Contents

S1 Local and global stability of a feasible equilibrium point

Ref. 14 introduced a Lyapunov function to grant the global stability of any feasible point under some constraints on interaction strengths. In the linear Lotka-Volterra version ($h = 0$), this theory works as follows. Let B be the interaction-strength matrix, i.e.,

$$B = \begin{bmatrix} \beta^{(P)} & -\gamma^{(P)} \\ -\gamma^{(A)} & \beta^{(A)} \end{bmatrix}.$$

Assuming that we can find a strictly positive diagonal matrix D, such that $DB+B^tD \succ 0$ (*59*), a Lyapunov function can be constructed to prove the global stability of any feasible equilibrium. In this case, the matrix B is called Lyapunov-diagonally stable. Practically, it is a complex task to find a suitable matrix D. A very strict condition would be to impose that the matrix B itself has to be positive definite.

On the other extreme, we can simply look at the local stability of a feasible equilibrium by looking at the Jacobian matrix. Assuming that $\tilde{P}$ and $\tilde{A}$ is a feasible equilibrium, the Jacobian is then given by

$$J = - \begin{bmatrix} \tilde{P}_1 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & \ddots & \ddots & & & \vdots \\ \vdots & \ddots & \tilde{P}_{S_{(P)}} & \ddots & & \vdots \\ \vdots & & \ddots & \tilde{A}_1 & \ddots & \vdots \\ \vdots & & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & \tilde{A}_{S_{(A)}} \end{bmatrix} \begin{bmatrix} \beta^{(P)} & -\gamma^{(P)} \\ -\gamma^{(A)} & \beta^{(A)} \end{bmatrix}.$$

Now, if the real parts of all eigenvalues are strictly negative, the feasible equilibrium is locally stable. The problem with this approach is that we have to evaluate the Jacobian and its eigenvalues for every possible feasible point. Fortunately, the concept of D-stability solves this issue (*15,60,61*). A matrix M is called D-stable if for every strictly positive diagonal matrix D, the real part of the eigenvalues of DM are strictly positive. Then, if the interaction-strength matrix B were to be D-stable, any feasible equilibrium is granted to be at least locally stable. Note that Lyapunov-diagonal stability implies D-stability.

Importantly, all these different conditions on stability are imposing constraints on the interaction strengths. Assuming the strength of the competition matrices $\beta^{(P)}$ and $\beta^{(A)}$ are set such that without mutualism any feasible equilibrium is stable, then these different notions of stability constrain the level of mutualistic strength that the system can handle before losing the stability of a feasible equilibrium. Intuitively, in the linear Lotka-Volterra case, the mutualistic interactions have to be "weaker" than competitive interactions to have the stability of a feasible equilibrium.

S2 Numerical test of the two conjectures for the global stability of feasible equilibrium

We provide details on the numerical simulations supporting our two conjectures related to the stability of a feasible equilibrium. Note that for a level of mutualistic strength of zero, i.e., $\gamma_0 = 0$, any feasible equilibrium point is automatically globally stable. Indeed, the competition matrices are chosen positive definite, thus according to Ref. 14, any feasible equilibrium point is globally stable.

To grant the global stability of any feasible equilibrium when $\gamma_0 > 0$, one can use the Lyapunov function developed in Ref. 14. However, in practice, it is a very difficult task to find the positive diagonal matrix D such that $DB + B^t D \succ 0$, and even more difficult to prove it for a full family of matrices. D-stability is a more relaxed condition (*15,60*), but as before, it is very difficult to prove that a given matrix is D-stable. However, we find that a very general result can be postulated. First, we need to formulate two conjectures:

Conjecture 1: if the interaction strength matrix M is Lyapunov stable, then M is D-stable.

Conjecture 2: if the interaction strength matrix M is Lyapunov stable, then any feasible equilibrium is globally stable.

Recall that for a given network, mutualistic trade-off, and interspecific competition strength, a matrix B is Lyapunov stable when the level of mutualistic strength is below a given critical value $\gamma_0 < \gamma_0^r$. However, there exists a stronger constraint on B within which the two conjectures can be proven. If we impose B to be positive definite, i.e., $B+B^t$ to be Lyapunov stable, then it has already been proven that B is D-stable (*15,60*). This proves our Conjecture 1. Moreover, following Ref. 14, there exists a Lyapunov function for any feasible equilibrium point. This proves our Conjecture 2. As for Lyapunov stability, the matrix B is positive definite if the level of mutualistic strength is below a critical value $\gamma_0 < \gamma_0^s$, which is stronger than the the critical value for Lyapunov stability ($\gamma^s < \gamma_0^r$). Thus, our two conjectures are partially proven and need to be numerically check only over a restricted range of mutualistic strength (from γ_0^s to γ_0^r).

For each of the observed networks, we explore the mutualistic trade-off of $\delta = 0$, 0.1 , 0.2, ... 1.5, interspecific competition of $\rho = 0.2$, and 10 equidistant steps in the level of mutualistic strength between the two critical values γ_0^s and γ_0^r with the following numerical simulations. For conjecture 1, at each step we generated 1000 samples of strictly positive diagonal matrices D and tested whether DB is still Lyapunov stable. During this test, we find no counter-example, thus our conjecture 1 seems to hold.

For conjecture 2, in each step from above, we chose 1000 different feasible equilibria and 1000 different initial points for the numerical integrator, and tested whether the numerical simulations converge to the chosen feasible equilibria. We find the numerical simulations always converged to the same chosen feasible equilibria, indicating that our conjecture 2 also holds.

If it would be analytically proven that the interaction-strength matrix B is Lyapunov-diagonally stable when it is only Lyapunov stable, our two conjectures would become theorems. Indeed, in this case we would have equivalence among Lyapunov stability, D-stability, and Lyapunov-diagonal stability. Additionally, Lyapunov-diagonal stability implies global stability for any feasible equilibrium point. Mathematically, our interaction-strength matrix B can be seen as a two-by-two block Z-matrix, where the diagonal block contains the positive definite mean-field competition matrices and the off-diagonal blocks are non-positive matrices. The equivalence of the three types of stability is known for Z-matrices (*40,60,61*), but remains an open question for block Z-matrices.

Finally, we run numerical simulations to show that the stability conditions derived within the linear Lotka-Volterra model imply also stability for our nonlinear dynamical model. What we have to show is that if B is Lyapunov stable, then B_{nl} is also Lyapunov stable for any positive plant and animal abundance. As for the two conjectures above, we explored the mutualistic trade-off of $\delta = 0, 0.1, 0.2, \ldots, 1.5$, interspecific competition of $\rho = 0.2$, and 10 equidistant steps in the level of mutualistic strength between zero and γ_0^r.

For each step we generate twice 1000 diagonal matrices where the elements are sampled from a uniform distribution between 0 and 1. Then, we multiply the two off-diagonal blocks of the matrix B by two of these random matrices. This procedure mimics the fact that in the nonlinear model the mutualistic interactions have a dominator greater than one. We find that all the generated matrices are Lyapunov stable. This indicates that the stability conditions derived in the linear version hold for the nonlinear version too.

S3 Critical value of mutualistic strength

The main consequence of our two conjectures for stability is that to determine the stability of a feasible equilibrium, it is enough to look at the eigenvalues of the interaction-strength matrix B. Thus, for computing the critical value in the level of mutualistic strength γ_0^r, we just need to find the critical value of γ_0 at which the real part of one of the eigenvalues reaches zero. Importantly, as shown in Fig. S1, we find that for any given mutualistic trade-off and interspecific competition, the higher the level of mutualistic strength, the smaller the real part of the eigenvalues of B.

Additionally, we find no particular pattern in how the critical level of mutualistic strength varies with the specific model parameterization. In Figures S2 and S3, we show how mutualistic trade-off δ and nestedness influence the critical value of mutualistic strength γ_0^r. For this, we randomize the observed networks and compute the slope of the linear regression between this level of mutualistic strength and nestedness. While in the majority of cases the slopes increase along with the mutualistic trade-off, we find that these slopes can be positive as well as negative, meaning that there is no general pattern.

S4 Stability and feasibility for strong mutualism

Our study of mutualistic networks is still limited to mutualistic interactions that are weaker than our critical value of mutualistic strength ($\gamma_0 < \gamma_0^r$). Recall that below this critical value, any feasible point is granted to be globally stable. Mathematically, we can disentangle the feasibility condition from the stability one, like in the two-species competition system. Here we explain how our dynamical system reacts to a mutualism that is stronger than our critical value of mutualistic strength—that corresponds to the strong mutualism regime (*6,39*). In the case where the handling time is zero ($h = 0$), the system usually blows up depending on the sign of the intrinsic growth rates (*43*). In the case where $h > 0$, it is always possible for any level of mutualistic strength to have both feasibility and local stability (*39*). Recall that any set of positive abundances can be turn into a feasible equilibrium point just by adjusting the intrinsic growth rates, such that the right side of Equation 2 vanishes.

Now, if we linearize the dynamics around this equilibrium point (A_i^*, P_i^*), we get the following linear Lotka-Volterra system:

$$\begin{cases} \frac{dP_i}{dt} = P_i \left(\tilde{\alpha}_i^{(P)} - \sum_j \beta_{ij}^{(P)} P_j + \sum_j \tilde{\gamma}_{ij}^{(P)} A_j \right) \\ \frac{dA_i}{dt} = A_i \left(\tilde{\alpha}_i^{(A)} - \sum_j \beta_{ij}^{(A)} A_j + \sum_j \tilde{\gamma}_{ij}^{(A)} P_j \right), \end{cases}$$

where $\tilde{\alpha}_i^{(A)} = \sum_j \beta_{ij}^{(A)} \tilde{A}_i - \sum_j \gamma_{ij}^{(A)} P_i^* / (1 + h \sum_j \gamma_{ij}^{(A)} P_i^*)^2$ and the linearized interaction strength $\tilde{\gamma}_{ij}^{(A)} = \gamma_{ij}^{(A)} / (1 + h \sum_j \gamma_{ij}^{(A)} A_i^*)^2$ (similar expressions hold for plants). In this linearized version, the mutualistic interaction strengths are now dependent on the abundances at the equilibrium point. Moreover, the linearized interaction strengths are inversely proportional to the square of these equilibrium abundances. Then, for any level of mutualistic strength, if the abundances at the feasible equilibrium are large enough such that all eigenvalues of

$$\tilde{B} = \begin{bmatrix} \beta^{(P)} & -\tilde{\gamma}^{(P)} \\ -\tilde{\gamma}^{(A)} & \beta^{(A)} \end{bmatrix}$$

have a strictly positive real parts, this feasible equilibrium is locally stable. This simple mathematical fact implies that now we cannot completely disentangle the feasibility conditions form the stability ones. There exists a proper subspace of the positive quadrant ($S \subset R^S_{\geq 0}$) such that when the feasible equilibrium is inside ($A_i^*, P_i^* \in S$), it is globally stable only inside the subspace S (*14,43*). In turn, this implies that if the intrinsic growth rates are too low, a feasible equilibrium may be unstable, and thus the domain of feasibility is also bounded by below. In general, we still have that for any level of mutualistic strength, the system can be at a feasible and locally stable equilibrium. However, it is more difficult to determine the complete set of conditions for feasibility and stability.

S5 Derivation of the structural vector

Here we provide the full analytic derivation of the structural vector. While the derivation is made with a linear version ($h = 0$) of the dynamical system (Equation 2), numerically we find that this is already a good approximation to the general solution (Fig. S4). Since we are interested in the existence of a feasible equilibrium point, i.e, $P_i^* > 0$ and $A_i^* > 0$ such that evaluated at that point the right side of Equation 2 vanishes, we have to solve the following linear system of equations:

$$\begin{bmatrix} \alpha^{(P)} \\ \alpha^{(A)} \end{bmatrix} = \left(\underbrace{\begin{bmatrix} \beta^{(P)} & 0 \\ 0 & \beta^{(A)} \end{bmatrix}}_{=C} - \underbrace{\begin{bmatrix} 0 & \gamma^{(P)} \\ \gamma^{(A)} & 0 \end{bmatrix}}_{=\Gamma} \right) \begin{bmatrix} P \\ A \end{bmatrix}. \tag{S1}$$

We do this by diagonalizing the matrix $C - \Gamma$ per block, and obtaining two independent systems of equations, one for the plants and one for the animals. This is achieved by multiplying by the matrix $T = 1 + \Gamma C^{-1}$ both left sides of the equation. This is the same technique as used in Ref. 6. The new equivalent system is given by:

$$\underbrace{\begin{bmatrix} \alpha^{(P)} + \gamma^{(P)}(\beta^{(A)})^{-1}\alpha^{(A)} \\ \alpha^{(A)} + \gamma^{(A)}(\beta^{(P)})^{-1}\alpha^{(P)} \end{bmatrix}}_{=\begin{bmatrix} \alpha_{eff}^{(P)} \\ \alpha_{eff}^{(A)} \end{bmatrix}} = \underbrace{\begin{bmatrix} \beta^{(P)} - \gamma^{(P)}(\beta^{(A)})^{-1}\gamma^{(A)} & 0 \\ 0 & \beta^{(A)} - \gamma^{(A)}(\beta^{(P)})^{-1}\gamma^{(P)} \end{bmatrix}}_{=\begin{bmatrix} \beta_{eff}^{(P)} & 0 \\ 0 & \beta_{eff}^{(A)} \end{bmatrix}} \begin{bmatrix} P \\ A \end{bmatrix} \tag{S2}$$

Using this mathematical transformation, we have moved from an observable parameterization space to an effective competition framework. Here, $\alpha_{eff}^{(P)}$ and $\alpha_{eff}^{(A)}$ are called effective intrinsic growth rates, and $\beta_{eff}^{(P)}$ and $\beta_{eff}^{(A)}$ are called effective competition rates. This is called so because the new linear system of equations, for the fixed point of plants and animals, is equivalent to the system of a purely competitive system within plants and within animals. Since the form of the equation is the same for the plants than for the animals, we will present the solution without the superscript (A) or (P) for the sake of clarity.

The system that we have to solve has the form $\alpha_{eff} = \beta_{eff} N$. However, here we have to keep in mind that we are interested in how the existence of feasible solutions, i.e., solutions with strictly positive species abundances ($N_i^* > 0$), are a function of α_{eff}. Since the matrix β_{eff} may not be symmetric, we solve the system using its singular value decomposition β_{eff} (*59*):

$$\beta_{eff} = \underbrace{\begin{bmatrix} | & | & & | \\ u^1_{eff} & u^2_{eff} & \cdots & u^S_{eff} \\ | & | & & | \end{bmatrix}}_{=U_{eff}} \underbrace{\begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \delta_S \end{bmatrix}}_{=D} \underbrace{\begin{bmatrix} - & v^1_{eff} & - \\ - & v^2_{eff} & - \\ & \vdots & \\ - & v^S_{eff} & - \end{bmatrix}}_{=V_{eff}}. \tag{S3}$$

The columns of matrix U_{eff} are composed of orthonormal vectors called the left singular vectors, the rows of matrix V_{eff} are composed of orthonormal vectors called the right singular vectors. The elements of the diagonal matrix D, called the singular values, are positive and can be ordered without loss of generality, such that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_S \geq 0$. Assuming that all singular values are strictly positive, i.e., the matrix β_{eff} is nonsingular, the solution for the equilibrium point N^* is unique and given by

$$N^* = \beta_{eff}^{-1}\alpha_{eff} = V_{eff}^{-1} D^{-1} U_{eff}^{-1}\alpha_{eff} = \sum_{k=1}^{S} v^k_{eff} \frac{1}{\lambda_k} < u^k_{eff} | \alpha_{eff} >, \tag{S4}$$

where $< u^i | \alpha_{eff} >$ denotes the scalar product between the vectors u^i and α_{eff}.

Now, we will explain how effective growth rates can be chosen such that all species have strictly positive abundances at the equilibrium point. This needs two extra assumptions on the matrix β_{eff}: all elements of matrices $\beta^t_{eff}\beta_{eff}$ and $\beta_{eff}\beta^t_{eff}$ should be positive. Then, using the Perron-Frobenius Theorem (theorem 8.4.4 in Ref. 60, this implies that all the elements of the singular vectors u^1 and v^1 are strictly positive. If we choose $\alpha_{eff} = u^1_{eff}$, then the equilibrium point, which is given by $N^* = \frac{1}{\lambda_1} v^1_{eff}$, is a feasible solution (i.e., $N_i^* > 0$). Note that any vector collinear to u^1_{eff} leads to a feasible solution, i.e., u^1_{eff} generates a vector of effective growth rate resulting in feasible solutions. Using effective growth rates on that specific vector, one can make any mutualistic network perfectly persistent (see Figure **??**). By imposing $N_i^* > 0$ to the solution in Equation S4, we derive the following inequality:

$$\frac{\sum_{k=1}^{S} < u_{eff}^k|\alpha_{eff} >< v_{eff}^k|\alpha_{eff} > - < u_{eff}^1|\alpha_{eff} >< v_{eff}^1|\alpha_{eff} >}{< u_{eff}^1|\alpha_{eff} >< v_{eff}^1|\alpha_{eff} >} \leq \frac{\lambda_2}{\lambda_1}. \tag{S5}$$

This inequality is the generalization of Equation 20 in Ref. 6 to the case where the competition matrix β_{eff} is not necessary symmetric. This inequality holds for plants and animals and tell us that the higher the collinearity of the effective intrinsic growth rates α_{eff} with the leading singular left u_{eff}^1 and right v_{eff}^1 vectors, the higher is the chance of obtaining a solution where all species are persistent. The left side of this inequality shown in Equation S4 is called deviation from the structural vector and the sentence above can be rephrased as: the lower the deviation, the higher the chance that all species are persistent.

Effective intrinsic growth rates ($\alpha_{eff}^{(P)}$ and $\alpha_{eff}^{(A)}$) are not the observable parameters of intrinsic growth rates ($\alpha^{(P)}$ and $\alpha^{(A)}$) of plants and animals. Thus, one has to move back from the effective competition framework to the observable parameter space. This can be achieved by using the inverse of the matrix $T = 1 + \Gamma C^{-1}$ on the left and right leading singular vectors of $\beta_{eff}^{(P)}$ and $\beta_{eff}^{(A)}$:

$$\begin{bmatrix} \alpha_L^{(P)} \\ \alpha_L^{(R)} \end{bmatrix} = T^{-1} \begin{bmatrix} (u_{eff}^{(P)})^1 \\ (u_{eff}^{(A)})^1 \end{bmatrix} \qquad \text{and} \qquad \begin{bmatrix} \alpha_R^{(P)} \\ \alpha_R^{(A)} \end{bmatrix} = T^{-1} \begin{bmatrix} (v_{eff}^{(P)})^1 \\ (v_{eff}^{(A)})^1 \end{bmatrix} \tag{S6}$$

This transformation generates results in a left and right vector of intrinsic growth rates for both the plants and the animals—what we call the structural vector. As in the effective framework, the higher the collinearity of the intrinsic growth rates with the vectors generated by these left and right structural vectors, the higher the number of persistent species.

S6 Resampling model

To study the effect of nestedness on the structural stability of a given network, we generate alternative network architectures using a resampling model. Traditionally, the Erdős-Rényi model, the swap (or fixed) model, and the probabilistic model have been used. However, as shown in Figure S8, these models generate networks with a very restricted range of nestedness.

Here, we use a resampling procedure that is able to generate a large range of nestedness. This is important in order to generate significantly different nested architectures, and in order to obtain better statistical estimates of the effect of nestedness on structural stability. This procedure is based on the matching-centrality statistical model (*46*). Moreover, these restrictive generative rules are biologically justified by the constraints imposed by the phylogeny (*34*).

The matching-centrality model aims to infer the probability of a link between two species (p_{ij}, i is an animal, and j is a plant) by assuming that the species are characterized by a latent trait of centrality (v_i^* and f_j^*) and a latent trait a matching (v_i and f_j). The model is given by

$$\text{logit}(p_{ij}) = -\kappa(v_i - f_j)^2 + \phi_1 v_i^* + \phi_2 f_j^* + m. \tag{S7}$$

In this way, centrality traits quantify variability in degree, while matching traits quantify the assortative mating structure. The parameters κ, ϕ_1, and ϕ_2 are positive scaling parameters that give the importance of the contributions of the terms. Although these latent traits are a priori unknown, they can be estimated from the network itself. Then, based on their estimation, the probability of a link between all pairs of plants and animals is estimated. Thus, a new network can simply be generated by drawing randomly the links based on those estimated interaction probabilities. The expected number of links, as well as the expected degree of the sampled network are equal to the ones of the observed networks. However, this model, which can be viewed as a probabilistic version of the swap model, also generates a restricted range of nestedness. To relax this constraint, we generalize the model by introducing a control parameter, that we call temperature T, that modulates the level of stochasticity in the model:

$$\mathrm{logit}(p(T)_{ij}) = \frac{1}{T}\left(-\kappa(v_i - f_j)^2 + \phi_1 v_i^* + \phi_2 f_j^*\right) + m(T). \tag{S8}$$

Here, the intercept $m(T)$ is adjusted for each temperature value such that the expected number of links is equal to the observed one. When the temperature goes to infinite, our model converges to the Erdős-Rényi model, when the temperature goes to zero, the system freezes in the most probable configuration predicted by our model.

As it can be observed in Figure S4, when varying the temperature, our model can reproduce a larger range of nestedness compared to the ones generated by alternative sampling models. Importantly, our model predicts also a maximum and a minimum level of nestedness that the system can reach, when generative rules (represented by the matching and centrality traits) are introduced.

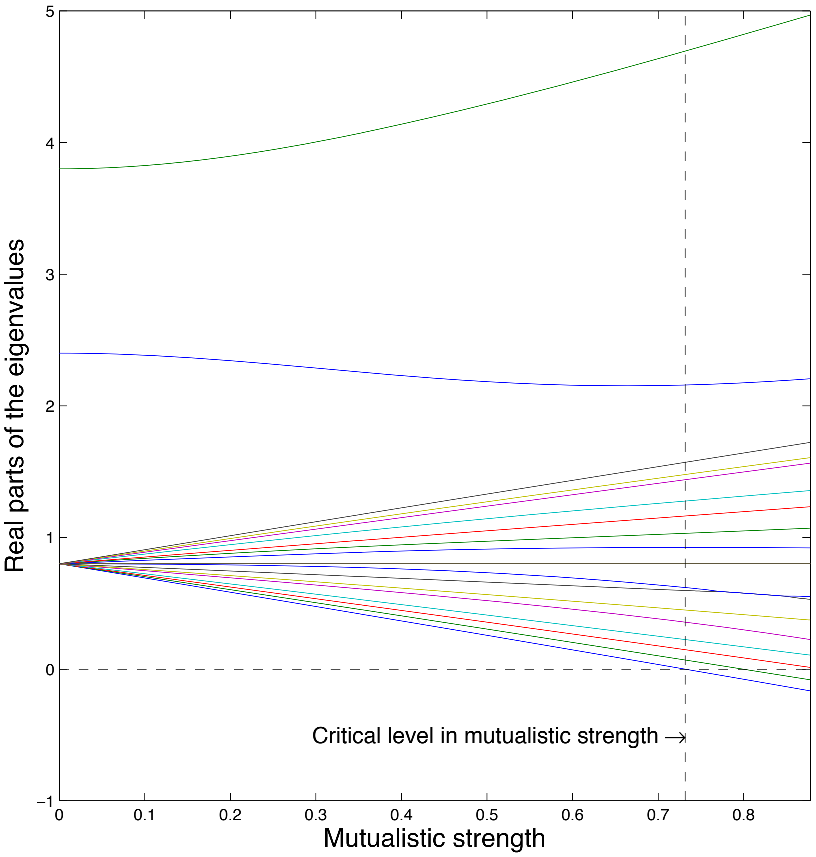

Fig. S1: Real part of the eigenvalues of matrix B as a function of the mutualistic strength (γ_0). Here we use the empirical network located in Yakushima Island, Japan (see Table S1), a mutualistic trade-off $\delta = 0.5$, and an interspecific competition $\rho = 0$. The vertical dashed line represents the critical value of mutualistic strength γ_0^r above which at least one of the eigenvalues has a negative real part.

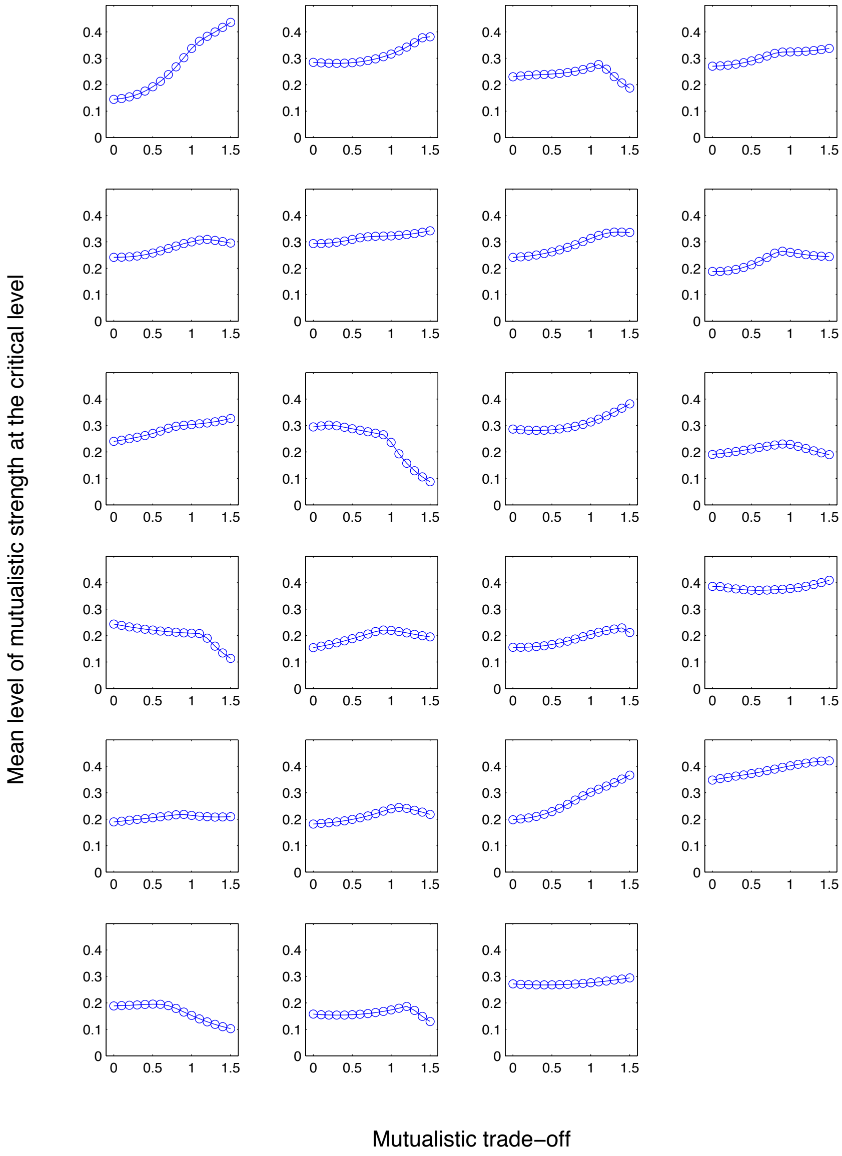

Fig. S2: Mutualistic trade-off and critical value of mutualistic strength. For each of 23 observed mutualistic networks (see Table S1), this figure illustrates how the mutualistic trade-off influences the maximum level of mean mutualistic strength that the system can handle to guarantee the stability of any feasible equilibrium. One can see that, in the majority of cases, the mutualistic trade-off enhances the level of mutualistic strength.

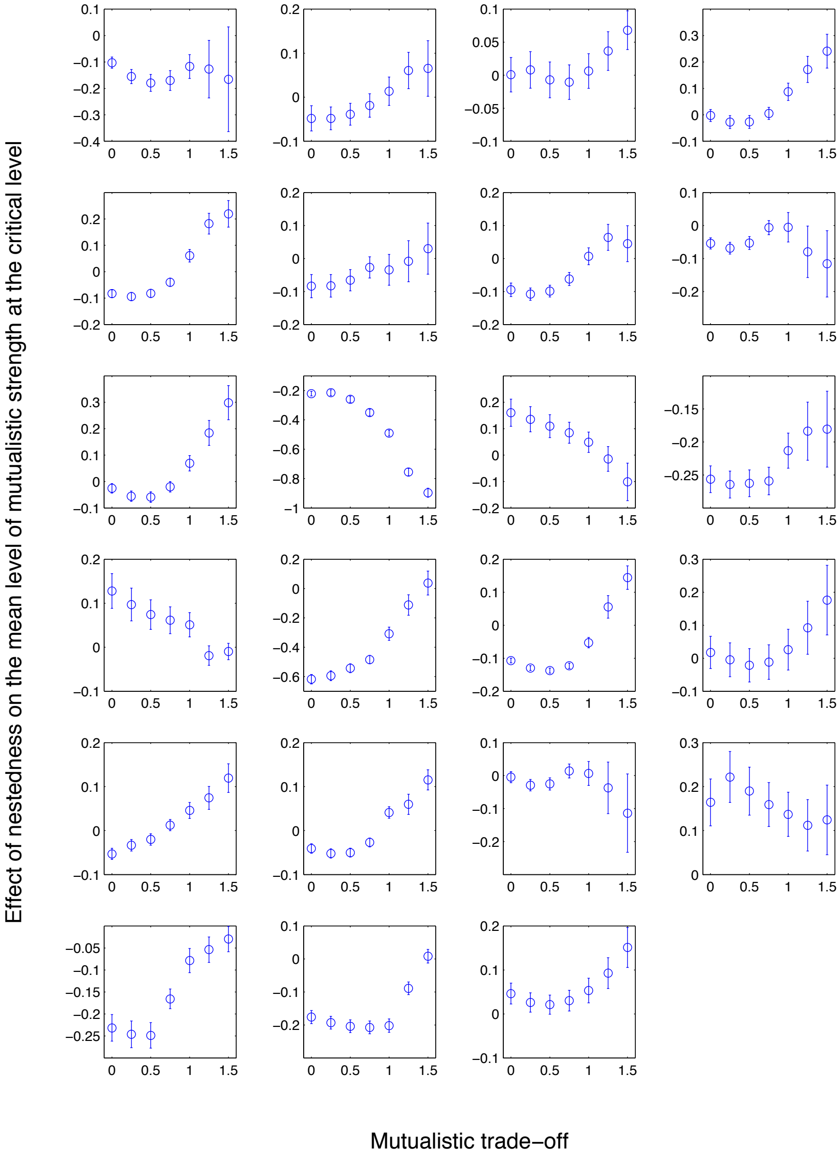

Fig. S3: Nestedness and critical value of mutualistic strength. For each of 23 observed mutualistic networks (Table S1), this figure illustrates, as function of the mutualistic trade off, the linear effect of nestedness on the maximum level of mean mutualistic strength that the system can handle to guarantee the stability of any feasible equilibrium. One can see that in the majority of cases, the slopes of this effect increase along with the mutualistic trade-off.

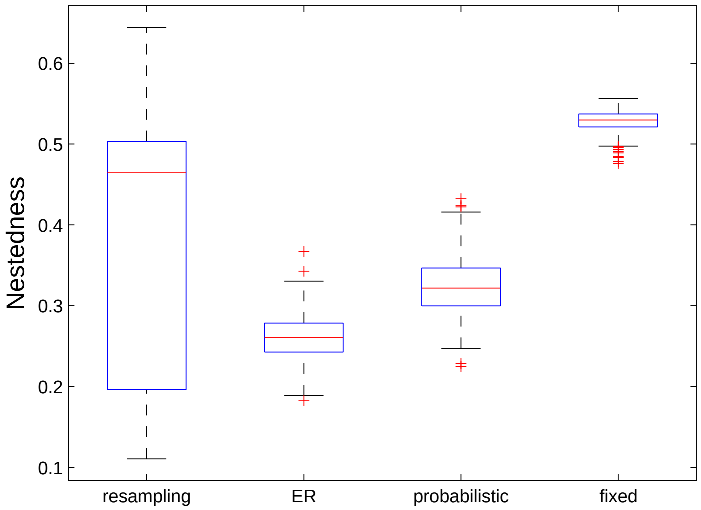

Fig. S4: Model-generated nested architectures. For the observed mutualistic network in the Snowy Mountains, Australia (see Table S1), the figure shows the different distributions of nestedness values generated by different models: our resampling model, the Erdős-Rényi model, the probabilistic model, and the fixed or swap model.

S7 Structural stability across observed mutualistic networks

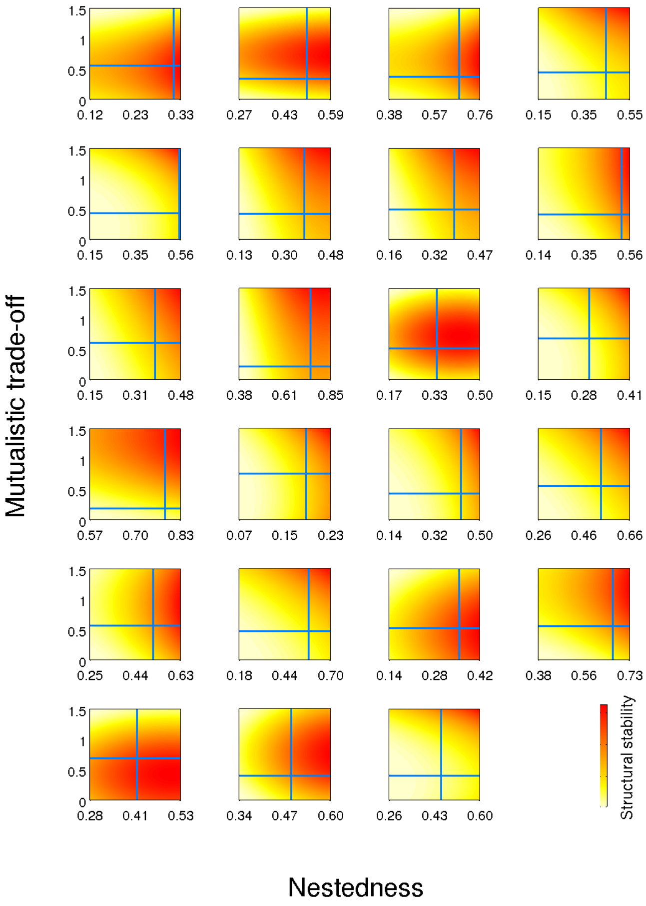

Fig. S5: Structural stability in all the observed networks. The darker the color, the larger the positive effect on the area of structural stability. The lightest regions correspond to architectures with a negative effect on this area. Dashed lines correspond to different values of mean strength level. Solid lines correspond to the observed values of nestedness and mutualistic trade-offs.

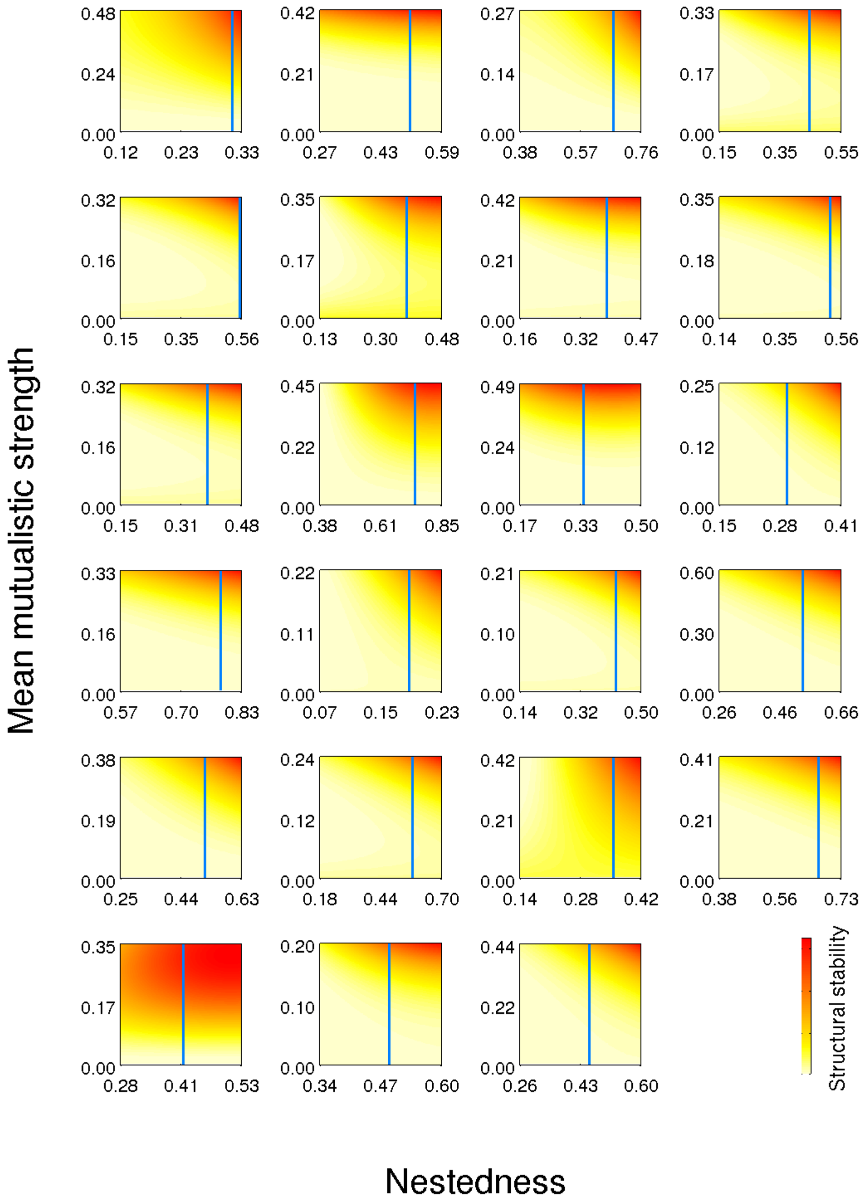

Fig. S6: Structural stability and nestedness. Keeping the observed mutualistic trade-off fixed, the figure shows the extent to which the observed nestedness (solid line) can modulate the area of structural stability (partial fitted residuals) across different values of mean mutualistic strength.

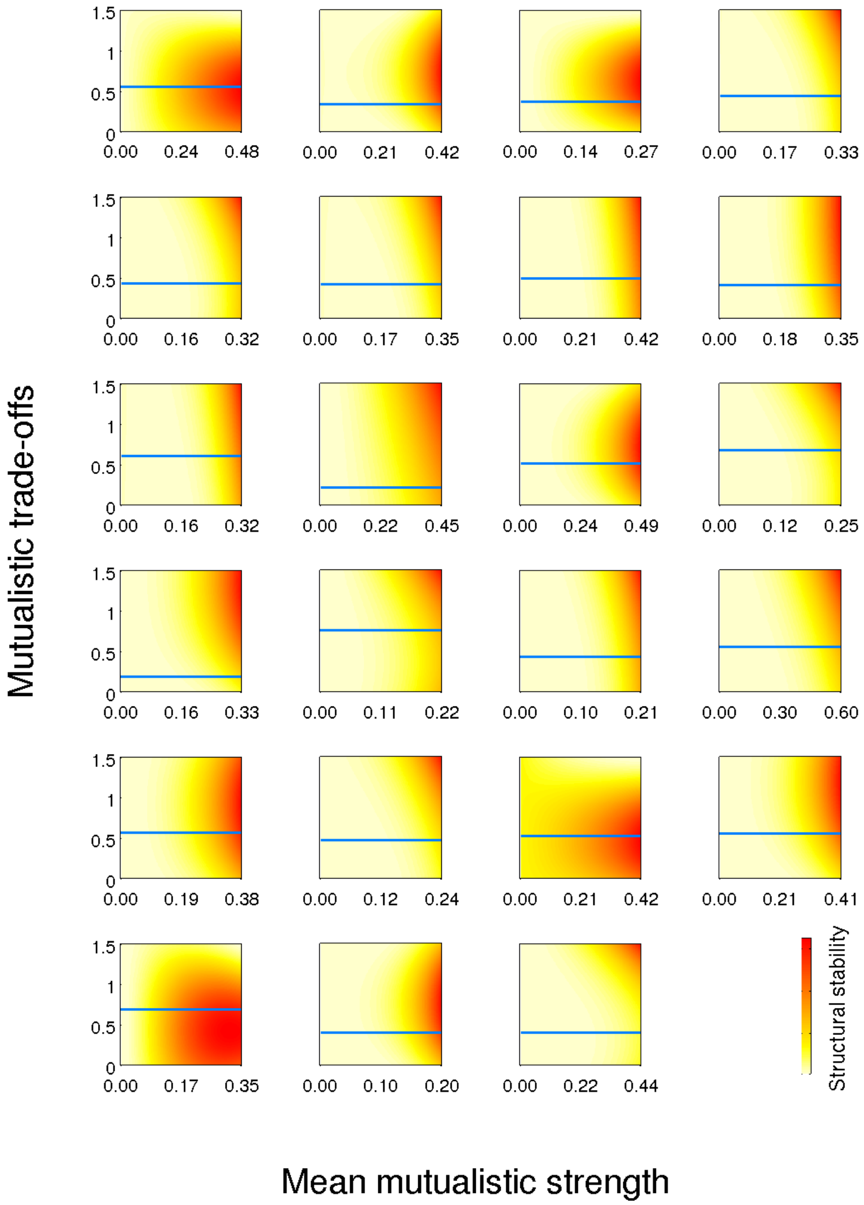

Fig. S7: Structural stability and mutualistic trade-offs. Keeping the observed nestedness fixed, the figure shows the extent to which the observed mutualistic trade-off (solid line) expands the area of structural stability (partial fitted residuals) across different values of mean mutualistic strength.

S8 Robustness of results to the choice of ρ and h

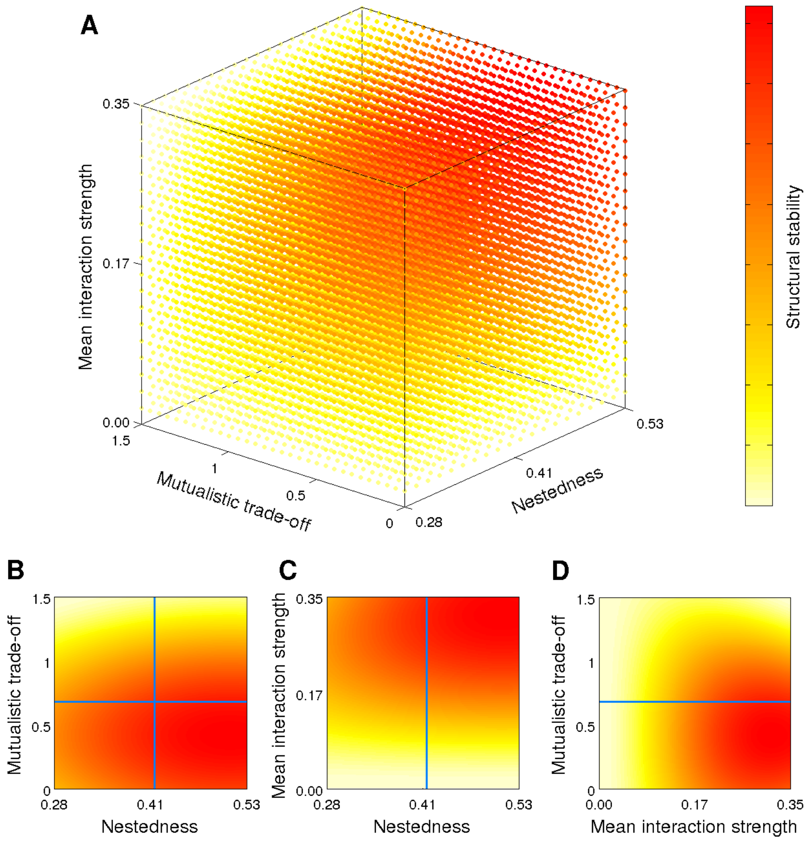

Fig. S8: Equivalent to Figure 5 in main text but where handling time is now sampled from a uniform distribution between 0.08 and 0.12 for each of the species.

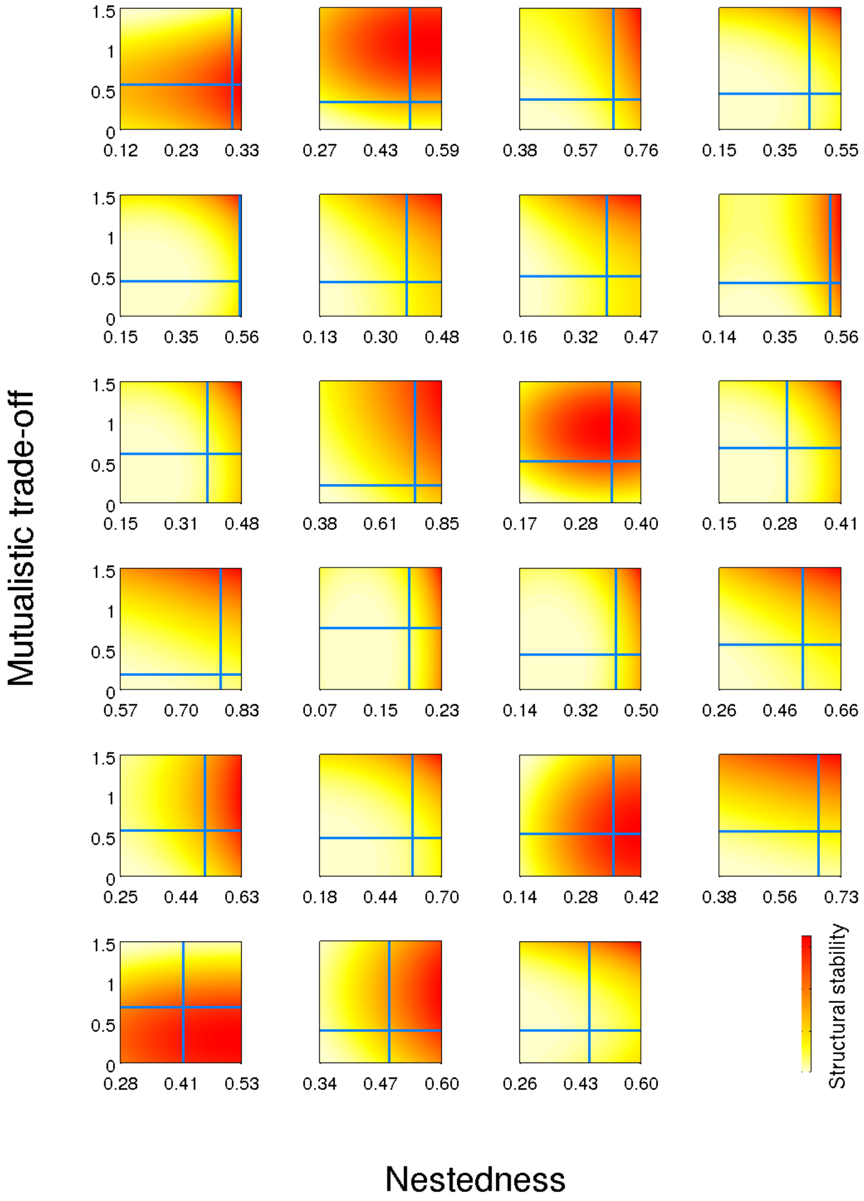

Fig. S9: Equivalent to Figure S5 in SM but with $\rho = 0.4$.

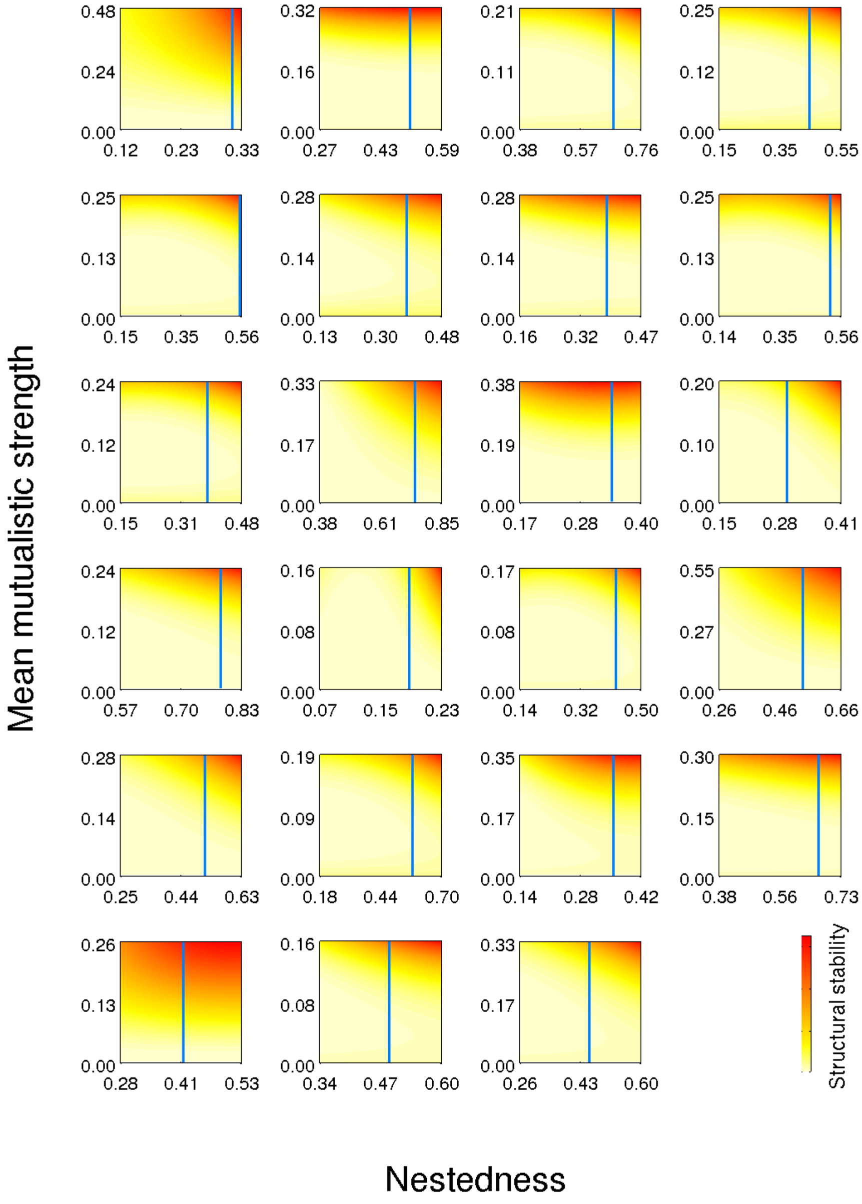

Fig. S10: Equivalent to Figure S6 in SM but with $\rho = 0.4$.

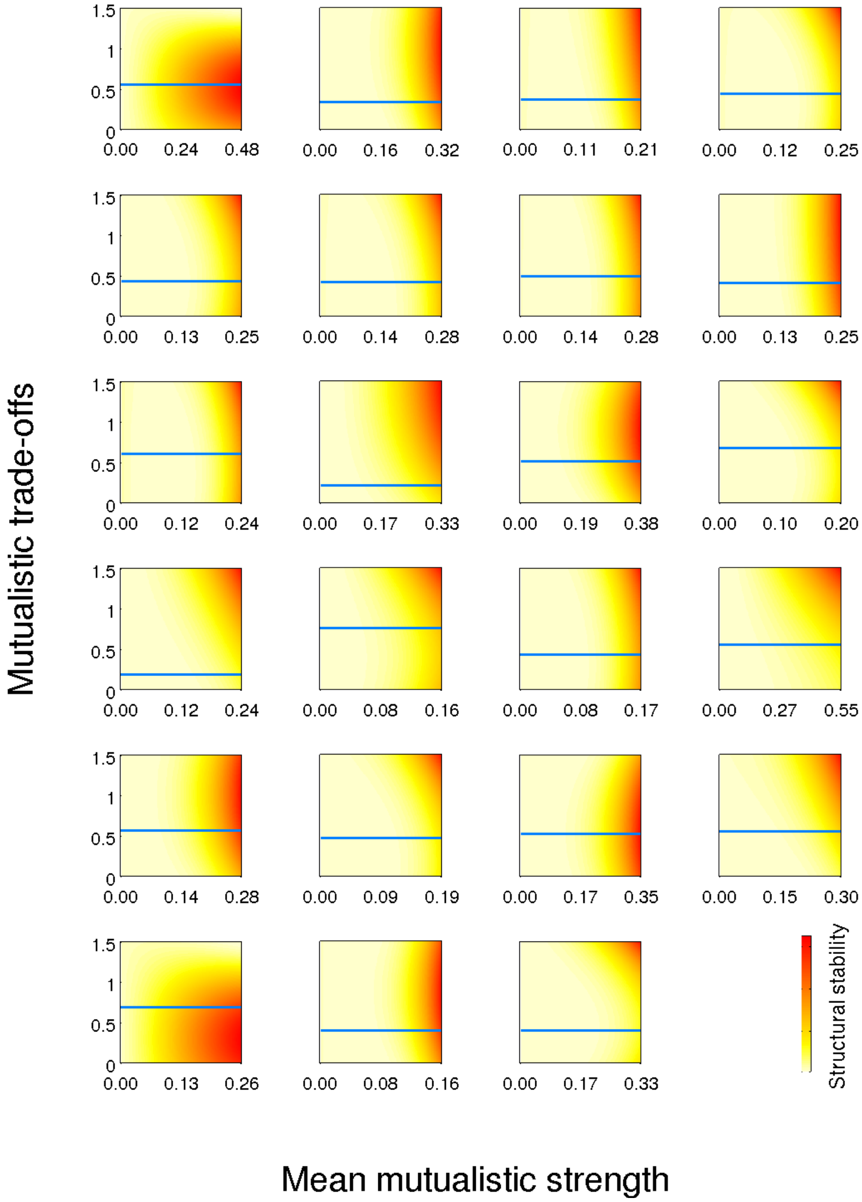

Fig. S11: Equivalent to Figure S7 in SM but with $\rho = 0.4$.

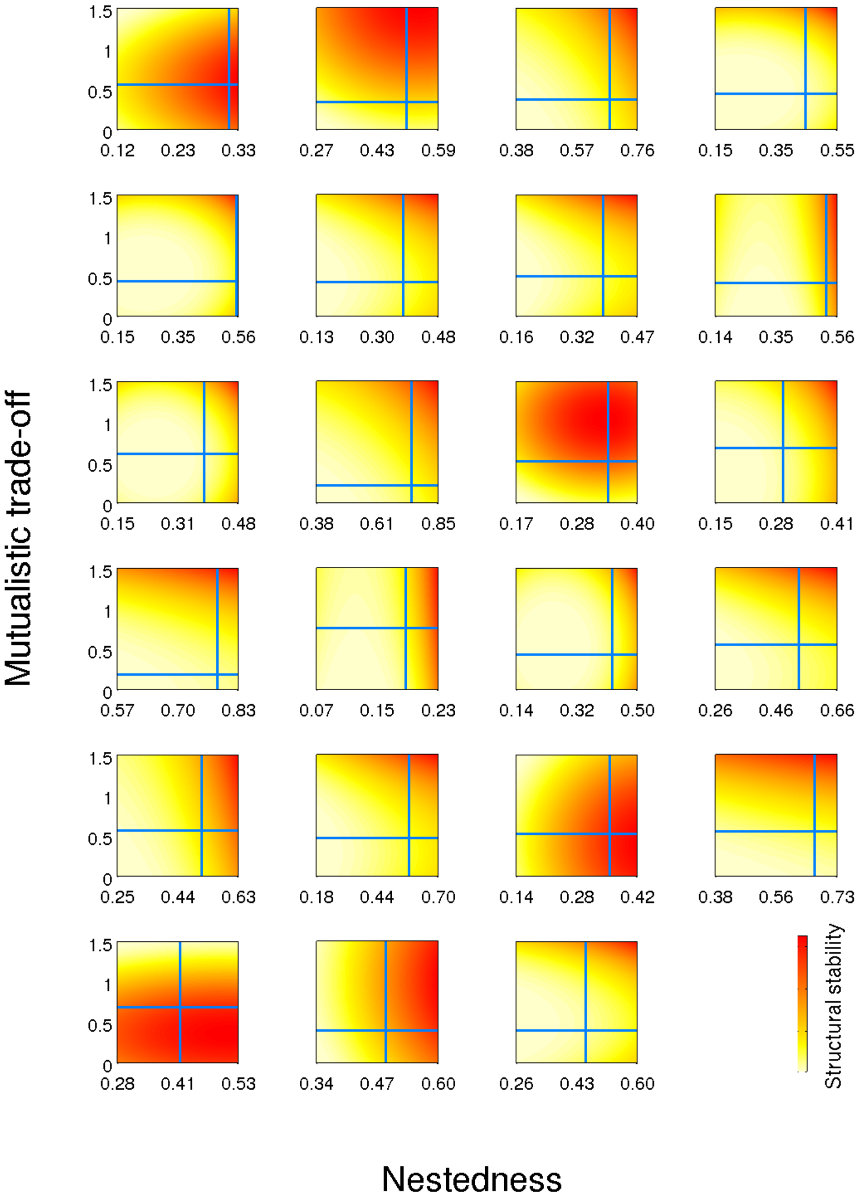

Fig. S12: Equivalent to Figure S5 in SM but with $\rho = 0.6$.

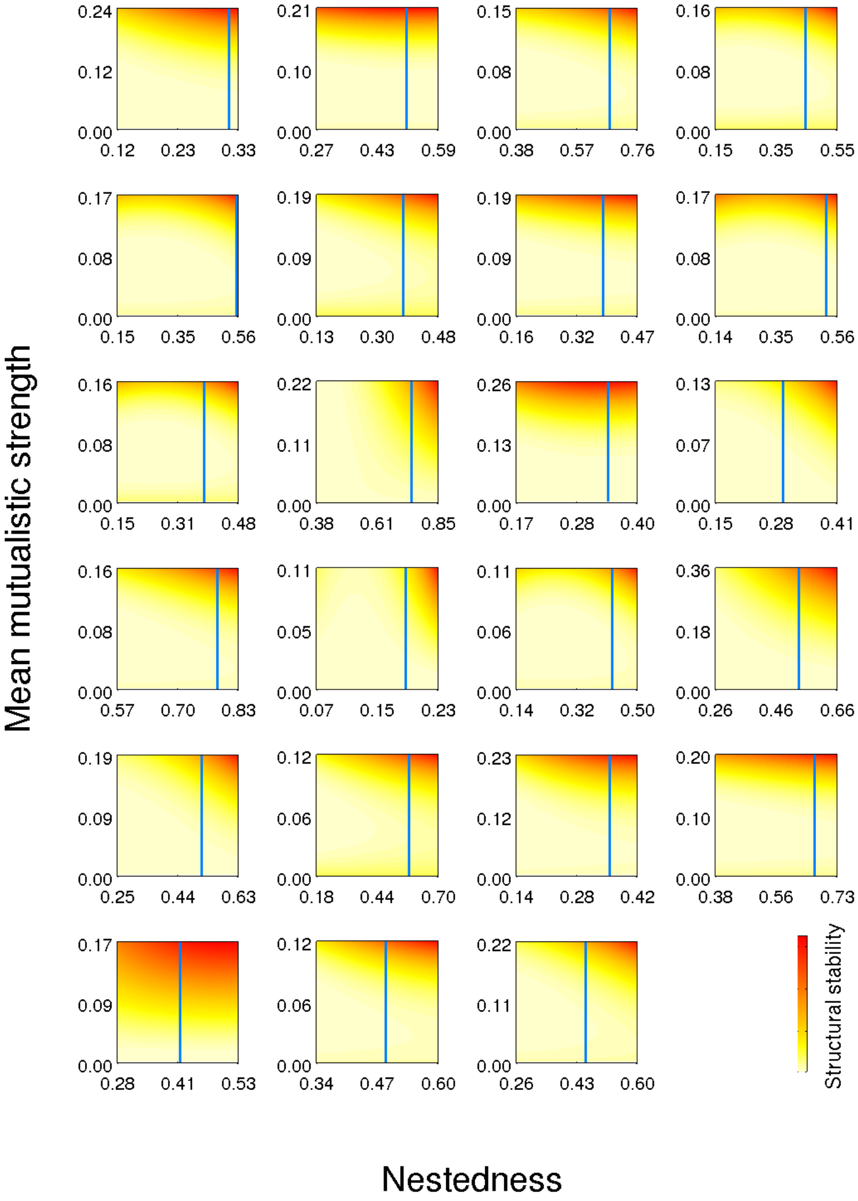

Fig. S13: Equivalent to Figure S6 in SM but with $\rho = 0.6$.

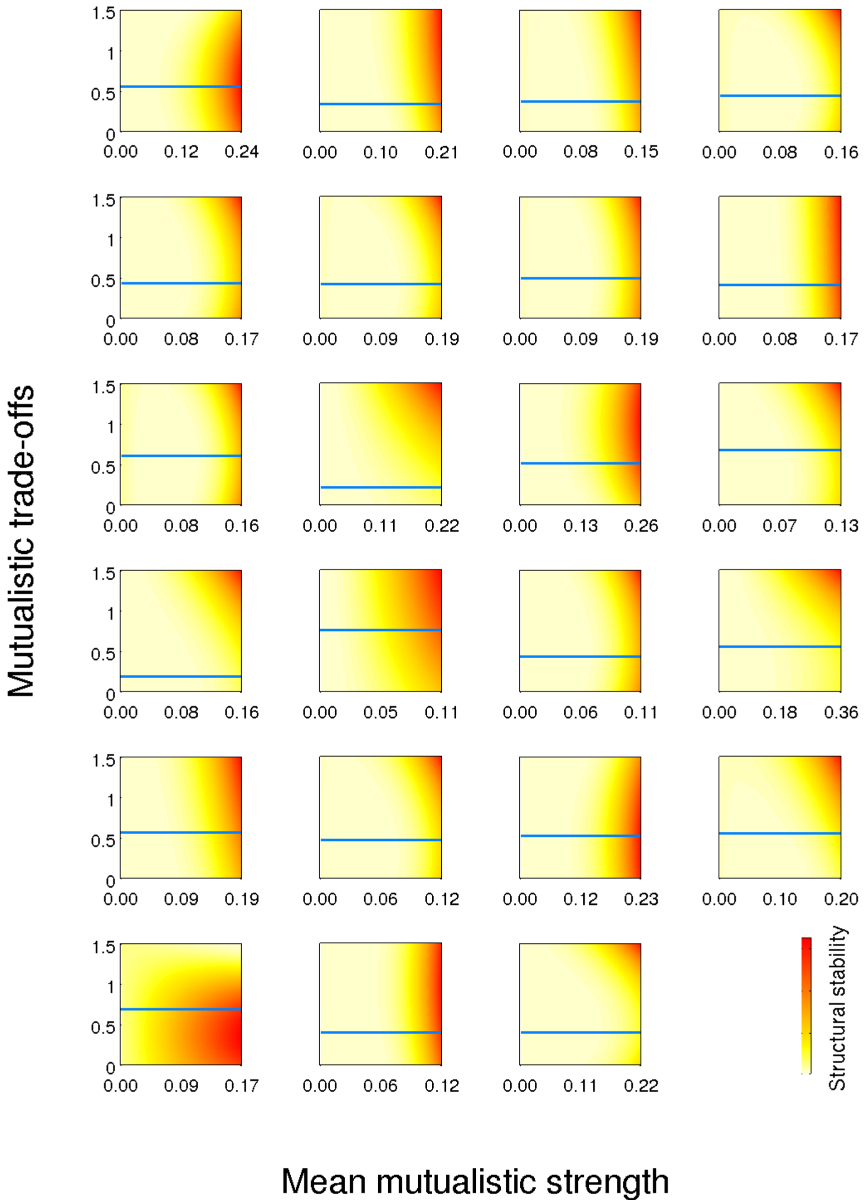

Fig. S14: Equivalent to Figure S7 in SM but with $\rho = 0.6$.

S9 Data

Network	Animals	Plants	Interactions	Location
`M_PL_004`	101	12	167	Central New Brunswick, Canada
`M_SD_001`	21	7	50	Princeton, Mercer, New Jersey, USA
`M_SD_002`	9	31	119	Mont Missim, Morobe Prov., New Guinea
`M_SD_003`	16	25	68	Caguana, Puerto Rico
`M_SD_004`	20	34	95	Cialitos, Puerto Rico
`M_SD_005`	13	25	49	Cordillera, Puerto Rico
`M_SD_006`	15	21	51	Fronton, Puerto Rico
`M_PL_006`	61	17	146	Hickling, Norfolk, UK
`M_PL_007`	36	16	85	Shelfanger, Norfolk, UK
`M_SD_008`	10	16	110	Mtunzini, South Africa
`M_SD_009`	18	7	38	Santa Genebra Reserve T1 SE, Brazil
`M_SD_010`	14	50	234	Tropical rainforest, Trinidad
`M_SD_011`	14	11	47	Calton, UK
`M_PL_013`	56	9	103	KwaZulu-Natal region, South Africa
`M_SD_012`	29	35	146	Santa Genebra reserve T2. SE, Brazil
`M_PL_017`	79	25	299	Bristol, UK
`M_SD_014`	17	16	121	Hato Raton, Sevilla, Spain
`M_PL_019`	85	40	264	Snowy Mountains, Australia
`M_PL_024`	18	11	38	Melville, Island, Canada
`M_PL_025`	44	13	143	North Carolina, USA
`M_PL_033`	34	13	141	Ottawa, Canada
`M_SD_020`	33	25	150	Nava Correhuelas, S. Cazorla, SE, Spain
`M_SD_023`	8	15	38	Yakushima Island, Japan

Table S1: Data set. We apply our study to the 23 quantitative mutualistic networks published at `www.web-of-life.es` at the time this analysis was conducted (see also Ref. 34). These networks represent diverse environmental and biotic conditions. Network identifiers consists on a combination of letters and a specific number: `M_PL` and `M_SD` stand for mutualistic networks of pollination and seed dispersal, respectively. The order of the networks matches the one used across the panels of Fig. 6.

References and Notes

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18. Any matrix B is called Lyapunov-stable if the real parts of all its eigenvalues are positive, meaning that a feasible equilibrium at which all species have the same abundance is at least locally stable. A matrix B is called D-stable if DB is a Lyapunov-stable matrix for any strictly positive diagonal matrix D. D-stability is a stronger condition than Lyapunov stability in the sense that it grants the local stability of any feasible equilibrium (*15*). In addition, a matrix B is called Lyapunov diagonally stable if there exists a strictly positive diagonal matrix D so that $DB + B^tD$ is a Lyapunov-stable matrix. This notion of stability is even stronger than D-stability in the sense that it grants not only the local stability of any feasible equilibrium point, but also its global stability (*14*). A Lyapunov–diagonally stable matrix has all of its principal minors positive. Also, its stable equilibrium (which may be only partially feasible; some species may have an abundance of zero) is specific (*57*). This means that there is no alternative stable state for a given parameterization of intrinsic growth rates (*55*). We have chosen the convention of having a minus sign in front of the interaction-strength matrix B. This implies that B has to be positive Lyapunov-stable (the real parts of all eigenvalues have to be strictly positive) so that the equilibrium point of species abundance is locally stable. This same convention applies for D-stability and Lyapunov–diagonal stability.

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21. All simulations were performed by integrating the system of ordinary differential equations by using the function ode45 of Matlab. Species are considered to have gone extinct when their abundance is lower than 100 times the machine precision. Although we present the results of our simulations with a fixed value of $\rho = 0.2$ and $h = 0.1$, our results are robust to this choice as long as $\rho < 1$ (*45*). In our dynamical system, the units of the parameters are abundance for N, 1/time for α and h, and 1/(time · biomass) for β and γ. Abundance and time can be any unit (such as biomass and years), as long as they are constant for all species.

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48. For a given network, we look at the association of the fraction of surviving species with the deviation from the structural vector of plants $\eta_{(P)}$ and animals $\eta_{(A)}$, nestedness N, mutualistic trade-off δ, and mean level of mutualistic strength $\bar{\gamma} = \langle \gamma_{ij} \rangle$ using the following binomial generalized linear model (*47*): $\mathrm{log\,it}(\mathrm{probabilityofsurviving}) \sim \log(\eta_A) + \log(\eta_P) + \bar{\gamma} + \bar{\gamma}^2 + \bar{\gamma} \cdot N + \bar{\gamma} \cdot N^2 + \bar{\gamma} \cdot \delta + \bar{\gamma}\delta^2$. Obviously, at a level of mutualism of zero ($\gamma = 0$), nestedness and mutualistic trade-off cannot influence the probability of a species to survive. We have to include an interaction between the mean level of mutualism and the nestedness and mututalistic trade-off. We have also included a quadratic term in order to take into account potential nonlinear effects of nestedness, mutualistic trade-off, and mean level of mutualistic strength. The effect of network architecture is confirmed by the significant likelihood ratio between the full model and a null model without such a network effect ($P < 0.001$) for all the observed empirical networks. The effect of network architecture on structural stability can be quantified by the partial fitted values defined as follows: $\mathrm{partialfittedvalues} = \hat{\beta}_1\bar{\gamma} + \hat{\beta}_2\bar{\gamma}^2 + \hat{\beta}_3\bar{\gamma} \cdot N + \hat{\beta}_4\bar{\gamma} \cdot N^2 + \hat{\beta}_5\bar{\gamma} \cdot \delta + \hat{\beta}_6\bar{\gamma} \cdot \delta^2$, where $\hat{\beta}_1 \cdots \hat{\beta}_6$ are the fitted parameters corresponding to the terms $\bar{\gamma}$ to $\bar{\gamma} \cdot \delta^2$, respectively.

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52. To estimate the mutualistic trade-off, δ, from an empirical point of view, we proceed as follows. First, there is available data on the frequency of interactions. Thus, q_{ij} is the observed number of visits of animal species j on plant species i. This quantity has been proven to be the best surrogate of per capita effects of one species on another (γ_{ij}^P and γ_{ij}^A) (*41*). Second, the networks provide information on the number of species one species interacts with (its degree or generalization level; k_i^A and k_i^P). Following (*41*, *50*, *51*), the generalization level of a species has been found to be proportional to its abundance at equilibrium. Thus, the division of the total number of visits by the product of the degree of plants and animals can be assumed to be proportional to the interaction strengths.

Mathematically, we obtain two equations, one for the effect of the animals on the plants and vice versa: $(q_{ij})/(k_i^P k_j^A) \propto \gamma_{ij}^P$ and $(q_{ij})/(k_i^A k_j^P) \propto \gamma_{ij}^A$. Introducing the explicit dependence between interaction strength and trade-off—for example, $\gamma_{ij}^P = \gamma_0/(k_i^P)^\delta$ and $\gamma_{ij}^A = \gamma_0/(k_i^A)^\delta$—we obtain $(q_{ij})/(k_i^P k_j^A) \propto (\gamma_0)/\left[(k_i^P)^\delta\right]$ and $(q_{ij})/(k_i^A k_j^P) \propto (\gamma_0)/\left[(k_i^A)^\delta\right]$. In order to estimate the value of δ, we can just take the logarithm on both sides of the previous equations for the data excluding the zeroes. Then, δ is simply given by the slope of the following linear regressions: $\log\left(\frac{q_{ij}}{k_i^P k_j^A}\right) = a^P - \delta \log(k_i^P)$ and $\log\left(\frac{q_{ij}}{k_i^A k_j^P}\right) = a^A - \delta \log(k_i^A)$, where a^P and a^A are the intercepts for plants and animals, respectively. These two regressions are performed simultaneously by lumping together the data set. The intercept for the effect of animals on plants (a^P) may not be the same as the intercept for the effect of the plants on the animals (a^A).

53. The competition matrix given by $\beta = \begin{bmatrix} 3 & 3.1 & 1 \\ 5 & 5 & 6 \\ 4 & 1 & 3 \end{bmatrix}$ is Lyapunov-stable but not *D*-stable.

Because Lyapunov diagonal stability implies *D*-stability, this matrix is also not Lyapunov–diagonally stable.

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