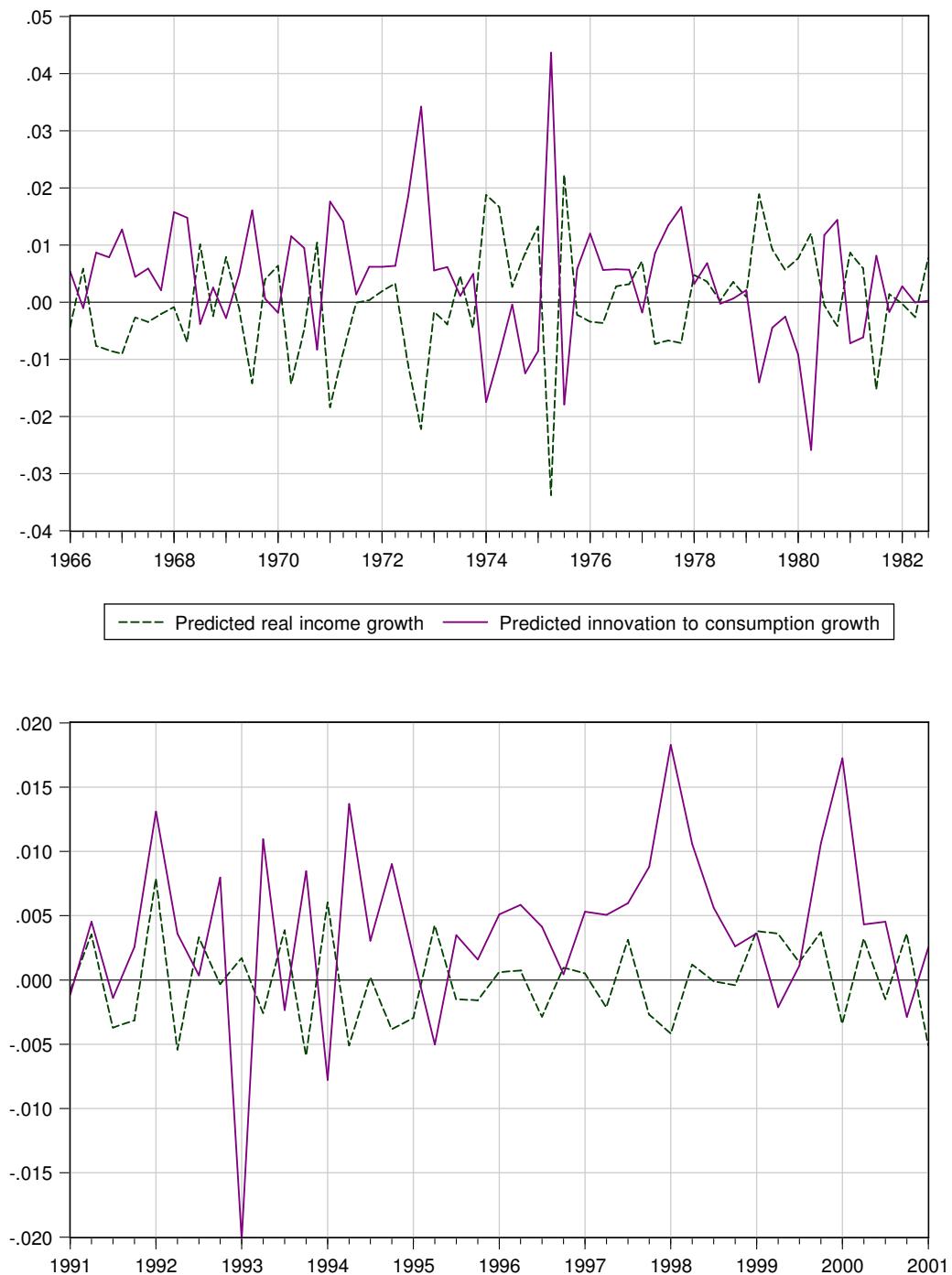


Figure 4.2: Innovations to Consumption Growth and Real Income Growth

The graphs compare the evolution of the filtered state estimates of the innovations to consumption growth, u_t , with the filtered state estimates for real income growth, Δy_t^r . The upper and lower graphs illustrate this relationship for the CPI model in the high- and low-inflation subperiods, respectively, corresponding to models (x)^{CPI} and (xii)^{CPI} in Table 4.3.

shocks of the same magnitude in the same quarters; the shocks are simply in opposite directions. This relationship is captured by the highly significant and negative $\rho_{vu} = -0.91$ of equation (x)^{CPI} in Table 4.3. On one hand, this clearly advocates against the statistical bias mentioned above and confirms that other consumption smoothers, such as precautionary savings motives, are at work in a highly uncertain environment. On the other hand, one should keep in mind the general rule that a statistically significant correlation does not necessarily imply a causal relationship between two variables. For this reason, the precautionary savings assumption needs to be analyzed in more detail in order to be able to exclude the possibility of any flawed inferences.

The lower graph also suggests a negative correlation between \hat{u}_t and $\Delta\hat{y}_t^r$ for the low-inflation CPI models, though this effect is less pronounced than in the high-inflation subperiod. In fact, the correlation seems to be first positive at the very beginning of the sample, as reflected by the co-movements in the estimated state series until the second quarter of 1992. After this point, the two series drift apart to present mostly opposite movements, even though the \hat{u}_t series is slightly smoother than the $\Delta\hat{y}_t^r$ series. Over the entire low-inflation subsample, the negative correlation of $\rho_{vu} = -0.28$ in (xii)^{CPI} is, however, not significant, such that precautionary savings do not seem to play an important role in periods experiencing low average inflation rates. This assumption, that precautionary savings are only present (or are more pronounced) in a particularly high inflationary environment, is thoroughly analyzed in Chapter 5.

4.4 Conclusion

In this chapter, I have developed and estimated two alternative ways of modeling money illusion as a signal extraction problem. In the first state-space model, which assumes the consumer to be completely money-illusioned, consumption growth is shown to be essentially unpredictable, following the implication of the standard REPIH model. This result pertains to the fact that the Kalman filter estimations rely almost entirely on the arbitrary assumptions the consumer has to make about the processes governing inflation and real income growth. The consequence of this maximal informational constraint is that the consumer's predictions of the "true" inflation are rather inaccurate.

At first glance, it seems surprising that a fully money-illusioned consumer ends up behaving the same way as a fully rational consumer facing no such informational

constraints. On the other hand, the money-illusioned consumer knows that even her best predictions of inflation are, at best, a rough approximation of the inflation's real evolution and that she is, thus, better off ignoring this variable in her consumption decisions rather than taking the risk of wrongly diverging from her optimal consumption path.

Once the consumer is given access to information about the evolution of the general price level, the picture changes dramatically. While, in this second setup, real income growth remains unobserved, the consumer might still suffer from some degree of money illusion due to deciding to not fully account for inflation in her consumption decisions. As expected, the estimations of the second state-space model clearly outperform those of the first one and also produce some interesting results. First, distinguishing periods with high inflation from periods with low average inflation not only shows that estimations over the whole sample potentially lead to biased conclusions, but also confirms that consumers behave differently in different inflationary environments.

Second, the estimation results are very different from the results obtained in Chapter 3, in particular with respect to the impact of inflation on real consumption growth. While the negative effect of current inflation on contemporaneous consumption growth is present in the high-inflation subperiod, it is not robust to different inflation measures in the low-inflation subperiod. Moreover, the positive nominal income effect of inflation on consumption growth attributed to [Branson and Klevorick \(1969\)](#) almost completely disappears when controlling for different inflation measures. This result lies in stark contrast to the results obtained in Chapter 3, indicating that the statistically significant positive effect of lagged inflation captured by the MICF could be due to other factors than money illusion as a rule of thumb.

Finally, the estimations of the second state-space model allow us to draw some interesting conclusions with regard to the relationship between real consumption growth and real income growth. The LC are found to have no significant impact on consumption growth, which excludes any excess sensitivity of consumption growth to income growth. On the contrary, the estimations suggest that consumption growth is rather smooth, particularly in the high-inflation period.

In this subperiod, the inertia in aggregate consumption growth is highly significant, persistent and attributable to factors other than HF. In particular, it is assumed that consumers increase their precautionary savings in times of greater uncertainty about the future evolution of inflation, which should be the case in the high-inflation period. This prudent behavior is typically induced by a consumer's

ambiguity aversion and can be modeled as a robust control problem. The following chapter explores this idea and shows that this aversion against high and volatile inflation can be modeled as an extension of the standard REPIH model.

Chapter 5

Seeking Robustness Against Money Illusion

This chapter considers a money-illusioned consumer who is aware of her confusion between nominal and real values and decides to account for it in her consumption decisions. The adopted modeling approach is to impose money illusion as a distortion to the benchmark REPIH model and to let the *robust* consumer choose the distorted model that yields the highest utility. Two robust consumption models are used to analyze the impact of money illusion on consumption: the closed-form robust control model, suggested by Luo (2008), and the model by Hansen et al. (1999), which also controls for habit formation. These models predict that robustness induces precautionary savings on the part of the consumer in order to hedge against the model misspecifications. Using quarterly U.S. data, the estimations show empirically that robustness is higher in the subperiod with high inflation rates than in the subperiod with low inflation rates. This result suggests that money illusion, measured via the degree of robustness, is more pronounced when inflation is high and, thus, has a negative impact on consumption growth. In consequence, the degree of robustness is not time-invariant, but instead varies in the degree of inflation uncertainty. Finally, assuming that robustness largely reflects the degree of uncertainty about inflation, money illusion can be interpreted as providing an economical intuition for the desire for robustness in the two selected robust consumption models.

Keywords: Robust control, money illusion, precautionary savings, robust permanent income hypothesis

JEL classification: C61, D11, D81, D91, E21.

5.1 Introduction

In Chapters 3 and 4, I have presented two possible ways to apprehend and model money illusion, defined as the consumers' confusion between nominal and real values. In particular, it was suggested that money illusion can be understood both as a time-saving rule-of-thumb behavior and as a signal extraction problem on the part of the consumer. Both concepts theoretically imply a non-neutrality of real consumption with respect to inflation. The empirical analyses performed in these chapters confirm that inflation is an essential determinant of consumption growth and suggest that consumers behave differently in periods with high and volatile average inflation than in periods with low and stable average inflation.

As a matter of completeness, the present chapter introduces a third and last way to model money illusion: namely, as a misspecification – or distortion – imposed on the benchmark consumption model. The benchmark model chosen in this chapter is the rational expectations–permanent income hypothesis (REPIH) model, which predicts that consumption growth follows a random walk (see Chapter 2). The intuition behind this alternative concept is that the representative consumer knows that she might, on occasion, misinterpret nominal values for real values, such that her illusion-free benchmark model becomes situationally inaccurate. To account for this possible imprecision, the consumer becomes more prudent with respect to the predictions of her reference model and decides to allow for some specification error in the purely rational model. This type of consumer, who is willing to account for misspecifications in the underlying consumption model, is said to be *robust* – or, in our case, robust to money illusion.

In addition to providing an additional method to model money illusion, the present chapter ties in perfectly with the previous chapters of this thesis and can be viewed as a synthesizing chapter. First, the two robust consumption models presented further below are an extension, or generalization, of the standard REPIH model presented in Chapter 2 and tested in Chapter 3. As is shown in Section 5.3, the REPIH represents, in fact, a particular case of the robust control permanent income hypothesis (henceforth, the RCPIH) model, in which the robustness parameter is set to equal zero.

Second, recall that the estimations in Chapter 4 suggest the presence of precautionary savings in periods characterized by high inflation rates or, more generally, high model uncertainty. In this context, the robust consumption models analyzed in this chapter not only provide a theoretical justification for precautionary savings

that can prevail even given the quadratic preferences assumed so far. Furthermore, robust control theory can be understood as a more general decision rule than the Kalman filter (used in Chapter 4), which allows us to better and more fully account for model uncertainty.¹

Robust control found its way into economics through the early works of [Dow and Werlang \(1992\)](#) and [Epstein and Wang \(1994\)](#) in finance. The real breakthrough of robust control methods in economics, however, was generated by a series of papers by Lars P. Hansen and Thomas J. Sargent.² They compiled their findings in a reference book ([Hansen and Sargent, 2008](#)), which is used as a guideline for this chapter and, in particular, for the [Hansen et al. \(1999\)](#) model estimated in Section 5.4.

The rest of this chapter is structured as follows: Section 5.2 shows, in a first step, that robustness, which can be understood as a particular form of ambiguity aversion, provides an ideal framework through which to analyze money illusion as a model misspecification. In a second step, it illustrates how robustness can be easily modeled as a simple static robust optimal control problem. The two sections that follow present two different extensions of the REPIH model that explicitly control for money illusion via some robustness parameter: Section 5.3 not only derives an alternative version of the closed-form RCPIH model suggested by [Luo \(2008\)](#), which establishes a direct link between robustness and consumption growth, but also emphasizes its limited appropriateness for empirical analysis. The last analysis, in Section 5.4, uses the robust version of the REPIH model developed by [Hansen et al. \(1999\)](#) to show that robustness is higher in periods experiencing high average inflation rates than in periods with low inflation, thus providing evidence in favor of money illusion in the high-inflation period. Section 5.5 concludes.

¹The Kalman filter is a typical example of the linear quadratic optimal control method that emerged in the late 1960s. Even though it is used extensively in modern economic studies and works well under full observation, it presents the particular drawback of being not fully robust to certain types of model perturbations, especially in the case of partial observation ([Williams, 2008](#), p. 4). Robust control theory, which originated in the 1980s engineering literature, directly addresses this drawback.

²Apart from [Hansen et al. \(1999\)](#), who introduced robustness into the PIH model, others have applied similar methods to inflation models ([Cogley and Sargent, 2005](#)), monetary policy models ([Cogley et al., 2008](#)) and asset prices ([Alonso and Prado, 2007](#)). An exhaustive literature review of the implications of ambiguity aversion for asset pricing and portfolio choices can be found in the work of [Guidolin and Rinaldi \(2013\)](#). Note, further, that [Giordani and Soderlind \(2004\)](#) provided a guideline for solving macroeconomic models with robust control methods, including various software components.

5.2 Modeling Robust Control

In a first step, this section provides a general intuition for robust control and further specifies how money illusion can be understood as a particular type of misspecification. In a second step, it shows how robustness can be modeled as a simple static min-max problem and solved by means of usual optimization techniques. This intuitive, two-step, sequential procedure is later extended to the dynamic setting of the RCPIH in Section 5.3.

5.2.1 Robustness as Aversion Against Money Illusion

In economics, robust control (or robustness) is closely related to the concept of ambiguity aversion and can, hence, be seen as a way of modeling model uncertainty. Ambiguity aversion, or Knightian uncertainty, pertains to the well-known [Ellsberg \(1961\)](#) paradox: If a decision maker (DM) faces two containers A and B, where A contains an equal number of white and black balls, but where the exact composition of B is unknown, the DM will opt for lottery A. This is a paradox because it violates the subjective expected utility theory, according to which the DM should be indifferent between the two lotteries ([Savage, 1954](#)). This example shows that people typically dislike uncertain environments and, thus, display some degree of ambiguity aversion. In this context, robust control can be interpreted as providing a solution to the Ellsberg paradox.

The idea of robustness is to find a decision rule for an ambiguity-averse agent who does not fully trust her reference model, which is typically assumed to have generated the data. In short, the “robust” DM is seeking for an alternative model that takes into account small specification errors. To address this concern for misspecification, the suggested decision rule is for the DM to consider a range of alternative but similar models and to opt for the model that yields the highest utility under the worst possible outcome. [Hansen and Sargent \(2008, p. 27\)](#) describe this particular problem as a two-player game between the DM, who wants to maximize her utility, and an evil player or “malevolent nature”, whose objective is to choose the worst possible model with the greatest distortion. In this setup, the DM decides over the control, while the malevolent nature has the distortion at its disposal.

Formally, the result of this ambiguity-robust behavior is a min-max decision rule, which is presented in Section 5.2.2. Note that this procedure – of optimizing a model while simultaneously hedging against the worst possible outcome – is a related but different way of thinking about model uncertainty in comparison to the Bayesian

approach, in which the DM must form a prior regarding the form of the model specification and considers the weighted average of all possible models.

The uncertain world faced by the robust DM is best illustrated by Figure 5.1, which is a simplified version of the “ambiguity circle” portrayed by Hansen and Sargent (2008, p. 12). Their interpretation of the ambiguity circle is that the robust DM, fearing misspecification, does not know the “true” data-generating model (f) and tries to find an approximating model (\tilde{f}) that is statistically close to f . She chooses her model from a range of statistically close alternative models (i.e., the distorted models), which all lie within the ambiguity circle. The size of the ambiguity circle increases with the degree of ambiguity aversion and is limited by some maximum misspecification η allowed by the DM. Similarly, η can also be interpreted as the maximum distortion that the malevolent nature can impose on the DM’s choice.

In the general procedure, the DM uses (relative) entropy as the distant measure between different models and to impose the restriction η on the size of the ambiguity circle.³ Note, however, that we will not be directly needing entropy in the two robust consumption models presented in Sections 5.3 and 5.4 in this chapter. While the conclusions of the former are valid for any degree of robustness, the latter is able to measure robustness indirectly via the subjective discount factor.

In this chapter, I suggest a related but more specific interpretation of Figure 5.1, specifically in terms of money illusion. For the rational consumer analyzed in Chapters 2 and 3, the true model f corresponds to the standard REPIH. The money-illusioned consumer, however, fears that f is an overly optimistic model in the sense that it does not allow for nominal values to have an impact on real consumption. To control for this error and adopt a more prudent behavior, the robust consumer imposes a distortion on the REPIH that captures her own money illusion. The consumer’s objective is to choose the distorted consumption model \tilde{f} that is equal to the standard REPIH with some money-illusion-induced specification errors. The RCPIH and the Hansen et al. (1999) models (presented in Sections 5.3 and 5.4) both potentially capture money illusion by allowing for such distortions in the benchmark REPIH models. In this interpretation, the maximum distortion η is proportional to the consumer’s fear of suffering from money illusion. If, on the contrary, the consumer does not suffer from money illusion or does not face any uncertainty about

³As Hansen and Sargent (2008, pp. 30-31) showed, relative entropy can be statistically measured with the expected value of the log-likelihood ratio. See also Olalla and Gómez (2011) for a more intuitively appealing definition of entropy.

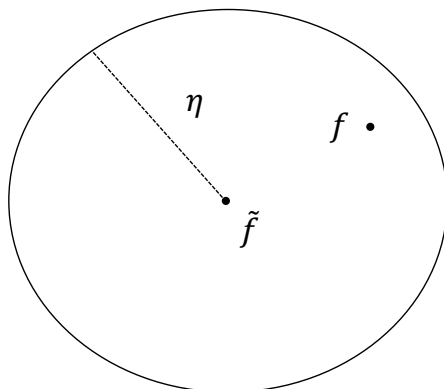


Figure 5.1: Ambiguity Circle

The ambiguity-averse DM concerned about misspecification does not know the true data-generating model f and chooses an approximating model \tilde{f} from a set of alternative models, limited by a maximum distortion of η . Source: Hansen and Sargent (2008, p. 12).

inflation, the ambiguity circle disappears, such that f and \tilde{f} coincide.

In light of the intuition behind money illusion (see Section 3.1.2) and the insights gained from the different empirical analyses (see Sections 3.4 and 4.3.1), it is reasonable to assume that the money-illusioned consumer opts for a higher degree of robustness (i.e., a higher η) in periods with higher and more volatile inflation rates than in periods experiencing low average inflation rates. This conjecture is, indeed, confirmed in Section 5.4, in which the robust PIH model developed by Hansen et al. (1999) is estimated over different periods.

The major drawback of modeling money illusion as a misspecification is that we cannot exclude the possibility that there are other misspecifications at work. In fact, it seems more plausible that the robust consumer seeks robustness against all possible kinds of misspecifications, such that the distortion η is not solely attributable to money illusion. Nonetheless, a few of the insights gained through the present dissertation allow us to assume that a non-negligible part of the specification error is due to money illusion. First, since the representative consumer is rational, deviations from the benchmark REPIH model should be only temporary. This excludes the presence of systematic or long-term biases due to misspecifications and suggests that, if robustness persists over a longer period, then it is induced by economic rather than statistical motives.

Second, recall that a positive impact of lagged consumption growth on current consumption growth does not necessarily only capture habit formation (HF), but also

indicates a particularly prudent behavior on the part of consumer (see Section 3.3.1). In consequence, whenever HF is controlled for in a model, then the corresponding coefficient also captures any effect stimulating precautionary savings (such as, for example, a greater uncertainty about the future evolution of income). Since the OLS estimations of the MICF in Chapter 3 and the Kalman filter estimations of the signal extraction problem in Chapter 4 both reveal an important negative inflation effect on consumption even when HF is controlled for, it seems that the consumer is affected by inflation-specific uncertainty. The negative inflation effect could possibly reflect an anticipated inflation effect via the real interest rate, but this effect is clearly rejected by both the literature and the data (see Section 3.3.2).

Finally, the estimation of the robust model of Hansen et al. (1999) in Section 5.4 shows that both HF and robustness are more pronounced in the subperiod with high average inflation than in the low-inflation subperiod. This clearly indicates that the increased robustness is due to factors other than the different uncertainty effects already captured by the HF coefficient. Since the superperiod in which the consumer presents a higher degree of robustness has been selected because it presents particularly high and volatile inflation rates, the assumption of this chapter is that at least part – if not all – of this increased robustness is due to the high inflation uncertainty of this period. If the consumer alters her consumption behavior only as a reaction to changing inflationary conditions, this violates homogeneity postulate of Patinkin (1965) and indicates that she suffers from money illusion (see Section 3.1.2).

The following section shows how robustness with respect to ambiguity aversion – or money illusion – can be modeled as a simple static robust control problem, which takes the form of a two-player game between the DM and the malevolent nature.

5.2.2 A Static Robust Optimal Control Problem

As an illustration of the above-presented modeling concept, suppose that the DM thinks that her model is subject to specification errors due to, for example, money illusion. In the static case, she faces the following max-min problem:⁴

$$\max_{\{c\}} \min_{\{\omega\}} \left\{ \mathbb{E} [u(c, s)] + \theta \omega^2 \right\}, \quad (5.1)$$

⁴This section uses the notation and intuition of Hansen and Sargent (2008, Ch. 1 and 2). In addition, it utilizes some elements of a guest lecture of Professor Yulei Luo (University of Hong Kong) on “Robust and Risk-sensitive Control”, which he kindly put at my disposal.

subject to the distorted linear constraint:

$$s = As_0 + Bc + C(\omega + \varepsilon), \quad (5.2)$$

where, for simplicity, preferences are quadratic with $u(c, s) = -(Qc^2 + Rs^2)$. c and s are the control and state variables, with s_0 representing the initial known state value. Q , R , A , B and C are nonzero parameters, and the shocks ε are normally distributed with a zero mean and a unitary variance. The specificity of this setup is the inclusion of ω , which represents the misspecification parameter distorting the mean of the innovation ε . Without concern for robustness or money illusion, $\omega = 0$ and the model reduces to a simple maximization problem, such that equation (5.2) becomes $s = As_0 + Bc + C\varepsilon$. In the presence of money illusion, however, equation (5.2) represents the range of distorted models from which the agent chooses.

This distortion parameter ω is controlled by the malevolent nature, which seeks to set the parameter as high as possible to minimize the agent's expected utility. We know from the previous section that ω is restricted by the arbitrary upper bound η , so that the distorted model \tilde{f} stays within the ambiguity circle and remains statistically close to the benchmark model f . The restriction η can be interpreted as the degree of money illusion that the consumer fears she is suffering from. In order to be robust against this money illusion, the consumer includes the "penalty" parameter $\theta > 0$ in the objective function. This parameter is inversely related to η and can be understood as a Lagrange multiplier attached to the distortion in the objective function.

To understand the interdependence between η and θ , note that the size of the ambiguity circle can be defined, in the dynamic setting, as $E_t \sum_{t=0}^{\infty} \omega_t^2 \leq \eta$ (Hansen et al., 1999, p. 27). In this case, $E_t \sum_{t=0}^{\infty} \omega_t^2$ measures the distance between the true and the distorted model. This distance augments with in the consumer's fear of suffering from money illusion, which is controlled for by the consumer by a lower penalty parameter θ . In times of decelerating inflation, for example, the consumer's money illusion decreases, reflected by a higher θ and an overall more robust model. Consequently, defining money illusion as a distortion in the reference model implies that both the maximum size of the ambiguity circle η and the degree of robustness θ become time dependent, and, respectively, increase and decrease with the degree of inflation uncertainty. This assumption differs from that of the existing literature on robustness, which typically assume the degree of robustness to be constant.

Before maximizing her objective function (5.1) with respect to the control variable, the agent first wants to minimize the distortion that her malevolent opponent can impose on the quality of her model. This is how the DM maximizes her utility, given the worst possible outcome. Inserting equation (5.2) into equation (5.1) and making use of $\mathbb{E}(\varepsilon) = 0$ gives the following minimization problem:

$$\begin{aligned} & \min_{\{\omega\}} \left\{ -\mathbb{E} \left[Qc^2 + R(As_0 + Bc + C(\omega + \varepsilon))^2 \right] + \theta\omega^2 \right\} \\ &= \min_{\{\omega\}} \left\{ -\mathbb{E} \left[Qc^2 + R((As_0 + Bc)^2 + 2(As_0 + Bc)C(\omega + \varepsilon) + C^2(\omega + \varepsilon)^2) \right] + \theta\omega^2 \right\} \\ &= \min_{\{\omega\}} \left\{ -[Qc^2 + R(As_0 + Bc)^2 + 2R(As_0 + Bc)C\omega + RC^2\omega^2 + RC^2] + \theta\omega^2 \right\}. \end{aligned}$$

The solution for the minimization problem yields the value of ω that causes the DM the least damage. We have:

$$\begin{aligned} -2RC(As_0 + Bc) - 2RC^2\omega - 2\theta\omega &= 0 \\ \omega &= \frac{RC(As_0 + Bc)}{\theta - RC^2}. \end{aligned}$$

Substituting this optimal ω back into equation (5.1) gives the robust objective function, which, reduced to a simple maximization problem, gives:

$$\begin{aligned} & \max_{\{c\}} \left\{ \mathbb{E} [u(c, s)] + \theta\omega^2 \right\} \\ &= \max_{\{c\}} \left\{ -\mathbb{E} \left[Qc^2 + R(As_0 + Bc + C(\omega + \varepsilon))^2 \right] + \theta\omega^2 \right\} \\ &= \max_{\{c\}} \left\{ -[Qc^2 + R(As_0 + Bc)^2 + 2RC(As_0 + Bc)\omega - (\theta - RC^2)\omega^2] \right\} \\ &= \max_{\{c\}} \left\{ - \left[Qc^2 + R(As_0 + Bc)^2 + 2 \frac{[RC(As_0 + Bc)]^2}{\theta - RC^2} - \frac{[RC(As_0 + Bc)]^2}{\theta - RC^2} \right] \right\} \\ &= \max_{\{c\}} \left\{ - \left[Qc^2 + \frac{R(\theta - RC^2)(As_0 + Bc)^2 + [RC(As_0 + Bc)]^2}{\theta - RC^2} \right] \right\} \\ &= \max_{\{c\}} \left\{ - \left[Qc^2 + \frac{[R(\theta - RC^2) + R^2C^2](As_0 + Bc)^2}{\theta - RC^2} \right] \right\} \\ &= \max_{\{c\}} \left\{ - \left[Qc^2 + \frac{R\theta}{\theta - RC^2} (As_0 + Bc)^2 \right] \right\}. \end{aligned}$$

The agent's optimal and robust control is then:

$$\begin{aligned}
& -2Qc - \frac{R\theta}{\theta - RC^2} 2ABs_0 - \frac{2B^2R\theta}{\theta - RC^2} c = 0 \\
c &= -\frac{ABR\theta}{Q(\theta - RC^2) + B^2R\theta} s_0.
\end{aligned} \tag{5.3}$$

As a matter of comparison, the result of the maximization problem without model misspecification (i.e., if $\omega = 0$) is straightforward and yields the following optimal control:

$$c = -\frac{ABR}{Q + B^2R} s_0. \tag{5.4}$$

While the parameters A , B , C , Q and R have no particular meaning in the present example, it is interesting to focus on the impact θ has on the agent's optimal control. In particular, notice that equation (5.3) converges to equation (5.4) when $\theta \rightarrow \infty$. In other words, as the DM increases the penalty (θ) imposed on the malevolent nature's distortion (ω), she reduces the size of the ambiguity circle (η) until the distortion completely disappears and the optimal robust control reduces to the optimal control without concern for robustness. The interpretation of the solution remains the same if we focus on robustness with respect to money illusion. In this case, θ increases as the fear of suffering from money illusion vanishes. In any case, the model without robustness can be seen as a special case of the static robust control problem.

Comparing both optimal controls, it appears that the optimal c is larger for the robust DM than for the standard optimizing agent. This can be interpreted as the result of the robust agent's need to offset the evil nature's distortion ω . The implication of this compensation is that, depending on the degree of ambiguity aversion (i.e., the value of θ), robustness can bring about large differences in the optimal decision rules of the DMs. This conclusion is naturally also valid in a dynamic context, such as that of the REPIH, as is shown in the next section.

5.3 A Closed-Form Robust Permanent Income Hypothesis Model

This section presents a tractable closed-form solution to the dynamic optimization problem of a representative robust consumer. The model, derived in detail in Section 5.3.2, and critically evaluated in Section 5.3.3, can be seen as an extension, or generalization, of the PIH analyzed in Chapter 2. According to the money illusion interpretation of robustness (see Section 5.2.1), the solution to this problem can be directly interpreted as the optimal consumption path for a money-illusioned consumer willing to account for money illusion as a distortion to her benchmark REPIH model. In order to derive the solution to the model, however, it is useful to first rewrite it as a univariate model, as suggested by Hansen and Sargent (2008, Ch. 2) and Luo (2008, pp. 380-381).

5.3.1 The Univariate REPIH Model

The idea of the univariate REPIH is to move from one control variable (c_t) and two state variables (A_t, y_t), as in Section 2.2, to one control variable and only one state variable (namely, the permanent income y_t^p). As a matter of notational simplicity, we define, in this section, $\beta = \frac{1}{1+\rho}$ as the discount factor and $R = 1 + r$, where $R > 1$, such that the case in which the discount rate equals the interest rate ($\rho = r$) implies $\beta R = 1$.

In order to derive the univariate PIH model, consider the original consumer problem, formerly described by equations (2.1), (2.2) and (2.3), which can be rewritten as

$$\max_{\{c_t\}_{t=0}^{\infty}} U_t = \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \quad (5.5)$$

subject to

$$A_{t+1} = R(A_t + y_t - c_t) \quad (5.6)$$

and

$$\lim_{j \rightarrow \infty} \left(\frac{1}{1+r} \right)^j A_{t+j} \geq 0, \quad (5.7)$$

where the control variable is real consumption c_t and the state variables are A_t and y_t , representing, respectively, assets and a general income process to be specified further below. The within-period utility function is quadratic, as described in

Chapter 2 (i.e., we have $u(c_t) = c_t - \frac{a}{2}c_t^2$).

Using dynamic programming techniques, I showed in Chapter 2 that the solution for this consumer problem is represented by the following Euler equation:

$$\mathbb{E}_t(c_{t+1}) = \frac{ac_t + (\beta R - 1)}{a\beta R},$$

which boils down to the optimal consumption path $\mathbb{E}_t(c_{t+1}) = c_t$ (equation (2.12)) in the case where $\beta R = 1$. Also, under the definition of permanent income,⁵ the consumption function becomes:

$$c_t = \beta(R - 1)y_t^p - \frac{1 - \beta R}{a}. \quad (5.8)$$

If $\beta R = 1$, the consumption function becomes $c_t = \frac{R-1}{R}y_t^p$, indicating that the agent consumes, in every period, a constant fraction of permanent income.

For the moment, assume that income y_t follows a random walk (RW). This assumption is rather restrictive, but it greatly simplifies the derivation of the closed-form solution for the robust consumption problem. As we shall see in Section 5.3.3, it can be relaxed for income to follow other processes. We have:

$$y_{t+1} = y_t + \varepsilon_{t+1}, \quad (5.9)$$

where the error terms are *iid* with a zero mean and a variance of σ_ε^2 . The advantage of this RW assumption is that, after substituting for equation (5.9) in the definition of permanent income (equation (2.18)), permanent income boils down to $y_t^p = A_t + \frac{R}{R-1}y_t$. Rewriting this as $A_t = y_t^p - \frac{R}{R-1}y_t$ and $A_{t+1} = y_{t+1}^p - \frac{R}{R-1}y_{t+1}$, we can use it to rewrite the above-presented standard REPIH model in terms of a single state variable: the permanent income y_t^p . The asset evolution equation (equation (5.6)) then becomes:

⁵In Section 2.2.2, permanent income is found to be equal to $y_t^p = A_t + \sum_{i=0}^{\infty} \left(\frac{1}{R}\right)^i \mathbb{E}_t(y_{t+i})$; see equation (2.17).

$$\begin{aligned}
A_{t+1} &= R(A_t + y_t - c_t) \\
y_{t+1}^p - \frac{R}{R-1}y_{t+1} &= R(A_t + y_t - c_t) \\
y_{t+1}^p &= R\left(y_t^p - \frac{R}{R-1}y_{t+1} - c_t\right) + Ry_t + \frac{R}{R-1}[y_t + \varepsilon_{t+1}] \\
y_{t+1}^p &= R(y_t^p - c_t) + y_t \left[R - \frac{R^2}{R-1} + \frac{R}{R-1} \right] + \frac{R}{R-1}\varepsilon_{t+1} \\
y_{t+1}^p &= R(y_t^p - c_t) + \frac{R}{R-1}\varepsilon_{t+1}. \tag{5.10}
\end{aligned}$$

If we further define $\zeta_{t+1} = \frac{R}{R-1}\varepsilon_{t+1}$, with $\zeta_t \sim iid(0, \sigma_\zeta^2)$, as the shock to permanent income, we can rewrite the consumer problem as a univariate PIH model with a single state variable y_t^p :

$$\max_{\{c_t\}_{t=0}^{\infty}} \mathbb{E}_t \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}$$

subject to

$$y_{t+1}^p = R(y_t^p - c_t) + \zeta_{t+1}. \tag{5.11}$$

The modifications to the original model presented in Chapter 2 do not alter the main economic implications of the REPIH. To see this, we can compute the optimal consumption resulting from the consumption function equation (5.8). In the case of $\beta R = 1$, we get $c_t = \frac{R-1}{R}y_t^p$, which can be extended to:

$$\begin{aligned}
c_{t+1} - \mathbb{E}(c_{t+1}) &= \frac{R-1}{R}[y_{t+1}^p - \mathbb{E}(y_{t+1}^p)] \\
c_{t+1} - c_t &= \frac{R-1}{R}[R(y_t^p - c_t) + \zeta_{t+1} - R(y_t^p - c_t)] \\
\Delta c_{t+1} &= \frac{R-1}{R}\zeta_{t+1} \tag{5.12} \\
\Delta c_{t+1} &= \varepsilon_{t+1}. \tag{5.13}
\end{aligned}$$

Equation (5.13) is identical to equation (2.22) and represents the main result of Hall (1978): that the PIH model under rational expectations implies that consumption follows a RW.

5.3.2 Incorporating Robust Control

The univariate REPIH derived in the previous section can be extended in such a way as to account for robustness concerns on the part of the consumer. Incorporating the robust control modeling method presented in Section 5.2, the dynamic programming problem for the robust control–permanent income hypothesis model (RCPIH) can be written as:

$$V(y_t^p) = \max_{\{c_t\}} \min_{\{\omega_t\}} \left\{ c_t - \frac{a}{2} c_t^2 + \beta \left[\theta \omega_t^2 + \mathbb{E}_t [V(y_{t+1}^p)] \right] \right\}, \quad (5.14)$$

subject to the following distorted budget constraint:

$$y_{t+1}^p = R(y_t^p - c_t) + \sigma_\zeta \omega_t + \zeta_{t+1}, \quad (5.15)$$

where $\theta > 0$ is the penalty parameter controlling for the degree of money illusion the robust consumer wants to control for in her benchmark model. σ_ζ is the volatility of permanent income, while ω_t represents the control variable of the malevolent nature distorting the mean of the shock to permanent income. The distortion (ω_t) can be interpreted as the degree of money illusion of which the consumer is aware but over which she has no control. For this reason, in comparison to the rational consumer from the simple REPIH setup, the money-illusioned consumer concerned about misspecification not only takes into account the distorted accumulation constraint, but also directly includes the product $\theta \omega_t^2$ in her Bellman equation. As in the static case analyzed in Section 5.2.2, this term penalizes any alternative candidate model that she would consider to be “too distant” from the reference model. It guarantees that the robust consumer chooses an approximating model in which money illusion is present but does not exceed some plausibility threshold.

Note that the RCPIH model summarized by this extended Bellman equation is essentially the same as that of Luo (2008), except for its interpretation in terms of money illusion and for the form of the quadratic utility function. In his analysis, Luo (2008) uses the same within-period utility function as Hall (1978), $u(c_t) = -\frac{1}{2}(\bar{c} - c_t)^2$, with \bar{c} being the bliss point level of consumption. As a matter of consistency with the previous section and with Chapter 2, however, I opt for the general formulation suggested by Romer (2006, p. 353), $u(c_t) = c_t - \frac{a}{2} c_t^2$, in which the bliss level of consumption is equal to $1/a$. Even though, compared to the first function, the latter formulation has very similar properties, it is intuitively more appealing because the consumer’s utility is zero when consumption is equal to

zero. The consequences of the different utility functions include, not only different derivations (presented in detail in Appendix D), but also slightly different value functions and optimal consumption paths. Apart from this, the solving procedure and notation adopted in this section are very close to those suggested by Luo (2008).⁶

The max-min structure of the problem in equation (5.14), which is one of the main contributions of the robust control theory, perfectly suits the intuition of the PIH model: The consumer wants to maximize her lifetime utility, given the worst possible distortion to her benchmark model. This rather prudent behavior, as we shall see below, implies that people who are averse towards model ambiguity or misspecification tend to have a lower consumption than rational consumers. For this reason, Hansen and Sargent (2008) saw robust consumption as a particular type of precautionary savings behavior. If the model misspecification controlled for by this model is interpreted as the presence of money illusion, this result implies that money illusion further contributes to aggregate consumption smoothing and provides an additional explanation for the negative inflation effect advocated by Deaton (1977) and extensively analyzed in Chapters 3 and 4.

Proposition 1 summarizes the solution to the robust control problem given by equations (5.14) and (5.15):

Proposition 1. *The value function for the RCPIH model takes the following quadratic form:*

$$V(y_t^p) = -A(y_t^p)^2 - By_t^p - C,$$

where A , B and C are defined as:

$$\begin{aligned} A &= \frac{a\theta(\beta R^2 - 1)}{2\beta R^2\theta - a\sigma_\zeta^2}, \\ B &= -\frac{2R\theta(\beta R^2 - 1)}{(2\beta R^2\theta - a\sigma_\zeta^2)(R - 1)}, \\ C &= \frac{2\beta R^4\theta^2(\beta R - 1)^2}{a(2\beta R^2\theta - a\sigma_\zeta^2)^2(R - 1)^2(\beta - 1)} + \frac{\sigma_\zeta^2[2R^2\theta(\beta R - 1) - \beta(\beta^2 R^2 - 1)]}{(2\beta R^2\theta - a\sigma_\zeta^2)^2(R - 1)^2(\beta - 1)} \\ &\quad + \frac{a\sigma_\zeta^4}{2(2\beta R^2\theta - a\sigma_\zeta^2)^2(\beta - 1)}. \end{aligned}$$

⁶One reason for following Luo (2008) to solve the RCPIH problem is not only that he was successful in finding an intuitively appealing, closed-form solution, but also that the underlying model can easily be extended and applied to other macroeconomic topics focusing on the relationship between consumption and income. See, for instance, Luo and Young (2010) and Luo et al. (2012, 2014).

The malevolent nature's optimal distortion is:

$$\omega_t = \frac{aR\theta\sigma_\zeta(\beta R^2 - 1)(2\beta R^2\theta - a\sigma_\zeta^2)}{(2\beta R^2\theta - a\sigma_\zeta^2)[(2\beta R^2\theta - a\sigma_\zeta^2) - a\sigma_\zeta^2(\beta R^2 - 1)] + 2\beta R^2\theta(\beta R^2 - 1)} y_t^p - \frac{\sigma_\zeta(\beta R^2 - 1)}{\beta\{\theta[2 + a(\beta R^2 - 1)] - a\sigma_\zeta^2\}}. \quad (5.16)$$

Proof. The detailed derivations are presented in Appendix D. See Section D.1 for the derivations of A, B and ω_t and Section D.2 for the derivation of C. \square

Proposition 1 shows that the robust consumer's (univariate) value function takes a quadratic form and is governed by five constant parameters: namely, the inverse of the bliss level of consumption a , the subjective discount factor β , the gross return R , the misspecification distance parameter θ and the variance of the shock to permanent income σ_ζ^2 . Even though the value function looks rather cumbersome, note that, if we assume that $\beta R = 1$ and define $\Phi = \frac{a\sigma_\zeta^2}{2\theta}$, it reduces to the following simpler form:

$$V(y_t^p) = -\frac{a(R-1)}{2(R-\Phi)} (y_t^p)^2 - \frac{R}{R-\Phi} y_t^p - \frac{\Phi^2}{2a(R-\Phi)^2(\beta-1)}.$$

Note that the parameter Φ is constant and can be interpreted as a measure of the impact of robustness on consumption (Luo, 2008). Applying the same transformations to the optimal distortion ω_t yields the following equation:

$$\omega_t = \frac{aR\theta\sigma_\zeta(R-1)(R-\Phi)}{(R-\Phi)[(R-\Phi) - \Phi(R-1)] + R(R-1)} y_t^p - \frac{R\sigma_\zeta(R-1)}{\theta[a(R-1) + 2(1-\Phi)]}.$$

We see that ω_t , which distorts the mean of the shock to permanent income and generates the ambiguity circle presented in Figure 5.1, depends on the state variable y_t^p and is restricted by the different robustness-dependent parameters. Recall that the robust consumer chooses, during her optimization process, the model in which ω_t causes her the least damage. The result of this dynamic optimization process is the consumer's optimal and robust consumption path, which is defined in Proposition 2.

Proposition 2. *The robust consumption function is:*

$$c_t = \frac{\beta R^2 - 1}{\beta R^2 - \Phi} y_t^p - \frac{(\beta R^2 - \Phi)(\beta R - 1) + (\beta R^2 - 1)\Phi}{a\beta R(\beta R^2 - \Phi)(R - 1)} \quad (5.17)$$

and the robust consumption growth is:

$$\Delta c_{t+1} = \frac{(\beta R - 1)(\beta R^2 - \Phi) + (\beta R^2 - 1)\Phi}{a\beta R(\beta R^2 - \Phi)} + \frac{(R - 1)(\beta R^2 - \Phi) - (\beta R^2 - 1)R}{\beta R^2 - \Phi} c_t + \frac{\beta R^2 - 1}{\beta R^2 - \Phi} \zeta_{t+1}, \quad (5.18)$$

where $\Phi = \frac{a\sigma_\zeta^2}{2\theta}$.

Proof. See Section D.3 in Appendix D. □

Equation (5.17) represents the final robust consumption function, in which consumption is a linear function of permanent income y_t^p and only depends on the constant parameters a , β , R and Φ . We see from equation (5.17) that the RCPIH model predicts that it is optimal for the consumer to consume, in each period, a constant fraction of permanent income y_t^p . This result is totally in line with the prediction of the standard REPIH scrutinized in Chapter 2.

To understand the exact implications of robustness for consumption, consider again the standard assumption that the discount rate equals the interest rate (i.e., $\beta R = 1$). In this case, the robust consumption function (5.17) melts down to the following simple equation, which is independent of the discount factor:

$$c_t = \frac{R - 1}{R - \Phi} y_t^p - \frac{\Phi}{a(R - \Phi)}. \quad (5.19)$$

Since $\Phi = \frac{a\sigma_\zeta^2}{2\theta}$ is decreasing in θ , the absence of money illusion as a misspecification ($\theta \rightarrow \infty$) implies $\Phi \rightarrow 0$. The corollary is that, without any concern for robustness, equation (5.19) boils down to the rational expectations consumption function scrutinized in Chapter 2: namely, $c_t = \frac{R-1}{R} y_t^p$.

Interpreting equation (5.19) as the optimal consumption function for the money-illusioned consumer, it appears that the effect of robust control is twofold, reflected by the two terms on the RHS. On one hand, the marginal propensity to consume out of permanent income y_t^p is higher for the money-illusioned consumer (provided that $\Phi < 1$) than for the standard permanent income consumer. On the other hand, however, concern about unfavorable outcomes makes the robust consumer save more in proportion to her aversion towards money illusion, as reflected by the second term on the RHS of equation (5.19), which lowers consumption as Φ increases. However, robustness only increases savings under the condition that $\Phi < R$, so that the second term of equation (5.19) remains negative. It can in point of fact be easily shown that

$\Phi < 1$, thus satisfying the conditions for both terms on the RHS to fit the suggested intuition of precautionary savings. For the proof, see Section D.4 in Appendix D or Luo et al. (2014, p. 7).

Since $\Phi < 1$, robustness in the RCPIH implies that the second term on the RHS of equation (5.19) has a negative effect on consumption that overshoots its positive effect on the MPC out of permanent income. Consequently, the negative term of equation (5.19) can be interpreted as a precautionary savings premium, which increases with the degree of aversion against money illusion. A major difference of robustness compared to “traditional” precautionary savings is that the latter stems from utility functions presenting convex marginal utilities.⁷ In the present case, however, precautionary saving occurs even with the quadratic preferences used throughout this present thesis.

Consider now the optimal consumption growth under robustness against money illusion (equation (5.18)). This is the main result of the RCPIH model and the most important equation for the present analysis, which seeks to deepen our understanding of the relationship between consumption growth and inflation. Equation (5.18) can easily be rewritten as the following AR(1) process:

$$c_{t+1} = \frac{(\beta R^2 - \Phi)(\beta R - 1) + (\beta R^2 - 1)\Phi}{a\beta R(\beta R^2 - \Phi)} + \frac{R(1 - \Phi)}{\beta R^2 - \Phi} c_t + \frac{\beta R^2 - 1}{\beta R^2 - \Phi} \zeta_{t+1}. \quad (5.20)$$

To simplify the interpretation and to stay in line with the literature and the previous chapters, we focus on the case for which $\beta R = 1$. Under this assumption, optimal consumption growth (equation (5.18)) becomes:

$$\Delta c_{t+1} = \frac{(R - 1)\Phi}{a(R - \Phi)} - \frac{(R - 1)\Phi}{R - \Phi} c_t + \frac{R - 1}{R - \Phi} \zeta_{t+1}, \quad (5.21)$$

and equation (5.20) reduces to the following AR(1) model:

$$c_{t+1} = \frac{(R - 1)\Phi}{a(R - \Phi)} + \frac{R(1 - \Phi)}{R - \Phi} c_t + \frac{R - 1}{R - \Phi} \zeta_{t+1}. \quad (5.22)$$

The simplicity of equations 5.21 and 5.22 not only allows for an intuitive interpretation, but also yields simple empirical models that can be easily estimated (see Sec-

⁷See Caballero (1990) for a complete analysis of the features and implications of precautionary savings. Of course, since the origin of the precautionary savings motive is different, the policy implications for robust consumers are different from those of precautionary savers.

tion 5.3.3 below).⁸ As expected, in the absence of robustness against money illusion (i.e., if $\Phi = 0$), consumption growth in equation (5.21) reduces to $\Delta c_{t+1} = \frac{R-1}{R}\zeta_{t+1}$, which corresponds to the standard RW implication of the REPIH derived in equations (2.22) and (5.13). Once concerns about money illusion are accounted for (i.e., with $0 < \theta < \infty$ and $0 < \Phi < 1$), consumption growth becomes more volatile relative to income than in the REPIH. The last term on the RHS of equation (5.21) shows that consumption growth becomes more sensitive to unanticipated shocks to permanent income. This feature deserves some particular attention because it embeds potential problems related to the values that Φ can take. For this reason, this feature is analyzed in detail in the next section.

The middle term on the RHS of equation (5.21) attributes an increasing role of past information, or past consumption, to consumption growth as robustness increases. Any predictive power of past consumption for Δc_{t+1} would, thus, not necessarily stem from some HF, but could instead be the result of the presence of aversion against money illusion or of other misspecifications. Consequently, the statistically significant coefficients of past consumption growth obtained in Section 3.4.2 could be partly attributable to concerns about money illusion (i.e., not only to HF) and further emphasize the importance of inflation as a determinant of consumption growth. This result is in line with the empirical results of Chapter 4, which indicate that there might be other consumption-smoothing mechanisms at work beyond solely HF.

Equation (5.22), apart from being simple in its form and intuitive in its interpretation, possesses the formidable advantage of being easily and readily empirically evaluated. This is done in the next section, which also shows that it incorporates various shortcomings stemming from the different assumptions made throughout this section.

5.3.3 Critical Evaluation of the RCPIH

The previous section showed that robustness, interpretable as the aversion against money illusion, can be neatly incorporated into the standard REPIH framework. Moreover, the derivation of the closed-form robust control problem has highlighted the fact that the REPIH model can be understood as a special case of the RCPIH model, in which the representative consumer is convinced that she does not confuse

⁸Note that this final equation is very similar to the one suggested by Luo (2008). Consequently, the limitations of this model (emphasized in Section 5.3.3) naturally apply to this paper and to the related literature mentioned in footnote 6.

nominal values for real values and confidently follows the benchmark model f as the “true” data-generating model (see Section 5.2.1). While the solution to the RCPIH problem is intuitively appealing, questions regarding whether it is an appropriate model to explain actual consumption data and whether it really outperforms the benchmark RW model remain. The present section highlights some limitations related to the solution of the closed-form model derived in Section 5.3.2 and critically evaluates its performance with actual U.S. data.

Under the benchmark assumption that $\beta R = 1$, the closed-form RCPIH model yields the optimal and robust consumption path described by equation (5.22), suggesting that consumption follows an AR(1) process in which the future value of consumption c_{t+1} is determined by current consumption c_t and the shock to permanent income ζ_{t+1} , as well as by a constant. Recall further that we assumed, in Section 5.3.1, that income (y_t) follows a simple RW (see equation 5.9), such that the shock to permanent income, $\zeta_{t+1} = \frac{R}{R-1}\varepsilon_{t+1}$, is a simple function of the innovation to income. Under this particular income process, equation (5.22) can be rewritten as:

$$c_{t+1} = \frac{(R-1)\Phi}{a(R-\Phi)} + \frac{R(1-\Phi)}{R-\Phi}c_t + \frac{R}{R-\Phi}\varepsilon_{t+1}, \quad (5.23)$$

which directly relates consumption to the innovation to real income, ε_{t+1} .

Equation (5.23) is a simple and intuitive consequence of the solution to the closed-form RCPIH model derived in the previous section. Not only does consumption depend on only a few parameters, but its theoretically appealing AR(1) structure renders it an easily testable model. Before evaluation the equation using real data, however, it is useful to first analyze the relationships between the different parameters and to define a plausible range for their values. To accomplish this, let us first consider the last term of equation (5.23), which embeds the positive impact of robustness with respect to money illusion, Φ , into the immediate response of consumption to unexpected income shocks, ε_{t+1} . This indicates that the consumption response to an unexpected income shock is the more aggressive as the fear of money illusion on the part of the robust consumer increases.

In fact, after pinning down arbitrary but realistic values for R , we can derive the exact relationship between the response of c_{t+1} and Φ , characterized by the multiplier of the income shock $\frac{R}{R-\Phi}$. Figure 5.2 depicts this impact of robustness on the immediate response of consumption to an innovation in income for three different values of R : namely $R = 1.01$, $R = 1.05$ and $R = 1.1$, corresponding to

a $\beta = 0.99$, $\beta = 0.95$ and $\beta = 0.91$, respectively. Since β is typically very close to one, Figure 5.2 is assumed to cover a wide range of realistic scenarios: For example, Hansen et al. (1999) arbitrarily set $\beta = 0.9971$. In the case for which $\beta R = 1$, this corresponds to an annual risk-free interest rate of 2.5%, after an adjustment for the effects of a geometric growth factor of 1.0033 (Hansen et al., 1999, p. 886). Hansen and Sargent (2008, Ch. 2) used, for comparison, the alternative specification of $\beta = 0.9995$. Note that, instead of arbitrarily setting β , it is also possible to estimate it within the Hansen et al. (1999) model. This is done in Section 5.4 and yields estimated values for β that oscillate around 0.994. In the $\beta R = 1$ setting, choosing a subjective discount factor close to one naturally implies that the gross return R is also very close to 1. As demonstrated further below, this restriction can be problematic when it comes to simulating equation (5.23) with actual data.

Figure 5.2 clearly shows that, for the money-illusioned consumer, the marginal impact of robustness on the consumption response $\frac{R}{(R-\Phi)^2}$ depends heavily on the value of Φ . In particular, the closer Φ and R are to one another, the greater the effect of robustness, as reflected by the exponential shape of the immediate consumption response. We see that the marginal effect of robustness is rather moderate when Φ takes values between 0 and 0.5, where the latter value implies that the immediate response of consumption is approximately twice as large as for the rational consumer setting $\Phi = 0$. Above this threshold, the marginal impact of robustness on the consumption response increases exponentially and rapidly produces consumption response values that are theoretically unrealistic. In light of this extremely sensitive relationship between robustness and the consumption response to an income shock, it seems reasonable to assume that the robustness parameter should lie within a low to medium range, such as, for example, the (0,0.5) interval.

As suggested by Figure 5.2, a Φ close to one implies unrealistic consumption responses: They would not be theoretically consistent with the goal of the robust consumer – namely, to choose a distorted model that remains statistically close to the benchmark REPIH model. Following the present interpretation, the consumption responses become unrealistic when the consumer has to choose a Φ that corresponds to unrealistic degrees of money illusion. Recall that, according to the definition of the robustness parameter, $\Phi = \frac{a\sigma^2}{2\theta}$ is inversely proportional to the penalty parameter θ , such that Φ approaches its upper bound (equal to one) when θ is particularly low. Yet, an excessively low θ allows the malevolent nature to impose a huge distortion on the approximating models from which the consumer can choose (i.e., the ambiguity circle becomes very large; see Section 5.2.1). This explains the shape of

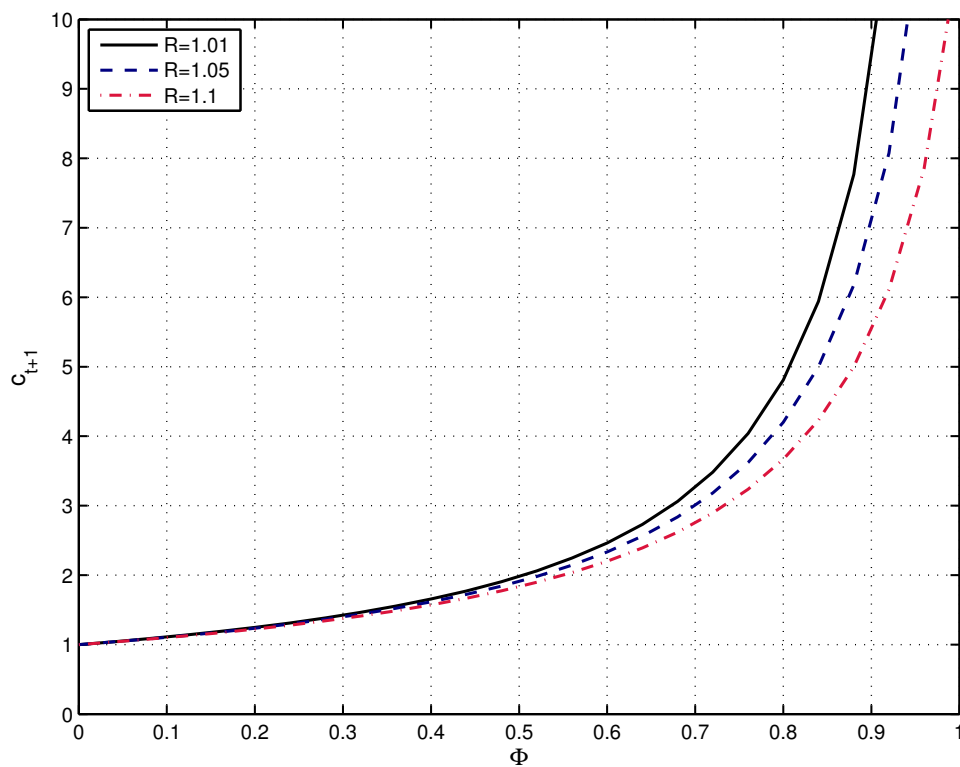


Figure 5.2: The Impact of Robustness on the Response of c_{t+1} to ε_{t+1}

The immediate response of consumption to an unanticipated one-unit change in income, ε , increases with the degree of robustness, $0 \leq \Phi < 1$. Under the assumption that $\beta R = 1$, the three pictured scenarios correspond to a β of 0.99 (solid line), 0.95 (dashed line) and 0.91 (dash-dot line), respectively.

the immediate responses of consumption in Figure 5.2, in which the consumer has to compensate for the huge distortions that occur when Φ is allowed to approach its upper bound.

Under the standard assumption that $\beta R = 1$, this intuition remains valid for any plausible value chosen for the subjective discount factor. Figure 5.2 also shows how this choice influences the consumption response to the income shock: For any given Φ , a higher β is associated with more sensitive consumption responses. Again, the closer Φ is to one, the more sensitive the response of consumption will be. Interestingly, this relationship implies that impatience has an effect on consumption similar to that of the fear of money illusion, such that it can be difficult *a posteriori* to distinguish one effect from the other.⁹ Of course, this is only valid within the RCPIH model when robustness is present. In the special case for which $\Phi = 0$, equation (5.23) reduces to the benchmark model of Hall (1978), $c_{t+1} = c_t + \varepsilon_{t+1}$, in which c_{t+1} reacts with a 1:1 ratio to a shock to ε_{t+1} and is independent from β . This is also visible in Figure 5.2, in which the immediate consumption response to a one-unit shock to income is equal to one when $\Phi = 0$ for any chosen R .

It is important to emphasize at this stage that the increased sensitivity of consumption to a one-unit unexpected shock to income, depicted in Figure 5.2, reflects only the immediate response of consumption and contains no information about its evolution over time. Of course, the standard PI consumer immediately and permanently achieves her new long-term level of consumption, since the response of consumption in the robustness-free equation $c_{t+1} = c_t + \varepsilon_{t+1}$ is immediate, constant and equal to one. In the case of an AR(1) process, the impulse response function (IRF) can be easily computed because its dynamics depend only on the AR coefficient. In this particular case, the response of consumption will rapidly return to its long-term value (equal to one), with the speed of this adjustment being determined by the “half-life responses” to the shock. Due to the autoregressive pattern of equation (5.23), we know that this will also be the case for the robust consumer. Note, however, that the convenient AR(1) structure of this reduced model nests several contradictions with respect to the values that the governing parameters can take.

To highlight this conceptual weakness, consider now the AR coefficient in equation (5.23), which we define as $\phi_1 = \frac{R(1-\Phi)}{R-\Phi}$. Since R is rather close to one, most conceivable estimates for ϕ_1 imply a very high degree of robustness for this equality

⁹This interdependence between β and Φ is used in Section 5.4 to estimation the presence of money illusion in aggregate U.S. data. See Hansen and Sargent (2008, Ch. 2) for a formal demonstration of the conditions under which impatience yields results equivalent to those of concerns about robustness.

to hold. For example, if we arbitrarily set $R = 1.0029$, as in Hansen et al. (1999), estimates of ϕ_1 as different as $\phi_1 = 0.1$ and $\phi_1 = 0.9$ would each require the robustness parameter to be very close to one (namely, equal to $\Phi = 0.99$ and $\Phi = 0.97$, respectively). If, on the contrary, Φ is expected to lie within the above-suggested acceptable range (say, below 0.5), the corresponding ϕ_1 would need to be greater than 0.997. In this case, however, the AR(1) consumption model would be very close to a simple RW, and the RCPIH could not be unambiguously disentangled from the benchmark REPIH model. As a result, it appears that the robust consumption model in equation (5.23) embodies foibles that will bring about difficulties in the estimation and interpretation. Worse still, this does not seem to provide a realistic alternative to the empirically appealing REPIH model.

As a concrete illustration of this problem and to determine the exact shape of the IRFs in our model, consider Figure 5.3, which shows the estimated IRF of consumption to a one-unit shock in income using the same quarterly U.S. data and samples as in the previous chapters.¹⁰ To avoid structural breaks (see Section 3.2.3), to better compare with the previous estimation results (Sections 3.4 and 4.3.1), and to investigate whether the consumer's concern for robustness varies with the inflation level, I report only the results for the high- and low-inflation subsamples, using both the IPD and the CPI as inflation indexes.

The plotted IRFs, which show the sensitivity of consumption to an unexpected income shock as well as the number of periods needed for consumption to return to its long-term value, are obtained as follows. First, I estimate the AR(1) model derived in equation (5.23) with OLS. This yields estimates for the different AR(1) coefficients, $\hat{\phi}_1$, as well as for the intercepts. In our case, the estimated $\hat{\phi}_1$ s are, respectively, 0.47 and -0.02 for the high and low IPD models and 0.44 and 0.26 for the high and low CPI models. While the intercepts are highly significant (at the 1% level) in all models, the estimated $\hat{\phi}_1$ s are significant at the 1% level for the two high-inflation models, significant at the 5% level for the low-inflation CPI model and not significant for the low IPD model.

Using these least squares estimates for ϕ_1 , we can calculate, for every model, the half-life response, defined as $h = \frac{\log(0.5)}{\log(|\hat{\phi}_1|)}$. For any starting response value (i.e., the intercepts in Figure 5.3), the half-life response indicates the number of periods needed for the consumption response to be reduced by half. Consequently, the half-life response produces the shape of the IRF and indicates the speed of adjustment of the endogenous variable after a shock in the error term. Finally, the starting response

¹⁰See Appendix A for a detailed description of the data.

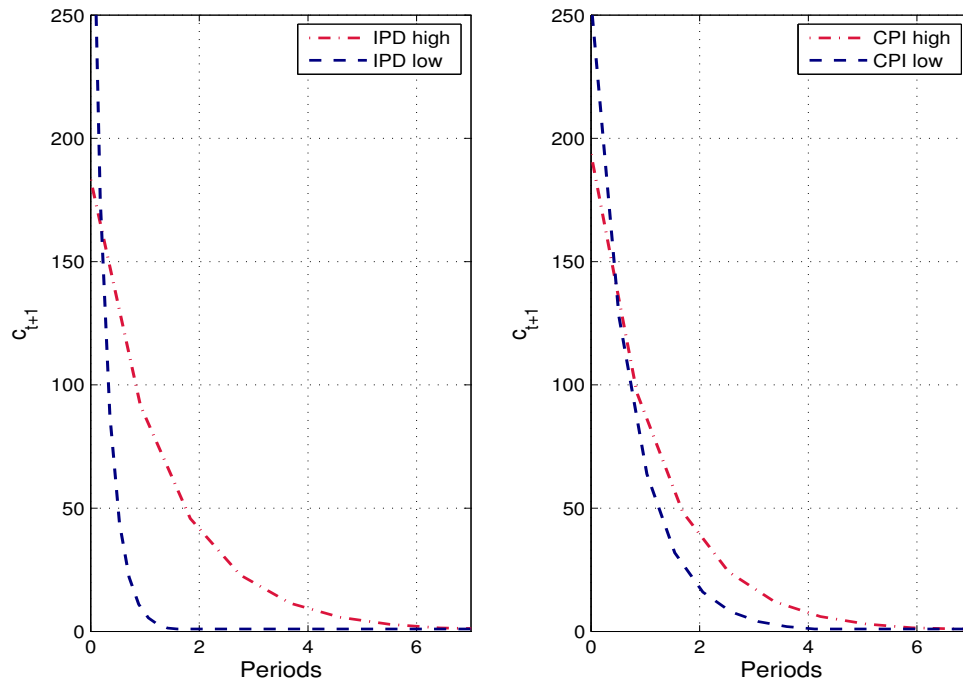


Figure 5.3: Impulse Response Functions of c_{t+1} to ε_{t+1}

The impulse response functions are obtained by estimating equation (5.23) with U.S. quarterly data. Consumption is deflated with IPD inflation (left graph) and CPI inflation (right graph) over two subperiods, one experiencing high inflation rates (dashed line) and the other one experiencing low inflation rates (dash-dot line). The high-inflation period is from 1966Q1 to 1981Q3 for the IPD model and from 1966Q1 to 1982Q3 for CPI, while the low-inflation period is from 1991Q1 to 2001Q1 for both deflators.

values in Figure 5.3 are obtained by premultiplying the (unitary) income shock with the corresponding multiplier $\frac{R}{R-\Phi}$. The robustness-capturing parameter Φ is calculated by means of the slope parameter $\hat{\phi}_1 = \frac{R(1-\Phi)}{R-\Phi}$ and the arbitrarily chosen $R = 1.0029$, corresponding to the benchmark $\beta = 0.9971$ of Hansen et al. (1999).

Looking at Figure 5.3, what is immediately clear are the unrealistic and excessive starting values of the response of consumption to an income shock. This poor result is valid for all four depicted models, since consumption immediately jumps to values above $c_{t+1} = 150$ when ε_{t+1} increases by one unit. The reason for this over-sensitivity of consumption is that the different $\hat{\phi}_1$ s are all well below the above-mentioned value of 0.997. These values lead to implausible Φ s that are all very close to one and, thus, are much higher than the suggested plausibility threshold. Note, however, that the excessive consumption responses can be reduced to plausible values when income is assumed to follow an AR(1) process instead of an RW (see equation (5.24) and its interpretation below).

Comparing the different starting values and shapes of the IRFs across the models, we can nonetheless observe the predicted impact of robustness on consumption. Disregarding the implausible starting response values, it appears that the IRFs diverge across the different subperiods but are similar for both inflation measures. In particular, the IRFs of the low-inflation models have a much higher starting values, but also have a faster speed of adjustment than their high-inflation counterparts. The extreme case is represented by the low-inflation IPD model, in which consumption reacts extremely aggressively to the income shock, but achieves its new long-run level after less than two quarters (due to a half-life response of $h = 0.18$ periods).

The observed differences between the subperiods, which result from the lower estimates for ϕ_1 s in the low-inflation periods (which yield higher Φ s), are in line with the predictions of the RCPIH model: namely, that robustness induces precautionary savings. According to the theory, as well as to the Kalman filter estimation results in Section 4.3.1, precautionary savings are expected to be higher in an overall uncertain environment. Similarly, concerns about money illusion are assumed to be higher within the high-inflation subperiods, induced by higher and more volatile inflation rates than those in the low-inflation period. In Figure 5.3, this phenomenon is reflected by the lower immediate consumption responses and the smoother adjustments in the high-inflation models. Note that this result is confirmed by the estimation of the robust model of Hansen et al. (1999) in Section 5.4.2, further supporting the idea that the consumer's concern about misspecification – and, hence, about money illusion – is higher in the high-inflation subperiod.

In light of the irreconcilable parameter values, it appears that the simple robust consumption model derived in Section 5.3 and summarized by the simple AR(1) model in equation (5.23) contains too many weaknesses to be able to efficiently describe actual U.S. consumption data. Without abandoning the still-appealing robust control framework, there are essentially three improvements that can be considered to improve the fit of the model. First, the standard assumption that $\beta R = 1$ could be relaxed, allowing for $\beta R > 1$. In this case, equation (5.20) does not reduce to equation (5.21), and the AR coefficient remains equal to $\phi_1 = \frac{R(1-\Phi)}{\beta R^2 - \Phi}$. Then, for a given β , setting R at high enough values allows for both ϕ_1 and Φ to remain within a more plausible range of values. For example, setting $\beta = 0.9971$ and $R = 1.5$ would be compatible with (ϕ_1, Φ) values of (0.88, 0.3), (0.75, 0.5) and (0.57, 0.7), respectively. However, since the relaxing of the $\beta R = 1$ assumption requires implausibly high risk-free interest rates in our model, other putative solutions might be more appropriate. Moreover, as shown in Section 5.4, alternative specifications of the robust consumption model can greatly improve the fit of the model, even when the assumption that $\beta R = 1$ is maintained.

Second, recall that we made the assumption, in Section 5.3.1, that income follows a RW, as described by equation (5.9). On one hand, this assumption was used to rewrite the standard REPIH model as a univariate model, which greatly simplified the derivation of the closed-form solution to the RCPIH model. On the other hand, especially with regard to the poor performance of the RCPIH in fitting actual data, this assumption might well be too restrictive and could be relaxed in favor of an AR(1) process, as was assumed, for example, in Chapter 4. Leaving out the constant, the income process can then be rewritten as $y_{t+1} = \gamma y_t + \varepsilon_{t+1}$. In this case, permanent income becomes $y_t^p = A_t + \frac{R}{R-\gamma} y_t$, such that equation (5.10) can be rewritten as $y_{t+1}^p = R(y_t^p - c_t) + \frac{R}{R-\gamma} \varepsilon_{t+1}$. We see that the altered income process has an impact only on the shock to permanent income. Consequently, if we newly define the permanent income shock as $\zeta_{t+1} = \frac{R}{R-\gamma} \varepsilon_{t+1}$, all the derivations of the closed-form solution to the RCPIH (presented in Section 5.3.2) remain valid under the new income process. Nonetheless, the new ζ has a crucial impact on the above-simulated consumption model under $\beta R = 1$ in equation (5.23), which now becomes:

$$c_{t+1} = \frac{(R-1)\Phi}{a(R-\Phi)} + \frac{R(1-\Phi)}{R-\Phi} c_t + \frac{R(R-1)}{(R-\Phi)(R-\gamma)} \varepsilon_{t+1}. \quad (5.24)$$

As we can see, changing the assumption with regard to the income process only

affects the income shock multiplier $\frac{R(R-1)}{(R-\Phi)(R-\gamma)}$, which now also depends on the parameter γ , which determines the evolution of income. Contrary to the above-presented RW case, which yields the implausible immediate consumption responses to income shocks depicted in Figure 5.3, the case in which income follows an AR(1) process can produce much more realistic consumption responses.

Compared to the shock multiplier under RW, $\frac{R}{R-\Phi}$, the multiplier under AR(1) dramatically lowers the immediate consumption response and, thus, yields much more realistic shapes for the consumption responses and IRFs than those depicted in Figures 5.2 and 5.3, respectively. For example, under the assumption that $R = 1.0029$ (see Hansen et al. (1999)) and with a robustness parameter as high as $\Phi = 0.999$, the immediate response of consumption to a unitary income shock would be lowered to 1.48, 2.46 and 7.25 for γ s equal to 0.5, 0.7 and 0.9, respectively, as compared to a consumption response of 256.6 in the RW case. Unfortunately, note that, even though the assumption that income follows an AR(1) is able to remove the problem pertaining to the implausible immediate responses of consumption to unexpected income shocks, there remains the problem that the coefficient of the autoregressive consumption process, equal to $\phi_1 = \frac{R(1-\Phi)}{R-\Phi}$ in equation (5.24), is the same as in equation (5.23). Consequently, the simulation of the model with actual U.S. data would still engender Φ values close to one, which are not reconcilable with realistic degrees of robustness.

The final improvement that can be envisaged for the RCPIH model presented in Section 5.3 is to add further elements to the robust control problem to increase the range of alternatives from which the representative consumer can choose. For example, Hansen et al. (1999) suggested a more general robust PIH model that is not only able to control for HF, different income processes and alternative production technologies, but is also able to precisely determine realistic degrees of robustness that actually fit to the U.S. consumption data, even when the standard assumption of $\beta R = 1$ is maintained. Since this last solution seems most promising, Section 5.4 essentially focuses on the robust consumption model of Hansen et al. (1999) in order to show that money illusion can be effectively modeled within the robust control framework.

5.4 The Hansen et al. (1999) Robust Consumption Model

The closed-form RCPIH model derived in Section 5.3 contains too many weaknesses to be efficiently used as an empirically robust consumption model in order to inves-

investigate whether money illusion is present in aggregate U.S. consumption data. The current section focuses on the robust PIH model developed by Hansen et al. (1999) (henceforth referred to as HST), which takes the form of a robust dynamic stochastic general equilibrium (DSGE) model (compared to all other models analyzed in this thesis, which can be considered as partial equilibrium models).

This more general setup not only provides a framework that can simultaneously take into account robustness, HF, investment and alternative income processes, but also allows for to use DSGE-specific tools for the analysis and estimation of the model.¹¹ The ability to include HF is a great improvement over the RCPIH model (derived in Section 5.3) because a significant impact of lagged consumption growth on current consumption growth captures, not only consumption habits, but also all other precautionary savings motives that are not related to inflation (see Section 5.2.1).

Section 5.4.1 briefly presents the HST model and the observational equivalence between robustness and the subjective discount factor. Section 5.4.2 estimates the model and shows that money illusion, measured with robustness, is higher in periods with high average inflation rates.

5.4.1 The Robust Consumer's Problem

HST's robust permanent income model uses the two-player game intuition suggested by the robust control theory (see Section 5.2.2) to derive the following consumer problem, which can be seen as an extension of the RCPIH model (presented in Section 5.3.2). The robust consumer faces the following Bellman equation:

$$V(k_t, y_t) = \max_{\{c_t\}} \min_{\{\omega_{t+1}\}} \left\{ -(s_t - b)^2 + \beta (\theta \omega_{t+1}^2 + \mathbb{E}_t [V(k_{t+1}, y_{t+1})]) \right\}, \quad (5.25)$$

¹¹Specifically, I have implemented the model in *Dynare*, an open-source software that is particularly convenient for handling DSGE models. A good starting point for programming HST's model is provided by the example in Barillas et al. (2010). I am also grateful to Professor Johannes Pfeifer (University of Mannheim) for his helpful advice. For further examples and information on *Dynare*, visit <http://www.dynare.org>.

subject to the following set of constraints:

$$s_t = (1 + \lambda)c_t - \lambda h_{t-1} \quad (5.26)$$

$$h_t = \delta_h h_{t-1} + (1 - \delta_h)c_t \quad (5.27)$$

$$c_t + i_t = \gamma k_{t-1} + y_t \quad (5.28)$$

$$k_t = \delta_k k_{t-1} + i_t. \quad (5.29)$$

In the objective function (5.25), b represents the bliss point level of consumption, corresponding to $b = \frac{1}{a}$ from the RCPIH (see equation (5.14) in Section 5.3.2). HST further defined s_t as the “household service stream” characterized by equations (5.26) and (5.27), where c_t is real consumption and h_{t-1} is a weighted average of current and past consumption. Note that $\delta_h \in (0, 1)$ and that if $\lambda > 0$, the consumer presents some degree of HF. According to equation (5.28), the consumer faces a linear production technology such that i_t is investment, y_t is the exogenous income process (to be defined further below), and k_t is the capital stock defined in equation (5.29). The constant parameters γ and δ_k are the marginal product and the depreciation factor of capital, respectively.

Recall that the robust consumer, fearing model misspecifications, can alternatively be seen as a money-illusioned consumer who wishes to control for his money illusion, especially in times of high inflation or high model uncertainty. Just as in the general case exposed in Section 5.2.2 and in the RCPIH in equation (5.14), the fear of specification errors is captured by the penalty parameter θ and the distortion parameter ω_{t+1} . While the malevolent nature imposes a maximum distortion, or maximum money illusion, on the set of optimal models from which the robust consumer can choose, the consumer can restrict the distortion with the penalty parameter to reflect the desired degree of robustness.

In their original setting, HST assumed that the income process y_t is governed by a permanent component \bar{y}_t and a transitory component \hat{y}_t , both of which follow a second-order autoregressive AR(2) process. Thus, they defined:

$$y_t = \mu_y + \bar{y}_t + \hat{y}_t, \quad (5.30)$$

where

$$\bar{y}_t = (\phi_1 + \phi_2)\bar{y}_{t-1} - \phi_1\phi_2\bar{y}_{t-2} + c_{\bar{y}}(\varepsilon_t^{\bar{y}} + \omega_t^{\bar{y}}) \quad (5.31)$$

$$\hat{y}_t = (\phi_1 + \phi_2)\hat{y}_{t-1} - \phi_1\phi_2\hat{y}_{t-2} + c_{\hat{y}}(\varepsilon_t^{\hat{y}} + \omega_t^{\hat{y}}). \quad (5.32)$$

The innovations $\varepsilon_t^{\hat{y}}$ and $\varepsilon_t^{\hat{i}}$ are standard normal *iid* shocks, and $\omega_t^{\hat{y}}$ and $\omega_t^{\hat{i}}$ are distortions to the means of the endowment processes that represent the misspecifications due to money illusion. The autoregressive coefficients ϕ_1 , ϕ_2 , α_1 and α_2 represent the persistence of the endowment process.¹²

Before the model is solved, it can be further simplified by solving the capital evolution equation (5.29) for i_t and substituting for it in equation (5.28). This gives:

$$c_t + k_t = (\delta_k + \gamma)k_{t-1} + y_t, \quad (5.33)$$

where the first term on the RHS yields a definition for the gross return on capital R :

$$R = \delta_k + \gamma, \quad (5.34)$$

which also coincides with the gross return on a risk-free asset. Finally, following Hall (1978), HST maintained the assumption that the subjective discount factor is equal to the risk-free interest rate (i.e., $\beta R = 1$).

The crucial contribution of HST is that they showed that there is no need to solve for the complete max-min problem of the robust consumer (as we do, for instance, for the RCPIH in Section 5.3.2). In fact, they showed that, for any degree of robustness, the solution is *observationally equivalent* to a solution of the same problem without robustness when the discount factor has been adjusted accordingly. Recall that the RCPIH already suggests that robustness has a similar impact to impatience on the consumption response to an income shock (see Figure 5.2).

Though HST first established this observational equivalence proposition, its precise definition was derived by Hansen et al. (2002). Formally, they showed that, for fixed values of all other parameters, every $(\hat{\beta}, \theta)$ pair is observationally equivalent to a corresponding $(\beta, 0)$ pair. In this context, for the $(\hat{\beta}, \theta)$ and $(\beta, 0)$ pairs to be observationally equivalent means that they yield both the same decision rule for (c_t, i_t) and can, thus, not be distinguished from one another. The result is a direct relationship between the subjective discount factor β and the equivalent *robust* discount factor $\hat{\beta}(\theta)$, which can be summarized by the following equation:¹³

¹²In this section, ϕ_1 should not be confused with the ϕ_1 of Section 5.3.3, which characterizes the robustness-induced persistence in consumption.

¹³The relationship in equation (5.35) is a simplification of the observational equivalence, which Hansen and Sargent (2008) showed to be equal to:

$$\hat{\beta}(\theta) = \frac{1}{R} - \frac{\sigma\alpha^2}{R-1},$$

$$\hat{\beta}(\theta) = \beta - \frac{\beta}{\theta(1-\beta)}. \quad (5.35)$$

According to this relationship between $\hat{\beta}$ and θ , an increase in the penalty parameter θ (i.e., a decrease in the distortion allowed by the robust consumer) leads to an increase in $\hat{\beta}$. In other words, the robust consumer has a higher $\hat{\beta}$ than the purely rational consumer. Moreover, letting $\theta \rightarrow \infty$ implies that $\hat{\beta} \rightarrow \beta$, which is the case for which the ambiguity circle disappears and the approximating robust model and the benchmark REPIH models coincide.

For our purpose, the observational equivalence in equation (5.35) provides the main tool to investigate for the presence of money illusion in aggregate consumption data. In contrast to HST, who assumed that the robust consumer presents a fear of misspecification that is time-invariant, the theoretical implications of money illusion exposed in Section 5.2.1 suggest that the degree of robustness depends on the degree of inflation uncertainty. Since a higher money illusion is accompanied by a higher degree of robustness, θ (and thus $\hat{\beta}$) should be higher in the high-inflation subperiod than in the low-inflation subperiod. This conjecture is verified in Section 5.4.2.

Since the degree of robustness is nested within the subjective discount factor, as suggested by the observational equivalence, the robust consumer's max-min problem reduces to a simple maximization problem, in which the distortion is set to $\omega_t = 0$. Attaching the Lagrange multipliers $2\beta^t\mu_{st}$ to equation (5.26), $2\beta^t\mu_{ht}$ to equation (5.27) and $2\beta^t\mu_{ct}$ to equation (5.34), respectively, we obtain the following FOC:

$$s_t : \mu_{st} = b - s_t \quad (5.36)$$

$$c_t : \mu_{ct} = (1 + \lambda)\mu_{st} + (1 - \delta_h)\mu_{ht} \quad (5.37)$$

$$h_t : \mu_{ht} = \beta\mathbb{E}_t(\delta_h\mu_{h,t+1} - \lambda\mu_{s,t+1}) \quad (5.38)$$

$$k_{t+1} : \mu_{ct} = \beta R\mathbb{E}_t(\mu_{c,t+1}), \quad (5.39)$$

as well as the constraints 5.26, 5.27 and 5.33. Since we assume $\beta R = 1$, equation (5.39) implies that μ_{ct} is a martingale. By equation (5.37), this, in turn, also implies that μ_{st} and μ_{ht} are martingales, leading to the precise definition of μ_{st}

where $\beta R = 1$, $\sigma = -\theta^{-1}$ is a risk-sensitivity parameter and α is some constant volatility parameter representing the forecast error in the following representation of the marginal utility of services: $\mu_{st} = \mu_{st-1} + \alpha\tilde{\epsilon}_t$. Substituting for R , σ and $\alpha = 1$ yields equation (5.35). See Hansen and Sargent (2008, pp. 231-234) for a detailed proof of the observational equivalence proposition.

(reported in Hansen and Sargent (2008, p. 231) and in footnote 13). As a matter of completeness, note that, in the absence of HF (i.e., if $\lambda = 0$), equation (5.26) implies that $c_t = s_t$, such that utility depends only on consumption. In this case, the FOC reduce to $\mu_{ct} = b - c_t$ and $\mu_{ct} = E_t(\mu_{c,t+1})$, resulting in the well-known RW hypothesis derived in Section 2.2.1, where the Euler equation (2.12) is equal to $c_t = \mathbb{E}_t(c_{t+1})$.

The just-derived $\omega_t = 0$ version of HST's permanent income model is estimated in Section 5.4.2 for the cases with and without HF. Recall that it does not exclude the presence of robustness, since it can be measured by way of the subjective discount factor β .

5.4.2 Estimation Results

The estimation results for HST's permanent income model, which is characterized by equations (5.26) to (5.34) and (5.36) to (5.39), are summarized in Table E.1 in Appendix E. They have been obtained using U.S. quarterly consumption and investment data, in which consumption is measured by the personal consumption expenditures of nondurables and services and investment is measured by the sum of durable consumption and gross private investment.¹⁴ Both series are expressed in per capita terms and in first log-differences. Using the same procedure as for the previous estimations, the nominal series have been deflated by two distinct inflation indexes: namely, the IPD and the CPI (see the data Appendix A). In order to draw inferences about the presence or not, in the data, of robustness against money illusion, the IPD and CPI models with and without HF have been estimated over two subperiods, one presenting high average inflation rates (1966Q1 to 1981Q3 for the IPD models and 1966Q1 to 1982Q3 for the CPI models) and the other presenting low average inflation rates (1991Q1 to 2001Q1 for both IPD and CPI).

The estimates reported in Table E.1 have been obtained by maximum likelihood using HST's original parameter estimates (reproduced in the "HST" columns) as initial values. The reason for this choice is that HST's estimates represent plausible benchmark values for each estimated model; moreover, this approach ensures that no bias is introduced by arbitrarily choosing alternative sets of initial values. Further, I followed HST's example, fixing b and δ_k to $b = 32$ and $\delta_k = 0.975$, and imposed the restriction $\beta R = 1$. Recall that it is the latter assumption that allows us to use the observational equivalence in equation (5.35) to investigate for the presence of

¹⁴The original series can be downloaded from the Federal Reserve Economic Data at <http://research.stlouisfed.org/fred2>.

money illusion. For this purpose, contrary to HST, I also estimated the parameter β in order to evaluate the robust consumer's behavior across the subperiods.

For model comparison, Table E.1 also reports the log-likelihood values resulting from the maximum likelihood estimation. Since these log-likelihood values can only be used to choose between two nested models (e.g., using a likelihood ratio test), they should be considered separately so as to discriminate only between the models with and without HF within each subperiod and inflation measure. According to the log-likelihood values, the preferred models are those that are highlighted in bold in Table E.1. Note that the table does not indicate the significance of each coefficient for several reasons. First, since HST did not provide them, we cannot use the individual significances for comparison with the HST's original results. Furthermore, the aim in this section is not to discriminate between the different parameters (since they are all part of the needed system of equations). Third, the ultimate significance of most parameters, though not their values, depends heavily on the initial values (as well as on the estimation routine) chosen for the estimation. Note, however, that the central coefficient of the present analysis, β , is highly significant in every reported model.

To add emphasis to the main result of the estimation of HST's model, Table 5.1 summarizes the estimates for β for the preferred models only. To avoid any confusion with regard to the observational equivalence, it is important to clarify that the reported estimates for the subjective discount factors represent the β s from the $(\beta, 0)$ pairs and not from their $(\hat{\beta}, \theta)$ equivalents. It is possible, however, to find the equivalent pairs by solving equation (5.35) for θ and calculating its exact value using the constant $\hat{\beta}$ and the estimated β s from Table 5.1. Note, however, that since the penalty factor θ is always positive, the assumed (fixed) $\hat{\beta}$ needs to be lower than the estimated β s. Consequently, the estimated β s from Table 5.1 suggest that HST's fixed $\beta = 0.9971$ is too high and should be set at lower values. Since it is assumed that money illusion is virtually absent in the low-inflation period (i.e., the consumer seeks no robustness in this period), the estimation results suggest that the benchmark value for β is approximately 0.9937 for the IPD model and 0.9941 for the CPI model.

The estimates for β reported in Table 5.1 clearly indicate that robustness is higher in the high-inflation subperiod than in the low-inflation subperiod, as suggested by the higher estimated values. Since this result is valid for both IPD and CPI models, it strengthens the intuition that inflation plays an important role for the robust consumer, independent of their choice of the inflation index. The obtained

Table 5.1: Estimates of β in the Selected Subperiods

	High Inflation		Low Inflation	
	IPD	CPI	IPD	CPI
β	0.9943	0.9944	0.9937	0.9941

The reported estimates for β provide from the preferred models of Table E.1, which summarizes the maximum-likelihood estimations of HST's robust consumption model. All β s provide from the estimation of the model with HF, except in the case of the IPD low-inflation model, which excludes HF.

difference in the estimated β s is central to our analysis because it clearly advocates in favor of the presence of money illusion as a distortion to the benchmark REPIH model. If there were no robustness, we would expect the discount factor to be the same across the different subperiods. However, the fact that the estimated β s are higher in the high-inflation subperiod endorses the assumption that model uncertainty is higher during this period and that the consumers have a higher willingness to hedge against the resulting misspecification in the benchmark model.

At this stage, it is important to recall that, while the presented estimation results concede little doubt about the higher degree of robustness in the high-inflation period, this is not necessarily attributable to money illusion alone. As a matter of fact, the indirect measure of money illusion adopted in this chapter does not exclude that the difference in robustness is due to other model distortions that occurred during the selected high-inflation subperiods. However, there is little doubt that a non-negligible part of the robustified misspecifications stem from inflation-related phenomena, especially when HF is directly controlled for in the model (see Section 5.2.1). Moreover, note that neither HST nor Hansen and Sargent (2008) ever provided any details regarding the source of the small misspecifications the robust consumer wants to hedge against. In this respect, the present analysis provides a first explanation by showing that robustness is closely related to inflation-related model uncertainty, which can be interpreted as money illusion whenever the consumer changes her consumption behavior in response to inflation (all other things held constant).

Another criticism one could level against the argument that robustness is more pronounced in the high-inflation subperiod is that the difference in the estimated β s in Table 5.1 are relatively small (and, thus, negligible) between the selected subpe-

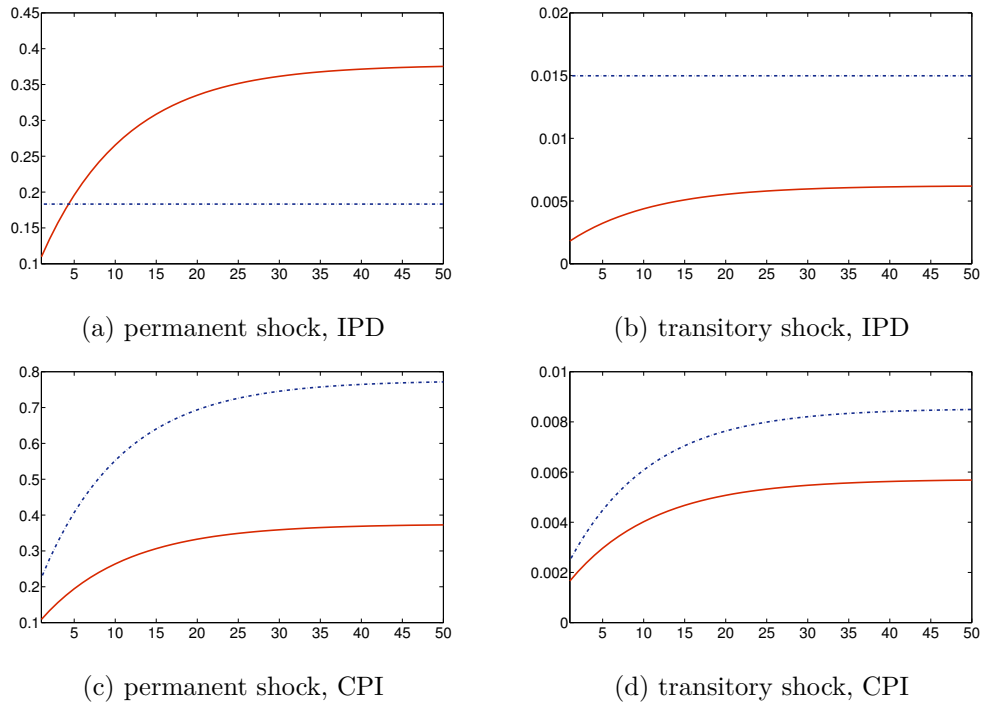


Figure 5.4: Impulse Response Function of Consumption to Income Shocks

Impulse response functions of consumption to shocks in persistent (\bar{y}) and transitory (\hat{y}) income. The red (solid) and blue (dash-dot) lines correspond to the preferred models in the high- and low-inflation subperiods, respectively. The X and Y axes represent, respectively, the number of periods and the consumption response in log deviations from the equilibrium.

riods. The fact that the differences in the subjective factor have, on the contrary, a large impact on consumption can be seen in Figure 5.4, which draws, for all preferred models, the IRFs of consumption to innovations in both components of the income process. While the absolute values of the consumption responses are not important for the present analysis, their relative values and shapes perfectly exemplify the insights that can be gained from the estimation of HST's robust consumption model.

Figure 5.4 depicts three features that are particularly noteworthy: two pertaining to the traditional permanent income theory and one *new* feature highlighting the effect of money illusion on consumption. First, looking at the IRFs for the IPD models in Figures 5.4a and 5.4b, we notice the different shapes of the high-inflation model (solid line) and the low-inflation model (dash-dot line). This difference is due to the fact that the preferred IPD model for the high-inflation period incorporates

HF, whereas the preferred model for the low-inflation period is a simple REPIH model. As we saw in Section 5.3.3, the IRF of the purely rational consumer is always flat because she immediately and fully adjusts her consumption after an income shock, such that her consumption instantly achieves its new long-term value. The consumer presenting some degree of HF, however, partially bases her current consumption on her past consumption, which explains the positive slope of the IRF induced by both permanent and transitory income shocks.

With regard to the shape of the IRF of consumption to income shocks, it is also interesting to highlight the apparently antagonistic predictions of HST's robust consumption model in Figure 5.4 and of the closed-form RCPIH model depicted in Figure 5.3. Due to the observational equivalence proposition, the former model implies IRFs that have exactly the same shape as the standard REPIH model (i.e., increasing under HF), whereas the latter model predicts a decreasing response of consumption over time. In fact, both models are fully consistent in that they both theoretically predict and empirically confirm that a concern for robustness activates precautionary savings on the part of the consumer.

The second feature that can be observed in Figure 5.4 is that the immediate consumption responses are much higher after permanent shocks than after transitory shocks. Comparing Figure 5.4a with Figure 5.4b and Figure 5.4c with Figure 5.4d, we notice that the persistent income shock contributes much more to consumption fluctuations than does the transitory shock. This is totally in line with the original idea of Friedman (1956) (see Section 2.2) and perfectly illustrates one central assumption made in Chapter 4: namely, that aggregate consumption is essentially affected by persistent shocks in aggregate income.

The last and most novel observation is the difference in the IRFs between the preferred models for the low-inflation subperiod (dash-dot line) and the high-inflation subperiod (solid line). Comparing the immediate response of consumption between the subperiods clearly shows that the consumption response is always lower in the high-inflation subperiod than in the low-inflation period. This result is particularly robust because it is valid for both shocks and for both inflation measures. Looking at the IRFs for the shocks with the greatest impact (i.e., the permanent income shocks in Figures 5.4a and 5.4c), we see that the values of the immediate consumption responses to the shocks are only approximately half as important in the high-inflation period as in the low-inflation period. This large difference between the subperiods clearly shows that the impact of robustness on consumption is much more pronounced than could be read from the relatively small differences in the

estimated β s in Table 5.1. In fact, even small deviations in the subjective discount factor can have large impacts on consumption.

The fact that the consumption responses to income shocks are significantly lower in the period with high average inflation rates than in the period with low average inflation rates allows us to draw two conclusions. First, robustness has a large negative impact on consumption and, hence, induces precautionary savings. This result summarizes the main findings by Luo (2008) and HST and can be explained by the fact that the robust consumer is more prudent than the rational one and reduces her consumption to prevent potential losses due to misspecifications.

Second, emphasizing the fact that at least some misspecifications are due to money illusion, the presented estimation results suggest that the robust consumer and the rational consumer are, in fact, the same money-illusioned consumer, whose degree of robustness varies with the inflation level. Following this line of thought, the money-illusioned consumer does not fear money illusion in the subperiod with low inflation rates and behaves according to the standard REPIH. During the period with high inflation rates, however, the same consumer fears that she might confuse nominal values for real values and decides to increase her savings in order to hedge against the potentially important negative impact of her money illusion. In light of the large negative effect of robustness on consumption during the high-inflation subperiod, the main insight to be gained from the empirical analysis of this chapter is that money illusion has a large negative impact on consumption during periods when inflation is particularly high.

5.5 Conclusion

This chapter has shown that money illusion can be effectively modeled as a form of misspecification imposed on the benchmark REPIH model. Contrary to Chapters 3 and 4, which consider different types of consumers that suffer from the distortions caused by their own money illusions, this chapter assumes that the money-illusioned consumer is aware of her confusion between nominal and real values and is able to fully account for it in her consumption behavior. To efficiently account for money illusion, this *robust* consumer considers a distorted version of the benchmark model in which the distortion is proportional to the degree of money illusion from which she assumes she is suffering.

To analyze the implications for the consumption of this robustness against money illusion, two different models are considered: a closed-form RCPIH model suggested

by Luo (2008) and a robust DSGE model developed by Hansen et al. (1999). Both models theoretically predict that robustness increases the consumer's precautionary savings, which can be interpreted as the consumer's buffer against the distortions engendered by her money illusion. Furthermore, despite certain empirical limitations of the RCPIH model, both robust consumption models empirically show that robustness is more pronounced in the selected subperiod characterized by high average inflation rates than in the subperiod with low inflation rates. Since the consumer only seeks robustness when she might, indeed, suffer from money illusion, this result suggests that money illusion is particularly pronounced when inflation is relatively high. Moreover, the estimations of the robust model by Hansen et al. (1999) demonstrate that the negative impact of robustness on consumption is very large during the high-inflation subperiod, revealing that money illusion has a potentially large negative impact on consumption.

In showing that the degree of robustness against money illusion is not time invariant and depends on the level of inflation, the estimation results suggest that the robust consumption model is a potentially more appropriate consumption model than the restrictive REPIH model, which is used as a benchmark throughout this thesis. Since the two models are nested, however, they coincide when inflation uncertainty is particularly low, as is the case in the selected low-inflation subperiod.

Despite these encouraging results, a major drawback of using the robust consumption model to analyze the impact of money illusion on consumption is that we address money illusion only indirectly as a source of model misspecification and distortion to the REPIH model. On one hand, the subperiods are selected in such a way as to ensure that they are characterized by two distinct inflationary environments (i.e., one period with high and volatile inflation rates, the other one with low and stable inflation rates). Further, recall that the inflation extraction problem estimated in Chapter 4 suggests that the money-illusioned consumer has a greater consumption smoothing in the high-inflation period. Moreover, all specification errors and sources of uncertainty not related to inflation are, typically, already captured by the HF coefficient, whenever HF is explicitly controlled for in the model. For example, it is plausible to assume that the increase in the HF coefficient during the high-inflation subperiods already captures the higher volatility in real GDP or the increased unemployment rates that prevailed during the 1970s in the U.S. These elements allow us to identify inflation uncertainty as the source of misspecification and to interpret robustness as a way to hedge against money illusion. Consequently, money illusion can be seen as providing an economically plausible justification for

the desire for robustness, which is justified only through statistical arguments in the robust models of [Hansen et al. \(1999\)](#) and [Luo \(2008\)](#).

On the other hand, we cannot exclude the possibility that the higher robustness in the high-inflation period reflects other model misspecifications that are related to inflation, but independent of money illusion. For example, the selected high-inflation superperiod was subject to important surges in the oil prices, which could have fostered the consumer's fear of misspecification during that period. Since it is unrealistic to expect to find at least two periods – or two countries – that have distinct inflation rates, but for which all other potential sources of uncertainty are the same, the robust consumption model could be improved in such a way that would allow us to better isolate the effect of money illusion from those of other distortions. For example, an interesting extension to the models analyzed in this chapter would be to use a *robust* Kalman filter to try to extract the inflation uncertainty from the other uncertainties. This challenging extension, described in detail in [Hansen and Sargent \(2008, Ch. 5, 17, 18\)](#), is left for future research.

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Appendix A

Data Description and Properties

A.1 U.S. Data Description

The estimations for the U.S. use quarterly time series, which have been calculated as the averages of the three monthly values within each specific quarter. The considered total sample period is from 1959Q1 to 2012Q1. Consumption (c_t) is measured as real personal consumption expenditures of nondurables and services per capita. Aggregate income (y_t) is measured as real disposable personal income per capita. The wealth variable (a_t) represents the ratio of household net assets to permanent income and is calculated as the ratio of financial assets to disposable income. Two distinct price indexes are used for the variable p_t capturing inflation: namely, the implicit price deflator for the consumption of non-durables and services (IPD) and the consumer price index (CPI). All the utilized time series are seasonally adjusted and expressed in logarithms. Since the series are seasonally adjusted, the growth rates are calculated as the difference from one quarter to the previous one.

Data for consumption, income, wealth and the IPD originate from the U.S. National Income and Product Accounts. The CPI is published by the U.S. Bureau of Labor Statistics. All original series are downloadable from the Federal Reserve Economic Data at <http://research.stlouisfed.org/fred2>.

A.2 Properties

Table A.1 gives the relevant Augmented Dickey-Fuller (ADF) test statistics for both the IPD-deflated and the CPI-deflated time series. In brackets are the *p-values* for the null hypothesis of the presence of a unit root.

Table A.1: Unit Root Tests of Selected U.S. Time Series

	c	Δc	y	Δy	a	Δa	p	Δp
IPD	-0.50 [0.98]	-5.93 [0.00]	0.17 [1.00]	-13.62 [0.00]	-1.01 [0.94]	-12.39 [0.00]	-1.45 [0.84]	-5.40 [0.00]
CPI	-2.36 [0.40]	-5.26 [0.00]	-1.55 [0.81]	-14.23 [0.00]	-0.73 [0.97]	-12.41 [0.00]	-1.13 [0.92]	-2.97 [0.04]

The statistics have been calculated over the entire sample period from 1959Q1 to 2012Q1. Small caps represent the variables expressed in logarithms. The difference operator Δ is the one-quarter log difference. “IPD” uses the the implicit price deflator for p and for the deflating of all other series, whereas “CPI” is based on the consumer price index. The reported statistics are the ADF test statistics, based on the Akaike information criterion, which include constants and linear trends for the series in levels and the constants only for the differenced series only. The numbers in square brackets are the [MacKinnon \(1996\)](#) one-sided p -values.

Looking at the first column for each variable, it appears that the null hypothesis of no unit root is strongly rejected for all series c_t , y_t , a_t and p_t . The considered time series are, thus, clearly non-stationary in their log-levels. However, all series are stationary (AR) processes in their one-quarter log-differences, since the null hypothesis is not rejected in the second lines. Consequently, to avoid any problems related to non-stationarity, all estimations throughout the thesis are based on the first-difference detrended time series Δc_t , Δy_t , Δa_t and Δp_t .

Appendix B

Estimations of the Inflation-Augmented RW Model

Table B.1: Linear IPD-Inflation Effect – Whole Sample

$$\Delta c_t = \alpha + \sum_{i=0}^L \lambda_i L^i (\Delta p_t) + \varepsilon_t$$

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)
α	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)
λ_0	-0.71*** (0.03)	-0.71*** (0.03)	-0.71*** (0.03)	-0.71*** (0.04)	-0.71*** (0.04)	-0.71*** (0.03)	-0.70*** (0.03)	-0.70*** (0.03)	-0.69*** (0.03)	-
λ_1	0.07 (0.03)	0.07*** (0.03)	0.07*** (0.03)	0.07*** (0.03)	0.07*** (0.03)	0.08*** (0.03)	0.08*** (0.03)	0.08*** (0.02)	0.08*** (0.02)	-
λ_2	0.07*** (0.03)	0.07*** (0.03)	0.07*** (0.03)	0.08*** (0.03)	0.08*** (0.03)	0.09*** (0.03)	0.09*** (0.03)	0.10*** (0.03)	0.10*** (0.03)	-
λ_3	-0.02 (0.04)	-0.02 (0.04)	-0.02 (0.04)	-0.02 (0.04)	-0.01 (0.04)	-0.01 (0.04)	-0.00 (0.03)	-0.01 (0.03)	-	-
λ_4	0.03 (0.03)	0.03 (0.03)	0.03 (0.03)	0.04 (0.03)	0.04 (0.03)	0.05* (0.03)	0.04 (0.03)	-	-	-
λ_5	0.03 (0.03)	0.03 (0.03)	0.03 (0.03)	0.04 (0.03)	0.04 (0.03)	0.04 (0.03)	-	-	-	-
λ_6	0.03 (0.03)	0.03 (0.03)	0.03 (0.03)	0.03 (0.03)	0.04 (0.03)	-	-	-	-	-
λ_7	0.02 (0.04)	0.02 (0.04)	0.02 (0.04)	0.02 (0.03)	-	-	-	-	-	-
λ_8	0.05 (0.03)	0.05* (0.03)	0.05 (0.03)	-	-	-	-	-	-	-
λ_9	0.04 (0.03)	0.03 (0.03)	-	-	-	-	-	-	-	-
λ_{10}	0.02 (0.03)	-	-	-	-	-	-	-	-	-
ε_{t-1}	0.53*** (0.07)	0.53*** (0.09)	0.51*** (0.07)	0.50*** (0.08)	0.50*** (0.08)	0.51*** (0.08)	0.49*** (0.08)	0.47*** (0.07)	0.46*** (0.07)	0.29*** (0.09)
\bar{R}^2	0.75	0.74	0.74	0.74	0.74	0.73	0.73	0.72	0.72	0.07
AIC	-7.87	-7.86	-7.85	-7.84	-7.85	-7.83	-7.82	-7.80	-7.81	-6.64
BIC	-7.65	-7.66	-7.67	-7.68	-7.71	-7.70	-7.70	-7.71	-7.73	-6.61
DW	1.77	1.80	1.77	1.77	1.74	1.76	1.74	1.75	1.72	2.02
F_p	50.48	54.06	58.86	63.98	72.27	80.40	91.47	106.52	133.07	16.42
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

The presented OLS estimates are calculated for quarterly U.S. data over the sample from 1959Q1 to 2012Q1. They rely on the IPD both as the deflator and as the inflation measure. In the equation, small caps denote the variables expressed in logarithms and L denotes the lag operator. {*, **, ***} indicate statistical significance of the coefficient at the {10, 5, 1} percent levels. In parenthesis are the estimated Newey-West heteroscedasticity and autocorrelation consistent (HAC) standard errors. The coefficient for ε_{t-1} is the estimated first-order moving-average (MA) coefficient of the error term in the equation. If it lies inside the unit circle, the MA(1) process is stationary. \bar{R}^2 , AIC, BIC and DW denote, respectively, the adjusted coefficient of determination, the Akaike information criterion, the Schwarz criterion and the Durbin-Watson statistic. F_p gives the statistic for the F-test of joint significance of the coefficients, while the number in squared brackets is the corresponding p-value. The highlighted model is the one chosen for the analysis and for Tables (3.1) and (3.2) in the text.

Table B.2: Linear CPI-Inflation Effect – Whole Sample

	$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \varepsilon_t$								
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)
α	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.00*** (0.00)
λ_0	-0.40*** (0.09)	-0.40*** (0.09)	-0.40*** (0.09)	-0.41*** (0.10)	-0.42*** (0.10)	-0.40*** (0.10)	-0.39*** (0.10)	-0.38*** (0.09)	-
λ_1	0.03 (0.09)	0.03 (0.09)	0.03 (0.09)	0.02 (0.08)	0.03 (0.08)	0.03 (0.08)	0.02 (0.08)	0.04 (0.07)	-
λ_2	0.08 (0.06)	0.08 (0.06)	0.08 (0.06)	0.10* (0.06)	0.10* (0.06)	0.11* (0.06)	0.12** (0.06)	0.15** (0.06)	-
λ_3	-0.17** (0.08)	-0.18** (0.08)	-0.19** (0.08)	-0.18** (0.09)	-0.17** (0.09)	-0.17** (0.08)	-0.17** (0.08)	-0.15** (0.07)	-
λ_4	0.05 (0.07)	0.06 (0.07)	0.07 (0.07)	0.09* (0.07)	0.09* (0.07)	0.10 (0.07)	0.10 (0.07)	-	-
λ_5	-0.00 (0.06)	-0.01 (0.06)	-0.01 (0.06)	0.03 (0.06)	0.03 (0.06)	0.04 (0.07)	-	-	-
λ_6	-0.03 (0.07)	-0.02 (0.07)	-0.01 (0.06)	0.03 (0.07)	0.03 (0.06)	-	-	-	-
λ_7	-0.02 (0.07)	-0.02 (0.07)	-0.02 (0.07)	0.01 (0.07)	-	-	-	-	-
λ_8	0.13 (0.08)	0.15* (0.08)	0.15* (0.08)	-	-	-	-	-	-
λ_9	0.02 (0.06)	0.03 (0.06)	-	-	-	-	-	-	-
λ_{10}	0.04 (0.05)	-	-	-	-	-	-	-	-
ε_{t-1}	0.47*** (0.07)	0.47*** (0.07)	0.45*** (0.07)	0.45*** (0.08)	0.45*** (0.08)	0.45*** (0.08)	0.44*** (0.07)	0.43*** (0.08)	0.41*** (0.07)
\bar{R}^2	0.38	0.37	0.37	0.35	0.35	0.35	0.35	0.34	0.16
AIC	-7.92	-7.91	-7.91	-7.90	-7.91	-7.91	-7.91	-7.90	-7.70
BIC	-7.70	-7.71	-7.73	-7.73	-7.76	-7.78	-7.80	-7.80	-7.66
DW	1.84	1.87	1.84	1.84	1.83	1.83	1.84	1.84	1.92
FP	11.06	11.75	12.79	13.29	15.05	17.05	19.54	22.18	42.68
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

All the notes for Table B.1 apply to this table except for the inflation measure (here: CPI).

Table B.3: Linear IPD-Inflation Effect – High-Inflation Subperiod

	$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i(\Delta p_t) + \varepsilon_t$													
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	(xii)	(xiii)	(xiv)
α	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.02*** (0.00)	0.02*** (0.00)	0.02*** (0.00)	0.02*** (0.00)	0.02*** (0.00)	0.02*** (0.00)	0.02*** (0.00)	0.02*** (0.00)
λ_0	-0.74*** (0.08)	-0.72*** (0.08)	-0.73*** (0.08)	-0.73*** (0.08)	-0.73*** (0.07)	-0.73*** (0.07)	-0.76*** (0.07)	-0.76*** (0.08)	-0.78*** (0.08)	-0.74*** (0.09)	-0.72*** (0.07)	-0.72*** (0.07)	-0.72*** (0.07)	-0.72*** (0.07)
λ_1	0.02 (0.09)	0.01 (0.09)	0.00 (0.09)	0.00 (0.09)	0.00 (0.09)	-0.01 (0.09)	0.00 (0.09)	0.02 (0.09)	0.00 (0.09)	0.05 (0.08)	0.05 (0.08)	0.05 (0.08)	0.05 (0.08)	0.05 (0.08)
λ_2	0.12 (0.09)	0.11 (0.09)	0.11 (0.09)	0.11 (0.08)	0.11 (0.08)	0.11 (0.08)	0.14* (0.08)	0.17** (0.07)	0.14* (0.07)	0.14* (0.07)	0.14* (0.07)	0.14* (0.07)	0.14* (0.07)	0.14* (0.07)
λ_3	-0.15* (0.07)	-0.16* (0.08)	-0.16*** (0.07)	-0.16*** (0.07)	-0.16*** (0.07)	-0.15* (0.07)	-0.12 (0.08)	-0.10 (0.07)	-0.10 (0.08)	-0.10 (0.07)	-0.10 (0.07)	-0.10 (0.07)	-0.10 (0.07)	-0.10 (0.07)
λ_4	0.07 (0.08)	0.08 (0.09)	0.07 (0.08)	0.07 (0.08)	0.07 (0.07)	0.08 (0.07)	0.10 (0.07)	0.10 (0.07)	0.10 (0.07)	0.10 (0.07)	0.10 (0.07)	0.10 (0.07)	0.10 (0.07)	0.10 (0.07)
λ_5	0.09 (0.08)	0.09 (0.08)	0.08 (0.08)	0.09 (0.06)	0.09 (0.06)	0.10* (0.05)	0.09 (0.06)	0.09 (0.06)	0.09 (0.06)	0.09 (0.06)	0.09 (0.06)	0.09 (0.06)	0.09 (0.06)	0.09 (0.06)
λ_6	0.00 (0.08)	0.00 (0.08)	0.00 (0.08)	0.03 (0.07)	0.03 (0.07)	0.03 (0.08)	0.03 (0.08)	0.03 (0.08)	0.03 (0.08)	0.03 (0.08)	0.03 (0.08)	0.03 (0.08)	0.03 (0.08)	0.03 (0.08)
λ_7	-0.05 (0.10)	-0.02 (0.10)	0.00 (0.10)	0.00 (0.08)	0.00 (0.08)	0.00 (0.08)	0.00 (0.08)	0.00 (0.08)	0.00 (0.08)	0.00 (0.08)	0.00 (0.08)	0.00 (0.08)	0.00 (0.08)	0.00 (0.08)
λ_8	-0.03 (0.13)	0.00 (0.12)	0.02 (0.12)	0.02 (0.12)	0.02 (0.12)	0.02 (0.12)	0.02 (0.12)	0.02 (0.12)	0.02 (0.12)	0.02 (0.12)	0.02 (0.12)	0.02 (0.12)	0.02 (0.12)	0.02 (0.12)
λ_9	0.05 (0.06)	0.07 (0.06)	0.07 (0.06)	0.07 (0.06)	0.07 (0.06)	0.07 (0.06)	0.07 (0.06)	0.07 (0.06)	0.07 (0.06)	0.07 (0.06)	0.07 (0.06)	0.07 (0.06)	0.07 (0.06)	0.07 (0.06)
λ_{10}	0.11* (0.06)	0.11* (0.06)	0.11* (0.06)	0.11* (0.06)	0.11* (0.06)	0.11* (0.06)	0.11* (0.06)	0.11* (0.06)	0.11* (0.06)	0.11* (0.06)	0.11* (0.06)	0.11* (0.06)	0.11* (0.06)	0.11* (0.06)
ε_{t-1}	0.44*** (0.09)	0.46*** (0.10)	0.48*** (0.09)	0.48*** (0.10)	0.48*** (0.10)	0.48*** (0.10)	0.49*** (0.10)	0.51*** (0.11)	0.48*** (0.10)	0.44*** (0.10)	0.46*** (0.10)	0.46*** (0.10)	0.46*** (0.10)	0.46*** (0.10)
R^2	0.75	0.75	0.75	0.75	0.76	0.76	0.76	0.76	0.76	0.75	0.75	0.75	0.75	0.75
AIC	-7.92	-7.91	-7.93	-7.96	-8.00	-8.02	-8.03	-8.03	-8.04	-8.01	-8.04	-8.04	-8.04	-8.04
BIC	-7.47	-7.51	-7.56	-7.62	-7.69	-7.75	-7.83	-7.83	-7.87	-7.88	-7.88	-7.94	-7.94	-7.94
DW	1.97	1.96	1.95	1.94	1.94	1.95	1.93	1.91	1.93	1.99	1.99	1.99	1.99	1.99
F_p	16.67	17.82	19.65	22.22	25.47	29.56	34.00	40.03	49.20	61.86	61.86	93.57	93.57	93.57
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

All the notes for Table B.1 apply to this table except for the sample period (here: from 1966Q1 to 1981Q3).

Table B.4: Linear CPI-Inflation Effect – High-Inflation Subperiod

	$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \varepsilon_t$										
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)
α	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)
λ_0	-0.66*** (0.14)	-0.64*** (0.14)	-0.63*** (0.14)	-0.62*** (0.13)	-0.62*** (0.12)	-0.62*** (0.13)	-0.62*** (0.12)	-0.66*** (0.12)	-0.73*** (0.10)	-0.68*** (0.12)	-0.66*** (0.09)
λ_1	-0.11 (0.19)	-0.11 (0.19)	-0.14 (0.19)	-0.14 (0.19)	-0.14 (0.19)	-0.13 (0.18)	-0.12 (0.19)	-0.04 (0.17)	-0.05 (0.17)	0.04 (0.12)	-
λ_2	0.26* (0.14)	0.25* (0.14)	0.22 (0.14)	0.22 (0.14)	0.22 (0.13)	0.25* (0.13)	0.24 (0.15)	0.25* (0.14)	0.17 (0.15)	-	-
λ_3	-0.18 (0.17)	-0.20 (0.16)	-0.20 (0.17)	-0.20 (0.17)	-0.20 (0.17)	-0.26* (0.15)	-0.26* (0.15)	-0.18 (0.13)	-	-	-
λ_4	0.19 (0.18)	0.20 (0.18)	0.19 (0.17)	0.19 (0.17)	0.20 (0.15)	0.18 (0.14)	0.16 (0.12)	-	-	-	-
λ_5	0.07 (0.12)	0.06 (0.12)	0.07 (0.12)	0.04 (0.12)	0.05 (0.13)	-0.02 (0.15)	-	-	-	-	-
λ_6	-0.20 (0.16)	-0.20 (0.15)	-0.13 (0.13)	-0.13 (0.12)	-0.13 (0.12)	-	-	-	-	-	-
λ_7	-0.03 (0.19)	0.00 (0.16)	0.04 (0.16)	0.01 (0.14)	-	-	-	-	-	-	-
λ_8	-0.17 (0.24)	-0.16 (0.24)	-0.05 (0.22)	-	-	-	-	-	-	-	-
λ_9	0.15 (0.15)	0.20 (0.13)	-	-	-	-	-	-	-	-	-
λ_{10}	0.10 (0.16)	-	-	-	-	-	-	-	-	-	-
ε_{t-1}	0.41*** (0.11)	0.42*** (0.11)	0.42*** (0.11)	0.41*** (0.11)	0.41*** (0.11)	0.40*** (0.12)	0.41*** (0.12)	0.43*** (0.12)	0.42*** (0.12)	0.40*** (0.12)	0.40*** (0.09)
R^2	0.55	0.55	0.55	0.55	0.56	0.56	0.57	0.57	0.56	0.56	0.56
AIC	-7.79	-7.81	-7.81	-7.84	-7.87	-7.89	-7.92	-7.92	-7.92	-7.92	-7.95
BIC	-7.36	-7.42	-7.45	-7.51	-7.58	-7.63	-7.69	-7.73	-7.76	-7.79	-7.85
DW	1.94	1.95	1.92	1.92	1.92	1.91	1.91	1.91	1.91	1.97	1.97
F_P	7.62	8.38	8.97	10.11	11.57	13.17	15.62	18.31	22.13	28.48	43.28
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

All the notes for Table B.1 apply to this table except for the inflation measure (here: CPI) and the sample period (here: from 1966Q1 to 1982Q3).

Table B.5: Linear IPD-Inflation Effect – Low-Inflation Subperiod

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \varepsilon_t$$

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
α	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)
λ_0	-0.62*** (0.10)	-0.65*** (0.10)	-0.66*** (0.09)	-0.64*** (0.11)	-0.67*** (0.09)	-0.67*** (0.08)
λ_1	0.11 (0.06)	0.13** (0.06)	0.16** (0.06)	0.19** (0.07)	0.22*** (0.07)	0.21*** (0.07)
λ_2	-0.10 (0.07)	-0.12* (0.06)	-0.09 (0.06)	-0.13** (0.06)	-0.11* (0.05)	-0.11* (0.05)
λ_3	0.12* (0.06)	0.10 (0.06)	0.07 (0.05)	0.05 (0.06)	0.05 (0.06)	0.08* (0.04)
λ_4	-0.21*** (0.05)	-0.18*** (0.05)	-0.21*** (0.06)	-0.20*** (0.07)	-0.26*** (0.07)	-0.30*** (0.05)
λ_5	0.02 (0.04)	0.05 (0.06)	0.05 (0.05)	0.11* (0.06)	0.17*** (0.06)	0.17*** (0.06)
λ_6	0.09* (0.05)	0.09* (0.05)	0.14*** (0.04)	0.07** (0.03)	0.08** (0.03)	0.06* (0.03)
λ_7	0.00 (0.07)	-0.04 (0.06)	-0.09 (0.05)	-0.10* (0.05)	-0.07 (0.05)	–
λ_8	0.10 (0.06)	0.14** (0.06)	0.14** (0.06)	0.11* (0.06)	–	–
λ_9	-0.11*** (0.04)	-0.10** (0.04)	-0.13*** (0.04)	–	–	–
λ_{10}	-0.11*** (0.04)	-0.09** (0.04)	–	–	–	–
λ_{11}	0.10* (0.06)	–	–	–	–	–
ε_{t-1}	0.17 (0.16)	0.17 (0.15)	0.10 (0.20)	0.21 (0.13)	0.11 (0.15)	0.05 (0.18)
\bar{R}^2	0.74	0.72	0.71	0.68	0.66	0.66
AIC	-9.12	-9.07	-9.05	-8.97	-8.93	-8.94
BIC	-8.54	-8.53	-8.55	-8.51	-8.51	-8.57
DW	1.90	1.93	1.95	2.00	2.00	1.98
F_P	9.59 [0.00]	9.50 [0.00]	9.82 [0.00]	9.44 [0.00]	9.62 [0.00]	10.68 [0.00]

All the notes for Table B.1 apply to this table except for the sample period (here: from 1991Q1 to 2001Q1).

Table B.6: Linear CPI-Inflation Effect – Low-Inflation Subperiod

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \varepsilon_t$$

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)
α	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)
λ_0	-0.34 (0.24)	-0.33 (0.24)	-0.34 (0.22)	-0.41* (0.20)	-0.44** (0.21)	-0.43** (0.20)	-0.52*** (0.19)
λ_1	0.25 (0.21)	0.24 (0.23)	0.30 (0.25)	0.37 (0.27)	0.33 (0.27)	0.33 (0.26)	0.23 (0.20)
λ_2	-0.48** (0.17)	-0.42** (0.17)	-0.51*** (0.17)	-0.48*** (0.18)	-0.48** (0.19)	-0.49** (0.21)	-0.34** (0.14)
λ_3	0.20 (0.18)	0.14 (0.20)	0.10 (0.18)	0.12 (0.18)	0.21 (0.16)	0.23 (0.16)	0.24 (0.16)
λ_4	-0.57*** (0.15)	-0.58*** (0.16)	-0.58*** (0.15)	-0.70*** (0.18)	-0.83*** (0.23)	-0.82*** (0.22)	-0.70*** (0.14)
λ_5	0.01 (0.16)	-0.01 (0.17)	0.10 (0.18)	0.28 (0.22)	0.28 (0.24)	0.29 (0.27)	—
λ_6	0.27 (0.17)	0.34* (0.17)	0.13 (0.16)	0.14 (0.19)	0.04 (0.16)	—	—
λ_7	-0.20 (0.17)	-0.31* (0.17)	-0.34* (0.17)	-0.22 (0.19)	—	—	—
λ_8	0.42* (0.23)	0.41 (0.24)	0.30 (0.18)	—	—	—	—
λ_9	-0.30** (0.15)	-0.37** (0.17)	—	—	—	—	—
λ_{10}	-0.19 (0.12)	—	—	—	—	—	—
ε_{t-1}	0.16 (0.14)	0.14 (0.14)	0.24** (0.11)	0.13 (0.14)	0.07 (0.19)	0.07 (0.18)	0.09 (0.16)
\bar{R}^2	0.45	0.45	0.39	0.37	0.36	0.38	0.35
AIC	-8.73	-8.74	-8.65	-8.62	-8.63	-8.68	-8.66
BIC	-8.19	-8.24	-8.19	-8.21	-8.26	-8.34	-8.36
DW	1.94	1.93	1.99	1.98	1.97	1.97	1.97
F_P	3.77 [0.00]	3.99 [0.00]	3.60 [0.00]	3.57 [0.00]	3.81 [0.00]	4.48 [0.00]	4.60 [0.00]

All the notes for Table B.1 apply to this table except for the inflation measure (here: CPI) and the sample period (here: from 1991Q1 to 2001Q1).

Table B.7: Nonlinear IPD-Inflation Effect – Whole Sample

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \sum_{j=0}^J \lambda_j^* L^j (\Delta p_t)^2 + \varepsilon_t$$

	(i*)	(ii*)	(iii*)	(iv*)	(v*)	(vi*)
α	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)
λ_0	-0.69*** (0.03)	-0.68*** (0.03)	-0.69*** (0.03)	-0.69*** (0.03)	-0.69*** (0.03)	-0.69*** (0.03)
λ_1	0.10*** (0.03)	0.10*** (0.03)	0.10*** (0.03)	0.10*** (0.03)	0.11*** (0.03)	0.10*** (0.02)
λ_2	0.10*** (0.03)	0.10*** (0.03)	0.10*** (0.03)	0.11*** (0.03)	0.11*** (0.03)	0.11*** (0.02)
λ_3	-0.00 (0.03)	0.00 (0.03)	0.01 (0.03)	0.01 (0.03)	0.01 (0.03)	0.01 (0.03)
λ_4	0.07** (0.03)	0.08** (0.03)	0.08*** (0.03)	0.08*** (0.03)	0.09*** (0.03)	0.09*** (0.03)
λ_5	0.07*** (0.02)	0.07*** (0.02)	0.07*** (0.02)	0.07*** (0.02)	0.07*** (0.02)	0.06*** (0.02)
λ_6	0.05** (0.02)	0.05** (0.02)	0.06** (0.02)	0.06** (0.03)	0.06** (0.03)	0.06** (0.03)
λ_7	0.04 (0.04)	0.04 (0.04)	0.04 (0.04)	0.04 (0.04)	—	—
λ_8	0.04 (0.03)	0.04 (0.03)	0.04 (0.03)	—	—	—
λ_9	0.02 (0.03)	0.02 (0.03)	—	—	—	—
λ_{10}	0.02 (0.03)	—	—	—	—	—
λ_0^*	-1.98*** (0.53)	-1.94*** (0.53)	-2.10*** (0.50)	-2.36*** (0.51)	-2.31*** (0.50)	-2.33*** (0.49)
λ_1^*	-1.89** (0.88)	-1.90** (0.84)	-2.06** (0.82)	-2.06*** (0.79)	-2.01*** (0.74)	-2.17*** (0.72)
λ_2^*	-1.23** (0.57)	-1.33** (0.54)	-1.28** (0.52)	-1.16** (0.47)	-1.17** (0.47)	-1.28*** (0.48)
λ_3^*	-2.51*** (0.64)	-2.51*** (0.64)	-2.40*** (0.61)	-2.36*** (0.63)	-2.21*** (0.65)	-2.07*** (0.67)
λ_4^*	-1.21** (0.57)	-1.15** (0.56)	-1.12** (0.56)	-1.00* (0.57)	-0.81 (0.62)	—
ε_{t-1}	0.46*** (0.07)	0.47*** (0.07)	0.45*** (0.07)	0.44*** (0.08)	0.44*** (0.08)	0.44*** (0.08)
\bar{R}^2	0.77	0.76	0.76	0.76	0.76	0.76
AIC	-7.94	-7.92	-7.92	-7.92	-7.92	-7.92
BIC	-7.64	-7.65	-7.66	-7.67	-7.69	-7.72
DW	1.84	1.88	1.84	1.84	1.81	1.81
F_P	40.40 [0.00]	41.97 [0.00]	44.68 [0.00]	47.04 [0.00]	50.29 [0.00]	54.45 [0.00]

All the notes for Table B.1 apply to this table.

Table B.8: Nonlinear CPI-Inflation Effect – Whole Sample

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \sum_{j=0}^J \lambda_j^* L^j (\Delta p_t)^2 + \varepsilon_t$$

	(i*)	(ii*)	(iii*)	(iv*)	(v*)	(vi*)	(vii*)
α	0.00*** (0.00)	0.00*** (0.00)	0.00*** (0.00)	0.00*** (0.00)	0.00*** (0.00)	0.00*** (0.00)	0.00*** (0.00)
λ_0	-0.22*** (0.05)	-0.21*** (0.05)	-0.21*** (0.05)	-0.20*** (0.05)	-0.20*** (0.05)	-0.19*** (0.05)	-0.18*** (0.05)
λ_1	0.23*** (0.07)	0.24*** (0.07)	0.24*** (0.06)	0.24*** (0.06)	0.24*** (0.06)	0.24*** (0.06)	0.23*** (0.06)
λ_2	0.18*** (0.07)	0.18*** (0.07)	0.19*** (0.07)	0.20*** (0.06)	0.20*** (0.06)	0.21*** (0.06)	0.22*** (0.06)
λ_3	-0.09 (0.08)	-0.09 (0.08)	-0.11 (0.08)	-0.10 (0.08)	-0.09 (0.08)	-0.10 (0.08)	-0.10 (0.07)
λ_4	0.12* (0.06)	0.13** (0.06)	0.14** (0.06)	0.15** (0.07)	0.14** (0.07)	0.14** (0.07)	0.14** (0.07)
λ_5	0.01 (0.05)	0.00 (0.05)	0.01 (0.05)	0.01 (0.06)	0.01 (0.06)	0.02 (0.06)	—
λ_6	-0.03 (0.05)	-0.02 (0.05)	-0.02 (0.06)	0.00 (0.06)	0.00 (0.06)	—	—
λ_7	-0.01 (0.07)	-0.01 (0.07)	-0.02 (0.07)	-0.00 (0.07)	—	—	—
λ_8	0.08 (0.08)	0.10 (0.08)	0.09 (0.08)	—	—	—	—
λ_9	-0.04 (0.06)	-0.03 (0.06)	—	—	—	—	—
λ_{10}	0.04 (0.05)	—	—	—	—	—	—
λ_0^*	-10.21*** (2.84)	-10.19*** (2.80)	-10.06*** (2.84)	-11.16*** (2.65)	-11.23*** (2.66)	-11.31*** (2.54)	-11.37*** (2.47)
λ_1^*	-9.85*** (3.76)	-10.19*** (3.67)	-9.83*** (3.57)	-9.90*** (3.42)	-9.93*** (3.40)	-10.08*** (3.39)	-10.00*** (3.39)
ε_{t-1}	0.43*** (0.07)	0.43*** (0.07)	0.41*** (0.07)	0.40*** (0.08)	0.41*** (0.07)	0.41*** (0.07)	0.40*** (0.07)
\bar{R}^2	0.43	0.43	0.42	0.42	0.42	0.42	0.42
AIC	-8.00	-7.99	-8.00	-8.00	-8.01	-8.01	-8.01
BIC	-7.75	-7.77	-7.78	-7.80	-7.83	-7.85	-7.87
DW	1.84	1.86	1.85	1.84	1.84	1.84	1.85
F_P	11.89 [0.00]	12.55 [0.00]	13.48 [0.00]	14.40 [0.00]	15.97 [0.00]	17.71 [0.00]	19.67 [0.00]

All the notes for Table B.1 apply to this table except for the inflation measure (here: CPI).

Table B.9: Nonlinear IPD-Inflation Effect – High-Inflation Subperiod

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i I^i(\Delta p_t) + \sum_{j=0}^J \lambda_j^* I^j(\Delta p_t)^2 + \varepsilon_t$$

	(i*)	(ii*)	(iii*)	(iv*)	(v*)	(vi*)	(vii*)	(viii*)	(ix*)	(x*)	(xi*)
α	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.02*** (0.00)	0.02*** (0.00)	0.02*** (0.00)
λ_0	-0.51*** (0.17)	-0.46*** (0.16)	-0.52*** (0.17)	-0.51*** (0.15)	-0.53*** (0.15)	-0.54*** (0.17)	-0.56*** (0.17)	-0.56*** (0.17)	-0.53*** (0.19)	-0.52*** (0.18)	-0.72*** (0.07)
λ_1	0.20 (0.16)	0.16 (0.15)	0.00 (0.09)	0.12 (0.15)	0.13 (0.15)	0.12 (0.15)	-	-	-	-	-
λ_2	0.23 (0.16)	0.18 (0.17)	0.11 (0.09)	0.19 (0.17)	0.19 (0.18)	-	-	-	-	-	-
λ_3	-0.10 (0.13)	-0.10 (0.13)	-0.14 (0.08)	-0.07 (0.14)	-0.10 (0.13)	-	-	-	-	-	-
λ_4	0.19 (0.19)	0.22 (0.18)	0.08 (0.09)	0.24 (0.17)	0.08 (0.07)	-	-	-	-	-	-
λ_5	0.09 (0.08)	0.08 (0.08)	0.08 (0.08)	0.09* (0.05)	0.09 (0.06)	-	-	-	-	-	-
λ_6	-0.03 (0.08)	-0.03 (0.08)	-0.02 (0.08)	-	-	-	-	-	-	-	-
λ_7	-0.08 (0.11)	-0.04 (0.10)	-0.03 (0.11)	-	-	-	-	-	-	-	-
λ_8	-0.02 (0.14)	0.01 (0.14)	0.02 (0.13)	-	-	-	-	-	-	-	-
λ_9	0.07 (0.05)	0.10* (0.05)	0.09 (0.05)	-	-	-	-	-	-	-	-
λ_{10}	0.11 (0.07)	-	-	-	-	-	-	-	-	-	-
λ_0^*	-5.81 (3.39)	-6.55** (3.15)	-5.03 (3.60)	-5.67* (2.64)	-4.97* (2.62)	-5.05* (2.76)	-4.53 (2.80)	-4.72* (2.82)	-4.67 (3.25)	-4.73 (3.37)	-
λ_1^*	-4.77 (5.13)	-4.21 (4.89)	-	-3.43 (4.66)	-3.41 (4.73)	-2.50 (4.61)	-0.06 (2.44)	-0.32 (2.44)	0.28 (2.26)	-	-
λ_2^*	-2.89 (2.75)	-1.87 (2.98)	-	-2.28 (3.03)	-2.06 (3.15)	2.63 (1.67)	2.91* (1.73)	2.64 (1.77)	-	-	-
λ_3^*	-0.78 (2.86)	-0.92 (3.23)	-	-1.73 (3.17)	-0.94 (3.25)	-1.34 (1.60)	-1.28 (1.62)	-	-	-	-
λ_4^*	-2.36 (3.55)	-3.19 (3.63)	-	-3.81 (3.54)	-	-	-	-	-	-	-
ε_{t-1}	0.41*** (0.08)	0.43*** (0.09)	0.42*** (0.10)	0.46*** (0.10)	0.44*** (0.09)	0.44*** (0.10)	0.45*** (0.10)	0.45*** (0.10)	0.42*** (0.10)	0.43*** (0.10)	0.46*** (0.10)
\bar{R}^2	0.74	0.74	0.75	0.75	0.76	0.75	0.75	0.75	0.75	0.75	0.75
AIC	-7.82	-7.82	-7.91	-7.93	-7.94	-7.97	-7.99	-8.01	-8.00	-8.03	-8.04
BIC	-7.21	-7.25	-7.47	-7.48	-7.53	-7.70	-7.76	-7.81	-7.83	-7.90	-7.94
DW	2.02	2.01	2.00	1.98	1.98	1.96	1.95	1.97	2.01	2.02	1.99
F_p	11.51 [0.00]	12.04 [0.00]	16.65 [0.00]	16.91 [0.00]	18.37 [0.00]	27.57 [0.00]	32.39 [0.00]	39.05 [0.00]	46.80 [0.00]	63.44 [0.00]	93.57 [0.00]

All the notes for Table B.1 apply to this table except for the sample period (here: from 1966Q1 to 1981Q3).

Table B.10: Nonlinear CPI-Inflation Effect – High-Inflation Subperiod

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \sum_{j=0}^J \lambda_j^* L^j (\Delta p_t)^2 + \varepsilon_t$$

	(i*)	(ii*)	(iii*)	(iv*)	(v*)	(vi*)	(vii*)	(viii*)	(ix*)	(x*)
α	0.01*	0.01*	0.01*	0.01*	0.01**	0.01**	0.01**	0.01*	0.01***	0.01***
	(0.01)	(0.01)	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
λ_0	-0.38	-0.37	-0.31	-0.30	-0.32	-0.31	-0.32	-0.42	-0.25	-
	(0.36)	(0.34)	(0.30)	(0.33)	(0.30)	(0.30)	(0.28)	(0.26)	(0.26)	-
λ_1	0.80*	0.80*	0.73*	0.76*	0.70*	0.67*	0.65*	0.58	-	-
	(0.42)	(0.41)	(0.42)	(0.41)	(0.39)	(0.39)	(0.39)	(0.35)	-	-
λ_2	-0.17	-0.18	-0.30	-0.23	-0.29	-0.35	-0.37	-	-	-
	(0.37)	(0.38)	(0.37)	(0.38)	(0.38)	(0.34)	(0.36)	-	-	-
λ_3	0.01	0.01	-0.13	0.06	-0.03	-0.09	-	-	-	-
	(0.38)	(0.37)	(0.30)	(0.40)	(0.43)	(0.35)	-	-	-	-
λ_4	-0.23	-0.23	0.25	-0.16	-0.23	-	-	-	-	-
	(0.65)	(0.64)	(0.19)	(0.64)	(0.57)	-	-	-	-	-
λ_5	-0.08	-0.08	-0.02	-0.07	-	-	-	-	-	-
	(0.17)	(0.16)	(0.12)	(0.17)	-	-	-	-	-	-
λ_6	-0.16	-0.16	-0.19	-0.09	-	-	-	-	-	-
	(0.18)	(0.17)	(0.16)	(0.15)	-	-	-	-	-	-
λ_7	-0.12	-0.12	-0.05	-0.07	-	-	-	-	-	-
	(0.18)	(0.15)	(0.15)	(0.14)	-	-	-	-	-	-
λ_8	-0.08	-0.07	-0.09	0.02	-	-	-	-	-	-
	(0.22)	(0.21)	(0.21)	(0.20)	-	-	-	-	-	-
λ_9	0.19	0.21*	0.17	-	-	-	-	-	-	-
	(0.12)	(0.11)	(0.11)	-	-	-	-	-	-	-
λ_{10}	0.02	-	-	-	-	-	-	-	-	-
	(0.15)	-	-	-	-	-	-	-	-	-
λ_0^*	-5.13	-5.21	-7.84	-6.76	-5.76	-6.16	-5.83	-4.29	-6.97	-12.66***
	(8.83)	(8.57)	(7.21)	(8.55)	(7.28)	(6.66)	(6.06)	(5.90)	(5.77)	(3.32)
λ_1^*	-25.75*	-25.87*	-22.40*	-25.50*	-23.00*	-21.93*	-21.65*	-19.97*	-7.67	-7.98
	(13.07)	(12.92)	(12.36)	(12.86)	(12.38)	(12.01)	(12.09)	(11.42)	(6.22)	(6.07)
λ_2^*	12.66	12.82	13.85	13.43	14.07	14.85*	15.32*	6.91*	8.02*	8.15*
	(8.68)	(8.80)	(9.02)	(9.25)	(8.79)	(8.55)	(8.69)	(4.05)	(4.57)	(4.46)
λ_3^*	-5.10	-5.25	-0.55	-6.27	-5.87	-4.38	-6.45*	-6.78**	-7.70**	-7.74**
	(10.02)	(9.73)	(8.18)	(10.45)	(11.06)	(9.49)	(3.30)	(3.23)	(3.52)	(3.65)
λ_4^*	13.84	13.85	-	11.61	10.33	4.83**	4.84**	4.97**	5.62**	5.32**
	(15.64)	(15.42)	-	(15.39)	(12.99)	(2.15)	(2.18)	(2.12)	(2.24)	(2.33)
ε_{t-1}	0.45***	0.45***	0.43***	0.43***	0.45***	0.44***	0.44***	0.44***	0.41***	0.39***
	(0.10)	(0.10)	(0.09)	(0.11)	(0.11)	(0.11)	(0.11)	(0.11)	(0.13)	(0.13)
\bar{R}^2	0.58	0.59	0.58	0.58	0.60	0.61	0.61	0.61	0.60	0.61
AIC	-7.81	-7.84	-7.84	-7.84	-7.93	-7.95	-7.98	-7.99	-7.98	-8.00
BIC	-7.22	-7.28	-7.31	-7.32	-7.54	-7.59	-7.65	-7.70	-7.72	-7.77
DW	1.92	1.92	1.96	1.90	1.91	1.92	1.92	1.90	1.91	1.92
F_P	6.35	6.88	7.12	7.15	10.06	11.15	12.59	14.02	15.29	17.92
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]

All the notes for Table B.1 apply to this table except for the inflation measure (here: CPI) and the sample period (here: from 1966Q1 to 1982Q3).

Table B.11: Nonlinear IPD-Inflation Effect – Low-Inflation Subperiod

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \sum_{j=0}^J \lambda_j^* L^j (\Delta p_t)^2 + \varepsilon_t$$

	(i*)	(ii*)	(iii*)	(iv*)	(v*)	(vi*)
α	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)
λ_0	-0.73*** (0.14)	-0.74*** (0.14)	-0.76*** (0.13)	-0.74*** (0.14)	-0.74*** (0.13)	-0.76*** (0.11)
λ_1	0.10 (0.16)	0.10 (0.15)	0.12 (0.12)	0.11 (0.13)	0.11 (0.13)	0.14** (0.06)
λ_2	-0.16 (0.11)	-0.15 (0.10)	-0.15 (0.10)	-0.11 (0.10)	-0.11 (0.07)	-0.10* (0.06)
λ_3	0.14 (0.10)	0.12 (0.08)	0.14* (0.08)	0.06 (0.06)	0.06 (0.06)	0.06 (0.06)
λ_4	-0.13 (0.14)	-0.12 (0.14)	-0.16** (0.06)	-0.19** (0.07)	-0.19*** (0.07)	-0.19*** (0.06)
λ_5	0.07 (0.08)	0.08 (0.07)	0.06 (0.05)	0.06 (0.05)	0.06 (0.05)	0.05 (0.04)
λ_6	0.12** (0.05)	0.16*** (0.04)	0.16*** (0.04)	0.15*** (0.04)	0.16*** (0.04)	0.16*** (0.04)
λ_7	-0.09 (0.06)	-0.12** (0.05)	-0.12** (0.05)	-0.10* (0.05)	-0.11** (0.05)	-0.10* (0.05)
λ_8	0.16** (0.07)	0.15** (0.07)	0.16** (0.07)	0.15* (0.07)	0.15** (0.06)	0.15** (0.06)
λ_9	-0.09** (0.04)	-0.10** (0.04)	-0.11** (0.04)	-0.13** (0.05)	-0.13** (0.05)	-0.13** (0.05)
λ_{10}	-0.05 (0.05)	—	—	—	—	—
λ_0^*	13.78 (8.12)	15.37 (9.45)	15.56 (9.39)	14.32 (9.06)	14.32 (8.88)	14.88* (8.08)
λ_1^*	2.02 (11.39)	2.93 (10.06)	1.06 (7.73)	2.59 (7.76)	2.51 (7.99)	—
λ_2^*	1.03 (6.04)	1.47 (5.84)	2.05 (5.02)	-0.18 (5.23)	—	—
λ_3^*	-5.33 (3.98)	-5.73 (3.69)	-6.54* (3.53)	—	—	—
λ_4^*	-1.36 (7.43)	-2.55 (6.70)	—	—	—	—
\bar{R}^2	0.69	0.69	0.70	0.69	0.71	0.72
AIC	-8.92	-8.94	-8.99	-9.00	-9.05	-9.10
BIC	-8.21	-8.28	-8.36	-8.42	-8.51	-8.59
DW	1.80	1.91	1.89	1.93	1.93	1.93
F_P	6.46 [0.00]	6.96 [0.00]	7.71 [0.00]	8.26 [0.00]	9.28 [0.00]	10.42 [0.00]

All the notes for Table B.1 apply to this table except for the sample period (here: from 1991Q1 to 2001Q1).

Table B.12: Nonlinear CPI-Inflation Effect – Low-Inflation Subperiod

$$\Delta c_t = \alpha + \sum_{i=0}^I \lambda_i L^i (\Delta p_t) + \sum_{j=0}^J \lambda_j^* L^j (\Delta p_t)^2 + \varepsilon_t$$

	(i*)	(ii*)	(iii*)	(iv*)	(v*)	(vi*)	(vii*)
α	0.01*** (0.00)	0.02*** (0.00)	0.02*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	0.01*** (0.00)
λ_0	0.30 (1.22)	-0.40 (1.17)	-0.46 (1.12)	-0.61 (1.05)	-0.54 (1.02)	-0.44 (1.08)	-0.33 (0.24)
λ_1	-0.75 (1.06)	-0.77 (0.99)	-0.43 (0.65)	-0.16 (0.76)	-0.30 (0.56)	0.25 (0.24)	0.24 (0.23)
λ_2	-0.40 (0.51)	-0.38 (0.54)	-0.48 (0.60)	-0.79 (0.53)	-0.53** (0.20)	-0.43** (0.18)	-0.42** (0.17)
λ_3	-0.61 (0.89)	-0.68 (0.91)	-0.48 (0.77)	0.14 (0.26)	0.19 (0.23)	0.14 (0.21)	0.14 (0.20)
λ_4	-0.27 (0.95)	-0.36 (0.87)	-0.71*** (0.20)	-0.60*** (0.18)	-0.61*** (0.16)	-0.58*** (0.16)	-0.58*** (0.16)
λ_5	0.17 (0.27)	0.13 (0.27)	0.07 (0.20)	0.03 (0.20)	0.04 (0.19)	-0.02 (0.18)	-0.01 (0.17)
λ_6	0.20 (0.19)	0.30 (0.20)	0.33* (0.18)	0.37** (0.18)	0.34* (0.17)	0.34* (0.17)	0.34* (0.17)
λ_7	-0.14 (0.17)	-0.25 (0.18)	-0.28* (0.15)	-0.36** (0.16)	-0.34** (0.16)	-0.31* (0.17)	-0.31* (0.17)
λ_8	0.35 (0.31)	0.36 (0.31)	0.41 (0.28)	0.43 (0.27)	0.44* (0.25)	0.41 (0.25)	0.41 (0.24)
λ_9	-0.29 (0.18)	-0.38* (0.20)	-0.41* (0.17)	-0.39** (0.18)	-0.39** (0.17)	-0.37** (0.17)	-0.37** (0.17)
λ_{10}	-0.20 (0.14)	-	-	-	-	-	-
λ_0^*	-34.47 (97.47)	24.10 (93.41)	25.86 (87.73)	29.34 (83.65)	22.39 (80.17)	9.01 (85.23)	-
λ_1^*	70.35 (67.31)	73.45 (62.41)	50.27 (36.44)	26.59 (43.10)	36.52 (27.21)	-	-
λ_2^*	-16.12 (33.11)	-15.81 (34.39)	-6.35 (33.05)	16.01 (30.43)	-	-	-
λ_3^*	48.14 (48.38)	50.52 (48.40)	38.04 (36.70)	-	-	-	-
λ_4^*	-23.49 (49.66)	-20.06 (45.52)	-	-	-	-	-
ε_{t-1}	0.21 (0.16)	0.11 (0.18)	0.07 (0.19)	0.09 (0.17)	0.10 (0.17)	0.14 (0.15)	0.14 (0.14)
\bar{R}^2	0.39	0.39	0.41	0.41	0.43	0.43	0.45
AIC	-8.56	-8.58	-8.62	-8.63	-8.67	-8.69	-8.74
BIC	-7.81	-7.87	-7.95	-8.01	-8.09	-8.15	-8.24
DW	1.99	1.96	1.96	1.94	1.94	1.93	1.93
F_P	2.48 [0.02]	2.60 [0.02]	2.85 [0.01]	3.00 [0.01]	3.32 [0.00]	3.53 [0.00]	3.99 [0.00]

All the notes for Table B.1 apply to this table except for the inflation measure (here: CPI) and the sample period (here: from 1991Q1 to 2001Q1).

Appendix C

Description of the Kalman Filter

The Kalman filter is a recursive procedure to compute the estimator of the state vector ξ at time t , based on all information available at time t . Within the two state-space models presented in Chapter 4, the Kalman filter is a method that can help separate real income growth from inflation. Formally, for the measurement equation $\tilde{y}_t = Ax_t + H'\xi_t$ and the state equation $\xi_t = \tilde{\mu} + F\xi_{t-1} + \tilde{v}_t$, the Kalman filter takes the following general form. Define:

$$\begin{aligned}\xi_{t|k} &= \mathbb{E}(\xi_t | \{\tilde{y}_i\}_{i=1}^k) \\ P_{t|k} &= \text{Var}(\xi_t | \{\tilde{y}_i\}_{i=1}^k),\end{aligned}$$

where $\xi_{t|t-1}$ denotes, for example, the expected value of ξ_t at time $t-1$, conditional on all information available up to $t-1$. $P_{t|t-1}$ is then the mean squared error (MSE) matrix of the forecast of $\xi_{t|t-1}$. The Kalman filter is a function that constructs the updated state estimate $\xi_{t|t}$ and the updated estimate covariance $P_{t|t}$ from $(\xi_{t-1|t-1}, P_{t-1|t-1}, \tilde{y}_t)$. Suppose that $\xi_{t-1|t-1}$ and $P_{t-1|t-1}$ are known and that

$$\begin{bmatrix} \xi_t \\ \tilde{y}_t \end{bmatrix} | \{\tilde{y}_i\}_{i=1}^{t-1}$$

is normally distributed. Then, following [Hamilton \(1994, pp. 380-384\)](#), the Kalman filter for the state-space model presented above is characterized by the following sequence of equations:

$$\xi_{t|t-1} = \tilde{\mu} + F\xi_{t-1|t-1} \quad (\text{C.1})$$

$$P_{t|t-1} = FP_{t-1|t-1}F' + Q \quad (\text{C.2})$$

$$\tilde{y}_{t|t-1} = A + H'\xi_{t|t-1} \quad (\text{C.3})$$

$$h_t = H'P_{t|t-1}H \quad (\text{C.4})$$

$$K_t = P_{t|t-1}Hh_t^{-1} \quad (\text{C.5})$$

$$\tilde{v}_t = \tilde{y}_t - \tilde{y}_{t|t-1} \quad (\text{C.6})$$

$$\xi_{t|t} = \xi_{t|t-1} + K_t\tilde{v}_t \quad (\text{C.7})$$

$$P_{t|t} = P_{t|t-1} - K_tHP_{t|t-1}. \quad (\text{C.8})$$

The first four equations represent the prediction step. In equation (C.1), the general state equation is used to produce the forecast $\xi_{t|t-1}$, which can be thought of as the predicted *a priori* state estimate. Using equation (C.2) for the predicted estimate covariance, which makes use of the assumption regarding its error term \tilde{v}_t , we can compute a forecast $\tilde{y}_{t|t-1}$ of the observed variables in equation (C.3). This forecast symbolizes the innovation MSE matrix, which is given by equation (C.4).

Equations (C.7) and (C.8) can be interpreted as the correction step yielding the updated *a posteriori* state estimate and its covariance, where K_t and \tilde{v}_t are defined in equations (C.5) and (C.6). Note that the K_t matrix is the so-called *Kalman filter gain*, which places a weight on the sensitivity of the filter to the measurements.

Appendix D

Solving the Robust Dynamic Programming Problem

D.1 Derivation of the Value Function

There are, fundamentally, two techniques to solve a robust control problem: “iteration” and “guess and verify”. In this section, I adopt the latter procedure, which consists of first formulating a particular guess for the form of the value function and then determining the exact values for all the terms of the guess. For our present problem, this technique implies extensive amounts of algebra, which I will go through step by step in this appendix.¹ Only if we show that there exists a solution for the guess and that it is unique is the guess correct in the sense of satisfying the Bellman equation.

To find the solution to the robust permanent income hypothesis model defined by equation (5.14) and (5.15), I follow [Luo \(2008\)](#), who suggests, as a guess for the value function $V(y_t^p)$, the following quadratic form $V(y_t^p) = -A(y_t^p)^2 - By_t^p - C$, where A , B , and C are yet-to-be-determined constant coefficients. Note that the quadratic form of the guess for the value function is rather intuitive, since the utility function is also quadratic and defined as $u(c_t) = c_t - \frac{a}{2}c_t^2$.

Substituting for this guess the Bellman equation for the robust consumer, equation (5.14) becomes:

¹Note that solving the problem with iteration also implies a guess for the value function and that this involves even more time and is even more algebra intensive.

$$-A(y_t^p)^2 - By_t^p - C = \max_{\{c_t\}} \min_{\{\omega_t\}} \left\{ c_t - \frac{a}{2}c_t^2 + \beta \mathbb{E}_t [\theta \omega_t^2 - A(y_{t+1}^p)^2 - By_{t+1}^p - C] \right\}, \quad (\text{D.1})$$

subject to the distorted budget constraint (5.15). This min-max optimization can now be solved using the standard procedures. Just as for the static case in Section 5.2.2, formally, it makes no difference whether we start with the minimization problem or with the maximization problem.

Substituting for y_{t+1}^p into the objective, the FOC for ω_t is:

$$\begin{aligned} \beta \mathbb{E}_t \{ 2\theta \omega_t - 2A [R(y_t^p - c_t) + \sigma_\zeta \omega_t + \zeta_{t+1}] \sigma_\zeta - B\sigma_\zeta \} &= 0 \\ 2\theta \omega_t - 2A\sigma_\zeta^2 \omega_t &= 2AR(y_t^p - c_t)\sigma_\zeta - B\sigma_\zeta \\ \omega_t &= \frac{B + 2AR(y_t^p - c_t)}{2(\theta - A\sigma_\zeta^2)} \sigma_\zeta. \end{aligned} \quad (\text{D.2})$$

Plugging equation (D.2) into equation (D.1) gives the following robust Bellman equation:

$$\begin{aligned} -A(y_t^p)^2 - By_t^p - C &= \max_{\{c_t\}} \left\{ c_t - \frac{a}{2}c_t^2 + \beta \mathbb{E}_t \left[\theta \left[\frac{B + 2AR(y_t^p - c_t)}{2(\theta - A\sigma_\zeta^2)} \sigma_\zeta \right]^2 \right. \right. \\ &\quad \left. \left. - A(y_{t+1}^p)^2 - By_{t+1}^p - C \right] \right\}, \end{aligned}$$

subject to the robust budget constraint:

$$\begin{aligned} y_{t+1}^p &= R(y_t^p - c_t) + \sigma_\zeta \left[\frac{B + 2AR(y_t^p - c_t)}{2(\theta - A\sigma_\zeta^2)} \sigma_\zeta \right] + \zeta_{t+1} \\ &= R(y_t^p - c_t) + \frac{AR\sigma_\zeta^2(y_t^p - c_t)}{\theta - A\sigma_\zeta^2} + \frac{B\sigma_\zeta^2}{2(\theta - A\sigma_\zeta^2)} + \zeta_{t+1} \\ &= \frac{B\sigma_\zeta^2}{2(\theta - A\sigma_\zeta^2)} + \frac{R\theta(y_t^p - c_t)}{\theta - A\sigma_\zeta^2} + \zeta_{t+1}. \end{aligned} \quad (\text{D.3})$$

Inserting equation (D.3) directly into the robust Bellman equation, we can derive the FOC for c_t :

$$\begin{aligned}
(1 - ac_t) - 2\beta\theta \left[\frac{B + 2AR(y_t^p - c_t)}{2(\theta - A\sigma_\zeta^2)} \sigma_\zeta \right] \left[\frac{2AR\sigma_\zeta}{2(\theta - A\sigma_\zeta^2)} \right] + \\
2A\beta \left[\frac{B\sigma_\zeta^2}{2(\theta - A\sigma_\zeta^2)} + \frac{R\theta(y_t^p - c_t)}{\theta - A\sigma_\zeta^2} \right] \left(\frac{R\theta}{\theta - A\sigma_\zeta^2} \right) + \\
\frac{B\beta R\theta}{\theta - A\sigma_\zeta^2} = 0.
\end{aligned}$$

Solving for c_t and rearranging and collecting terms leads to the following simplifications:

$$\begin{aligned}
(1 - ac_t) - \left(\frac{A\beta R\theta\sigma_\zeta^2}{\theta - A\sigma_\zeta^2} \right) \left[\frac{B + 2AR(y_t^p - c_t)}{(\theta - A\sigma_\zeta^2)} \right] + \\
\frac{2A\beta R^2\theta^2(y_t^p - c_t)}{(\theta - A\sigma_\zeta^2)^2} + \frac{AB\beta R\theta\sigma_\zeta^2}{(\theta - A\sigma_\zeta^2)^2} + \frac{B\beta R\theta}{\theta - A\sigma_\zeta^2} = 0 \\
c_t \left[\frac{2A\beta R^2\theta + a(\theta - A\sigma_\zeta^2)}{\theta - A\sigma_\zeta^2} \right] = 1 + \frac{2A\beta R^2\theta y_t^p (\theta - A\sigma_\zeta^2) + B\beta R\theta(\theta - A\sigma_\zeta^2)}{(\theta - A\sigma_\zeta^2)^2} \\
c_t = \frac{2A\beta R^2\theta}{a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta} y_t^p + \frac{(\theta - A\sigma_\zeta^2) + B\beta R\theta}{a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta}. \quad (\text{D.4})
\end{aligned}$$

In line with the theoretical foundations of the PIH, it appears that consumption corresponds to a constant fraction of permanent income y_t^p . However, we still need to determine the values of A , B and C from our guess in order to uniquely determine the value function and to confirm that the guess is correct. To do this, we first substitute equation (D.4) back into equations (D.3) and (D.2), then insert the optimal y_{t+1}^p and ω_t back into the robust Bellman equation. For y_{t+1}^p , we now have:

$$\begin{aligned}
\mathbb{E}(y_{t+1}^p) &= \frac{B\sigma_\zeta^2 + 2R\theta y_t^p}{2(\theta - A\sigma_\zeta^2)} - \frac{R\theta}{\theta - A\sigma_\zeta^2} c_t \\
&= \frac{B\sigma_\zeta^2 + 2R\theta y_t^p}{2(\theta - A\sigma_\zeta^2)} - \frac{R\theta}{\theta - A\sigma_\zeta^2} \left\{ \frac{2A\beta R^2\theta y_t^p + (\theta - A\sigma_\zeta^2) + B\beta R\theta}{a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta} \right\} \\
&= \frac{aR\theta}{a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta} y_t^p + \frac{aB\sigma_\zeta^2 - 2R\theta(1 + B\beta R)}{2[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]}. \quad (\text{D.5})
\end{aligned}$$

Similarly, plugging equation (D.4) into (D.2) yields the optimal distortion (i.e.,

the optimal worst-case distribution ω_t), which depends only on the state y_t^p :

$$\begin{aligned}
\omega_t &= \frac{B\sigma_\zeta + 2AR\sigma_\zeta y_t^p}{2(\theta - A\sigma_\zeta^2)} - \frac{2AR\sigma_\zeta}{2(\theta - A\sigma_\zeta^2)} \left\{ \frac{2A\beta R^2\theta y_t^p + (\theta - A\sigma_\zeta^2) + B\beta R\theta}{a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta} \right\} \\
&= \frac{\sigma_\zeta}{2(\theta - A\sigma_\zeta^2)} \left\{ \frac{aB(\theta - A\sigma_\zeta^2) + 2aAR(\theta - A\sigma_\zeta^2)y_t^p - 2AR(\theta - A\sigma_\zeta^2)}{a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta} \right\} \\
&= \frac{aAR\sigma_\zeta}{a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta} y_t^p + \frac{aB\sigma_\zeta - 2AR\sigma_\zeta}{2[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]}. \tag{D.6}
\end{aligned}$$

The next step is to substitute equations (D.4), (D.5) and (D.6) back into the value function (D.1) and to solve for A, B and C. Since this implies a lengthy equation, I first consider separately each element on the right-hand side (RHS) of equation (D.1) and rearrange them to fit the left-hand side (LHS) of equation (D.1).

For the first term, inserting equation (D.4) into $(c_t - \frac{a}{2}c_t^2)$ yields:

$$\begin{aligned}
&\frac{2A\beta R^2\theta y_t^p + (\theta - A\sigma_\zeta^2) + B\beta R\theta}{a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta} - \\
&\frac{a}{2[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2} \left\{ 2A\beta R^2\theta y_t^p + (\theta - A\sigma_\zeta^2) + B\beta R\theta \right\}^2 = \\
&- \frac{2aA^2\beta^2 R^4\theta^2}{[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2} (y_t^p)^2 + \frac{2A\beta^2 R^3\theta^2[2AR - aB]}{[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2} y_t^p + \\
&\frac{(\theta - A\sigma_\zeta^2)[a(\theta - A\sigma_\zeta^2) + 4A\beta R^2\theta] + B\beta^2 R^2\theta^2(4AR - aB)}{2[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2}. \tag{D.7}
\end{aligned}$$

For the second term, we make use of equation (D.6) and get:

$$\begin{aligned}
\beta\theta\omega_t^2 &= \frac{\beta\theta\sigma_\zeta^2}{4[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2} [2aARy_t^p - (2AR - aB)]^2 \\
&= \frac{\beta\theta\sigma_\zeta^2}{4[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2} [4a^2A^2R^2(y_t^p)^2 - 4aAR(2AR - aB)y_t^p + (2AR - aB)^2] \\
&= \frac{a^2A^2\beta R^2\theta\sigma_\zeta^2}{[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2} (y_t^p)^2 - \frac{aA\beta R\theta\sigma_\zeta^2(2AR - aB)}{[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2} y_t^p + \\
&\frac{\beta\theta\sigma_\zeta^2(2AR - aB)^2}{4[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2}. \tag{D.8}
\end{aligned}$$

The third term on the RHS is obtained by making use of equation (D.5):

$$\begin{aligned}
-A\beta\mathbb{E}[(y_{t+1}^p)^2] &= -\frac{A\beta}{4[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2} \left\{ 2aR\theta y_t^p + [aB\sigma_\zeta^2 - 2R\theta(1 + B\beta R)] \right\}^2 \\
&= -\frac{a^2 A\beta R^2 \theta^2}{[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2} (y_t^p)^2 - \frac{aA\beta R\theta[aB\sigma_\zeta^2 - 2R\theta(1 + B\beta R)]}{[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2} y_t^p - \\
&\quad \frac{A\beta[aB\sigma_\zeta^2 - 2R\theta(1 + B\beta R)]^2}{4[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2}. \tag{D.9}
\end{aligned}$$

The second-to-last term on the RHS of the value function in (D.1) follows directly from equation (D.5):

$$-B\beta\mathbb{E}(y_{t+1}^p) = -\frac{aB\beta R\theta}{a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta} y_t^p - \frac{B\beta[aB\sigma_\zeta^2 - 2R\theta(1 + B\beta R)]}{2[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]}. \tag{D.10}$$

The next step is to collect the appropriate terms from equations (D.8), (D.9) and (D.10) to fit the value function $V(y_t^p) = -A(y_t^p)^2 - By_t^p - (1 - \beta)C$, after βC has been subtracted from both sides and to solve for A , B and C . For A , we now have:

$$\begin{aligned}
-A(y_t^p)^2 &= \left\{ \frac{-2aA^2\beta^2 R^4\theta^2 + a^2 A^2\beta R^2\theta\sigma_\zeta^2 - a^2 A\beta R^2\theta^2}{[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2} \right\} (y_t^p)^2 \\
A[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2 &= aA\beta R^2\theta[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta] \\
A &= \frac{a\theta(\beta R^2 - 1)}{2\beta R^2\theta - a\sigma_\zeta^2}. \tag{D.11}
\end{aligned}$$

Equation (D.11) is the final definition of A because it depends only on the different parameters of the model. Collecting the terms for B and making use of equation (D.11), we find the exact value for B :

$$-B = \frac{2A\beta R^2\theta[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta] - aB\beta R\theta[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]}{[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]^2}$$

$$\begin{aligned}
B[a\beta R\theta - a(\theta - A\sigma_\zeta^2) - 2A\beta R^2\theta] &= \frac{2a\beta R^2\theta^2[\beta R^2 - 1]}{2\beta R^2\theta - a\sigma_\zeta^2} \\
B \frac{a\beta R\theta[2\beta R^2\theta - a\sigma_\zeta^2 + aR\sigma_\zeta^2 - 2\beta R^3\theta]}{2\beta R^2\theta - a\sigma_\zeta^2} &= \frac{2a\beta R^2\theta^2[\beta R^2 - 1]}{2\beta R^2\theta - a\sigma_\zeta^2} \\
B &= -\frac{2R\theta(\beta R^2 - 1)}{(2\beta R^2\theta - a\sigma_\zeta^2)(R - 1)}. \tag{D.12}
\end{aligned}$$

Finally, using equations (D.11) and (D.12) and collecting the remaining terms allows us to derive the definition of C . Since its derivation implies a great deal of algebra, it is presented separately in Section D.2.

Not only does finding the exact values for A , B and C give us the exact form of the value function $V(y_t^p)$, but knowing A and B also allows us to derive the robust consumption function, the consumption growth (both derived in Section D.3) and the following optimal distortion ω_t . Substituting for A and B in equation (D.6) yields, after simplification:

$$\begin{aligned}
\omega_t &= \frac{aAR\sigma_\zeta}{a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta} y_t^p + \frac{aB\sigma_\zeta - 2AR\sigma_\zeta}{2[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta]} \\
&= \frac{aR\theta\sigma_\zeta(\beta R^2 - 1)(2\beta R^2\theta - a\sigma_\zeta^2)}{(2\beta R^2\theta - a\sigma_\zeta^2)[(2\beta R^2\theta - a\sigma_\zeta^2) - a\sigma_\zeta^2(\beta R^2 - 1)] + 2\beta R^2\theta(\beta R^2 - 1)} y_t^p - \\
&\quad \frac{\sigma_\zeta(\beta R^2 - 1)}{\beta\{\theta[2 + a(\beta R^2 - 1)] - a\sigma_\zeta^2\}}.
\end{aligned}$$

D.2 Derivation of C

Recall that, in Section 5.3.2, I began with the conjecture that the value function of the Bellman equation (5.14) for the robust permanent income consumer is equal to $V(y_t^p) = -A(y_t^p)^2 - By_t^p - C$. After finding the exact values for A and B in equations (D.11) and (D.12), we can insert these into each last term on the RHS of equations (D.7) through (D.10). Adding these terms up and setting them equal to $C(\beta - 1)$ allows us to solve for C .

Before deriving each component of C , it is helpful to first solve for some recurrent elements of equations (D.7) to (D.10). In particular, consider the following equalities:

$$\begin{aligned}
\theta - A\sigma_\zeta^2 &= \frac{\beta R^2 \theta (2\theta - a\sigma_\zeta^2)}{2\beta R^2 \theta - a\sigma_\zeta^2} \\
a(\theta - A\sigma_\zeta^2) + 2A\beta R^2 \theta &= a\beta R^2 \theta \\
a(\theta - A\sigma_\zeta^2) + 4A\beta R^2 \theta &= \frac{a\beta R^2 \theta [2\theta(2\beta R^2 - 1)]}{2\beta R^2 \theta - a\sigma_\zeta^2} \\
4AR - aB &= \frac{2aR\theta(\beta R^2 - 1)(2R - 1)}{(2\beta R^2 \theta - a\sigma_\zeta^2)(R - 1)} \\
2AR - aB &= \frac{2aR^2 \theta(\beta R^2 - 1)}{(2\beta R^2 \theta - a\sigma_\zeta^2)(R - 1)} \\
aB\sigma_\zeta^2 - 2R\theta(1 + B\beta R) &= -\frac{2R^2 \theta(\beta R - 1)}{R - 1}.
\end{aligned}$$

For the C -term of $(c_t - \frac{a}{2}c_t)$ (i.e., equation (D.7)), inserting A and B and making use of the needed above-mentioned pre-calculations yields:

$$\begin{aligned}
&\frac{(\theta - A\sigma_\zeta^2)[a(\theta - A\sigma_\zeta^2) + 4A\beta R^2 \theta] + B\beta^2 R^2 \theta^2 (4AR - aB)}{2[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2 \theta]^2} = \\
&\frac{2R^2 \theta [(\beta^2 R^2 - 1) - 2\beta R^2(\beta R - 1)]}{a(2\beta R^2 \theta - a\sigma_\zeta^2)^2 (R - 1)^2} - \frac{\sigma_\zeta^2}{2(2\beta R^2 \theta - a\sigma_\zeta^2)^2}. \tag{D.13}
\end{aligned}$$

Then, the part of $\beta\theta\omega_t^2$ in equation (D.8) that is not dependent on y_t^p becomes, after simplification:

$$\frac{\beta\theta\sigma_\zeta^2(2AR - aB)^2}{4[a(\theta - A\sigma_\zeta^2) + 2A\beta R^2 \theta]^2} = \frac{\theta\sigma_\zeta^2(\beta R^2 - 1)^2}{\beta(2\beta R^2 \theta - a\sigma_\zeta^2)^2 (R - 1)^2}. \tag{D.14}$$

The last term of equation (D.9) can be rewritten as:

$$-\frac{A\beta[aB\sigma_\zeta^2 - 2R\theta(1 + B\beta R)]^2}{4[a(\theta - A\omega_\zeta^2) + 2A\beta R^2 \theta]^2} = -\frac{\theta(\beta R^2 - 1)(\beta R - 1)^2}{a\beta(2\beta R^2 \theta - a\sigma_\zeta^2)(R - 1)^2}. \tag{D.15}$$

Extracting the C -term of equation (D.10) and substituting for A and B gives:

$$-\frac{B\beta[aB\omega_\zeta^2 - 2R\theta(1 + B\beta R)]}{2[a(\theta - A\omega_\zeta^2) + 2A\beta R^2 \theta]} = \frac{2R\theta(\beta R^2 - 1)(\beta R - 1)}{a(2\beta R^2 \theta - a\sigma_\zeta^2)(R - 1)^2}. \tag{D.16}$$

We can now combine and simplify equations (D.13) through (D.16). First, adding equations (D.15) and (D.16) yields:

$$\frac{\theta(\beta R^2 - 1)(\beta^2 R^2 - 1)}{a\beta(2\beta R^2\theta - a\sigma_\zeta^2)(R - 1)^2}. \quad (\text{D.17})$$

Second, summing up equation (D.14) and the second term of equation (D.13) gives:

$$\frac{\sigma_\zeta^2 \left\{ 2\theta[2\beta R^2(\beta R - 1) - (\beta^2 R^2 - 1)] + a\beta\sigma_\zeta^2(R - 1)^2 \right\}}{2\beta(2\beta R^2\theta - a\sigma_\zeta^2)^2(R - 1)^2}. \quad (\text{D.18})$$

Third, adding (D.17) and (D.18) and rearranging terms yields:

$$\frac{2R^2\theta^2(\beta R^2 - 1)(\beta^2 R^2 - 1)}{a(2\beta R^2\theta - a\sigma_\zeta^2)^2(R - 1)^2} + \frac{\sigma_\zeta^2[2R^2\theta(\beta R - 1) - \beta(\beta^2 R^2 - 1)]}{(2\beta R^2\theta - a\sigma_\zeta^2)^2(R - 1)^2} + \frac{a\sigma_\zeta^4}{2(2\beta R^2\theta - a\sigma_\zeta^2)^2}. \quad (\text{D.19})$$

Then, combining the first term of the RHS of (D.19) with the first term of equation (D.13) gives, after some simplifications:

$$\frac{2\beta R^4\theta^2(\beta R - 1)^2}{a(2\beta R^2\theta - a\sigma_\zeta^2)^2(R - 1)^2}. \quad (\text{D.20})$$

Finally, summing up (D.20) with the remaining terms of (D.19) allows us to find the precise and final definition of C :

$$\begin{aligned} C(\beta - 1) &= \frac{2\beta R^4\theta^2(\beta R - 1)^2}{a(2\beta R^2\theta - a\sigma_\zeta^2)^2(R - 1)^2} + \frac{\sigma_\zeta^2[2R^2\theta(\beta R - 1) - \beta(\beta^2 R^2 - 1)]}{(2\beta R^2\theta - a\sigma_\zeta^2)^2(R - 1)^2} + \\ &\quad \frac{a\sigma_\zeta^4}{2(2\beta R^2\theta - a\sigma_\zeta^2)^2} \\ C &= \frac{2\beta R^4\theta^2(\beta R - 1)^2}{a(2\beta R^2\theta - a\sigma_\zeta^2)^2(R - 1)^2(\beta - 1)} + \frac{\sigma_\zeta^2[2R^2\theta(\beta R - 1) - \beta(\beta^2 R^2 - 1)]}{(2\beta R^2\theta - a\sigma_\zeta^2)^2(R - 1)^2(\beta - 1)} \\ &\quad + \frac{a\sigma_\zeta^4}{2(2\beta R^2\theta - a\sigma_\zeta^2)^2(\beta - 1)}. \end{aligned}$$

D.3 The Robust Consumption Function and Consumption Growth

To precisely define the robust consumption function, we can substitute A and B from equations (D.11) and (D.12) into the previously derived optimal robust consumption equation (D.4):

$$\begin{aligned}
c_t &= \frac{1}{a(\theta - A\sigma_\zeta^2) + 2A\beta R^2\theta} \\
&\quad \left\{ \frac{2a\beta R^2\theta^2(\beta R^2 - 1)y_t^p}{2\beta R^2\theta - a\sigma_\zeta^2} + \theta - \frac{a\theta\sigma_\zeta^2(\beta R^2 - 1)}{2\beta R^2\theta - a\sigma_\zeta^2} + \frac{2\beta R^2\theta^2}{(2\beta R^2\theta - a\sigma_\zeta^2)(1 - R)} \right\} \\
&= \frac{2\theta(\beta R^2 - 1)}{2\beta R^2\theta - a\sigma_\zeta^2} y_t^p + \frac{(2\beta R^2\theta - a\sigma_\zeta^2)(\beta R - 1) + a\sigma_\zeta^2(\beta R^2 - 1)}{a\beta R(2\beta R^2\theta - a\sigma_\zeta^2)(1 - R)} \\
&= \frac{\beta R^2 - 1}{\beta R^2 - \Phi} y_t^p - \frac{(\beta R^2 - \Phi)(\beta R - 1) + (\beta R^2 - 1)\Phi}{a\beta R(\beta R^2 - \Phi)(R - 1)}, \tag{D.21}
\end{aligned}$$

where $\Phi = \frac{a\sigma_\zeta^2}{2\theta}$.

Using equation (D.21) and the non-distorted budget constraint, as in equation (5.11), we can derive the precise consumption growth for the robust consumer:

$$\begin{aligned}
\Delta c_{t+1} &= \frac{\beta R^2 - 1}{\beta R^2 - \Phi} (y_{t+1}^p - y_t^p) \\
&= \frac{\beta R^2 - 1}{\beta R^2 - \Phi} (R - 1)y_t^p - \frac{\beta R^2 - 1}{\beta R^2 - \Phi} R c_t + \frac{\beta R^2 - 1}{\beta R^2 - \Phi} \zeta_{t+1} \\
&= (R - 1) \left[c_t + \frac{(\beta R^2 - \Phi)(\beta R - 1) + (\beta R^2 - 1)\Phi}{a\beta R(\beta R^2 - \Phi)(R - 1)} \right] - \frac{\beta R^2 - 1}{\beta R^2 - \Phi} R c_t + \frac{\beta R^2 - 1}{\beta R^2 - \Phi} \zeta_{t+1} \\
&= \frac{(\beta R - 1)(\beta R^2 - \Phi) + (\beta R^2 - 1)\Phi}{a\beta R(\beta R^2 - \Phi)} + \frac{(R - 1)(\beta R^2 - \Phi) - (\beta R^2 - 1)R}{\beta R^2 - \Phi} c_t + \\
&\quad \frac{\beta R^2 - 1}{\beta R^2 - \Phi} \zeta_{t+1}.
\end{aligned}$$

D.4 Proof that $\Phi < 1$

Starting with the consumer problem in equation (D.1), the second-order condition for the minimization of the Bellman equation with respect to ω_t , under the known constant A (see equation D.11) and the assumption that $\beta R = 1$ yields:

$$\begin{aligned}
2\beta\theta - 2A\beta\sigma_\zeta^2 &> 0 \\
1 &> \frac{a\sigma_\zeta^2(R - 1)}{2R\theta - a\sigma_\zeta^2} \\
1 &> \frac{\Phi(R - 1)}{R - \Phi} \\
1 &> \Phi.
\end{aligned}$$

Appendix E

Estimation Results for the Hansen et al. (1999) Model

Table E.1: Estimation of the Robust Consumption Model of Hansen et al. (1999)

	HST		IPD High		IPD Low		CPI High		CPI Low	
	HF	No HF	HF	No HF	HF	No HF	HF	No HF	HF	No HF
β	0.9971	0.9971	0.9943	0.9941	0.9940	0.9937	0.9944	0.9943	0.9941	0.9940
$\hat{\rho}_h$	0.68	—	0.68	—	0.60	—	0.69	—	0.67	—
λ	2.44	—	2.44	—	2.43	—	2.44	—	2.44	—
α_1	0.81	0.90	0.81	0.88	0.94	0.90	0.82	0.95	0.82	0.93
α_2	0.19	0.24	0.19	0.23	0.23	0.24	0.20	0.23	0.21	0.24
ϕ_1	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	1.00
ϕ_2	0.70	0.45	0.70	0.43	0.79	0.45	0.54	0.34	0.76	0.46
μ_y	13.71	13.59	13.71	13.60	13.71	13.59	13.71	13.59	13.71	13.59
$c_{\bar{y}}$	0.16	0.17	0.16	0.17	0.18	0.20	0.20	0.17	0.20	0.19
$c_{\bar{y}}^*$	0.11	0.10	0.11	0.18	0.13	0.10	0.05	0.20	0.19	0.11
log-Lik	—	—	-200.83	-301.48	-248.82	-181.89	-303.48	-318.85	-235.87	-251.43

This table reports the maximum likelihood estimates for the model parameters defined in equations (5.36) to (5.39) and (5.33), using quarterly U.S. consumption and investment data. The columns under “HST” report HST’s original estimates for the period from 1970Q1 to 1996Q3 for a fixed β under habit formation (HF) and without habit formation (no HF) (see Table 2 in their paper). IPD and CPI indicate the deflators. The “high” inflation subperiods are from 1966Q1 to 1981Q3 and from 1966Q1 to 1982Q3 for IPD and CPI, respectively. The “low” inflation subperiod is from 1991Q1 to 2001Q1 for both the IPD and CPI model. Log-Lik reports the maximum of the log-likelihood function.