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Helping At-Risk Students Add Up: Motivational Lessons for Students in High School Mathematics

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Mathematics, which can be easily related to the real-world, is often taught as a separate subject that is taught one hour, or less, of each school day. In this context, the students cannot possibly be expected to relate mathematics to their own lives. In fact, they perceive the mathematics that they complete in class as work and strive to leave it behind them as soon as class has ended. (Midkiff, 1993, p. 7)

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Foreword

According to *Teaching in America*, a Carnegie Corporation report shows that nineteen million adolescents in the United States are considered at-risk students, students whose home conditions and backgrounds are such an obstacle that they are in danger of failing and/or dropping out of school (Morrison, 2000, p. 201). These students are facing a variety of situations such as minority and low socioeconomic statuses, low levels of proficiency in the English language, dysfunctional families, unstable home lives, high drug or community crime rates, and teenage pregnancies. These factors do not say that these students are certainly going to fail or drop out of school. Some do overcome these situations, but as they face these difficulties, school often becomes significantly less of a priority for many at-risk students. The teacher in the classroom is often the one who contends with students who care little about learning, and must somehow find a way to connect with them and motivate them to learn.

Mathematics may be one of the most complicated subjects to teach to at-risk learners. Finding people in the United States who see the value of learning mathematics is difficult; as a result, many people lack basic mathematics skills needed to solve problems in everyday life. It is no secret that some individuals perceive America as being behind many industrialized countries in science and mathematics. One study shows that fourteen out of twenty countries scored higher in mathematics and science than the United States (Ornstein & Lasley, 2000, p. 91). This

problem is only magnified for students who are at-risk. Many of these other countries do not test all of their students, nor does everyone have to go to school. These factors make the problem seem greater than it really is. Nonetheless, many of these countries do have different strategies for teaching mathematics to their students that seem to work more efficiently than some strategies teachers use in American high schools. Educators agree that we need to do something to help solve this crisis.

Research suggests that weak motivation lies at the root of these students' difficulties in school. Any motivation they do have is often extrinsic instead of intrinsic. They feel they have bigger issues to deal with than school. This lack of intrinsic motivation is generally the major difference between those who are at-risk and other learners in the classroom. The at-risk students also spend less time engaged in any given activity, meaning they generally do not learn as much (Middleton, 1995, p. 254). Students with intrinsic motivation typically have a desire to learn or excel, so the methods teachers use in reaching highly motivated students are not quite as important for these students. Educators should focus on finding a method that motivates lower-achieving pupils to succeed and at the same time does not neglect higher-achieving students.

The methods teachers currently use in the classroom fall into two groups, direct and indirect instruction. Direct teaching methods involve teacher-centered methodologies, including lecture, questioning, discussion, and practice and drill. Indirect methods are student-centered activities such as role-playing, simulations and games, inquiry and discovery (such as experimentation), and independent projects (Morrison 2000, pp. 523, 526).

Though teachers tend to use direct instruction most often in mathematics classrooms, indirect methods are the most effective way to motivate high school students to learn mathematics, especially students considered by educators to be at-risk. Educators acknowledge

the need for a change from current methods, and recognize specific techniques that help students learn mathematics. Using a variety of these indirect strategies, one may develop lesson plans, which improve students' achievement in mathematics and motivate them to learn and appreciate mathematics outside of the classroom.

A Need for Change

In the 1989-90 school year, only about fifty percent of juniors and seniors in high school had mastered eighth-grade mathematics (Englelmann, Carnine, & Steely, 1991, p. 292). In 1996 only sixteen percent of seniors met twelfth-grade performance standards (Ornstein & Lasley, 2000, p. 90). These numbers indicate that current approaches to teaching these students are not working. Somehow, the majority of American students graduating from high school are not gaining the mathematics education they need, and schools are sending students out into the workforce unprepared to handle basic mathematics problems. This information was reported for all students in the U.S.; certainly the problem is much worse among students who are already at-risk for failing or dropping out of school. Part of the difference between our students and those in countries such as Japan, South Korea, Thailand, and Israel appears to be in the techniques educators use in their classrooms (Ornstein & Lasley, 2000, p. 91).

Researchers have conducted numerous studies to try to determine these differences, and the answer seems to lie in the manner in which teachers are using class time. According to one study, Japanese students spend only forty-one percent of class instructional time practicing routine procedures as opposed to the ninety-six percent of class instructional time that U.S. students spend on the same types of activities (Maccini & Gagnon, 2000, ¶ 3). Studies show that these other countries use methods that emphasize higher mental processes, while schools in the United States tend to emphasize knowledge-based learning (Englemann, Carnine, & Steely,

1991, p. 292). If the United States began following the models of teaching set forth by other high-achieving countries, then perhaps the United States could close the gap in test scores. High-achieving countries seem to be using approaches in the classroom that focus on students and allow them to construct their own knowledge. In the United States, on the other hand, teachers often tell students what they should learn or memorize. As a result, students often forget information after taking a test.

Direct methods of teaching, though sometimes necessary, emphasize learning for knowledge. Using indirect instruction, teachers not only help students gain knowledge but involve them in thinking that is more creative and utilizes higher cognitive processes. The National Council of Teachers in Mathematics has begun to realize the necessity of teaching students to think instead of simply repeating memorized facts and information. The recently revised *Principles and Standards for School Mathematics* (NCTM, 2000) places emphasis on connecting mathematics to other disciplines. The standards advocate the implementation of instructional approaches that focus on the student (Maccini & Gagnon, 2000, ¶ 22). Some teachers already realize the importance of using indirect teaching methods, and many of these same teachers feel they do not have the time or knowledge needed to put these methods into practice. Yet, if students do not learn through lecture as well as they could through indirect instruction, then the time taken for indirect instruction is a worthwhile endeavor.

Why Are Indirect Methods More Effective?

Indirect methods more effectively assist at-risk students as well as other students in learning information, remembering it longer, and applying the information to life situations. Ultimately, indirect methods utilize cognitive processes that require the students to become more involved is her or his own learning. Benjamin Bloom's Cognitive Taxonomy breaks down the

process of cognitive thinking into six levels – knowledge, comprehension, application, analysis, synthesis, and evaluation (as cited in Ornstein & Lasley, 2000, p. 99). Direct instruction leaves students at the lower end of this taxonomy, but indirect teaching methods often cover the entire range of thinking. The rationale for trying to teach to the higher levels is that trying to teach students everything they will need to know in life is almost impossible, but a teacher can teach the students to think, which will be far more valuable in the end. In mathematics, this means helping students develop problem-solving and reasoning skills and helping them see the connection between the mathematics they learn in school and the uses of mathematics outside of the classroom.

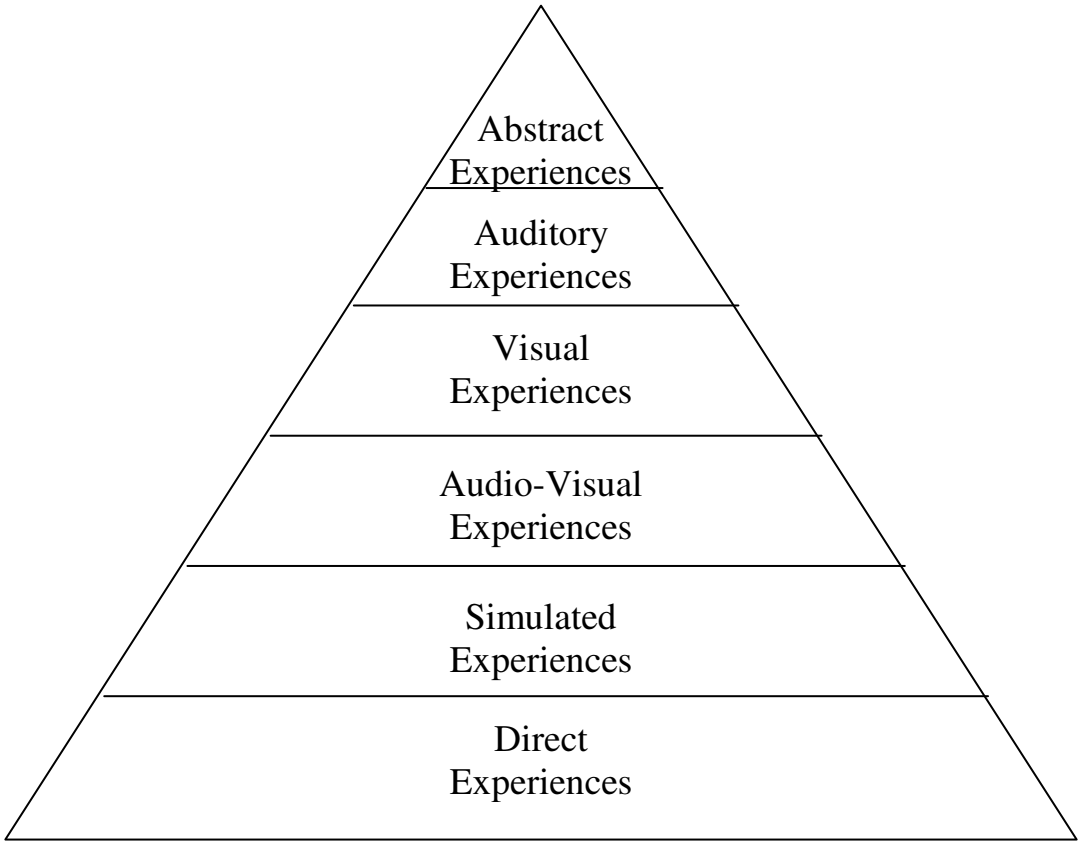
By allowing students to make these connections, teachers provide students with a reason to learn, something many at-risk students lack. These connections help students improve mathematical scores, improve motivation, and recognize situations in which they need to exercise certain mathematics skills (Bottage & Hasselbring, 1993, p. 557; Maccini & Gagnon, 2000, ¶ 42). One study cited in *Strategies for Effective Teaching* found in that "teachers who taught for meaning . . . rather than focusing on basic skill development that was not contextualized . . . [achieved] higher levels of student engagement and academic success" (Ornstein & Lasley, 2000, p. 48). The obvious inference is that indirect methods help increase students' understanding of mathematical concepts.

If teachers in the United States began using indirect methods that place students in contexts outside of mathematics, scores and achievement would increase significantly. Kay Toliver, a teacher in an inner-city school, understands the significance of these methods. Students who had previously disliked mathematics come out of her classroom excited about learning the subject. Her classes examine the mathematics in bridges, buildings, trade and

commerce, and other places in the world around them (Toliver & Skolnik, 1994, ¶ 4). Her ideas about teaching model the social learning theory of motivation, which is a combination of the behaviorist, cognitive, and humanistic theories of motivation. The social learning theory says that if students do not value the information they are learning, they will not be motivated to learn (Woolfolk, 1998, p. 375). Unfortunately, many students in the United States do not see the importance of learning mathematics; without motivation to learn, they have little incentive to do well. Indirect methods often incorporate experiences or situations from other contexts so that students have an easier time finding the value in the activities they are doing in the mathematics classroom.

In addition to seeing mathematics as a valuable resource for solving problems, successful students display interest and involvement in learning mathematics. Schiefele and Csikszentmihalyi (1994, p. 263) found that across all subject areas, interest was highly correlated with intrinsic motivation. Activities that have more personal meaning and involvement will help increase the students' intrinsic motivation so that learning is both valued and enjoyed. Most learners can only tolerate around ten to fifteen minutes of lecture before losing interest because they are not actively involved in their own learning (Ornstein & Lasley, 2000, p. 173). Many of them consider lectures boring and dull. Teachers might do well to consider Edgar Dale's Cone of Experiences (see figure 1) when forming their lesson plans. His idea is that people are more likely to remember first the information that they experience directly and next the information they gain through simulated experiences. From there to the top of his cone are audio-visual information, visual, and information they hear. Last, his cone shows that people are only likely to remember ten percent of the information they read (Kuykendall, 1991, table VI).

Figure 1: "Edgar Dale's Cone of Experiences"
(Kuykendall, 1991, table VI)



Direct instruction tends to focus solely on methods at the top of Dale's Cone, where the student is least involved in her or his own learning experience. When teachers apply indirect methods in the classroom, students become involved and interested in learning. Students receiving active teaching score higher on post and maintenance tests than those who receive instruction by traditional methods (Engelmann & Carnine, 1991, p. 295). As a result, when at-risk students become more involved, they forget they are learning and have fun. They retain the information more efficiently and, once again, their motivation to learn increases.

Evidence also suggests that intrinsic motivation to learn increases when teachers provide variety in the presentation of material and make the tasks fun. Variety helps to avoid boredom and retain attention. Indirect strategies provide educators with options for presenting material.

Educational Psychology cites these as two of the four main strategies to increase motivation in students (Woolfolk, 1998, p. 429). If students are not bored in the classroom, they are prone to enjoy learning and have fun. Then, students' attitudes about mathematics will improve, and positive attitudes increase students' scores on cognitive tests (Tomic, 1989, p. 345). Applying indirect methods of teaching in the classroom provides many advantages, but many teachers not aware of these methods do not incorporate them into their curricula.

Examples of Indirect Teaching Strategies

Though the ways to use indirect instructional approaches in high school mathematics classes are endless; manipulatives, simulations, games, and cooperative learning techniques seem to be the most common. Many of these strategies are very beneficial for teaching students because they have many of the characteristics mentioned above. J. A. Middleton's "Model of Academic Intrinsic Motivation" in figure 2 (1995, p. 256) shows how teachers who emphasize these techniques in their lesson plans in order to increase their students' interest have better success in boosting student motivation (Middleton 1995, p. 275). Each method contributes a unique property to the classroom learning experience and has a high success rate among students that educators consider at-risk.

The basic reason that utilizing manipulatives in mathematics lessons works so well among at-risk students is because the concrete objects portray abstract ideas that students may otherwise have difficulty understanding. They illustrate a wide array of mathematical relationships and students can physically arrange these objects and explore relationships on their own (Quinn, 1998, ¶ 5). A teacher could consider many items as a manipulative for demonstrating mathematical concepts. Some companies have developed and marketed tools specifically designed for teaching mathematics, yet conventional items can often serve the same

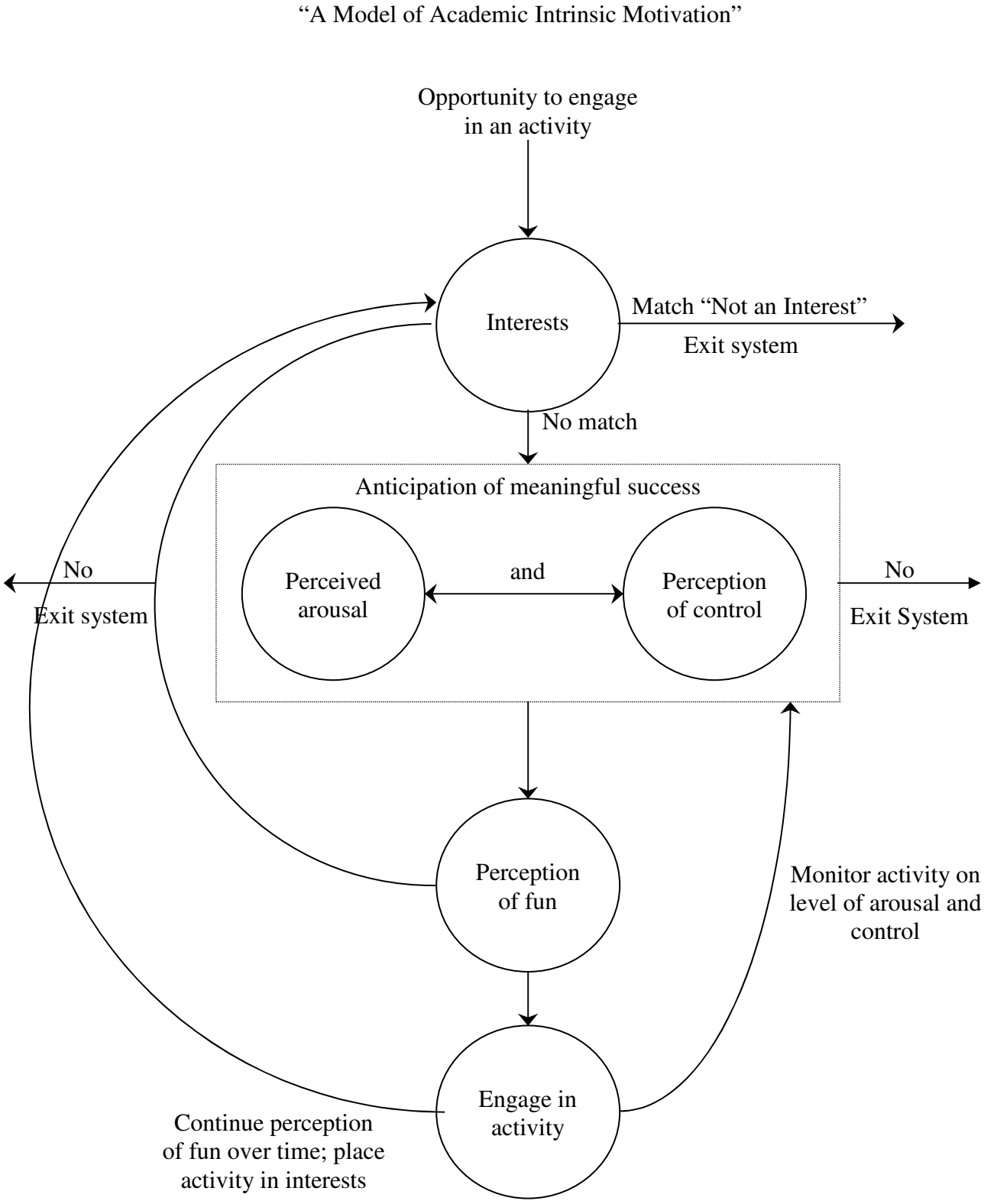


Figure 2. A model of academic intrinsic motivation. From: "A Study of Intrinsic Motivation in the Mathematics Classroom: A Personal Constructs Approach," by J. A. Middleton, 1995, *Journal for Research in Mathematics Education vol 26*(3), p. 256.

purpose just as easily. Some examples of manipulatives are toothpicks, coins, base-ten blocks, algebra tiles, counters, cotton balls, spinners, balances, dice, and paper cutouts of shapes. These teaching aids are effective across grade and development levels. What this means is that in a classroom with a mix of at-risk students, low achievers, high achievers, and children with learning disabilities, all students have a chance to excel. The students are then equipped to move across Piaget's Levels of Cognitive Development from the concrete level to semi-concrete and finally to the abstract level that is used in most mathematical processes (Maccini & Gagnon, 2000, ¶ 34). Understanding these concepts helps increase the confidence level of at-risk students and their motivation to learn.

Along with understanding, the retention rate of students improves when they use manipulatives. Problem-solving performance is significantly higher among students when they use manipulatives compared with baseline measures of performance without using manipulatives. Hands-on experiences also allow students to generalize to other settings more effectively (Maccini & Gagnon, 2000, ¶ 35). Results show that manipulatives are advantageous for teachers who are trying to illustrate how mathematics applies to situations their students will face throughout life. These tasks provide meaningful learning experiences for the students while fostering their critical thinking skills, truly training them to be self-sufficient individuals in mathematics.

Another effective method for the mathematics classroom is the use of simulations and games. Simulations are replications of the real-world situations involving skills or processes designed to create a learning experience for students (Ornstein & Lasley, 2000, p. 248). Games, in the context of an educational environment, relate to subject matter and portray a concept or provide a fun way to remember the concept by using goals and rewards as incentives.

Obviously, both methods are valuable because they involve the active participation of students. Simulations and games have particular value for mathematics. Seven out of eight studies have found that games improve mathematics achievement, and twelve out of fourteen studies have shown that students have greater interest in activities involving simulations or games. Unquestionably, simulations and games enhance students' retention rate over time (Randel, Moris, Wetzel, & Whitehill 1992, p. 269). With a greater interest and involvement among students, this teaching strategy provides teachers with a tool to strengthen the motivation to learn among at-risk students.

Simulations and games may increase motivation as well. They relate the ideas of study and fun. Most students typically separate the two, not realizing that a person can actually have fun while learning. At times, the students do not even recognize that they have been learning until long after the task is completed. Teachers can help students formulate these connections by discussing the implications of the games in pre- and/or post- activity discussions. In addition, during the activity students are often required to exercise many skills, just as someone might expect them to do in the workplace. Topics come alive for at-risk students when teachers can anticipate having them find joy in learning mathematics.

Finally, cooperative learning is an efficient form of indirect teaching. The beauty of this methodology is teachers can apply it in combination with other types of teaching styles. As students work in smaller groups, they can learn from each other in ways that a teacher may not be able to provide for them. The setting is not as high risk as whole-class instruction for lower-achieving students. This process reduces the stress and anxiety about mathematics that many students may experience when asked to respond individually. One study showed that students within a small group setting asked 240 times more questions than they did when the class met as

a whole (Ornstein & Lasley, 2000, p. 320). This is not surprising; a group setting provides fewer opportunities for other students to criticize them for a wrong answer or a "stupid question." Pressure that students feel from their peers diminishes, and their understanding of concepts is increased. Students often answer questions from classmates in ways that teachers cannot because a student that just learned the concept may have a better insight into what is giving another student difficulty. Moreover, people generally learn more by teaching than by having someone teach them. They may formulate connections that they would otherwise not make and reinforce their own understanding of the topic. Cooperative learning provides chances to form positive classroom relationships, which is the type of interaction many at-risk students need. Cooperative learning as a whole aids in boosting their attitudes, and thus their motivation, to learn.

Implementing Indirect Teaching Strategies

In conclusion, all of the information from research seems to point in one direction. Indirect instruction increases students' level of skill significantly and allows them to apply what they learn in the classroom to situations occurring outside of the classroom. These strategies – such as the use of manipulatives, simulations and games, and cooperative learning techniques – provide the type of environment needed to implement hands-on experiences into the classrooms. When teachers begin to use the techniques that researchers have proven to work, students will have the potential to surpass teachers' expectations for achievement in mathematics.

Observations at Bowling Green High School in Bowling Green, KY

School Context

Bowling Green High School is a city school with a wide variety of students. About 1000 students attend the school. The school is very ethnically diverse with African Americans, Asian Americans, Latin Americans, Bosnians, and other minority groups. The social classes of the students range from a lower socioeconomic status to upper middle class. Very few students are in the middle range of the middle class. The reason is that the school's district is in both middle and lower-class areas, while the parents of students from higher-class areas often pay tuition for them to go to this school for the ethnic diversity. Students can take many different types of classes such as remedial, Advanced Placement, and vocationally oriented subjects. Computer labs exist throughout the school, and the library is expansive, containing many resource books, leisure books, and eight computers for online research using either the Internet or periodical searches, and many tables so that several classes at one time can come in to do research.

There is a medium level of security. Visitors must sign in and wear a pass, and faculty and staff monitor the halls, lunchroom, and bathroom areas during lunch and class changes. Some parent involvement exists, though it could be much higher.

February 20, 2001

Algebra I – Special Education:

Many of the students in the three classes I observed had trouble with basic mathematics concepts. Much of the class time was spent reviewing fundamental processes. Each class was small, which helped the teacher have more time to work with individual students. The students in the first period class were somewhat motivated already, but many of them still needed the extra push to keep them motivated and to see the usefulness of what they were learning. The second period class was less motivated, although the students could be convinced to participate in class the majority of the time. The fourth period class was an entirely different case. This class consists of four male students that had many problems outside school. The teacher had an extremely difficult time motivating these students or even getting them to complete their classwork. In addition, one student was far above the other three in mathematics and was often bored with the material the teacher was presenting; however, he was not motivated to work alone on topics that are more complex. The class as a whole could not handle subject matter of greater difficulty. All of the learners in each class were at-risk because of a variety of reasons, and many had learning disabilities or difficult lives at home.

The teacher worked with the fourth period students on reviewing for a test on fractions and proportions during the period. She went over the review worksheets with them, having the students answer aloud in class. For the last part of the review, they worked alone to finish the worksheets. Many of the students were having trouble. Those that were not having difficulty tried to explain how to do the problems to the others. Sometimes that helped the students having trouble, but other times it did not, and at that point the teacher would try to explain the concept

again. Students that found the task too difficult tended to be off task quite often to avoid having to admit that they could not do the work.

I noticed that the teacher had a very good reputation with the students. Many of them felt comfortable in her classroom when they did not enjoy other classes or teachers. She would joke around with them and talk about things going on in their lives. Often this light conversation gave her a chance not only to find out about their interests, but it opened the door to find out how things were going in their home lives so that she could connect with them more effectively.

February 22, 2001

Algebra I – Special Education:

The teacher allowed me to take over the instruction of the class to teach the lesson on graphing ordered pairs on this day (see lesson on p. 62). Each class was a shortened period to allow for an extra period on character education that day. The lessons seemed to go well.

During the first two periods, the students became excited about what they were doing and seemed to enjoy the class. One advantage of the lesson was that students who caught on quickly wanted to find numerous places on the map while the others were working without my having to prompt them. This gave them practice and helped them keep from being bored with the lesson. They also helped the other students that had trouble reading the map or plotting points, but I quickly learned that I had to be careful to facilitate my questions in such a way that the quicker students did not do the work for those who were having trouble. Most students were able to make the connection between finding points on a map to plotting points on a Cartesian coordinate system. They brainstormed many uses of both maps and graphs, and realized the importance of these tools in their own lives through the discussion. The fourth-period class was a different case. Though they did more work than usual, they still were off-task and mostly

uninterested in the topic. These students are going to be more difficult to motivate in any way, especially intrinsically. Overall, the lesson was a success, but problems still exist in motivating the students to learn.

Study Skills – Special Education:

The study skills class was mainly a study hall for students who needed extra help or time on homework, and gave them the opportunity to touch base with the special education teacher. Most students in the school did not have the option to take a study hall unless they had shown a great need for one. The teacher also worked with the students on study skills, time management, and test anxiety.

My main observation in this class was the comment that one student made about his math class. He is in a regular Basic Algebra I class and came in very frustrated about his mathematics teacher. He complained that he did not understand the material and had asked the teacher for help. He felt that the teacher had belittled him because the teacher kept commenting about how easy the problems were and how he should be able to do them. He was honestly making the effort and still did not understand, and the teacher did not try to explain the concept in another way that might help him understand it more.

<u>February 23, 2001</u>

Basic Algebra II:

There were 22 students in this class, mostly sophomores and juniors. A few of them had IEPs for learning disabilities, though there were no students with physical disabilities. It was a collaborative class with the special education teacher, though she could not be there that day because of IEP meetings. The class contained a wide variety of ethnic minority groups such as

African Americans, Hispanics, Bosnians, and Asians. All seemed to speak English well, including the two Bosnian students that immigrated recently as refugees. The social class of the students was somewhat important because of the stark contrast between some of the students. There seemed to be no middle ground. Students were either from lower or upper economic classes. Motivation to learn ranged from very high to very low. It seemed that the at-risk students were mostly the ones with low levels of motivation, though some of those students at-risk had higher motivation levels.

The teacher went over the homework with the students before beginning the lesson. He had the students help him solve problems that others had asked questions about. He then took the homework up and proceeded with the lesson. He guided the classroom through each process, and after explaining the lesson, he gave the students a worksheet to work on in class and finish for homework. Some students did not participate during the class. They slept through the beginning of the class and then talked during the time they were supposed to be working. The teacher wanted to keep the atmosphere relaxed, so during the lesson he only reprimanded those who were disruptive. During the classwork time, he did not mind the students who were talking so long as they were also working. If they were not working, he asked to see what they had finished. He helped students who needed it individually, and some students helped each other. It was clear that the teacher expected them to complete the worksheet for the next class, or they would receive a low grade.

I felt that many of the students that were not participating in class might be more motivated if they were involved in more indirect methods of learning. For the past two days, all they had done was go over homework, listen to a lecture on new material, and begin a new worksheet. I had the opportunity to grade the worksheets they were supposed to complete for the

day. Most of the students who finished their worksheets had all or most problems correct. Those who did not complete the worksheets, often had incorrect answers. This told me that the reason they had not finished was not because they did not want to, but because they had not understood the concept. A few had begun the worksheet and finished problems correctly; however, they had failed to answer the remainder of the problems. Those students most likely found the task boring, so they did not care to carry out the assignment.

February 26, 2001

Basic Algebra I – Special Education:

On Friday, only the second period class had worked on the assignment of plotting given points to form a picture. They had done a decent job with the activity, though many had not finished it completely or correctly. One student had finished, so the teacher gave him another picture to work on while the other students finished the ones they had started. I watched the teacher explain how to do the task during first and fourth periods. She used many analogies in her explanations so that students might have something to which they could relate the concepts. They seemed to enjoy the task and worked very diligently to complete it. Very few students were off task in each of the periods, and they were able to complete the assignment correctly.

When the teacher asked the fourth-period class to recall what I had taught them on Thursday, they had trouble remembering. It came back to them quickly with a little prompting from her, but they still had not completely grasped the concept of plotting points on a coordinate plane. She did not lecture except for a few minutes, and then she let the students practice their skills.

Study Skills – Special Education:

I tutored the student that had complained about his math teacher on Thursday. We worked on his homework sheet dealing with point-intercept form. The student was not getting a simple explanation of the concept, so I began to use more concrete examples in explaining it to him, and he began to understand more. His confidence level was still low, and though he was doing the problems with very little help from me, he did not think he could do them on his own.

Basic Algebra II:

The students reviewed for their test during this period. The teacher again went over homework and then gave a short lecture on each of the types of problems that would be on the test. Most students were bored and uninterested. Many laid their heads down on their desks and others worked on unrelated assignments. The students who did not understand the material were the most disruptive. When the collaborative teacher realized that the disruptive students probably did not understand the worksheet, she began helping them. I was bored just watching the class. I graded the homework again, and many students had done better, but there were still quite a few low scores. The teacher seemed to make no attempt to use any other methods to reach the students other than lecture and practice and drill.

March 5, 2001

Basic Algebra I – Special Education:

On Friday, the first-period students had begun a test over graphing on Friday, and were completing it on this day. Most of them were doing fairly well on the test, though it was taking them a while to complete. The majority of the points for the test consisted of drawing one of the pictures on graph paper given specific points.

The second period class had completed their test the previous week, so they began a new lesson. The lesson was still on graphing, but the teacher began to give more complex lessons, talking generally about slope and the other quadrants since previous activities had only focused on the first quadrant. She lectured for most of the period, giving students notes to copy from the board. She used many analogies so that the students would remember the definitions that she gave to them. At the end of the period, she assigned them ten problems over the material to complete in class. The students clearly seemed bored with the lesson and the problems. I really must wonder how many of the definitions they will remember and if they mean anything to the students.

Though both of these classes were covering the same material, the teacher modified her lessons to fit the pace of each individual class. The first class needed more time for their tests, while the students in the second class did fine on the test without taking the extra day to complete it.

Algebra Lesson Plans for At-Risk High School Students

The lesson plans, located at the end of this document, were designed to motivate high school students to learn and appreciate mathematics so that students' achievement both in and out of the classroom may improve. These lessons use a variety of indirect strategies including manipulatives, cooperative learning activities, and games and simulations. The techniques used are based on research about motivating at-risk students as well as on observations of at-risk students in a classroom setting. The overall goal of the lessons is for students to recognize mathematics in the world around them and understand the value of their learning experiences in the classroom, thus increasing the students' motivation to learn. They teach students how to solve problems, reason, and apply their knowledge to a variety of situations. These three skills are valuable for many situations outside of the classroom setting.

Analysis of the Effectiveness of the Lesson Plans

To test how well the lessons in this thesis worked, the author taught a sample of them to high school students enrolled in a summer school algebra program trying to raise their grades enough to pass the course they had taken that school year. Six of the sample lesson plans were taught over a span of five days – "Investigating Properties of Quadratic Equations", "A Systematic Approach to Solving Word Problems", "Using Algebra Tiles and the A-B-C Method to Factor Quadratic Equations", "Understanding Direct and Inverse Relationships", "Using Right Triangle Algebra for Practical Purposes", and "Investigating Properties of Absolute Value Equations." The lesson "The Basics of Graphing Points and Lines" was also tested during the observation stages of the study and will be discussed later in this section for comparison and analysis purposes. The following is an account of the lessons taught to the students and an analysis of their effectiveness.

School Context

Trimble County High School is located in a rural area where many students live and work on farms. The majority of students come from families of low or lower-middle-class socioeconomic statuses, though a few belong to the upper middle class. Only about 350-400 students attend the school, which is not very ethnically diverse – with most students being white, while a few are Hispanic or African American. The variety of classes has grown in the past several years; however, most of the classes offered are those required by state curriculum. English is the only Advanced Placement class offered, though the school does offer a number of college preparatory classes. About half of the students attend vocational school in the neighboring county during their junior and senior years. The school has a resource classroom for those needing special education and two high quality computer labs in the school. The library is

a decent size, and contains many resource books, leisure books, and computers for online research using either the Internet or periodical searches, and contains many tables so a couple of classes may use it for research at the same time.

There is a medium level of security. Visitors must sign in and wear a pass, and people monitor the halls, lunchroom, and bathroom areas during lunch and class changes. Some parent involvement exists, though it could be much higher. The school operates on a fifty-minute class schedule, with seven periods in a day.

Summer School Context

The summer school setting was much different from the usual school setting. Students attended half a day most days and a full day on other days for four weeks. During this time, the students worked on lessons they had trouble with throughout the year and took tests over those lessons to replace the grades they had received for them during the school year. The students completed summer school classes at their own pace, and several students moved to a second class after completing the first course. Most of the students belong to families with low socioeconomic status. Some of the students were in summer school because they were not motivated to learn the material or do their work during the regular school year, while others simply had a difficult time with the subject.

Each day three to five students who were in summer school to raise their Algebra I grade participated in the lessons. Two of the students came into the class during the last week because they had been working on another course. One of the students simply had trouble understanding the material. The other students had some difficulty with the subject, but their main problem was a lack of motivation to learn. All of the students were at-risk for failing or dropping out of school. The reason they were attending summer school was that they received very low grades

for the year in Algebra. They were in a classroom with geometry students, and the regular teacher devoted her time to helping those students. The context was much different from the usual classroom setting since there were so few students and teachers could allot more time for individual instruction. The motivational techniques researched by the author still applied to the lessons, although there were both advantages and disadvantages to teaching the students in the altered environment using these techniques.

Because of time constraints, the students had to get through as much of the material as possible each day. The author slightly modified the lessons for the study by cutting out certain activities, thus reducing the amount of time needed to teach each lesson. However, for each lesson the writer was careful to cover the basic components of the different concepts with the students. The reader must also keep in mind that this was not the first time the students had seen the concept. A classroom teacher had presented the subject matter to them once before during the course of the regular school year, though they had obviously had much difficulty with the topic the first time around. What follows is an account of each lesson taught to the students and the successes and problems with each lesson as related to the study of motivating at-risk high school students to learn in Algebra.

Analysis of Lesson Plans

The first lesson that the students participated in was "Investigating Properties of Quadratic Equations." The students did not seem very interested in the material they were learning, but seemed to understand it at the time. One of the major successes in this lesson was the students stayed involved and on task throughout the entire class time. The regular classroom teacher was very surprised at how well the students behaved and participated. A drawback was that students were not able to go into the computer lab to work with the software as had been

anticipated because school employees were rearranging and setting up new computers. The students had a more difficult time seeing the connections between the graphs and the parameters because they had to constantly re-graph equations instead of having each problem printed out and labeled with its equation as they worked. This caused them to become frustrated and want to quit working, but they continued when someone encouraged them.

Another confusing aspect of the lesson for the students was understanding the CALC ZERO function and determining the left and right bounds of the equation to calculate the zeros of the equation. While working the problems, each of the students was generally able to find one of the zeros, but became confused when finding the second solution. This caused even more frustration when working the word problems because they were struggling with both setting up the problem and finding the solution to the quadratic equation. An unexpected of the lesson was that students began to help one another while working these problems. This led to better understanding on the part of each student.

Since the students had such a difficult time with the word problems, the next lesson was "A Systematic Approach to Solving Word Problems," an effort to help them develop this skill for later lessons and for situations outside of the classroom. The setback of teaching this lesson was the time constraint. Learning to solve word problems takes time, which was scarce. They seemed to catch on very well, once they understood that a word problem was not something they can just solve right away. When they began to see the key words, they seemed to do much better. They still became very frustrated and confused at times, and they still did not understand why they would need to know how to work word problems later in life.

The human-knot activity was a huge success. The students had fun during the activity, and afterward they were immediately able to pick out the similarities between the activity and

solving word problems. They also understood the example of how mathematics can be like a foreign language. The lesson ended with a discussion in which students worked as a class to solve distance word problems on the board as a class. Overall, the lesson was very successful and improved the students' skills for solving word problems.

The factoring lesson plan is the longest of all of the lesson plans, so the students only completed the first day of the lesson, which introduced the most basic concepts of factoring. After doing a few examples together as a class, the students worked on expanding five problems and factoring ten problems. Instead of graphing the figures on graph paper, they simply manipulated the algebra tiles to help them work each problem. At first, they had difficulty understanding the concept of area as it related to arranging the blocks. They kept making shapes that were not rectangles. Once they understood that they were looking for a rectangle, they solved each problem without trouble, with the exception of the problems with more than one x unit by x unit square. On those problems, they needed a little assistance, but still caught on quickly. The students were able to help each other through the problems so that together they figured out solutions to the more difficult problems.

The students did seem to have trouble understanding the connection between the answer they found using the algebra tiles and factoring without the manipulative. This connection likely would have come while performing the A-B-C method of factoring, if the teacher had allowed time for this in the day's lesson. The students were also distracted from what they were doing at times by the geometry students in the classroom who were making fun of them for playing with blocks. This problem would not have been an issue in a normal classroom setting where all of the students would have been doing the same task. This distraction also may have kept the students from making the needed connections. Until the geometry students began making

unnecessary comments, the algebra students had reacted very positively to the algebra tiles. Afterward became embarrassed and did not want to work with them. Since these dilemmas were unrelated to the lesson, the overall success of the lesson was high.

The students exhibited a very positive attitude toward the lesson "Understanding Direct and Inverse Relationships," and they seemed to have fun during the activity. The lesson sparked their interest enough that they even wanted to investigate what the relationships would be if the farmer decided to build two silos. One student would explain to the others why they should approach the problem a certain way, while the other students would respond with whether they agreed and why or why not. Using the manipulative definitely aided the students' understanding of the concept because they were very surprised at how some of the dimensions were related to each other.

The students did have some trouble with the formulas and understanding how to use them to find the volume of the silo. This took some time, so the students did not get to the part of the lesson where they were to convert the model dimensions to the actual size, nor did they have time to write the proposal to the farmer. Through discussion, the students did demonstrate their knowledge of how using direct and inverse relationships could be useful outside of the classroom. A final setback of the lesson was that there were a couple of times when one person in the group was doing all of the work while the others talked. This could have been avoided had the students known they would be graded for their participation. Because of the way the regular teacher calculated the summer school scores, there was no peer evaluation for this activity.

The same was true for the lesson on right triangle algebra. This time, however, the instructor was prepared for this barrier and took steps to make sure all students were involved. For example, the teacher chose students to do the measuring and computations so that each

student did each job at least twice. The students were already fairly competent in using the Pythagorean theorem, so they went through that part of the lesson very quickly. They had a harder time using the similar triangles. They enjoyed the change of going outside so they did not become quite as frustrated when working through the computations.

A major problem during the lesson was that after the students got back inside, two of the students did not want to work any longer. The other two students worked very diligently, while the first two simply refused to do anything else for the rest of the day. One of these students had just come into the algebra class on the day of the previous lesson and had given the instructor some discipline problems at that time. This student seemed to lead the other in misbehaving. Unfortunately, the only discipline measure the teacher had available was to kick the student out of summer school, an action which summer school teachers use only as a last resort. Instead of taking this measure for this particular student, the teacher chose to ignore the behavior and complete the lesson with the other two students. In a normal classroom setting, the instructor would have had more options available to discipline the students, and the students may have been less likely to refuse to participate.

The final lesson the students took part in was "Investigating Properties of Absolute Value Equations." This lesson did not go as well as most of the others did. It was the end of the last week of summer school, so the students were very restless and did not want to work well at all. They did not listen during the original explanation of how to solve the problems using the spaghetti strands. One of the students thought she understood the concept from earlier in the year, and instead of listening she tried to work the problems without using the spaghetti. The problem was that she slightly mixed up the steps for solving them, so all of the problems were incorrect. However, she was motivated to do the work. The two students that had given the

teacher problems during the previous lesson refused to do their work again, but this time it seemed to be more out of frustration. They began trying the problems, but quickly became confused and quit trying since they had not listened to the explanation. They did not even try after the teacher tried explaining it to them individually a second time. The fourth student did listen to the second explanation. After working through the first two problems with the instructor, the student was able to complete almost all of the first ten problems on the handout.

A teacher could improve this lesson by offering the students a list of steps they should take to solve the problems with the spaghetti strands. The biggest difficulty for the students was knowing which order to do each step was, so a handout would give guidance while working the problems. Student understanding also may have increased had there been time to get to the part of the lesson where the students graphed the equations. This visual aid may have made the topic more clear to them. Overall, the implementation of the lesson needs improvement, but as the difference in the understanding of the students who worked demonstrated, the manipulatives seem to help more than a simple explanation of how to solve the problems does.

Conclusion

With minor exceptions, the lessons seemed to be a success. The largest setbacks were caused by factors related to the fact that this was not a normal classroom setting –major constraints due to time, a lack of discipline options, and a difference in grading procedures. These issues were obstacles to effective teaching. This conclusion is based in part on the success of the graphing lesson, which was taught in a normal classroom setting to three different classes. This lesson went much more smoothly because the obstacles just listed were not a factor when teaching the graphing lesson (See the observation entry for February 22, 2001). Another reason for this conclusion is the difference in how the summer school students acted while they were under direct instruction. The students were constantly off task, talking and cutting up with one another. They did very little work, and did not seem to care if they finished summer school. One student was an exception. He worked diligently in both circumstances and tried to stay on task, though it was very difficult for him when the others were not working. This was a stark contrast to the students' behavior when participating in a lesson using indirect instruction instead of completing worksheets full of practice and drill problems. Comparatively, during the sample lessons the students were much more on task and interested in the mathematical concepts, suggesting that indirect methods are the most effective way to motivate high school students to learn mathematics, especially students considered by educators to be at-risk for failing or dropping out of school.

Each indirect strategy used in the lesson plans added something unique to the classroom experience by allowing the learners to become more directly involved in classroom activities. The lessons were based on research of indirect teaching methods and observations of at-risk students' behavior in a classroom setting. The indirect methods used in the lessons helped the

sampled students learn, remember, and apply information both in and out of the classroom more effectively than when they were taught by direct instructional methods. By helping students make connections to areas outside of mathematics, these lessons helped the teacher provide students with a reason to learn. The variety of activities in the lessons worked to keep students from becoming as bored and uninterested as they were while under direct instruction, thus the lessons increased their intrinsic motivation to learn. The combined strategies work together to help motivate high school students to learn in algebra more effectively than traditional teaching methods.

Research-Based Lesson Plans

Lesson Plan #1 – A Systematic Approach to Solving Word Problems

Researched-Based Techniques Used in this Lesson to Motivate At-Risk Students

- Higher mental processes elicited among students:
  Critical thinking skills of analyzing, evaluating, categorizing, and comparing; problem-solving skills; and decision-making skills

- Activities that involved student-initiated learning:
  The Human-Knot activity – students determine the steps of problem solving on their own and apply them to mathematics

- Problem solving and reasoning skills developed:
  Working in a logical manner, planning an approach, organizing information, following through to the end of the problem, and explaining the reasoning for each step as they work

- Activities related to Edgar Dale's Cone of Experiences:
  Direct experiences – the Human-Knot activity; simulated experiences – solving problems students may see outside of the classroom; audio-visual experiences – discussing cartoon; audio experiences – class discussions; and abstract experiences – reading problems and writing answers

- Variety in strategies used:
  Doing the Human-Know activity, participating in class discussions, working in groups, using an advanced organizer (cartoon), and solving problems from contexts outside of the classroom

- Simulations or games used:
  The Human-Knot activity including a post-activity discussion

- Tasks that involved cooperative learning:
  Doing the Human-Knot activity, identifying key-words, brainstorming steps for solving problems, and working problems
  The tasks included opportunities for peer tutoring and a peer evaluation activity to hold the students accountable for being on task

- Connections to uses of mathematics outside of the classroom:
  Necessity of using problem solving skills in the work force, dealing with mathematics anxiety, understanding the language of mathematics, and any other topics included in class discussion of how problem solving is necessary outside of the classroom

Subject: Algebra I
Content: Word Problems

Content Connections of Lesson:
NCTM Standards: Students should . . .

- Compute fluently and make reasonable estimates
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving
- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics
- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

KY Learning Goals and Academic Expectations: Students should . . .
1.2 – make sense of material they read; 1.5-1.9 – use mathematical ideas and procedures; 2.8 – understand math procedures; 4 – become responsible group members; 5.1 – use critical thinking skills; 5.5 – use problem solving processes; 6.1 – connect knowledge to different subject areas; 6.2 – use what they know to acquire new knowledge

Kentucky Core Content for Assessment Connections: Students should. . .
MA-H-4.3.1 – write and solve linear equations describing real-word situations; MA-H-4.3.2 understand how formulas, tables, graphs, and equations relate to each other; MA-H-4.3.5 – show how equations and graphs are examples of real world quantities

KY New Teacher Standards: The teacher should . . .
I – focus on academic expectations, develop ability to apply knowledge, integrate concepts across disciplines, motivate and actively involve students, address diversity, use technology, include assessment strategies, encourage students to be resourceful, creative, and flexible; II – communicate with and challenge students, establish shared expectations for responsibilities, support inquiry, create a media-rich environment; III – communicate expectations for learning, link learning with prior knowledge, model the skill to be learned, use multiple

teaching strategies and actively engage students in learning experiences, address diversity, use questioning strategies, use multiple perspectives and viewpoints, provide practice, identify misconceptions and provide guidance, link learning with future roles; VIII – communicate skills and core concepts, apply methods of inquiry, utilize technology, connect knowledge to real life situations; IX – use a variety of software, use the computer to access the Internet, design lessons that use technology, use computers for learning activities

Time: Two or three 1 ½ hr blocks

Objectives:
A. Students will know and apply the steps for solving word problems.
B. Students will explain how mathematics is like a different language.
C. Students will be able to translate distance, work, and investment word problems from English to mathematical symbols.
D. Students will be able to set up and solve distance, work, and investment word problems.
E. Students will work together cooperatively in groups to solve a problem.
F. Students will actively participate in class discussions.
G. Students will explain how mathematical word problems are useful in real-life.
H. The students will exhibit a positive attitude toward the lesson.

Rationale:
For students to understand that mathematics is like a different language, and they must read problems carefully in order to understand them and be able to use the techniques of solving them in settings outside of the classroom.

Strategies:
A. Use a Calvin and Hobbes cartoon from Sept. 3, 1989, as an advanced organizer to inform the students that using problem solving techniques in word problems will relieve some anxiety.
B. Have students discuss how they will use word problems once they are no longer in school. Make sure they understand that they will have to come up with their own equations to solve in situations outside of the classroom instead of someone handing the solutions to them.
C. The knot activity is to develop their problem-solving skills and discuss the similarities between solving it and solving word problems. It also encourages the students to work together to reach a common goal by leading and following other people.
D. Having the students think about mathematics as a different language helps them use their knowledge of how cultures interact and see how that applies to the 'world' of mathematics.
E. Having students brainstorm steps for solving word problems aids their thinking about the process.
F. The posterboards the students make help them pinpoint the mathematical process they should be doing when they see certain keywords in problems. The teacher should hang the posters in the room for the students to reference in the future.
G. Having students help each other work through example problems helps them learn the process of solving word problems and allows the students to benefit from peer tutoring.
H. The peer evaluation activity holds students accountable for the work they do in their groups and encourages responsibility and participation throughout the lesson.

Activities:

A. Show the Calvin and Hobbes cartoon and have the students relate the feeling Calvin had to how many people feel when told to solve a word problem.
B. Have students recall what previous knowledge about solving equations and using formulas. Discuss how they will be using equations when they are no longer in school.
C. Have students do the Human Knot Activity. Students stand shoulder-to-shoulder in a circle and grab someone's right hand (not the person beside them). Next, have them grab the left hand of a different person (also not the person beside them). Without letting go of each other's hands, the students should unravel the knot into the original circle. They can let their hands pivot, and can go between, over, or under each other's hands.
D. After the students have successfully 'untied' the knot, discuss the implications of the knot, keeping in mind the relationship with word problems. For example, how did they feel before and after untying the knot, how did they begin, and what process did they use?
E. In discussion, have students relate the knot activity to solving word problems.
F. Discuss as a class how mathematics is a different language, the important concepts in translating a language, and relate these concepts to translating from English to mathematics.
G. Divide students into 5 groups and have them write out basic steps for solving word problems.
H. When students have completed writing out their steps for solving word problems, have them discuss these steps as a class. Then, give students the handout "Simple Steps for Solving Word Problems." Talk about why each step is important.
I. Assign each group one of the following categories: addition, subtraction, multiplication, division, and equals. Using markers and a half sheet of posterboard, have each group list examples of keywords that are synonyms of the assigned word.
J. Work through examples of translating English phrases to math together as well as solving distance, work, and investment word problems.
K. Students practice solving word problems on the Internet using "Math Drills Homepage" at http://www.saab.org/mathdrills/sdv.cgi to practice distance, speed, and time word problems.
L. Group members will anonymously give each member of their group 0-2 points each for the following categories and explain the score they give: work ethic, contribution to the group, sensitivity to other group members.

Materials:

A. Overhead projector
B. Comic Strip transparency
C. "Steps for solving word problems" transparency and handouts
D. Example problems
E. 5 half sheets of posterboard
F. Markers
G. Computers with access to the Internet

Assessment

A. Class participation
B. Group participation
C. Problem solving ability
D. Peer assessment
E. Test over solving word problems

Scoring Rubric

Class Discussion Participation

9-10 points	Full participation in discussion. Communicates well and demonstrates a full understanding of the topics discussed.
7-8 points	Participates in discussion. Communicates idea to the class and seems to have an understanding of the topics discussed.
5-6 points	Mostly uninterested and bored during discussion. Communicates some ideas to the class, but demonstrates little understanding of the topics discussed.
0-5 points	No participation. Shows no understanding of the topics discussed.

Participation in group work and class discussion:

9-10 points	Participates fully. Contributes several ideas to peers during group activities, and is willing to follow others' direction when needed.
7-8 points	Participates mostly. Contributes ideas during group activities, and mostly follows others' direction.
5-6 points	Somewhat participates. Contributes little to group work during activities.
0-4 points	Does not participate in their group.

Problem Solving:

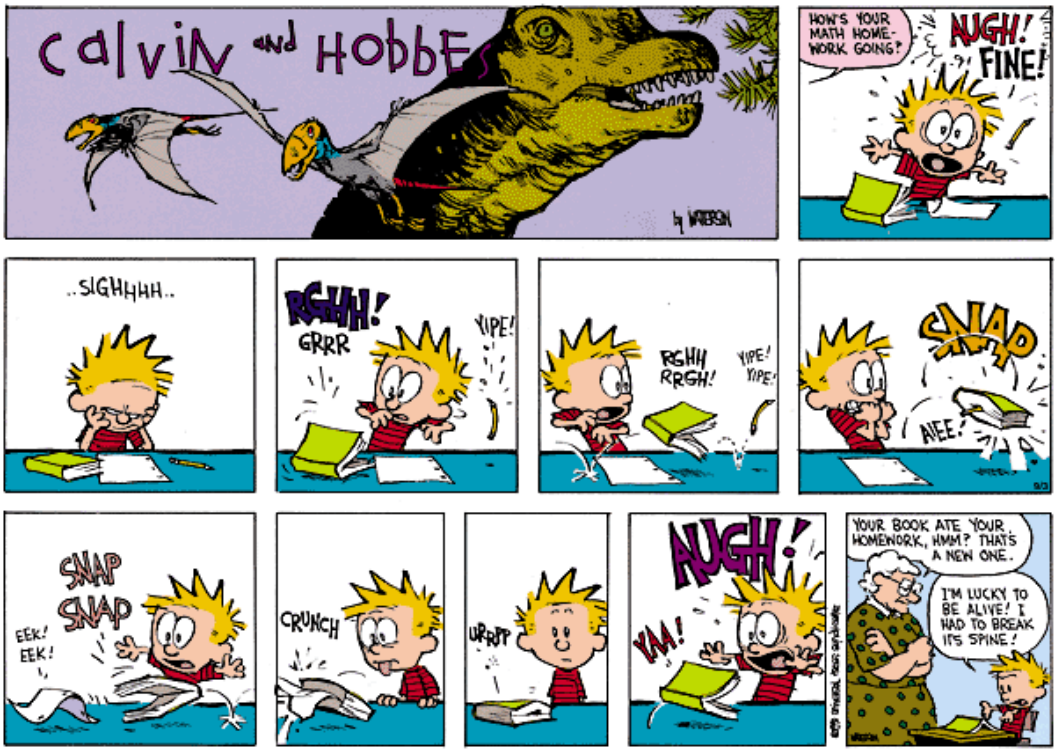
9-10 points	Attempts all problems, and finishes most word problems correctly with appropriate steps shown.
7-8 points	Attempts all problems with appropriate steps shown, but few or none are complete correctly.
5-6 points	Attempts all problems, but few or none are completed correctly. Does not show appropriate steps.
0-4 points	No attempt made to solve the problems.

Peer Evaluation:

0-6 points	Points determined by peer evaluation. The total number of points a student receives is the average of each of the evaluations given to them by their peers.

Scoring Guide

32-36 – A
29-31 – B
25-28 – C
22-24 – D
0-21 – F



From Watterson, Bill. (1989 September 3). *Calvin and Hobbes*. Retrieved October 31, 2000 from the World Wide Web: http://www.ucomics.com/calvinandhobbes/viewch.cfm.

Simple Steps for Solving Word Problems

1.) Read Problem ENTIRELY First
    Don't try to start solving before knowing what you are being asked to do!

2.) Make a Plan
    Make any charts, tables or graphs
    Draw and label pictures
    Label variables with what they stand for

3.) Look for Key Words and Set Up Appropriate Equations
    Certain words indicate that you should perform a certain mathematical operation
    These words help you to decide what equations to use

4.) Work in an Organized Manner
    Explain your reasoning as you go, then if you have to start over or you become lost, you can understand what you did.

Once you have completed the first three steps, it is just a matter of doing the math. The hard part is over because the rest is something that you have seen before.

Remember! Even if you cannot finish the problem, you can receive partial credit for anything you have written. If you have nothing down, I cannot give you any points!

5.) Be Sure to Finish the Problem!
    Many students do all of the work but forget to answer the question. Be sure you tell what your answer means in terms of the problem.

Practice, Practice, Practice!

Like everything that you learn, solving word problems becomes easier the more you do it.

In-class examples for translating English to math:

the difference between 6 and z	6 - z
16 percent of y	y(16/100)
10 fewer than x (this is the backwards one!)	x-10
3 more than the total of x and 8	(x + 8) + 3
5 increased by a factor of w equals 45	5w = 45

(Come up with more examples in class if students are having trouble)

In-class examples of distance problems:
A 555-mile, 5-hour plane trip was flown at two speeds. For the first part of the trip, the average speed was 105 mph. Then the tailwind picked up, and the remainder of the trip was flown at an average speed of 115 mph. For how long did the plane fly at each speed?

Answer: Go through each step. *First, read the entire problem aloud in class.*

Make a plan: Such as this chart, using d = r*t .

t → the time for the first part of the trip.	(5 - t) hours → time for the second part (total minus first part)
r = 105 mph → rate for the first part	r = 115 mph → rate for the second part

Look for key words and set up appropriate equations:

d = 105t miles → distance for the first part	555 - d miles → distance for second part (total minus first part)

Work in an organized manner:

Since the d is the distance for the distances add to 555 miles solve for d in both equations. Then set them equal to each other. Solve for t, and find that t = 2 hrs, which is the time spent on the first part of the trip.

Finish the Problem:

Since t = 2 hrs for the first part of the trip, the answer is "The plane flew for two hours at 105 mph and 3 hours at 115 mph."

Examples to work through with students during class.

1.) A motorboat goes 25 miles downstream in the time it goes 15 miles upstream. The current flows at 5 miles per your. What is the boat's rate in still water?

2.) John invests a sum at 4% per year. After half a year, he receives $20 interest. Find the investment.

3.) A press can print one day's newspapers in 4 hours. A high-speed press can do the job in half that time. How fast can both presses together do the job?

Set up and solve the following problems.[1]

1. With the wind, an airplane travels 1120 miles in seven hours. Against the wind, however, it takes eight hours. Find the rate of the plane in still air and the velocity of the wind.

2. One pipe can fill a tank in 5 hours. A second can fill it in 3 hours. How long will it take both pipes together to fill the tank?

3. Two cyclists start at the same time and travel in opposite directions. One travels at 3 miles per hours faster than the other. In 3 hours, they are 81 miles apart. What is the speed of the slowest cyclist?

4. A man invests $10, 000, part at 4 per cent per year, and the remainder at 6 per cent. From this he receives $500 annually. How much does he invest at each rate?

5. A job can be done by 8 men in 3 hours, or by 15 boys in 5 hours. How long would it take 3 men and 25 boys together?

6. Two space rockets are launched. The first one with speed of 6000 mph was launched. The second one with speed of 7000 mph was launched 20 minutes later.

7. A mortgage company invests $35, 000 at 8% per year. How much must it invest at 10% per year for an annual income of $5000 from both investments?

8. A man contracts to build a road in 72 days, a job requiring 60 men. The man hires 50 men who work for a while until he realizes that he must hire 30 more to finish on time. How many days do these 30 men work?

9. Two cars, one averaging 58 mph and one averaging 29 mph, start from city A to go to a city B. The slower car arrived 4 hours after the other car to city B. Find the distance between city A and city B.

10. Mr. Turner invested a sum of money at 7%, and twice that sum at 3%. His yearly return was $390. How much did Mr. Turner invest at each rate?

11. One machine labels 1200 cans in one hour, and a second labels 900 cans in an hour. If the faster machine starts 1 hour before the slower, how long will it take to label 8200 cans?

12. Mrs. Able invests $7000 at 4%. Mr. Able wishes to invest enough at 6% for their combined annual income to total $1000. How much should he invest?

[1]Problems adapted from Saab, E. (1998). *Math Drills Homepage.* Retrieved October 24, 2000, from http://www.saab.org/mathdrills/sdv.cgi; Stapel, E. (2000). *Purplemath – Your algebra resource*. Retrieved November 2, 2000, from http://www.purplemath.com); and from Dolciani, M. P., Berman, S. L., & Freilich, J. (1965). *Modern algebra: Structure and method book one*. Boston, MA: Houghton Mifflin Company.

Test Objectives:

A. Students will state and apply the steps to solving mathematical word problems.
B. Students will describe why working word problems in steps is a helpful technique.
C. Students will defend their opinion of how reading a mathematical problem is like reading a foreign language.
D. Students will set up and solve word problems dealing with distance, investment, and work.
E. Students will demonstrate their knowledge of how people use mathematical word problems in real-life.

Test Questions:

1. Carefully read the example below. Then answer the questions that follow in parts "a" and "b".

   A 555-mile, 5-hour plane trip was flown at two speeds. For the first part of the trip, the average speed was 105 mph. Then the tailwind picked up, and the remainder of the trip was flown at an average speed of 115 mph. For how long did the plane fly at each speed?

Answer:
d = r*t .
t → the time for the first part of the trip.

r = 115 mph → rate for the second part

r = 105 mph → rate for the first part

d = 105t miles → distance for the first part

555 - d miles → distance for second part (total minus first part)

$$d = r*t$$
$$555 - 105t = 115(5 - t)$$
$$10*t = 20$$
$$t = 2$$

2 hours at 105 mph

   a.) Label each step of the example with problem solving steps that are there, and tell what steps are missing from the person's answer.
   b.) Describe why each of these steps are a helpful techniques in solving word problems.

In problems 2 – 7 involving distance, work, and investment, apply your knowledge of problem solving steps to set up and solve the following problems. Be sure to label each step as you work the problem.

2. An executive drove from home at an average speed of 30 mph to an airport where a helicopter was waiting. The executive boarded the helicopter and flew to the corporate offices at an average speed of 60 mph. The entire distance was 150 miles; the entire trip took three hours. Find the distance from the airport to the corporate offices.

3. Working together, person A and person B painted a fence in 8 hours. Last year, person B painted the fence by himself. The year before, person A painted it by himself, but took 12 hours less than person B took. How long did person A and person B take, when they were painting alone?

4. A total of $6,000 in invested into two simple interest accounts. The annual simple interest rate on one account is 9%; on the second account, the annual simple interest rate is 6%. How much should be invested in each account so that both accounts earn the same amount of annual interest?

5. Two people were working on your car. One can complete the given job in six hours, but the new person takes eight hours. They worked together for two hours, but then the first person left to help another mechanic on a different job. How long will it take the new person to finish your car?

6. A car and a bus set out a 2 p.m. from the same point, headed in the same direction. The average speed of the car is 30 mph slower than twice the speed of the bus. In two hours, the car is 20 miles ahead of the bus. Find the rate of the car.

7. An investor deposited an amount of money into a high-yield mutual fund that returns a 9% annual simple interest rate. A second deposit, $2,500 more than the first, was placed in a certificate of deposit that returns a 5% annual simple interest rate. The total interest earned on both investments for one year was $475. How much money was deposited in the mutual fund?

Give a one to two paragraph response to the following questions.

8. You have just received a new bookshelf from your grandmother for Christmas. However, you must assemble it, and the directions are in Spanish. You have had Spanish I in high school and know a little about the language, but you still have trouble reading it. Do you think your dilemma of building the bookshelf can be compared to solving a mathematical word problem? Write one or two paragraphs defending your position.

9. Briefly, explain three instances when being able to solve a word problem may be helpful in real-life.

10. Using one of the instances you came up with in number nine, write your own example of a word problem and then solve it.

Lesson Plan #2 – Understanding Direct and Inverse Relationships

Researched-Based Techniques Used in this Lesson to Motivate At-Risk Students

- Higher mental processes elicited among students:
  Critical thinking skills of organizing, evaluating, and comparing; problem-solving skills; creative-thinking skills; and decision-making skills

- Activities that involved student-initiated learning:
  Observing how different widths and heights affect volume and surface area – students learn the meaning of direct and inverse relationships through investigation

- Problem solving and reasoning skills developed:
  Working in a logical manner, planning an approach, organizing information, following through to the end of the problem, and explaining the reasoning for each step as they work

- Activities related to Edgar Dale's Cone of Experiences:
  Direct experiences – Constructing a "silo" from paper; simulated experiences – putting themselves in the place of the farmer and writing a letter to him; audio experiences – class discussions; and abstract experiences – computing dimensions of the silo

- Variety in strategies used:
  Constructing a product, participating in class discussions, working in groups, writing, and solving problems from contexts outside of the classroom

- Manipulatives used:
  Paper and dry material

- Simulations or games used:
  Farmer scenario of building the silo from paper including pre- and post-activity discussions

- Tasks that involved cooperative learning:
  Making the silo
  The tasks included opportunities for peer tutoring and a peer evaluation activity to hold the students accountable for being on task

- Connections to uses of mathematics outside of the classroom:
  Necessity of using problem solving skills in situations outside of the classroom, understanding how direct and inverse relationships help students know how to maximize or minimize space and/or resources for projects they may incur throughout life

Subject: Algebra I
Content: Direct and Inverse Relationships[2]

Content Connections of Lesson:
NCTM Standards: Students should . . .

- Understand meanings of operations and how they relate to one another
- Compute fluently and make reasonable estimates
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Analyze change in various contexts
- Analyze characteristics and properties of two-and three-dimensional geometric shapes and Develop mathematical arguments about geometric relationships
- Use visualization, spatial reasoning, and geometric modeling to solve problems
- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof
- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements
- Organize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Use the language of mathematics to express mathematical ideas precisely
- Recognize and use connections among mathematical ideas
- Recognize and apply mathematics in contexts outside of mathematics
- Create and use representations to organize, record, and communicate mathematical ideas

KY Learning Goals and Academic Expectations: Students should . . .

1.3 – make sense of what they observe; 1.5-1.9 – use mathematical ideas and procedures; 1.10 – organize information; 1.11 – write to communicate ideas; 2.8 – understand mathematical procedures; 2.9 – understand dimension concepts; 2.10 – understand measurement concepts; 2.11 – understand change concepts; 3 – develop into self-sufficient individuals; 4 – become responsible group members; 5.1 – use critical thinking skills; 5.3 – organize information; 5.4 – use a decision making process; 5.5 – use problem solving processes; 6.1 – use existing knowledge to acquire new knowledge; 6.2 – expand existing knowledge by making connections

KY Core Content for Assessment Connections: Students should . . .

MA-H-1.2.1 – perform operations with real numbers in problem solving situations; MA-H-1.2.2 – simplify real number expressions; MA-H-1.3.4 – understand ratio and proportion; MA-H-2.2.6 – calculate volume and surface area; MA-H-2.3.1 – solve geometry problems using algebra; MA-H-2.3.4 – understand how change in dimensions affect volume and

[2]Lesson Plan adapted from Gerver, R. Ph.D., Sgroi, R. Ph.D., Carter C., Hansen M., Moilna D., & Westegaard S. PH.D. (1997). *South-Western algebra: An integrated approach.* Cinncinati, OH: South-Western Educational Publishing.

surface area; MA-H-4.1.5 – apply direct and inverse variations; MA-H-4.2.4 – create tables of numerical values of functions; MA-H-4.2.8 – use direct and inverse variation

KY New Teacher Standards: The teacher should . . .

I – focus on academic expectations, develop ability to apply knowledge, integrate concepts across disciplines, motivate and actively involve students, address diversity; II – communicate with and challenge students, establish shared expectations for responsibilities, support inquiry; III – communicate expectations for learning, link learning with prior knowledge, use multiple teaching strategies and actively engage students in learning experiences, use questioning strategies, provide practice, link learning with future roles; VIII – communicate skills and core concepts, apply methods of inquiry, connect knowledge to real life situations

Time: Three 1 ½ hour blocks

Objectives:

A. The students will demonstrate their understanding of direct and inverse relationships through the proposal they write to the farmer.
B. The students will explore the concept of direct and inverse relationships.
C. The students will work cooperatively in a group.
D. The students will use substitution modify the formula for volume.
E. The students will explain direct and inverse relationships that occur outside of the classroom.
F. Students will exhibit positive attitudes toward the lesson.

Rationale:

This lesson is designed to be an investigative activity where students can model the direct and inverse relationships between numbers to increase their understanding of the concept.

Strategies:

A. Before class, assign the students in the class to groups of 4 or 5, equally distributing them according to race, gender, and ability level. Keep unmotivated students in separate groups.
B. The paper and dry material are tangible items so the students can see how the relationships change instead of having only an abstract idea in their heads.
C. The activity should arouse the students' curiosity as to why the product of the base and the height do not change and how they could find the greatest volume of the silo, increasing their motivation to learn the concept.
D. The group work builds team membership skills and the students can learn from one another.
E. Making a table of the information helps the students to organize what they know so that they can draw conclusions from the data later.
F. The silo activity gives the students an example of how direct and inverse relationships show up in everyday life.
G. Discuss direct and inverse relationship using these measurements as examples.
H. Manipulating the formula for volume shows students how there can be various approaches for solving problems.
I. The writing activity gives the students a chance to combine skills from other subjects with mathematics, and appeals to students who have an interest in writing.

J. Discuss other situations that require maximum or minimum use of resources and/or space.
K. The peer evaluation activity holds students accountable for the work they do in their groups and encourages responsibility and participation throughout the lesson.

Activities:

A. The students are to pretend a farmer has 93.5 square yards of material to construct the sides of a silo and wants to construct a silo with the largest volume possible. He has asked for help in determining the largest volume possible using the given material. Have them determine why it is appropriate to use an 8-½ inch × 11 inch sheet of paper to explore this problem.
B. Have the students work in their groups to make silos of varying width and height, exploring how the relationships change in relation to one another as they change the measurements. They should begin by rolling the paper so that the height is 11 inches, taping it, and then filling it with the dry material. They should measure the dry material using the measuring cups to determine the volume and record the circumference of the base, the height, and the volume in a table. The students should record any observations they have about the relationship of the width and height to the volume.
C. The students should repeat the activity by rolling the paper the other direction, and cutting and taping it to form wider and taller silos until they have 10 different measurements.
D. The students should find the product of the circumference of the base and the height for each of the silos and discuss as a class why it remains constant. Discuss inverse relationships and in relation to this activity. Expand the concept to other relationships. Then they should discuss what would happen if the relationship were a direct relationship. Give the students the example of how the volume changes as the height changes. Talk about other examples.
E. Discuss how to find the volume of a cylinder and the circumference of a circle.
F. Have students rewrite the formula for the volume of a cylinder in terms of the circumference and the height.
G. Have the students find the volume of each of their silos using this new formula.
H. Have the students write up a proposal to send to the farmer of what dimensions he should use to build his silo and why. The students should write in terms of the real material that the farmer would use instead of just the dimensions they found with the sheet of paper.
I. Group members will anonymously give each member of their group 0-2 points each for the following categories and explain the score they give: work ethic, contribution to the group, sensitivity to other group members.

Materials:

A. 8 ½ × 11 inch sheet of paper for each student
B. Dry filling material such as macaroni, beans, or popcorn kernels
C. Measuring cups for each group
D. Ruler
E. Scissors
F. Tape

Assessment

A. Understanding the problem farmer's problem and the solution
B. Explanation of the proposal

Scoring Rubric

Understanding the Problem

4 points	Complete understanding of the problem
3 point	Misinterprets minor part of the problem
2 points	Misinterprets major part of the problem
1 point	Completely misinterprets problem
0 points	No attempt

Planning a Solution

4 points	Plan that could lead to a correct solution with no arithmetic errors and neatly displays any diagrams necessary
3 point	Substantially correct procedure with minor omission or procedural error or diagram is unclear
2 points	Partially correct procedure but with major fault or missing necessary diagram(s)
1 point	Totally incorrect plan and/or missing necessary diagram(s)
0 points	No attempt

Reasoning

4 points	Clearly explained the reasons for the decisions made throughout the problem
3 points	Reasons not clearly explained, but work suggests sound reasoning
2 points	Partly correct reasoning, or correct reasoning used for only part of the problem
1 point	Reasoning is incorrect
0 points	No reasoning is evident from the work

Getting an Answer

3 points	Correct solution and answer labeled correctly
2 points	Copying error; computational error; partial answer for a problem with multiple answers; no answer statement; or answer labeled incorrectly
1 point	Wrong answer based on an inappropriate plan
0 points	No answer

Scoring Guide

14-15 points – A
12-13 points – B
11 points – C
9-10 points – D
0-8 points – F

Lesson Plan #3 – Using Right Triangle Algebra for Practical Purposes

Researched-Based Techniques Used in this Lesson to Motivate At-Risk Students

- Higher mental processes elicited among students:
  Critical thinking skills of analyzing, evaluating, and comparing; problem-solving skills; and decision-making skills

- Problem solving and reasoning skills developed:
  Working in a logical manner, planning an approach, using indirect approaches to a problem, following through to the end of the problem, and explaining the reasoning for using indirect methods to solve problems

- Activities related to Edgar Dale's Cone of Experiences:
  Direct experiences – Determining height; simulated experiences – designing a road system; and audio experiences – class discussions

- Variety in strategies used:
  Designing a road system, determining height, participating in class discussions, and working in groups

- Manipulatives used:
  Cardboard and rulers

- Simulations or games used:
  Designing a road system and determining height including pre- and post-activity discussions for both activities

- Tasks that involved cooperative learning:
  Designing the road system and determining height
  The tasks included opportunities for peer tutoring

- Connections to uses of mathematics outside of the classroom:
  Being able to determine the height of extremely tall objects and constructing roads

Subject: Algebra I
Content: Right Triangle Algebra

Content Connections of Lesson:
NCTM Standards: Students should . . .

- Compute fluently and make reasonable estimates
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Use visualization, spatial reasoning, and geometric modeling to solve problems
- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Make and investigate mathematical conjectures
- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics
- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

KY Learning Goals and Academic Expectations: Students should . . .

1.5-1.9 – use mathematical ideas and procedures; 2.7 – understand number concepts; 2.8 – understand mathematical procedures; 2.9 – understand dimensionality concepts; 2.10 – understand measurement concepts; 4 – become responsible group members; 5.1 – use critical thinking skills; 5.4 – use a decision-making process; 5.5 – use problem-solving processes; 6.1 – connect knowledge from different subjects

KY Core Content for Assessment Connections: Students should . . .

MA-H-1.2.1 – perform mathematical operations to solve problems; MA-H-1.3.4 – understand ratio and proportion; MA-H-2.1.4 – apply ratio measures; MA-H-2.2.3 – determine height using methods of indirect measurement; MA-H-2.2.4 – use Pythagorean relationships to solve problems; MA-H-2.2.5 – apply the concept of similarity; MA-H-2.3.2 – apply algebra to solve problems involving geometry; MA-H-4.2.6 – determine solutions to quadratic equations

KY New Teacher Standards: The teacher should . . .

I – focus on academic expectations, develop ability to apply knowledge, integrate concepts across disciplines, motivate and actively involve students, address diversity; II – communicate with and challenge students, establish shared expectations for responsibilities,

show flexibility and modify classroom processes to support instruction, support inquiry; III – communicate expectations for learning, link learning with prior knowledge, model the skill to be learned, use multiple teaching strategies and actively engage students in learning experiences, use questioning strategies, provide practice, link learning with future roles; VIII – communicate skills and core concepts, apply methods of inquiry, connect knowledge to real life situations

Time: Two 1 ½ hour blocks

Objectives:
A. The students will state the Pythagorean theorem.
B. The students will apply the Pythagorean theorem to situations that require them to use it to find the measure of sides of a triangle.
C. The students will work together cooperatively in a group.
D. The students will use ratios to solve problems involving similar triangles.
E. The students will discuss how they can use right triangles and algebra together to solve problems in areas other than pure mathematics.
F. The students will exhibit a positive attitude toward the lesson.
G. The students will use different concepts in mathematics together to solve problems.
H. The students will work together cooperatively in a group.

Rationale:
This lesson actively involves the learner in classroom activities to develop the students' interest in the subject.

Strategies:
A. The example of a road is a practical example of a time when people may need to use the Pythagorean theorem since roads that are perpendicular to one another allow the best visibility for drivers.
B. The activity and working together motivates students because they are getting to make something while it reinforces the concept of the Pythagorean theorem.
C. Working together as a class develops team membership skills and increases the motivation to learn.
D. The discussion helps the students see connections between the concepts they are learning and the activities they are performing.
E. Before class, assign the students in the class to groups of 4 or 5, equally distributing them according to race, gender, and ability level. Make sure that any unmotivated students are not in groups together.
F. Combining skills shows students how different mathematics skills are interrelated.
G. Each of the activities the students do actively involve them in their own learning.

Activities:
A. Introduce the concept of the Pythagorean theorem to students.
B. Discuss why it might be important for 2 roads to meet at a 90 degree angle.

C. Have students take two strips of cardboard that symbolize a road to construct an intersection where the roads meet perpendicular to each other. The students should use the Pythagorean theorem instead of measuring the angles between the roads.
D. Discuss why a road construction crew would find it necessary to use the Pythagorean theorem to make roads perpendicular to one another instead of measuring the angle between the roads.
E. Have the students work together to develop a model of an entire road system by combining each student's intersection.
F. Discuss other uses of the Pythagorean theorem outside of the classroom.
G. Introduce the idea of using right triangles and ratios between similar triangles to determine an unknown side of a triangle.
H. Have the students go outside and work in their groups to determine the approximate height of the schools' flagpole, the school itself, a tree, and other tall objects by measuring shadows and comparing them to the shadows and height of known objects or people and then using the ratio between the similar triangles.
I. Give the students a variety of problems involving the Pythagorean theorem and similar right triangles.

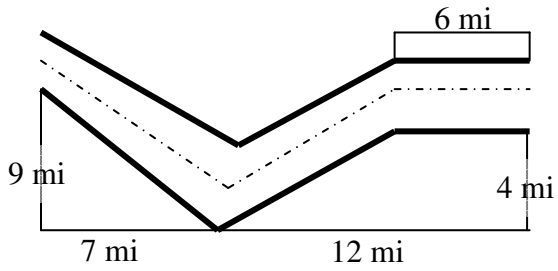
Materials:
A. 2 strips of cardboard per student (all of various lengths)
B. Ruler
C. Masking Tape
D. Wide open floor space
E. Yard stick or measuring tape
F. Tall objects such as a flag pole, a tree, and the school building
G. Handout involving problems with right triangles

Assessment
A. Solving problems on the handout involving right triangles using what the students learned

Problems

1. A 25-foot ladder is to touch a wall at a point 20 feet above the ground. How far away from the wall should the foot of the ladder be places?

2. Find the length of the road pictured below to the nearest tenth of a mile.



3. Find the length of each line segment given points P(3, 10), Q(3, 4), and R(11, 4)
   $\overline{PQ}, \overline{QR}, and \overline{PR}$

4. Given right triangles ΔABC ~ ΔPQR with ∠A and ∠P as right angles, if AB = 12, BC = 24, AC = 20, and PQ = 8, what are QR and PR?

5. You need to know the height of your house to string Christmas lights, but your neighbor has borrowed your ladder and is now out of town. Pretend that your height is 5 ½ feet and your shadow at the same time is 9 ½ feet. How could you find how high your house is if you know that the measure from the base of the house to the shadow of the lower part of the roof is 17 ft and 3 ½ inches.

Lesson Plan #4 – Occurrences of Trigonometry in Commonplace Machines

Researched-Based Techniques Used in this Lesson to Motivate At-Risk Students

- Higher mental processes elicited among students:
  Critical thinking skills of analyzing, evaluating, and comparing

- Activities that involved student-initiated learning:
  Exploring the sine function using sound waves

- Problem solving and reasoning skills developed:
  Working in a logical manner, planning an approach, organizing information, and following through to the end of the problem

- Activities related to Edgar Dale's Cone of Experiences:
  Simulated experiences – exploring sine waves with the computer program; audio-visual experiences – watching and discussing the lesson included with the computer program; audio experiences – class discussions; visual experiences – graphing problems with the graphing calculator; and abstract experiences – solving problems involving trigonometry

- Variety in strategies used:
  Playing the computer program, using graphing calculators, participating in class discussions, working in groups, and solving problems

- Technology used:
  Computer program and graphing calculators

- Simulations or games used:
  The computer program including a post-activity discussion, as well as a discussion along with the program

- Connections to uses of mathematics outside of the classroom:
  Sounds waves, machine movements, automobiles, and pendulums

Subject: Algebra I
Content: Trigonometry

Content Connections of Lesson:
NCTM Standards: Students should . . .

- Understand meanings of operations and how they relate to one another
- Compute fluently and make reasonable estimates
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements
- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics
- Use representations to model and interpret physical, social, and mathematical phenomena

KY Learning Goals and Academic Expectations: Students should . . .
1.3 – make sense of things they observe; 1.5-1.9 – use mathematical ideas and procedures; 1.16 – use computers to collect information; 2.8 – understand mathematical procedures; 2.10 – understand measurement concepts; 3 – develop into self-sufficient individuals; 4 – develop ability to become responsible group members; 5.1 – use critical thinking skills; 6.1 – connect knowledge from different subjects; 6.2 – expand knowledge by making connections with new knowledge

KY Core Content for Assessment Connections: Students should . . .
MA-H-1.1.1 – apply real numbers to real world and mathematical situations; MA-H-1.2.2 – simplify real number expressions; MA-H-1.3.4 – understand ratio and proportion; MA-H-2.1.5 – apply right triangle trigonometric measures

KY New Teacher Standards: The teacher should . . .
I – focus on academic expectations, develop ability to apply knowledge, integrate concepts across disciplines, motivate and actively involve students, accommodate students at different levels of performance, address diversity, use technology; II – communicate with and challenge students, support inquiry, modify classroom processes as situation demands, create a media-rich environment; III – communicate expectations for learning, link learning with prior knowledge, model the skill to be learned, use multiple teaching strategies and actively engage students in learning experiences, use questioning strategies, provide practice, link learning with future roles; VIII – communicate skills and core concepts, apply methods of inquiry, utilize technology, connect knowledge to real life situations; IX – use a variety of

software, design lessons that use technology, apply research-based instructional practices, use computers for learning activities

Time: Two 1 ½ hour blocks

Objectives:
A. Students will use a computer game to learn how trigonometry is useful in life applications.
B. Students will learn the formulas for the sine, cosine, and tangent functions of right triangle trigonometry.
C. Students will be able to explain the amplitude and frequency of sound waves as functions of time.
D. Students will draw right triangles and verify the length of the sides and the measure of the angles using right triangle trigonometry.
E. Students will graph the sine, cosine, and tangent functions on a graphing calculator and explain their differences and similarities.
F. Students will exhibit positive attitudes toward the lesson.

Rationale:
This goal of this lesson is to teach students the basic concepts of trigonometric functions and how they are useful in life applications.

Strategies:
A. The computer program discusses the sine wave and six "machines" – a weight on a string, a gadget, a seesaw, a cylinder in an automobile engine, a pendulum, and music – which move like a sine wave. This helps students see actual applications of trigonometry and how it could be useful to them outside of the classroom.
B. Discussing each one of the "machines" instead of simply reading about them on the program, allows the students to ask questions and go further in depth about the concept.
C. At the end of the program, the students explore the sine wave as a part of sound waves in music, learning about amplitude as a function of time as well as the frequency of the waves.
D. The computer activity sets the tone for the rest of the discussion of trigonometry so that students are more interested in the topic and have an increased motivation to learn the basics of trigonometry.
E. Graphing the functions helps students see how they vary from one another as well as how they relate to each other.
F. The acronym helps students remember the formulas, which students often find to be confusing.
G. Having the students draw the triangles and find the angle and side relationships reinforces the concept and helps them see why there are more than one way to find the measure of a side or an angle.
H. Having the students find the angle measure in two different units of measure helps them see that trigonometric functions are a ratio relationship independent of the unit of length.

Activities:

A. Have students play the game "Six 'Machines'" on the CD-ROM *Solid Gold Gnarly Math*. Walk through the program with the students discussing the various topics presented in it.
B. At the end of the program, the students should explore different sound waves and how the sine function varies depending on the frequency and amplitude of the wave.
C. When the students finish playing the computer program, begin a discussion about the trigonometric functions of cosine and tangent. Talk about their relationship to one another.
D. Using a graphing calculator, have the students plot the graphs of sine, cosine, and tangent simultaneously on a Cartesian coordinate plane and discuss their differences.
E. Show the students the acronym "SOHCAHTOA" to help them remember the formulas for each of the trigonometric functions.
F. Explain how the trigonometric functions relate to angles and sides in right triangles. Have the students draw a series of right triangles (at least five), each with varying angle measures for the other two angles. Then, have them measure the lengths of sides with a ruler in both centimeters and inches and then measure the angles with a protractor.
G. Beside each triangle, the students should verify their measurements using trigonometry. For each of the following cases, there should be at least one triangle that verifies the measurement:
    – A triangle where they use trigonometry to find the length of the hypotenuse in both inches and centimeters
    – A triangle where they use trigonometry to find the length of the "opposite" side in inches
    – A triangle where they use trigonometry to find the length of the "adjacent" side in centimeters
    – A triangle where they find the measure of each of the two angles other than the right angle of a triangle using both centimeters and inches to measure the sides and use trigonometry to find the angle measurement. (This is to verify to the students that the measure of the angle when using any unit of measurement for the sides.)
H. The students should use two different trigonometric functions to find the same angle for each of two of the triangles, and then use two different trigonometric functions to find the same side on each of two different triangles.
I. As a class, discuss the answers for what the students found and have them explain how they can get the same answer by working a problem two different ways.
J. Discuss whether it mattered which unit of measure the students used when finding the measure of the angles.

Materials:

A. A computer for each student
B. A copy of the CD-ROM *Solid Gold Gnarly Math* for each student
C. A graphing calculator for each student
D. Typing paper
E. Rulers with both inches and centimeters
F. Protractors

Assessment

A. Finding measures of angles and sides of the triangles they draw using trigonometry

Lesson Plan #5 – The Basics of Graphing Points and Lines

Researched-Based Techniques Used in this Lesson to Motivate At-Risk Students

- Higher mental processes elicited among students:
  Critical thinking skills of analyzing, categorizing, organizing, and comparing

- Activities that involved student-initiated learning:
  Finding places and points on the map – the more places a student desires to find, the more practice that students receives in finding coordinates

- Problem solving and reasoning skills developed:
  Working in a logical manner, organizing information, and following through to the end of the problem

- Activities related to Edgar Dale's Cone of Experiences:
  Direct experiences – finding places on a map; simulated experiences – solving problems students may see outside of the classroom; audio experiences – class discussions; visual experiences – using the computers; and abstract experiences – reading and organizing information

- Variety in strategies used:
  Finding places on a map, participating in class discussions, working in groups, solving problems from contexts outside of the classroom, using the Internet, using magazines and newspapers, finding data, organizing information, and drawing activity for practice

- Manipulatives used:
  Maps

- Technology used:
  The Internet

- Simulations or games used:
  Finding places on a map including a post-activity discussion

- Tasks that involved cooperative learning:
  The map activity
  The tasks included opportunities for peer tutoring

- Connections to uses of mathematics outside of the classroom:
  Using maps, creating news articles in journalism, gathering and making inference from statistics, and any other topics included in class discussion of how graphing is necessary outside of the classroom

Subject: Algebra I
Content: Graphing

Content Connections of Lesson:
NCTM Standards: Students should . . .

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- Use mathematical models to represent and understand quantitative relationships
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Create and use representations to organize, record, and communicate mathematical ideas
- Use representations to model and interpret physical, social, and mathematical phenomena

KY Learning Goals and Academic Expectations: Students should . . .

1.1 – use reference tools to find information they need; 1.2 – make sense of material they read; 1.3 – make sense of things they observe; 1.5-1.9 – use mathematical ideas and procedures; 1.10 – organize information; 1.13 – communicate ideas with visual aids; 2.7 – understand number concepts; 2.8 – understand mathematical procedures; 2.13 – understand statistics; 3 – become self-sufficient individuals; 4 – become responsible group members; 5.1 – use critical thinking skills; 5.3 – organize information; 6.2 – use what they know to acquire new knowledge

KY Core Content Connections: Students should . . .

MA-H-2.3.1 – solve real-world geometry problems using algebra; MA-H-3.2.1 – analyze and interpret results from data

KY New Teacher Standards: The teacher should . . .

I – focus on academic expectations, develop ability to apply knowledge, integrate concepts across disciplines, motivate and actively involve students, address diversity; II – communicate with and challenge students, establish shared expectations for responsibilities; III – communicate expectations for learning, link learning with prior knowledge, model the skill to be learned, use multiple teaching strategies and actively engage students in learning experiences, use questioning strategies, provide practice, link learning with future roles; VIII – communicate skills and core concepts, apply methods of inquiry, connect knowledge to real life situations

Time: Two 1 ½ hr blocks

Objectives:
A. Students will use various applications from outside of the classroom of graphing and coordinate systems.
B. The students will exhibit a positive attitude toward the lesson.
C. Students will learn how to graph points and lines.
D. Students will graph a given point.
E. Students will graph a given line.
F. Students will read and find places on a map using a coordinate system.
G. Students will design a graph given a set of data.
H. Students will work cooperatively in a group.

Rationale:
This lesson is designed to be an introduction to a unit on graphs so students see the value in learning the skill of graphing in real-life situations and can apply the skill when they need to.

Strategies:
A. Divide the students into groups of four or five students so that they may work together.
B. Using maps of the students' hometown or state is very useful in helping them to see the value of reading a map. They can learn about the area in which they live as opposed to just finding places that they have never heard of or know little about them.
C. The maps serve as an aid in remembering how to graph points on a coordinate plane, as well as helping students see one of the uses of a coordinate system.
D. Having students find something of their own helps ensure that one student is not completely controlling the group.
E. The Graphing Fun activity gives students the practice and drill they need to master the concept, while providing a fun activity for them to complete and something of which they can be proud.
F. Looking for graphs in magazines and newspapers shows students real-life situations that use graphs.
G. The students come up with an explanation of how to graph information on their own. This method allows the students an opportunity to construct their own knowledge and helps them have a better understanding of the concept.
H. When the students construct their own graph, they are able to choose data that they are interested in and see how mathematics can relate to their interests.
I. The discussion allows students to see the variety of data that graphs can portray within the context of conversations surrounding the students' interests. The teacher will introduce them to analyzing information using a graph.

Activities:
A. Give each group a map (preferably all the same map, but different maps could work), which is marked off into a coordinate grid. Explain how students can use the map to find certain places or streets using the indexes and grid system.
B. Have the students find certain places on the map as a group and record the names of the places with their corresponding coordinates.

C. Assign each of the students a place to find individually, such as their home. The students many help one another.
D. Explain how to graph an ordered pair. Use this notation to have the students find different locations on the map and tell the class what is located there.
E. Talk about maps and the people who use them. Discuss different professions that use maps. Expand to ask the students about other possible uses of coordinate systems.
F. Explain the Cartesian coordinate plane, and how it relates to the coordinate graph system. Discuss what the origin is, how to graph positive and negative numbers, and (x, y) notation.
G. Have the students plot points on graph paper.
H. The students will complete the Graphing Fun activity where they graphing given points on a grid sheet to make a picture.
I. Have the students look for examples of line graphs in newspapers or magazines.
J. Discuss the properties of these graphs. Have the students come up with an explanation for how the data was graphed in each case.
K. Using old magazines or newspapers, the Intranet, textbooks, library books, or other media available to the students, have them find two sets of data about something that they are interested in such as a hobby or career. One set should be percentages and the other set should be numbers. The data can be population sizes, money rates, stocks, weather, etc.
L. Using the data they found, students will construct table of each of the data. They should plot the set of data on graph paper as a line graph.
M. As a class, students will discuss the implications of the graphs they made, and analyze any conclusions they can draw from them.

Materials:
A. Enough maps for every four or five students to have one
B. Chalkboard
C. Graphing Fun Handouts
D. Old Newspapers and magazines
E. Access to the Internet
F. Other books or textbooks either in the class or the library
G. Graph paper
H. Straightedges

Assessment
A. How the student's picture turns out in the graphing fun activity.
B. If the students correctly complete their assignment of finding data and graphing it

Lesson Plan #6 – Properties and Applications of Linear Equations

Researched-Based Techniques Used in this Lesson to Motivate At-Risk Students

- Higher mental processes elicited among students:
  Critical thinking skills of analyzing, evaluating, and comparing; problem-solving skills; creative-thinking skills; and decision-making skills

- Activities that involved student-initiated learning:
  The string activity – students investigate and discover the properties of lines

- Problem solving and reasoning skills developed:
  Working in a logical manner, planning an approach, organizing information, following through to the end of the problem, and explaining the reasoning for how the student worked the problem

- Activities related to Edgar Dale's Cone of Experiences:
  Direct experiences – finding the slope of a ramp; simulated experiences – designing a ramp; audio experiences – class discussions; and visual experiences – string activity and computer program

- Variety in strategies used:
  Doing the string activity, measuring the slope of a ramp, designing a ramp, participating in class discussions, working in groups, and playing a computer game

- Manipulatives used:
  String and graph paper

- Technology used:
  Computer game

- Simulations or games used:
  Building a ramp including a pre-activity discussion and playing a computer game

- Tasks that involved cooperative learning:
  Building a ramp and determining the slope of a ramp
  The tasks included opportunities for peer tutoring and a peer evaluation activity to hold the students accountable for being on task

- Connections to uses of mathematics outside of the classroom:
  Construction, ski slopes, topological maps, and any other topics included in class discussion of how problem solving is necessary outside of the classroom

Subject: Algebra I
Content: Graphing Linear Equations

Content Connections of Lesson:
NCTM Standards: Students should . . .

- Compute fluently and make reasonable estimates
- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Analyze change in various contexts
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Make and investigate mathematical conjectures
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

KY Learning Goals and Academic Expectations: Students should . . .

1.3 – make sense of what they observe; 1.5-1.9 – use mathematical ideas and procedures; 2.8 – understand mathematical procedures; 2.11 – understand mathematical change concepts; 4 – be responsible group members; 5.1 – use critical thinking skills; 5.3 – organize information to develop understanding; 5.5 – use problem-solving processes; 6.1 connect knowledge from different subjects; 6.2 – use what they know to acquire new knowledge; 6.3 – expand knowledge by making connections

KY Core Content for Assessment Connections: Students should . . .

MA-H-2.1.2 – apply spatial relationships such as parallelism and perpendicularity; MA-H-2.1.4 – describe properties of and apply ratio measures including slope; MA-H-2.2.7 – apply formulas for the slope of a line; MA-H-4.1.4 – identify linear functions from graphs and equations; MA-H-4.2.2 – graph the equation of a line; MA-H-4.2.5 – determine the slope and intercepts of a linear function; MA-H-4.3.1 – write and solve linear equations; MA-H-4.3.2 understand how equations and graphs of functions relate to one another; MA-H-4.3.3 – demonstrate how slope shows rate of change in linear functions from real-world situations

KY New Teacher Standards: The teacher should . . .

I – focus on academic expectations, develop ability to apply knowledge, integrate concepts across disciplines, motivate and actively involve students, address diversity; II – communicate with and challenge students, establish shared expectations for responsibilities, support inquiry; III – communicate expectations for learning, link learning with prior knowledge, model the skill to be learned, use multiple teaching strategies and actively engage students in learning experiences, use questioning strategies, provide practice, link learning with future roles; VIII – communicate skills and core concepts, apply methods of inquiry,

connect knowledge to real life situations; IX – use variety of software, design lessons that use technology, apply research-based instructional practices, use computers for learning activities

Time: Three 1 ½ hour blocks

Objectives:
A. The students will write linear equations in both slope intercept form and point-intercept form.
B. The students will explain the properties of lines.
C. The students will explain the concept of slope as a way of expressing a rate of change.
D. The students will apply their knowledge of slope and linear equations.
E. The students will work cooperatively in groups.
F. The students will draw conclusions from their investigation of lines using string and graph paper.
G. The students will exhibit a positive attitude toward the lesson.
H. The students will use a computer game to practice their skills and become confident in working problems dealing with linear equations.

Rationale:
The goal of this lesson is for students to investigate the properties of a line to understand the concept of slope and how to write a linear equation.

Strategies:
A. The discussion of applications where slope is used in life helps the students see its importance generates more interest in the concept.
B. Before class, assign the students in the class to groups of 4 or 5, equally distributing them according to race, gender, and ability level. Make sure that any unmotivated students are not in groups together.
C. Having the students investigate the properties of lines helps them to develop their understanding of the concept and see why the properties exist.
D. The students will have a better understanding of slope-intercept and point-slope forms of linear equations if they play an active role in finding the formulas.
E. Determining the slope of the wheelchair ramp and designing their own ramp are activities where students can be involved and see first-hand a way in which slope is used in life.
F. The peer evaluation activity holds students accountable for the work they do in their groups and encourages responsibility and participation throughout the lesson.
G. The computer game at the end of the lesson provides a review for the students and allows those having more difficultly to practice the skills that they have the most difficulty concerning the topic.

Activities:
A. Discuss what slope means and places where they might see applications of slope such as a ski slope, roofs, ramps, mountains, and topological maps.
B. Use the examples the class came up with to explain slope as a rate of change and come up with more examples of this such as distance vs. time.
C. Have the students draw a Cartesian coordinate system on a piece of graph paper.

D. Using a piece of string have the students work in groups to investigate the problems on the handout about the number of lines that can be drawn given certain conditions.
E. Discuss the answers to the problems and the reasons behind the answers.
F. Have the students determine what information is necessary to write an equation in slope-intercept form.
G. Discuss how they might determine the equation of a line given only two points.
H. Have the students determine how to find the slope of a line given two points.
I. Take the students to the nearest wheelchair ramp in the school and have them work in groups to determine its slope.
J. Have the students work in groups and use graph paper to draw a wheelchair ramp for a building. According to building code standards the slope has to be less than or equal to 0.125. The building they are designing it for it 2 feet from the ground, the edge of the building is 5 feet away, and the people who own the building do not want the ramp to extend more than 1 foot past the edge of the building.
K. Group members will anonymously give each member of their group 0-2 points each for the following categories and explain the score they give: work ethic, contribution to the group, sensitivity to other group members.
L. Play *Mathville SkiQuations* by Ingenuity Works Inc. The computer game takes the students from plotting points to writing linear equations working at the student's own pace.

Materials:
A. String
B. Graph paper
C. Wheelchair ramp
D. Yardstick or measuring tape
E. Handout
F. Computer for each student with the software *Mathville SkiQuations* by Ingenuity Works Inc. installed on it

Assessment
A. Points for working cooperatively in a group
B. Success of drawing a wheelchair ramp model

Using the string and your Cartesian coordinate system, determine the number of lines that you can draw given each condition.

1. Lines with a slope of 6 and a y-intercept of 5
2. Lines with y-intercept of 3
3. Lines with a slope of 6
4. Lines through the point (-2, 4)
5. Lines through the points (3, 2) and (-5, 1)
6. Lines through the points (-1, 6), (3, 2), and (4, -4)
7. Lines through the points (-3, -4), (0, 0), (1, 2), and (2, 6)
8. Lines perpendicular to $y = x + 3$
9. Lines parallel to $y = x + 3$
10. Lines perpendicular to $y = x + 3$ through the point (1,2)
11. Lines parallel to $y = x + 3$ through the point (1, 2)
12. Lines through the point (-6 4) with a slope of 2

Cooperative Learning Scoring Rubric

Work Ethic

3 points	Consistently and actively works toward group goals.
2 points	Works toward group goals with occasional prompting
1 point	Works toward group goals only when prompted
0 points	Does not work toward group goals

Contribution

3 points	Consistently and actively contributes knowledge, opinions, and skills
2 points	Contributes to the group with occasional prompting.
1 point	Contributes to the group only when prompted.
0 points	Does not contribute to the group

Planning a Solution

4 points	Plan that could lead to a correct solution with no arithmetic errors and neatly displays any diagrams necessary
3 point	Substantially correct procedure with minor omission or procedural error or diagram is unclear
2 points	Partially correct procedure but with major fault or missing necessary diagram(s)
1 point	Totally incorrect plan and/or missing necessary diagram(s)
0 points	No attempt

Getting an Answer

3 points	Correct solution and answer labeled correctly
2 points	Copying error; computational error; partial answer for a problem with multiple answers; no answer statement; or answer labeled incorrectly
1 point	Wrong answer based on an inappropriate plan
0 points	No answer

Scoring Guide

12-13 points – A
10-11 points – B
9 points – C
7-8 points – D
0-6 points – F

Lesson Plan #7 – Solving Systems of Equations with Two Unknowns

Researched-Based Techniques Used in this Lesson to Motivate At-Risk Students

- Higher mental processes elicited among students:
  Critical thinking skills of analyzing, evaluating; problem-solving skills; and decision-making skills

- Activities that involved student-initiated learning:
  Learning a new concept from books or other resources

- Problem solving and reasoning skills developed:
  Working in a logical manner, planning an approach, organizing information, following through to the end of the problem, and explaining the reasoning for each step as they work

- Activities related to Edgar Dale's Cone of Experiences:
  Direct experiences – teaching the method to another class member; audio experiences – class discussions; abstract experiences – reading explanations of the concept, solving problems, and the variety of experiences students include in teaching their lesson

- Variety in strategies used:
  Participating in class discussions, working in groups, solving problems, and participating in any strategies included in the lessons that the students prepare

- Manipulatives used:
  Any that the students use in their lessons

- Technology used:
  The Internet, graphing calculators, and any technology that the students use in their lessons

- Simulations or games used:
  The competition between groups to solve a system of equations using each of the different methods

- Tasks that involved cooperative learning:
  Learning a new concept, teaching it to other students, and playing the game at the end of the lesson. The tasks included opportunities for peer tutoring and a peer evaluation activity to hold the students accountable for being on task

- Connections to uses of mathematics outside of the classroom:
  Necessity of being able to read technical material for understanding in the work force and understanding the language of mathematics

Subject: Algebra I
Content: Systems of Equations[3]

Content Connections of Lesson:
NCTM Standards: Students should . . .

- Represent and analyze mathematical situations and structures using algebraic symbols
- Build new mathematical knowledge through problem solving
- Apply and adapt a variety of appropriate strategies to solve problems
- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Create and use representations to organize, record, and communicate mathematical ideas

KY Learning Goals and Academic Expectations: Students should . . .
2.8 – understand mathematical procedures; 4 – become responsible group members; 5.3 – organize information to change understanding of a concept

Kentucky Core Content for Assessment Connections: Students should . . .
MA-H-4.1.2 – identify differences among expressions and equations; MA-H-4.1.3 – understand systems of linear equations; MA-H-4.2.2 – graph the equation of a line; MA-H-4.2.3 – solve systems of linear equations

KY New Teacher Standards: The teacher should . . .
I – focus on academic expectations, develop ability to apply knowledge, integrate concepts across disciplines, motivate and actively involve students, address diversity, use technology; II – communicate with and challenge students, establish shared expectations for responsibilities, support inquiry, create a media-rich environment; III – communicate expectations for learning, use multiple teaching strategies and actively engage students in learning experiences, use questioning strategies, provide practice, identify student misconceptions; IV – use multiple assessment strategies; VIII – communicate skills and core concepts, apply methods of inquiry, utilize technology

Time: Four 1 ½ hour blocks

Objectives:

A. Students will solve two equations with two unknowns using each of the four methods of graphing, substitution, and addition.
B. Students will exhibit positive attitudes toward the lesson.
C. Students will work together cooperatively to learn a new concept in math.
D. Students will plan and teach a new method to another student.
E. Students will read an explanation of a concept for understanding and comprehension.

[3]Lesson Plan adapted from Grant, C. A. & C. E. Sleeter. (1998). "Solving Two Equations with Two Unknowns." *Turning on Learning: Five Approaches for Multicultural Teaching Plans for Race, Class, Gender, and Disability* (2nd ed.). Upper Saddle River, NJ: Merrill Publishing, 102-105.

Rationale:
The goal of this lesson is for students to learn to solve a system of equations by working together and interacting cooperatively in a group.

Strategies:
A. Divide students into four groups with equal numbers of students. Separate students who associate with each other often as well as students at the same skill levels.
B. Assign to each group one of the following four methods for solving equations with two unknowns: graphing, substitution, addition, and determinants.
C. Learning the concept from a textbook develops skills students need in the work force of reading an explanation and comprehending the information.
D. Once the students feel confident about using the method their group is learning, the teacher should check their problems to be sure they understand them.
E. The teacher should check each group's lesson plan to see how they will teach the methods and then make copies of the plan for each student in the group.
F. Divide the students into four new groups with the equal numbers of students of different ethnic backgrounds and gender in each group. Each group should have one or two students who can do one of each of the four methods. Separate students who associate with each other often and students at the same skill level.
G. Students use graphing calculators to check their answers to gain an understanding of how to use the technology.
H. On the day after the students play the game, give the students a quiz on which they must demonstrate their abilities to solve the equations to check their understanding of the concept.
I. Students will give peer evaluations to understand the importance and helpfulness of each group member.

Activities:
A. Give students each a number and a letter 1 a through 1 d, 2 e-h, 3 a-d, 4 e-h, 5 a-d, 6 e-h. The number denotes the student's expert groups and the letter denotes their home group. Assign to each of the six expert groups one of the following four methods for solving equations with two unknowns. Have the students read the section in their textbook relating to their topic and try the problems to which they can find answers in the back to help them learn to work the problems.
B. Begin a brief discussion explaining why it is important to be able to read, explain, and understand material in the work force. They may not always know the information their employer expects them to know, and learning on their own could make them more valuable to their companies. Explain to the students that they are going to learn a new concept from their book by reading and helping each other.
C. The students should get in their expert groups. Tell students to use their textbooks, the supplementary books, and lessons on the Internet to learn how to use their group's assigned method of solving two equations with two unknowns. Groups one and two are to learn solving them by graphing, groups three and four are to learn the method of substitution, and groups five and six should work on the elimination method. They should work together to apply the method to solve various problems in the books and to make sure that everyone in the group can solve the equations using the given method. The students should see that he problems build on everything they have learned about linear equations up until now.

D. Once the students feel confident about using the method their group is learning, the teacher should check their problems and explanations to be sure they understand them.
E. When the teacher feels that the students understand the method, the group can begin to design a mini-lesson, which they will use to teach the other students. The lesson should have four parts – objectives, materials, procedure, and evaluation – to allow each student to contribute to the plan. The students may choose to explain the method and ask the other students to do problems, or assign homework and a quiz – the group should decide. The students should make sure that each group member could teach the lesson they have created.
F. In the student's home groups, each student in the group should teach his or her method for solving equations to the other group members. The students should follow their lesson plans. Note that the students from different groups with the same topic will have to collaborate, but they could have two different ways of explaining the method, which may help some students who do not understand it explained a certain way.
G. The students should make sure all of the members of the groups could perform the three methods until each group is confident that all members understand each of the methods.
H. Show the students how to solve a system of equations using a graphing calculator.
I. The students will stay in their home groups, and use their expert group number as their assigned number.
J. The teacher chooses a number and the student from each group with that number comes to the board. The teacher reads the problem and the students must write and solve the problem using the method specified by the teacher. Their group may help them, but the chosen student must be the one to write the answer. After each of the students in the round have finished the problem, then all students need to check the answer on their graphing calculators.
K. The groups can earn four, three, two, or one point for solving the problem first, second, third, or fourth, respectively. Give no points to a group for a problem solved incorrectly. All groups are required to keep score, and each group must work to earn 50 points.
L. The groups peer evaluate each other. Each student anonymously gives each of the students in both of their groups a score from 0-10 based on their performance in the group, 10 being the highest and 0 being the lowest. Two points are possible for each of five categories: cooperation with the group, respect for other group members, helpful assistance, tactfulness in giving feedback, appreciation of other group members.
M. The students will take a quiz over the three methods.

<u>Materials</u>:
A. Textbooks
B. Supplementary books/textbooks with explanations on how to solve two equations with two unknowns
C. Pencil and paper
D. Chalkboard
E. Computers with access to the Internet

<u>Assessment</u>
A. Quiz over the methods
B. Group assessment
C. Peer assessment

Scoring Rubric

Ability to solve the equations

16-20 points	Student sets up and answers problems correctly. Student demonstrates understanding of all methods.
11-15 points	Student is able to partially make connections between methods and is able to use most, but not all of the methods.
6-10 points	Student somewhat understands how to work the problems, but has trouble finding a correct answer. Student is able to use one or two methods correctly.
0-5 points	The student uses incorrect procedures without understanding the concepts related to solving the problem.

Ability to interact and cooperate

24-30 points	Provided many ideas, assisted other partners, gave and accepted feedback tactfully and graciously.
17-23 points	Participated in group work. Gave feedback in an unoffending way but accepted it reluctantly.
9-16 points	Showed some participation in group work. Feedback was given somewhat offensively and was not accepted very willingly.
0-8 points	Little or not participation in group work. Student was openly rude when giving feedback and refused to accept feedback from others.

Ability to plan a lesson

16-20 points	Good plan for teaching the method. Lesson clearly achieves objectives and provides plenty of examples and practice for the learner.
11-15 points	Plan for teaching method is present. Lesson achieves objectives and provides sufficient examples and practice for the learner.
6-10 points	Plan for teaching method is incomplete. Lesson somewhat achieves objectives and provides some examples and practice for the learner.
0-5 points	Contains no plan for teaching the method. Lesson does not achieve objectives and provides few or no examples or practice for the learner.

Ability to teach a lesson

16-20 points	Student gives a complete explanation with coherent and unambiguous explanations. All students in the group understand the method and are able to solve problems successfully.
11-15 points	Parts of student's explanation are unclear or inconsistent. Most group members fully understand how to use the method to solve problems.
6-10 points	Student's explanation is unclear or inconsistent. Most group members do not fully understand how to use the method and have much difficulty using the method to solve problems.
0-5 points	Student's explanation is not understandable or not present. No group member can use the method to solve problems.

Peer Evaluation

0-10 points	Points determined by peer evaluation activity. The total number of points a student receives is the average of each of the evaluations given to them by their peers.

Scoring Guide

A – 90-100
B – 80-89
C – 70-79
D – 60-69
F – 0-59

Lesson Plan #8 – Using Statistics to Manage a Successful City

Researched-Based Techniques Used in this Lesson to Motivate At-Risk Students

- Higher mental processes elicited among students:
  Critical thinking skills of analyzing, prioritizing, evaluating, and comparing; problem-solving skills; creative-thinking skills; and decision-making skills

- Activities that involved student-initiated learning:
  Playing the computer game and researching real-estate prices, population growth, and sales tax rates on the Internet and in periodicals

- Problem solving and reasoning skills developed:
  Planning an approach, organizing information, and explaining the reasoning for decisions concerning the students' cities

- Activities related to Edgar Dale's Cone of Experiences:
  Direct experiences – pricing houses; simulated experiences – solving problems students may see outside of the classroom and building a city using the computer program; audio experiences – class discussions; visual; finding information on the Internet and observing patterns; and abstract experiences – reading periodicals, making graphs, and doing computations

- Variety in strategies used:
  Using a computer program, participating in class discussions, working in a group, gathering data, making observations from the data, and solving problems from contexts outside of the classroom

- Technology used:
  Computer game and the Internet

- Simulations or games used:
  The computer program simulation including pre- and post-activity discussions

- Tasks that involved cooperative learning:
  Playing the computer program, brainstorming, finding data, making observations about data collected, and pricing houses
  The tasks included opportunities for peer tutoring and a peer evaluation activity to hold the students accountable for being on task

- Connections to uses of mathematics outside of the classroom:
  Necessity of understanding statistics to run a city, using mathematics to find a home, observing population growth, comparing sales tax rates, and any other topics included in class discussion of how mathematics relates to government

Subject: Algebra I
Content: Statistics (averages and lines of best fit)

Content Connections of Lesson:
NCTM Standards: Students should . . .

- Compute fluently and make reasonable estimates
- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Analyze change in various contexts
- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- Select and use appropriate statistical methods to analyze data
- Develop and evaluate inferences and predictions that are based on data
- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics
- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

KY Learning Goals and Academic Expectations: Students should . . .

1.1 – use reference tools; 1.2 – make sense of materials they read; 1.3 – make sense of materials they observe; 1.5-1.9 – use mathematical ideas and procedures; 1.10 – organize information; 1.11 – write for different purposes; 1.16 – use computers; 2.7 – understand number concepts; 2.8 – understand mathematical procedures; 2.11 – understand mathematical change concepts; 2.13 – understand statistics; 2.33 – use services in their community; 2.28 – demonstrate skills that lead to success in future schooling and work; 3 – become self-sufficient individuals; 4 – become responsible group members; 5.1 – use critical thinking skills; 5.3 – organize information; 5.4 – use a decision making process; 5.5 – use problem-solving processes; 6.1 – connect knowledge from different subjects; 6.3 – expand understanding of existing knowledge by making connections

KY Core Content for Assessment Connections: Students should . . .

MA-H-3.1.2 – recognize that curve fitting is a method of describing and predicting from a set of data; MA-H-3.2.1 – analyze, interpret results, make decisions, and draw conclusions based on a set of data; MA-H-3.2.2 – plot data and select an appropriate curve of best fit; MA-H-

3.2.3 – organize and display data; MA-H-3.3.4 – use data and curve of best fit to make and defend predications; MA-H-4.2.2 – graph the equation of a line; MA-H-4.2.3 – solve a system of linear equations; MA-H-4.3.1 – write linear equations describing real-world situations; MA-H-4.3.4 show how changes in parameters affect graphs of functions.

KY New Teacher Standards: The teacher should . . .

I – focus on academic expectations, develop ability to apply knowledge, integrate concepts across disciplines, motivate and actively involve students, address diversity, use technology, include school and community resources; II – communicate with and challenge students, establish shared expectations for responsibilities, support inquiry, create a media-rich environment; III – communicate expectations for learning, link learning with prior knowledge, model the skill to be learned, use multiple teaching strategies and actively engage students in learning experiences, use questioning strategies, guides students to discuss moral, ethical, or social issues, provide practice, link learning with future roles; VIII – communicate skills and core concepts, apply methods of inquiry, utilize technology, connect knowledge to real life situations; IX – use a variety of software, use the computer to access the Internet, design lessons that use technology, apply research-based instructional practices, use computers for learning activities

Time: Four 1 ½ hour blocks

Objectives:

A. Students will work cooperatively in a group.
B. Students will exhibit positive attitudes toward the lesson.
C. Students will use mathematics to make a simulation of a successful city.
D. Students will list important aspects of a successful city.
E. Students will demonstrate knowledge of sales tax rates through discussion and explain how sales tax rates affect a city and individuals.
F. Students will describe how a change in parameters of a linear function affects its graph.
G. Students will compute three different types of averages – mean, median, and mode.
H. Students will compare and contrast each of the three types of averages.
I. Students will gain experience in pricing houses.
J. Students will explain why housing prices differ between various areas of a city.
K. Students will collect data.
L. Students will make a scatter plot of data they have collected.
M. Students will determine a line of best fit.
N. Students will make predictions about a set of data using a line of best fit.

Rationale:

This lesson helps students understand how people use statistics outside of the classroom for their own benefit and for the benefit of a city government.

Strategies:

A. Before class, assign the students in the class to groups of 4 or 5, equally distributing them according to race, gender, and ability level. Make sure that any unmotivated students are not in groups together.
B. The students play the computer simulation initially to get an idea of what is important to having a successful city and to become familiar with the program for later in the lesson.
C. The students discuss the important features of a city as a class to make sure that all students are on the same page and to share their ideas with one another.
D. The students do their brainstorming in a small group setting to discourage students from feeling intimidated or ignored in a large group discussion.
E. The sales tax activity helps students understand why taxes are important to a city and to see how mathematics and economics play a role in a city's success.
F. The real estate project gives students experience in pricing houses, while helping them to learn about the three different types of averages and how they compare and contrast to each other. The students also learn why prices vary on houses throughout a city.
G. The population activity serves to teach students the importance of mathematics in predicting population growth and decline to cities while helping them learn how to determine a line of best fit for a set of data.
H. Developing a city is a way for students to use their knowledge from the lesson and to discover how a background in mathematics helps them be more successful in designing their city. The students can make something of which they are proud and work together cooperatively as a group to make decisions about their city, just as leaders of a community might do.
I. The peer evaluation activity holds students accountable for the work they do in their groups and encourages responsibility and participation throughout the lesson.

Activities:

A. Have the students play the computer simulation *SimCity 3000* by Electronic Arts Inc. just enough to learn how to play the game and see what aspects are important to have a successful city. Each group of students should work together to make decisions about how to build their city. They should pay attention to the aspects that seem most important to making a city successful for businesses, industries, individuals, and farms.
B. Have the students brainstorm these important features in their groups after they gain some experience with the game and then each group shares their ideas with the class.
C. Have the students make a graph comparing the money spent by the city's residents (x-axis) and the sales tax revenue created by their expenditure (y-axis). Have them make observations about the graphs for sales taxes of 5 percent, 6 percent and $8^1/_2$ percent and find where the graphs intersect. (The students should realize that they only intersect when the cost is zero, so the greater the sales tax, the more taxes they pay.)
D. Have the students discuss the advantages and disadvantages of having each of the different sales tax rates.
E. Have the students work in groups to gather a list of prices for a two bedroom house in different areas of the nearest large city. They should also record the type of building structures, parks, and schools that are in each area. They can use the Internet, newspapers, real estate magazines, or even phone interviews as resources.

F. After gathering their data, the students should find the mean, median, and mode for the price of a two bedroom house in each area of the city, as well as the mean, median, and mode for the price of a two bedroom house in the city overall.
G. As a class, discuss the students' findings and the difference between mean, median, and mode.
H. Using any resources available to the students, have them work in groups to find past and current population statistics for their city and decide if the population has increased of decreased through the years and by how much it has changed each year.
I. Show the students how to find a line of best fit. Then, have the students create a scatter plot of their population data and find their own line of best fit for their graph.
J. The students should speculate about why the population has changed over the years and write a paragraph on their conclusions.
K. Have the students use the equation from their line of best fit to predict the population in 10 years if it continues at the current rate. Have the students decide as a group what this could mean for the future of their city and how the city could change to accommodate the trend or to try and change the trend.
L. Using what they have learned, the students should play *SimCity* again. This time each group is to design a city that is not in debt, and explain on paper why they included different features in their simulation so their city would be successful.
M. The groups peer evaluate each other. Each student anonymously gives each of the students in both of their groups a score from 0-10 based on their performance in the group, 10 being the highest and 0 being the lowest. Two points are possible for each of five categories: cooperation with the group, respect for other group members, helpful assistance, tactfulness in giving feedback, appreciation of other group members.

Materials:
A. Computers for each group of students that have access to the Internet and the computer program *SimCity 3000* by Electronic Arts Inc. installed on it
B. Newspapers
C. Real estate magazines
D. Graph paper
E. Rulers

Assessment
A. Being able to use knowledge to create a successful city and explain the reasoning
B. Working cooperatively in a group

Scoring Rubric for Creating City

Understanding the Problem

8-10 points	Complete understanding of the problem
6-7 points	Misinterprets minor part of the problem
4-5 points	Misinterprets major part of the problem
2-3 points	Completely misinterprets problem
0-1 points	No attempt

Planning a Solution

8-10 points	Plan leads to a successful city (not in debt)
6-7 points	Plan that could lead to a successful city
4-5 points	Plan that neglects certain aspects of creating a certain city
2-3 points	Totally incorrect plan
0-1 points	No attempt

Reasoning

8-10 points	Clearly explained the reasons for the decisions made throughout the problem
6-7 points	Reasons not clearly explained, but work suggests sound reasoning
4-5 points	Partly correct reasoning, or correct reasoning used for only part of the problem
2-3 points	Reasoning is incorrect
0-1 points	No reasoning is evident from the work

Work Ethic

15-20 points	Consistently and actively works toward group goals.
10-14 points	Works toward group goals with occasional prompting
5-9 points	Works toward group goals only when prompted
0 points	Does not work toward group goals

Peer Evaluation

0-10 points Points determined by peer evaluation. The total number of points a student receives is the average of each of the evaluations given to them by their peers.

<u>Scoring Guide</u>

54-60 points – A
48-53 points – B
42-52 points – C
36-41 points – D
0-35 points – F

Lesson Plan #9 – Investigating the Properties of a System of Inequalities

Researched-Based Techniques Used in this Lesson to Motivate At-Risk Students

- Higher mental processes elicited among students:
  Critical thinking skills of analyzing and comparing; problem-solving skills; creative-thinking skills; and decision-making skills

- Activities that involved student-initiated learning:
  Investigating properties of a system of inequalities using the geoboard activity and deciding how to find a point on the geoboard with the fewest number of guesses

- Problem solving and reasoning skills developed:
  Working in a logical manner, planning an approach, organizing information, and explaining why graphing the system is more effective than just guessing points

- Activities related to Edgar Dale's Cone of Experiences:
  Direct experiences – the geoboard activity; audio experiences – class discussions; and visual experiences – graphing the systems of inequalities students create

- Variety in strategies used:
  Doing the geoboard activity, participating in class discussions, working in groups, and graphing problems

- Manipulatives used:
  Geoboard

- Simulations or games used:
  The geoboard activity including a post-activity discussion

- Tasks that involved cooperative learning:
  Doing the geoboard activity and graphing equations
  The tasks included opportunities for peer assessment

Subject: Algebra I
Content: Systems of Inequalities

Content Connections of Lesson:
NCTM Standards: Students should . . .

- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Use visualization, spatial reasoning, and geometric modeling to solve problems
- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others
- Use the language of mathematics to express mathematical ideas precisely
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Create and use representations to organize, record, and communicate mathematical ideas
- Use representations to model and interpret physical, social, and mathematical phenomena

KY Learning Goals and Academic Expectations: Students should . . .

1.3 – make sense of what they observe; 1.5-1.9 – use mathematical ideas and procedures; 2.7 – understand number concepts; 2.8 – understand mathematical procedures; 4 – become responsible group members; 5.1 – use critical thinking skills; 5.3 – use problem-solving processes; 6.2 – use what they know to acquire new knowledge

KY Core Content for Assessment Connections: Students should . . .

MA-H-1.3.3 – understand how to use order relations to solve problems; MA-H-4.1.2 – recognize differences between equations and inequalities; MA-H-4.2.1 – solve linear inequalities; MA-H-4.3.2 – understand how formulas and graphs of functions relate

KY New Teacher Standards: The teacher should . . .

I – focus on academic expectations, develop ability to apply knowledge, motivate and actively involve students, address diversity; II – communicate with and challenge students, establish shared expectations for responsibilities, support inquiry; III – communicate expectations for learning, link learning with prior knowledge, model the skill to be learned, use multiple teaching strategies and actively engage students in learning experiences, use questioning strategies, provide practice; VIII – communicate skills and core concepts, apply methods of inquiry

Time: 1 ½ hour block

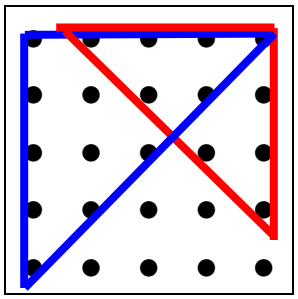
Objectives:
A. The students will solve a system of inequalities.
B. The students will exhibit a positive attitude toward the lesson.
C. The students will work together cooperatively in a group.
D. The students will develop their concept of solving systems of inequalities.
E. The students will show enjoyment in classroom activities.

Rationale:
The students will enjoy challenging one another with the geoboard problems, which will increase their motivation to learn about inequalities.

Strategies:
A. Divide the students into pairs before class.
B. The activity arouses the students' interest in learning the concept and motivates them to solve the problems.
C. The geoboard model helps the students see that more than one point can satisfy an inequality and how the areas overlap.
D. Comparing the system of inequalities to a system of equations helps the students build their understanding of the new concept from a concept with which they are already familiar.
E. Learning to graph the inequalities helps the students see the relationship between each inequality.
F. Working together allows the students to learn from one another and increases the motivation to learn.

Activities:
A. Hand out a geoboard and a red and blue rubberband to each student. Explain to the students that the geoboard represents the first quadrant of the coordinate plane and each peg along the bottom (x- and y-axes) represents one unit.
B. Each student picks a point on their geoboard and writes it on a piece of paper where their partner cannot see. Then, the students take turns trying to guess their partner's point. Whoever guesses the point first is the winner. They should keep track of the number of tries it took to report to the class. They should do this several times.
C. When everyone has had the chance to complete several rounds, the teacher should poll the class to find out who had the least number of guesses and who had the most. See if the students can devise a way narrow the choices down some given their knowledge of inequalities. They should realize that writing one inequality would narrow it down. Then, ask if they could narrow it down further (using a system of inequalities will help them do so).
D. Have the students repeat the activity, but this time the student should write two true inequalities of the form $y > a*x + b$ and $y < -c*x + d$ where a, b, c, and d are not equal to each other. Then, they should give the inequalities to their partner to help them figure out the point.
E. As a class, allow the students to discuss different methods for finding the point. If no one thought of it, show the students that by graphing the lines their partner gave them, and

stretching the rubberbands to include all of the "true" area for each line, it created a box in which the point would lie (see below).



F. Have the students connect graphing the inequalities on the geoboard to graphing them on a coordinate plane. Show the students the two overhead examples provided with the supplemental textbook resources. Ask them to predict what the region might look like if there were more than two inequalities. Discuss what happens if the shaded regions do not overlap.

G. Have them transfer their knowledge to graph paper and do the following problems:

1.) $\begin{matrix} x+y \le 6 \\ x-y<1 \end{matrix}$ 2.) $\begin{matrix} y-3x<6 \\ y-2x>1 \end{matrix}$ 3.) $\begin{matrix} x+y \le 5 \\ x \ge 1 \end{matrix}$ 4.) $\begin{matrix} y<\frac{1}{2}x \\ y>\frac{1}{2}x-3 \end{matrix}$ 5.) $\begin{matrix} x>y \\ y>0 \end{matrix}$

Materials:

A. Geoboard
B. Rubber bands
C. Graph paper

Assessment

A. Ability to correctly solve a system of equations

Lesson Plan #10 – Investigating Properties of Quadratic Equations

Researched-Based Techniques Used in this Lesson to Motivate At-Risk Students

- Higher mental processes elicited among students:
  Critical thinking skills of analyzing, evaluating, and comparing; and problem-solving skills

- Activities that involved student-initiated learning:
  Graphing the equations and drawing conclusions about the parameters of the graphs

- Problem solving and reasoning skills developed:
  Working in a logical manner, planning an approach, organizing information, following through to the end of the problem, and explaining the reasoning for using graphing to solve quadratic equations

- Activities related to Edgar Dale's Cone of Experiences:
  Simulated experiences – solving problems students may see outside of the classroom; audio experiences – class discussions; visual experiences – using a computer to compare graphs; and abstract experiences – reading problems and writing answers

- Variety in strategies used:
  Using computers and graphing calculators, working in pairs, and solving problems from contexts outside of the classroom

- Technology used:
  Graphing calculators and computers

- Tasks that involved cooperative learning:
  Graphing equations on a calculator and uploading the graphs onto a computer
  The tasks included opportunities for peer

- Connections to uses of mathematics outside of the classroom:
  Necessity of solving quadratic equations outside of the classroom and any other topics included in class discussion of how solving quadratic equations is necessary outside of the classroom

Subject: Algebra I
Content: Graphing Quadratic Equations

Content Connections of Lesson:
NCTM Standards: Students should . . .

- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Make and investigate mathematical conjectures
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics
- Create and use representations to organize, record, and communicate mathematical ideas
- Use representations to model and interpret physical, social, and mathematical phenomena

KY Learning Goals and Academic Expectations: Students should . . .
1.3 – make sense of what they observe; 1.5-1.9 – use mathematical ideas and procedures; 1.10 – organize information; 1.16 – use technology to collect information; 2.8 – understand mathematical procedures; 2.11 – understand mathematical change concepts; 4 – become responsible group members; 5.1 – use critical thinking skills; 5.3 – organize information to develop understanding; 5.5 – use problem solving processes; 6.1 – connect knowledge with different subjects; 6.3 – use what they know to acquire new knowledge

KY Core Content for Assessment Connections: Students should . . .
MA-H-4.1.4 – identify quadratic functions from graphs and equations; MA-H-4.2.5 – determine the intercepts of a quadratic function; MA-H-4.2.6 – determine approximate solutions to quadratic equations; MA-H-4.3.2 – understand how graphs and equations relate to one another; MA-H-4.3.4 – show how changes in parameters affect graphs of functions

KY New Teacher Standards: The teacher should . . .
I – focus on academic expectations, develop ability to apply knowledge, integrate concepts across disciplines, motivate and actively involve students, address diversity, use technology; II – communicate with and challenge students, establish shared expectations for responsibilities, create a media-rich environment, support inquiry; III – communicate expectations for learning, link learning with prior knowledge, model the skill to be learned, use multiple teaching strategies and actively engage students in learning experiences, use questioning strategies, provide practice, link learning with future roles; VIII – communicate skills and core concepts, apply methods of inquiry, utilize technology, connect knowledge to real life situations; IX – use a variety of software, use the computer to support instruction, design lessons that use technology, explore and use technology resources, use computers for learning activities

Time: One 1 ½ hour block

Objectives:
A. Students will use a graphing calculator to graph equations.
B. Students will use a graphing calculator with a computer.
C. Students will use a graphing calculator to solve quadratic equations by graphing.
D. Students will compare and contrast quadratic equations to the equation $y = x^2$.
E. Students will determine the properties of quadratic equations.
F. Students will display interest in learning about quadratic equations.
G. Students will solve real-life word problems that require them to solve quadratic equations by graphing.
H. Students will explain the practical applications of solving quadratic equations in life.
I. Students will work cooperatively with other students.
J. The students will exhibit a positive attitude toward the lesson.

Rationale:
The students will be able to examine properties of quadratic equations using graphing calculators so they focus on understanding the concept instead of being over-involved with computing the solutions and drawing the graphs.

Strategies:
A. Working in pairs allows the students to benefit from a peer tutoring situations and prevents individual students from falling behind in the class discussion because they have fallen behind the rest of the students in the activities.
B. The use of technology, such as graphing calculators, is interesting to many students regardless of the activity the students are performing.
C. Incorporating the use of computers with the graphing calculators provides students with a skill that they can use in other subjects when needing to display a graph in a document.
D. Graphing the equations and making comparisons teaches students about the basic properties of quadratic equations and develops their familiarity with them for later discussions.
E. Discussing intercepts introduces the idea of solving quadratic equations and what solving them really means.
F. Using the calculators to introduce solving quadratic equations allows the students to become familiar with the process before learning to graph them by hand so that they are not learning too many concepts at once. This is also a more precise method and is likely more interesting to the students than graphing them by hand.
G. Discussing problems with this method shows students the need to learn other methods of solving quadratic equations that they will learn in later lessons.
H. Finding problems that require students to solve quadratic equations shows them the need for knowing the concept when they are outside of the classroom.

Activities:
A. Demonstrate how to use a USB cable and the appropriate software (Texas Instrument calculators are recommended) to connect their calculator to the computer, upload a graph, and insert the graph into a document.

B. Students and their partners will graph each set of equations listed on their handout on the same pair of axes using a graphing calculator and compare each of the graphs.
C. Students will upload their graphs into the computer.
D. Students will insert the graphs into a document and list their observations about how changing the parameter of a quadratic function changes its graph.
E. The class will discuss their findings and the teacher will lead them into a discussion about x- and y-intercepts.
F. Students will use their graphing calculators to find solutions to various quadratic equations using the ZOOM and TRACE functions on their calculators, as well as using the built in functions to find the solutions.
G. Students will discuss problems with the graphing method of solving quadratic equations.
H. As a class, the students will look at various problems where they would need to know how to solve a quadratic equation outside of the classroom.

Materials:
A. Graphing Calculators for each student
B. A Computer for each pair of students
C. USB connection and appropriate software for the graphing calculator and a computer
D. Handout
E. Chalkboard
F. Overhead projector with a hook-up to display a graphing calculator

Assessment
A. Handout where students solve equations and word problems by graphing quadratic equations.

Graph each of the following sets on the same pair of axes using your graphing calculator. Compare the graphs in each set and write down any observations that you make.

1. $y = x^2$; $y = x^2 + 3$; $y = x^2 - 1$
2. $y = x^2$; $y = (x + 2)^2$; $y = (x - 3)^2$
3. $y = x^2$; $y = {}^1/_2x^2$; $y = 2x$
4. $y = x^2$; $y = -x^2$; $y = -x^2 + 3$; $y = -(x - 3)^2$

Graph the following on your graphing calculator and use the ZOOM and TRACE functions to find their solutions.

1. $x^2 - 4x = 0$
2. $x^2 - 16 = 0$
3. $x^2 + 1 = 0$
4. $x^2 + 2x + 1 = 0$

Solve the following problems by graphing the equation on a graphing calculator and finding its solution.

1. To cover two floors completely takes 690 square feet of carpet. One floor is 3 feet longer than it is wide. The other floor is 2 feet wider than the first is long, and its length is twice the length of the first. Find the dimensions of the floor.

2. To make room for a barbecue pit, a man cuts the area of his garden in half by subtracting equal amounts from its length and width. If the garden originally is 30 feet by 40 feet, by how much should he reduce each dimension?

3. A person shoots a ball into the air using a slingshot from ground level at a speed of 110 ft/s. After how many seconds will the ball be 150 feet above ground?
   $h = {}^1/_2at^2 + vt + z$, where a is the acceleration due to gravity (-32 ft/s^2), v is the speed of the object at the release, and z is the initial height. (Hint: rewrite the equation in the form of $y = ax^2 + bx + c$ after substituting in the given numbers.)

Lesson Plan #11 – Using Algebra Tiles and the A-B-C Method to Factor Quadratic Equations

Researched-Based Techniques Used in this Lesson to Motivate At-Risk Students

- Higher mental processes elicited among students:
  Critical thinking skills of analyzing, evaluating, and comparing; problem-solving skills; and decision-making skills

- Activities that involved student-initiated learning:
  Doing the balance activity, exploring with algebra tiles to formulate a method for factoring and solving quadratic equations, finding relationships between factored and un-factored forms of an equation, and exploring job opportunities where solving a quadratic equation could be helpful

- Problem solving and reasoning skills developed:
  Working in a logical manner, planning an approach, organizing information, following through to the end of the problem, and explaining the reasoning for each step as they work

- Activities related to Edgar Dale's Cone of Experiences:
  Direct experiences – doing the scale activity, manipulating algebra tiles to factor equations; simulated experiences – solving problems students may see outside of the classroom; audio experiences – class discussions; visual experiences – seeing the geometric model for factoring and finding jobs on the Internet; and abstract experiences – reading problems and writing answers

- Variety in strategies used:
  Doing the balance activity, manipulating algebra tiles, participating in class discussions, working in groups, researching on the Internet, practicing a systematic method for factoring, and solving problems from contexts outside of the classroom

- Manipulatives used:
  Algebra tiles and the factoring box in the a-b-c method

- Technology used:
  The Internet

- Tasks that involved cooperative learning:
  Doing the balance activity and manipulating algebra tiles
  The tasks included opportunities for peer tutoring

- Connections to uses of mathematics outside of the classroom:
  Necessity of solving quadratic equations in job situations and any other topics included in class discussion of how solving quadratic equations is necessary outside of the classroom

Subject: Algebra I
Content: Factoring Quadratic Equations[4]

Content Connections of Lesson:
NCTM Standards: Students should . . .

- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Use visualization, spatial reasoning, and geometric modeling to solve problems
- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving
- Apply appropriate techniques, tools, and formulas to determine measurements
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

KY Learning Goals and Academic Expectations: Students should . . .
2.8 – understand various mathematical procedures; 2.11 – understand mathematical change concepts; 4 –become responsible group members; 5.1 – use critical thinking skills; 5.3 – organize information to develop or change understanding of a concept; 6.2 – use what they know to acquire new knowledge and develop new skills; 6.3 – expand understanding of existing knowledge by making connections

KY Core Content for Assessment Connections: Students should . . .
MA-H-1.1.1 – apply real numbers to real-world and mathematical situations; MA-H-2.3.1 – solve real-world geometry problems using algebra; MA-H-2.3.2 – apply algebra to solve problems involving geometric figures in a coordinate plane; MA-H-2.3.3 – understand how figures are geometrically and algebraically related; MA-H-2.3.4 – understand how change in dimension of geometric shape affects area; MA-H-4.1.2 – understand differences between expressions and equations; MA-H-4.2.6 – determine approximate solutions to quadratic equations; MA-H-4.2.7 – factor polynomial expressions

KY New Teacher Standards: The teacher should . . .
I – focus instruction on academic expectations, develop ability to apply knowledge, motivate and actively involve learner, address diversity; II – establish importance of responsibility, motivate and encourage inquiry, promote self-control and self-discipline, encourage responsibility; III – communicate expectations, use cooperative learning experiences, use questioning strategies, provide practice, link learning to future roles; VIII – connect knowledge to real-life

[4]Lesson ideas adapted from Stapel, E. (2000). *Purplemath – Your algebra resource*. Retrieved November 2, 2000, from http://www.purplemath.com

Time: Three 1 ½ hour blocks

Objectives:
A. Students will use algebra tiles to factor quadratic equations.
B. Students will transfer the use of algebra tiles to grid paper to factor quadratic expressions.
C. Students will use the a-b-c method of factoring to solve a quadratic equation.
D. Students will write all quadratics equations in standard form.
E. Students will use practical applications of factoring.
F. Students will solve word problems of real-life situations using factoring.
G. Students will work cooperatively in a group.
H. Students will exhibit positive attitudes toward the lesson.
I. Students will explore various properties of factoring.

Rationale:
The goal of this lesson is for students to realize that they can have fun learning math and that math has practical applications for the real world. The lesson should challenge, motivate, and actively involve the learner.

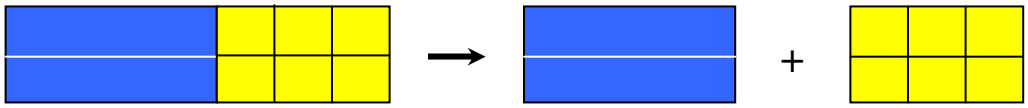
Strategies:
A. The scale activity is designed to reinforce the idea that the same amount must be taken from each side to keep the scale balanced, just as when working with equations they must perform the same operation to both sides to keep the two sides equivalent.
B. Before class, assign the students in the class to groups of 4 or 5, equally distributing them according to race, gender, and ability level. Make sure that any unmotivated students are not in groups together.
C. Introduce the algebra tiles to the students. Compare a 3 by 3 unit square to each of the algebra tiles so that they understand what the algebra tiles represent. These serve as a manipulative for learning to factor that the students can refer to during the unit.
D. Introduce the term "quadratic expression" and "quadratic equation" and let students determine the difference between them.
E. Students explore the concepts with the algebra tiles to construct their concept of factoring.
F. Research and discuss jobs and situations that require the use of quadratic equations. This activity gives students a context for why the rest of the lesson is important to them.
G. The a-b-c method of factoring gives students another alternative to factoring that uses a manipulative, but is more abstract than the first method. This strategy is a form of scaffolding to help the students learn to factor on their own.

Activities:

Day One

A. In their groups, have the students use balance scales to put enough weights on each side so that the scale balances, but they must not have the same combination of weights on each side. Let them experiment with taking certain weights off each side of the scales. Ask them what they observe. For example, have them explain what happens if they take off five pounds from one side and ten pounds from the other.

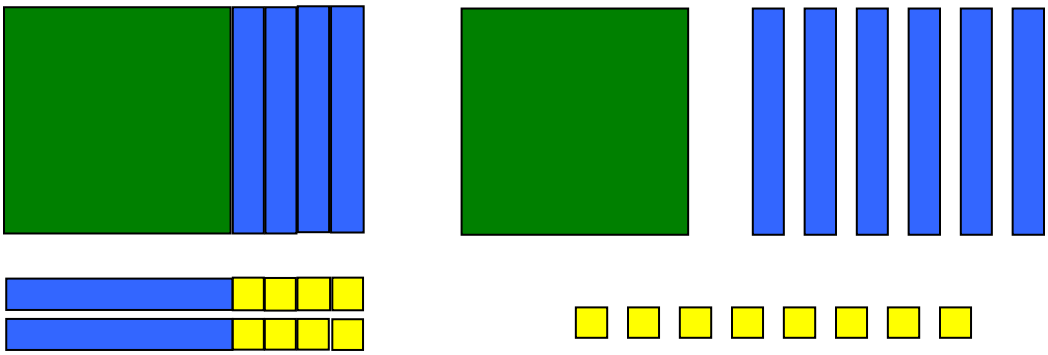
B. Have the students work in groups with the algebra tiles to demonstrate the distributive property. For example, $2(x + 3) = 2(x) + 2(3) = 2(x) + 6$ with the algebra tiles is as follows:



C. Have the students work in their groups with the algebra tiles to compare finding the area of a 3 by 3 unit square to finding the area of a rectangle with dimensions $(x + 4)$ by $(x + 2)$. They should eventually rearrange them to show that they also represent $x^2 + 6x + 8$.

Example of how to use algebra tiles:

$$(x + 4)(x + 2) = x^2 + 6x + 8$$



D. Have the students work in groups and try $(x + 2)(x +3)$ and $(x + 5)(x + 1)$. Discuss the solutions.
E. Have them factor these expressions using the same method: $x^2+9x+14$ and $3x^2+10x +3$
F. Have the students do several problems on their own using the algebra tiles and drawing the figures on graph paper.

Day Two

A. Introduce the remaining algebra tiles. Have them form a square or rectangle showing the following: $a^2 + 2ab + b^2$ and discuss the type of figure it forms. Introduce the term for this type of equation, "perfect square trinomial". Have the students form a square or rectangle that shows $a^2 – 2ab + b^2$ (also a perfect square) and show that $(a – b)(a + b) = a^2 + ab – ab – b^2 = a^2 – b^2$ (the difference of squares).
B. Introduce the term "quadratic equation" (an equation where the greatest degree of any of the terms is two). Have students give examples and non-examples of a quadratic equation. Explain that the standard form of a quadratic is "$ax^2 + bx + c = 0$, where a, b, and c are simply numbers. Have students explain the difference between quadratic equations and quadratic expressions.
C. Have the students work in their groups to discover how to solve an equation using this method. Have them solve the previous examples and check their work by substituting their

answers in the original equation. Discuss the answers as a class and how to put quadratic equations in standard form.

D. Escort the students to the computer lab. Have each use the Internet to find word problems dealing with other subject areas that would require them to use a quadratic equation. The teacher make offer some suggestions such as having them search for the keywords "quadratic equations" or by visiting websites like the following:
   http://forum.swarthmore.edu/teachers/high/
   http://www.iln.net/
   http://www.purplemath.com/
   http://www.yahooligans.com/tg/
E. Have the students use what they find to come up with a list of jobs that may use quadratic equations and have them find an average salary for the jobs.
F. After they finish, have each group present their findings to the class.

Day Three

A. Have the students work in groups and experiment more with the algebra tiles to see if they can come up with a method to factor a quadratic. Do they see any relationships between the numbers? What are they? Brainstorm a list of what they come up with on the board for the class.
B. Explain the a-b-c method to factoring a quadratic using an example. Have the students check the answer using their algebra tiles.

   Explanation of the a-b-c method:
   The idea is to multiply *a* and *c* and then find factors of the product *ac* that add up to *b*. Use the equation $2x^2 + x - 6 = 0$ for practice. Have the students list all the factors of -12 and decide on the combination that gives them a sum of one when added. Show them the box method. Put the first term in the upper right-hand corner and the last term in the bottom left-hand corner. Then put the factors in the other two corners (it doesn't matter which order) with an *x*. Take the greatest common factor from each row and column and use the sign closest to it. Then the answer is on perimeter of the box as shown below.

$2x^2 + x - 6 = 0$ ➔

	2	x
2x	4x	$2x^2$
-3	-6	-3x

➔ $(2x - 3)(x + 2)$

C. Give students the following problems one at a time, have them work them, then discuss the answers as a class allowing the students to explain the process: $x^2 + 2x + 1 = 0$, $r^2 - 10r + 21 = 0$, $3z^2 + 2z - 1 = 0$. Do as many examples as needed until it seems that the class understands the concept.
D. Ask the students to factor this expression $6v^3 + 26v^2 + 8v = 0$. What is different about it? Lead students to discover that they must first factor out common terms before using the a-b-c- method. Then have them try $8y^2 - 48y = 216$. Lead them to discover that this must first be in standard form before they can solve it.
E. Explain how to factor the difference of two squares. Have them do examples.

F. Give the students the factoring handout. Review the steps on this handout for factoring equations.
G. Have the students work word problems involving factoring.

Materials:
A. Eight sets of algebra tiles including sixteen 1 unit by 1 unit squares, ten 1 unit by *x* unit rectangles, three *x* unit by *x* unit squares, three *a* unit by *a* unit squares, four *a* unit by *b* unit rectangles, and three *b* unit by *b* unit squares.
B. Computers
C. Balance Scale
D. Set of small weights
E. Factoring steps handout
F. Graph paper

Assessment
A. Solving problems using factoring on handout
B. Class participation
C. Cooperation in group
D. Effort

Steps for Factoring Expressions

1. If there is a common factor, find the greatest common factor.

2. If an expression is a binomial, is it the difference of two squares? If so, you can factor it.

3. If an expression is a trinomial, assume that it is the product of two binomials, and search for them using the a-b-c method.

4. If an expression is neither a binomial nor a trinomial, see if you can group the terms and find a common factor. If so, factor as in steps 2 and 3.

5. After factoring, write down *all* of the factors, including monomial factors.

6. Multiply the factors to check your work. The product should be the original expression.

Factor the following, by using a geometric model. Show your work on graph paper.

1. $x^2 + 12x + 27$
2. $x^2 + 8x + 7$
3. $x^2 + 5x + 6$
4. $x^2 + 11x + 18$
5. $x^2 – 9x + 8$

Set up an equation and solve.

1. A bullet left a gun at 1600 feet per second. In how many seconds did the bullet hit the balloon 4656 feet directly overhead if the height *h* is given by the equation $h = r*t – 16t^2$ where r = rate and t = time?

2. Find two consecutive odd integers the sum of whose squares is 202.

3. A builder decorates 36 square meters of a courtyard with a triangular garden whose base is 1 meter longer than its height. Find the base and height of the triangular garden.

4. The plowed area of a field is a rectangle 80 feet by 120 feet. The owner plans to plow an extra strip of uniform width on each side of the four sides of the field, in order to double the plowed area. How many feet should he add to each dimension?

5. To support the weight of a building, an engineer determines that its foundation must have an area of 496 square feet bearing on the soil. The building is to be rectangular, and its foundation, 2 feet thick. If the inside of the foundation is two times the inside width, find these dimensions.

Scoring Rubric

Participation in group work and class discussion:

5 Participates fully. Contributes at least two ideas to the group during research and participates in class discussion.

2 Somewhat participates. Contributes little to the group and does not say much during class discussion.

0 Does not participate in class activities.

Group work and research:

5 Worked well in groups. All members helped find information to be presented before the class and contributed in trying to solve the factoring problems.

2 Some group members participated while others were off-task most of the time.

0 No group interaction. Members worked individually or not at all.

Effort/Interest:

5 Shows much effort and interest in the tasks. Good attitude toward the topic.

2 Some effort or interest shown, but also seem somewhat uninterested.

0 Show no effort or interest. Seems unmotivated and tuned out.

Homework:

5 Attempts all problems, and finishes 3 out of 5 factoring problems and 3 out of 5 word problems correctly with appropriate steps shown.

3 Attempts all problems few or none complete correctly.

0 No attempt made to solve the problems.

Scoring Guide

A – 18-20
B – 16-17
C – 14-15
D – 12-13
F – 0-11

Lesson Plan #12 – Investigating Properties of Absolute Value Equations

Researched-Based Techniques Used in this Lesson to Motivate At-Risk Students

- Higher mental processes elicited among students:
  Critical thinking skills of analyzing and comparing; and problem-solving skills

- Activities that involved student-initiated learning:
  Researching jobs and discovering a method to solve absolute value equations using manipulatives

- Problem solving and reasoning skills developed:
  Organizing information and making inferences

- Activities related to Edgar Dale's Cone of Experiences:
  Direct experiences – using the number lines and researching jobs; simulated experiences – solving problems students may see outside of the classroom; audio experiences – participating in class discussions; visual experiences – observing graphs; and abstract experiences – reading problems and writing answers

- Variety in strategies used:
  Using manipulatives for solving absolute value equations, graphing equations, participating in class discussions, working in groups, researching on the Internet, and solving problems from contexts outside of the classroom

- Manipulatives used:
  Spaghetti strands and the number lines

- Connections to uses of mathematics outside of the classroom:
  How absolute value relates to distances, a rain gauge, stocks, and any other topics included in class discussion

Subject: Algebra I
Content: Solving Absolute Value Equations[1]

Content Connections of Lesson:
NCTM Standards: Students should . . .

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- Understand meanings of operations and how they relate to one another
- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Solve problems that arise in mathematics and in other contexts
- Make and investigate mathematical conjectures
- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

KY Learning Goals and Academic Expectations: Students should . . .

1.1 – use reference tools to find information; 1.3 – make sense of things they observe; 1.5-1.9 – use mathematical ideas and procedures; 1.16 – use computers to collect information; 2.7 – understand number concepts; 2.8 – understand mathematical procedures; 3 – develop into self-sufficient individuals; 4 – develop ability to become responsible group members; 5.1 – use critical thinking skills; 6.1 – connect knowledge from different subjects; 6.2 – expand knowledge by making connections with new knowledge

KY Core Content for Assessment Connections: Students should . . .

MA-H-1.2.2 – simplify real number expressions; MA-H-4.1.4 – identify functions from graphs and equations; MA-H-4.2.4 – create tables of numerical values of functions; MA-H-4.3.2 – understand how formulas, tables, graphs, and equations of functions relate to each other

KY New Teacher Standards: The teacher should . . .

I – focus on academic expectations, develop ability to apply knowledge, integrate concepts across disciplines, motivate and actively involve students, address diversity, use technology; II – communicate with and challenge students, establish shared expectations for responsibilities, support inquiry, create a media-rich environment; III – communicate expectations for learning, link learning with prior knowledge, model the skill to be learned, use multiple teaching strategies and actively engage students in learning experiences, use questioning strategies, provide practice, link learning with future roles; VIII – communicate skills and core concepts, apply methods of inquiry, utilize technology, connect knowledge to real life situations; IX – use a variety of software, use the computer to access the Internet,

[1]Lesson Plan adapted from Gerver, R. Ph.D., Sgroi, R. Ph.D., Carter C., Hansen M., Moilna D., & Westegaard S. PH.D. (1997). "Absolute Value and the Real Number System." *South-Western Algebra: An Integrated Approach*. Cinncinati, OH: South-Western Educational Publishing, 424.

design lessons that use technology, apply research-based instructional practices, use computers for learning activities

Time: Two 1 ½ hour blocks

Objectives:
A. Students will use the number lines to solve absolute value equations.
B. Students will transfer the use of number lines to a method for solving absolute value equations that does not use manipulatives.
C. Students will state a definition of absolute value.
D. Students will explain the concept of absolute value.
E. Students will give examples of where solving absolute value equations is used outside of the classroom.
F. Students will work cooperatively in a group.
G. Students will investigate the properties of absolute value equations.
H. The students will graph and recognize the graph of an absolute value equation.
I. The students will compare and contrast the graph of an absolute value equation with linear and parabolic graphs.
J. Students will exhibit positive attitudes toward the lesson.

Rationale:
This lesson should teach students not only how to solve an absolute value equation, but what solving it actually means so that they are more likely to remember how to solve the equations.

Strategies:
A. Students explore the concept of absolute value using the manipulatives to construct their own knowledge about solving absolute value equations.
B. The number lines helps the students see what absolute value really is.
C. Before class, assign the students in the class to groups of 4 or 5, equally distributing them according to race, gender, and ability level. Make sure that any unmotivated students are not in groups together.
D. The groups allow the students to learn from one another and combine their knowledge to form their understanding of the concept.
E. Looking at the graphs of the equations helps the students further understand the concept appealing to different types of learners.
F. Using the Internet to find places where they may come across problems involving absolute value outside of the classroom helps the students see the usefulness of what they are learning.

Activities:
A. The students should draw two number lines from –25 to 25 on a sheet of paper making sure that the marks between numbers are an equal distance apart and cut each of the number lines out. They should label one of the number lines the zero finder and the other should be labeled the solution finder.
B. The teacher should lead the students in a discussion about the meaning of absolute value using examples of where students would find this concept outside of the classroom such as driving a car, a rain gauge, or stocks.

C. The teacher should walk the class through how to solve an absolute value equation using the two number lines and two pieces of spaghetti using the example of $|2x - 3| = 5$. To solve the equation, they should solve it as if there were no absolute value bars, and then align the answer on the solution finder with the zero on the zero finder. Since they are looking for a number that is 5 units away from $2x - 3$, they should place a strand of spaghetti on -5 and $+5$ on the zero finder. The answer is where the spaghetti crosses the solution finder.
D. Have the students check the answers.
E. Have the students work in their groups to answer the other problems on their handout.
F. Have students discuss why this method works and why there are always two answers unless the equation is set to zero in the original problem.
G. See if students can come up with a method as a group of how to solve the problems without using the manipulatives and come up with one method for them to use as a class.
H. Have the students examine the graphs of absolute value equations by graphing the equations on the handout by using a table of values and then plotting the points. They should compare them to linear and parabolic graphs and to each other.
I. Escort the students to the computer lab. Have each use the Internet to find word problems dealing with other subject areas that would require them to use absolute value equations. The students use the following website and keywords to help them:
    http://www.iln.net/ –use key words " 'word problems' AND 'absolute value equations' " and look for application problems at the end of the lesson on absolute value equations
J. Have the students use what they find to come up with a list of jobs that may use absolute value equations.
K. The students should present their findings to the class.

Materials:
A. Computers with access to the Internet
B. Paper
C. Scissors
D. Rulers
E. Handout

Assessment
A. Absolute value handout on solving absolute value equations

Solve the following absolute value equations using the solution finders as a guide.

1. $|x + 1| = 5$
2. $|-(x + 1)| = 5$
3. $-|x + 1| = 5$
4. $|x + 2| = 7$
5. $|x + 6| = 14$
6. $|x + 10| = 2$
7. $|x – 7| = 11$
8. $|2x - 3| - 4 = 3$
9. $3|4x + 8| = 48$
10. $|x^2 - 4x - 5| = 7$

Graph the following equations by making a table of values and then plotting the points on a pair of axes. For 11 –14 compare and contrast the graphs to both a linear equation and a quadratic equation. For 16 – 18 compare and contrast the graphs to each other.

11. $y = |x|$
12. $y = |x – 3|$
13. $y = |x + 5|$
14. $y = |4x – 6|$
15. $y = |x + 2|$
16. $y = |x| + 2$
17. $y = -|x + 2|$
18. $y = -|x| + 2$

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Appendix: Online Resources for Teachers

Beaverton Schools (Beaverton, OR). (1997, January 23). Vose math resource links. *Vose School Education Resources Page*. Retrieved February 9, 2001, from http://www.teleport.com/~vincer/math.html#math

This site contains a list of links to many math websites containing math lesson plans, news, history, puzzles, and other resources. These links provide a source of ideas and information for forming lessons.

Drexel University. (n. d.). *The Math Forum: Teacher's Place*. Retrieved November 2, 2000, from http://forum.swarthmore.edu/teachers/high

This site offers lesson plans, ideas and resources, Internet activities, and math software archives for high school math teachers.

ERIC Clearinghouse on Information & Technology, Syracuse University. (n. d.). *AskERIC: Education Information with a Personal Touch*. Retrieved February 9, 2001, from http://ericir.syr.edu

This web site is a compilation of over 3000 educational resources on a variety of issues. This site contains a search engine to search for ideas for my lesson plans. It also contains information on how to integrate mathematics with real-life applications.

Future Graph Inc. (2000). *Interactive Learning Network.* Retrieved December 4, 2000, from http://www.iln.net

This site offers many lessons, problems, videos, etc for middle school to college students in a wide variety of subjects so that students can interact with math and understand its real-life

applications. It covers topics such as tutoring and study tips, which could be helpful in dealing with low-achieving students.

Great Lakes Collaborative. (1998, July 14). *Explorer*. Retrieved February 9, 2001, from http://unite.ukans.edu/explorer-db/browse/static/Mathematics/index.html

This is a site containing math lesson plans and resources pertaining to real-life applications. They may help give me ideas for my lessons plans.

Learning Network Inc. (2001, January 22). *Funbrain.com*. Retrieved February 9, 2001, from http://www.funbrain.com

This web page includes links to math games for students of all ages. Students can have fun learning math through playing games, which helps increase motivation to learn, and I can incorporate these games into some of my lessons.

North Central Regional Educational Laboratory. (2000, December 18). *Pathways to School Improvement.* Retrieved February 9, 2001, from http://www.ncrel.org/sdrs

This site contains extensive information about education issues including information about math and at-risk students. I can use this resource for a variety of sources within my thesis including research information and lesson plan ideas.

Yahoo! Inc. (n. d.). *Yahooligans! Teacher's Guide.* Retrieved January 29, 2001, from http://www.yahooligans.com/tg

This web site is a search engine of teacher resources powered by Yahoo!