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A Coupled PDE Model for the Morphological Instability of a Multi-Component Thin Film During Surface Electromigration

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A COUPLED PDE MODEL FOR THE MORPHOLOGICAL INSTABILITY OF A MULTI-COMPONENT THIN FILM DURING SURFACE ELECTROMIGRATION

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A COUPLED PDE MODEL FOR THE MORPHOLOGICAL INSTABILITY OF A MULTI-COMPONENT THIN FILM DURING SURFACE ELECTROMIGRATION

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This thesis is dedicated to my family.

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CONTENTS

LIST OF FIGURES

LIST OF TABLES

A COUPLED PDE MODEL FOR THE MORPHOLOGICAL INSTABILITY OF A MULTI-COMPONENT THIN FILM DURING SURFACE ELECTROMIGRATION

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In this thesis a model involving two coupled nonlinear PDEs is developed to study instability of a two-component metal film due to horizontal electric field and in a high-temperature environment similar to operational conditions of integrated circuits. The proposed model assumes the anisotropies of the diffusional mobilities for two atomic species, and negligible stresses in the film. The purpose of the modeling is to describe and understand the time-evolution of the shape of the film surface. Toward this end, the linear stability analysis (LSA) of the initially planar film surface with respect to small shape perturbations is performed. Such characteristics of the instability as the perturbation growth rate ω and the cut-off wave number are studied as functions of key physical parameters.

CHAPTER 1

INTRODUCTION

Directed motion of adsorbed atoms[1] on crystal film surface in response to applied electric current is called surface electromigration [**1**]. The electromigration phenomena was first discovered in 1861, then in 1960s was recognized as one of the reasons for failure of the integrated microelectronic circuits (ICs). Designing reliable advanced microcircuits was the main motivation for advancing further research in this area. Two main parameters in designing circuits that should be considered due to electromigration problem are the current density and temperature.

Basically there are two situations in regard to circuit malfunction. In the first situation known as open circuit the current flow in the conducting material would be reduced because of a void created by depletion of matter. In the second situation, electromigration causes the atoms of a conducting material to accumulate and create a hillock which leads to an electrical connection (a short circuit) between two metal lines. Both situations are shown in Figure 1.0.1.

1.1. Problem Statement

Influence of substrate wetting and surface electromigration on a thin film stability and morphology dynamics has been studied in [**5**] for single atomic species diffusing on a surface (a single-component metal film), which is formulated as a single nonlinear PDE. Following [**2**], we will investigate, through an elaborate mathematical model, the effects of electromigration on morphological stability and

[1]An adsorbed atom (adatom) is an atom that has been adsorbed on a crystal surface from a vapor phase or from an atomic beam and it has not yet been incorporated into a solid thus, it is mobile on a surface.

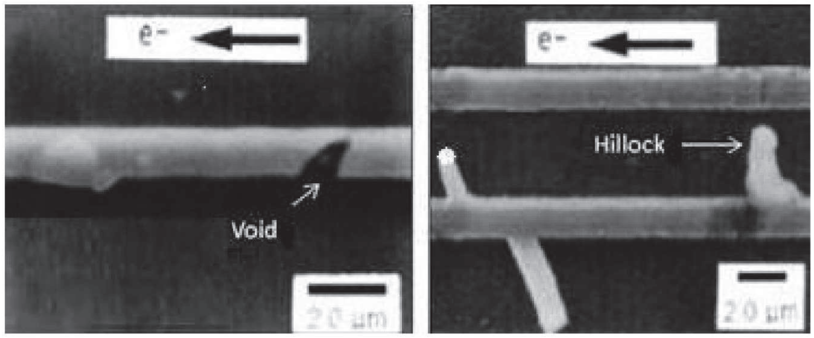


FIGURE 1.0.1. Damage to circuites caused by enhanced atomic flux on circuit surface: **Left:** Void developed by electromigration at the grain boundary. **Right:** Electromigration piles up atoms of a conductor and cause a hillock. (From: W.D. Nix, et al., 1992)

dynamics of a surface of a thin (thickness ~ few nanometers), multicomponent (specifically, two-component) metal or semiconductor film. A typical system is Al_xCo_{1-x} thin film, or a Si_xGe_{1-x} thin film interconnection. Here x stands for concentration of Al or Si atoms in the alloy (compound); the concentrations of both atomic species sum up to 1 at any given time, $x + 1 - x = 1$. Figure 1.1.1 shows agglomeration of $Si_{0.82}Ge_{0.18}$ semiconductor film into particles. This is the result of the instability of the film surface, in this case, caused by mechanical stress in the film.

As in [**5**], we assume anisotropies of the surface diffusional mobilities $M_i(\theta)$, $i = A$, or $i = B$, where A and B are two atomic species, and surface orientation independence (isotropic) and therefore constant surface energies γ_i. Here $\theta(s)$ is the surface orientation angle, that is, the angle that the unit normal to the film surface makes with the vertical coordinate axis z (Figure 1.1.2). This angle is a function of

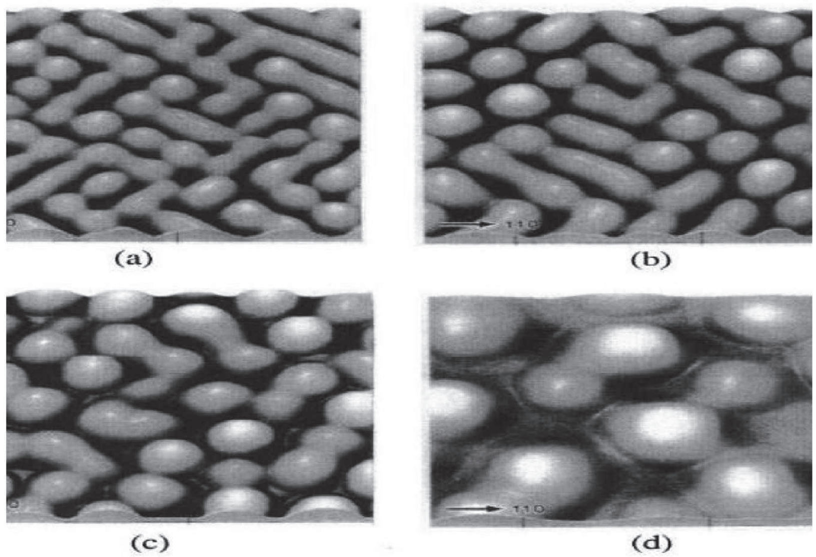


FIGURE 1.1.1. AFM images of 10nm-thick $Si_{0.82}Ge_{0.18}$ alloy films annealed at 850 C for (a) 1 min, (b) 5 min, (c) 20 min, and (d) 2 h (From: H. Gao and W.D. Nix, 1999)

the position on the surface (assumed an open plane curve), described by the arclength variable s. We will only consider the case of the *horizontal* electric field E (which rules out the surface turning onto itself), and thus at any fixed time the plane curve is the graph of a function $z = h(x)$. The goal of the modeling is to describe and understand the time-evolution of this graph. Thus we think of the curves on the film surface as a function of not only x, but also time t: $z = h(x,t)$. Next, $h(x,t)$ will be determined from an initial-boundary value problem for a certain parabolic PDE. In Figure 1.1.2, n is the normal vector, E_{loc} is the local electric field (the projection of E on the surface), and j is the atomic flux vector.

For illustration, Figure 1.1.2 (b) depicts a sitiation where the electric field is oriented in a manner that the surface will be planarized by downhill flow of atoms. However, in Figure 1.1.2 (a) the orientation of the electric field makes the surface

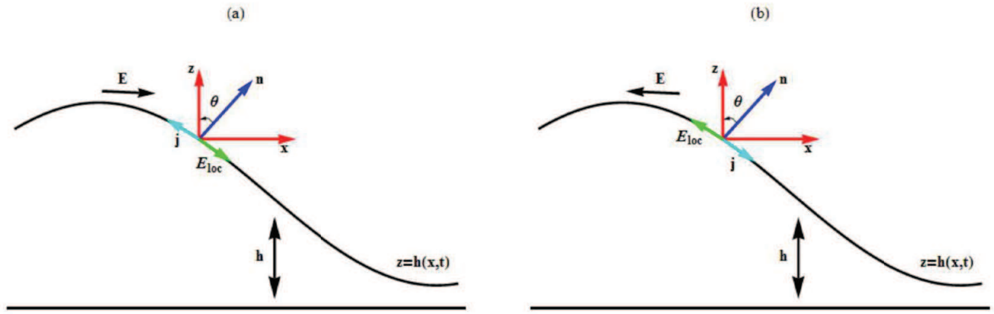


FIGURE 1.1.2. Sketch of the metal film curve $h(x,t)$ in the horizontal, constant electric field E. E_{loc} is the projection of E on the surface. The atom's flux j on the surface is in the direction opposite to the direction of E_{loc}. (a) $E_{loc} = E\cos(\theta)$, (b) $E_{loc} = -E\cos(\theta)$.

less planar, resulted from uphill flow of atoms. The origin of atoms' flow in this situation is the momentum transfered from energetic electrons (through scattering), the effect is termed electromigration.

To simplify the model, we ignore stresses of all kinds in the film, including compositional and heteroepitaxial stresses. We also assume the post-deposition scenario, when the surface's shape and the film thickness change by natural, high-temperature surface diffusion of surface atoms (which arises due to non-uniformity of the surface chemical potential μ along the surface), and by electromigration-assisted surface diffusion (See Fig. 1.1.2).

Following [**2**] the normal velocity of the atoms on the surface is given by:

$$V = -\Omega\left(\frac{\partial J_A}{\partial s} + \frac{\partial J_B}{\partial s}\right), \tag{1.1.1}$$

where $\Omega = const.$ is the atomic volume and J_A and J_B are the surface diffusion fluxes of components A and B. Thus the PDE for $h(x,t)$ is:

$$h_t = V/\cos\theta, \quad \cos\theta = \left(1 + h_x^2\right)^{-1/2}. \tag{1.1.2}$$

Accounting for the two contributions to the surface diffusion as listed above, we write:

$$J_i = -\frac{\nu D_i}{kT} C_i(s,t) M_i(\theta) \frac{\partial \mu_i}{\partial s} - \frac{\nu D_i}{kT} \alpha E_0 q C_i(s,t) M_i(\theta) \cos\theta, \tag{1.1.3}$$

where $C_i(s,t)$ is the component i of the fraction of species on the surface, D_i is the component i of the diffusivity, The number ν is the density of all species on the surface, kT is the Boltzmann's factor, E_0 the magnitude of the applied electric field, q the effective charge of adatoms, and $\alpha = \pm 1$ is used to select either stabilizing or destabilizing action of the field for the chosen combination of the vertical or horizontal orientation of the field and the mobilities $M_i(\theta)$. The surface composition $C_B(s,t)$ on $z = h(x,t)$ must be determined as part of the solution. It evolves according to:

$$\delta \frac{\partial C_B}{\partial t} + C_B V = -\Omega \frac{\partial J_B}{\partial s}, \tag{1.1.4}$$

where $0 \leq \delta \leq 1$ is the thickness of the surface layer and quantifies the "coverage". After the distribution $C_B(s)$ is determined from eq. (1.1.4) at any fixed time, the distribution of A-species, C_A, is simply determined from $C_A = 1 - C_B$.

Finally, the surface chemical potential (μ_i) and the atoms mobility ($M_i(\theta)$) are given by:

$$\mu_i = \Omega\gamma_i\kappa + \mu_i^0\left(C_i\right), \quad \kappa = -h_{xx}\left(1 + h_x^2\right)^{-3/2}, \tag{1.1.5}$$

where γ_i are the constant surface energies, as mentioned above,

$$M_i(\theta) = \frac{1 + \beta_i \cos^2\left[N_i(\theta + \phi_i)\right]}{1 + \beta_i \cos^2\left[N_i\phi_i\right]}, \quad \beta_i = const., \quad N_i = const., \quad \phi_i = const. \tag{1.1.6}$$

and $\mu_i^0\left(C_i\right)$ are the thermodynamic contributions to the chemical potentials, written using the regular solution model of the mixture as [**3, 4**]

$$\mu_i^0\left(C_i\right) = kTln\frac{C_i}{1 - C_i}. \tag{1.1.7}$$

We linearize $\mu_i^0\left(C_i\right)$ about the reference concentration $C_i = \frac{1}{2}$ [**2**] and obtain

$$\mu_i^0\left(C_i\right) \approx -2kT + 4kTC_i. \tag{1.1.8}$$

Note that eq. (1.1.6) is dimensionless. The physical origin of the anistropic mobility is that atomic diffusion is impeded along some crystal planes, but is facilitated along other planes. The parameters in eqs. (1.1.5)-(1.1.6) are described in [**5**].

Next, we will simplify this complicated system of equations by changing from variable s to variable x. Using $\partial/\partial s = (\cos\theta)\partial/\partial x = \left(1 + h_x^2\right)^{-1/2}\partial/\partial x$ and $\theta = arctan\left(h_x\right)$, we obtain:

$$h_t = -\Omega\left(\frac{\partial J_A}{\partial x} + \frac{\partial J_B}{\partial x}\right), \tag{1.1.9}$$

$$J_i = \frac{\nu D_i}{kT} M_i\left(h_x\right)\left(1+h_x^2\right)^{-1/2}\left[-C_i(x,t)\frac{\partial \mu_i}{\partial x} - \alpha E_0 q C_i(x,t)\right], \tag{1.1.10}$$

$$\delta\frac{\partial C_B}{\partial t} = -\left(1+h_x^2\right)^{-1/2}\left[C_B h_t + \Omega\frac{\partial J_B}{\partial x}\right], \tag{1.1.11}$$

where

$$\mu_i = -\Omega\gamma_i h_{xx}\left(1+h_x^2\right)^{-3/2} - 2kT + 4kTC_i, \tag{1.1.12}$$

and

$$M_i\left(h_x\right) = \frac{1+\beta_i\cos^2\left[N_i(arctan\left(h_x\right)+\phi_i)\right]}{1+\beta_i\cos^2\left[N_i\phi_i\right]}. \tag{1.1.13}$$

Next, we choose h_0 as the length scale and h_0^2/D_B as the time scale (where h_0 is the height of as-deposited planar film) and write the dimensionless counterparts of Eqs. (1.1.9) and (1.1.11), where we use same notations for dimensionless variables (still, $i = A, B$):

$$h_t = \frac{\partial}{\partial x}\{\left(1+h_x^2\right)^{-1/2}[M_A\left(h_x\right)(1-C_B(x,t))(S_A\frac{\partial \mu}{\partial x} - P_A\frac{\partial C_B}{\partial x} + T_A) \tag{1.1.14}$$

$$+M_B\left(h_x\right)C_B(x,t)(S_B\frac{\partial \mu}{\partial x} + P_B\frac{\partial C_B}{\partial x}) + T_B]\},$$

$$\frac{\partial C_B}{\partial t} = -\left(1+h_x^2\right)^{-1/2}[QC_B h_t - \tag{1.1.15}$$

$$\frac{\partial}{\partial x}\left\{M_B\left(h_x\right)\left(1+h_x^2\right)^{-1/2}C_B(x,t)\left(\left(S_B\frac{\partial \mu}{\partial x} + P_B Q\frac{\partial C_B}{\partial x}\right) + T_B Q\right)\right\}\Bigg],$$

where

$$\mu = -\gamma_i h_{xx}\left(1+h_x^2\right)^{-3/2}, \tag{1.1.16}$$

and $M_i(h_x)$ is given by eq. (1.1.13). The dimensionless parameters are:

$$S_i = \frac{\Omega^2 \nu \gamma_i D_i}{D_B k T h_0^2}, T_i = \frac{\Omega \nu D_i \alpha E_0 q}{D_B k T}, Q = \frac{h_0}{\delta}, G_i = \frac{\gamma_S}{\gamma_i}.$$

and

$$P_i = \frac{4\Omega \nu D_i}{D_B h_0}.$$

Eqs. (1.1.14) and (1.1.15) form a system of two coupled, highly nonlinear parabolic PDEs for the height (h) of the film above the substrate and the concentration of B-species (C_B). (Recall that the concentration of A-species is $C_A = 1 - C_B$.)

CHAPTER 2

LINEAR STABILITY ANALYSIS

2.1. Linearization of The Problem

Film surface is considered as a dynamical system (a system whose state changes with time). In order to study the stability of the film surface under small perturbation, by writing $M(h_x) = M(0) + M'(0)h_x$, we will linearize $M(h_x)$ about $h_x = 0$ (a planar surface). $M_i(0)$ and $M_i'(0)$ will be calculated from Eq. (1.1.13) for given β_i, N_i and ϕ_i (see [**5**]). Then:

$$\frac{\partial M_i(h_x)}{\partial x} = \frac{\partial M_i(h_x)}{\partial h_x} h_{xx} = M_i'(0)h_{xx}. \tag{2.1.1}$$

Then, we substitute μ (Eq. (1.1.16)), into eqs. (1.1.14) and (1.1.15) and after performing all differentiations, we take $h(x,t) = h_0 + \xi(x,t)$ and $C_B(x,t) = C_B^0 + \hat{C}_B(x,t)$, where $\xi(x,t)$ and $\hat{C}_B(x,t)$ are *small* perturbations of the base state h_0=const. (planar as-deposited surface) and C_B^0=const. (initial concentration of B atomic species on this surface). To linearize the resulting equations for ξ and $\hat{C}_B$ we omit all nonlinear contributions, such as $\xi^2, \xi\xi_x, \xi\hat{C}_B, \hat{C}_B\hat{C}_{B_x}$ etc.

Next, we have to choose the perturbation function. Thus, in the linear PDEs for ξ and $\hat{C}_B$ we assume:

$$\xi(x,t) = Ue^{\omega(k)t}e^{ikx}, and \tag{2.1.2}$$

$$\hat{C}_B(x,t) = Ve^{\omega(k)t}e^{ikx}$$

respectively, where U, V are constant unknown amplitudes, ω is the growth rate to be determined and k is the wavenumber. After substituting these forms we have the following equations emerging from eqs. (1.1.14) and (1.1.15):

$$V\left(k^2 M_A(0)P_A - k^2 C_B^0 M_A(0)P_A - k^2 C_B^0 M_B(0)P_B - ikM_A(0)T_A + ikM_B(0)T_B\right) \tag{2.1.3}$$

$$+U\left(-\omega - k^4 M_A(0)S_A + k^4 C_B^0 M_A(0)S_A - k^4 C_B^0 M_B(0)S_B - k^2 M_A'(0)T_A + k^2 C_B^0 M_A'(0)T_A\right.$$

$$\left. - k^2 C_B^0 M_B'(0)T_B\right) = 0$$

$$V\left(-\omega - k^2 Q C_B^0 M_B(0)P_B + ikQM_B(0)T_B\right) \tag{2.1.4}$$

$$+U\left(-Q\omega C_B^0 - k^4 C_B^0 M_B(0)S_B - k^2 Q C_B^0 M_B'(0)T_B\right) = 0$$

Equations (2.1.3) and (2.1.4) form the linear homogeneous algebraic system for unknown amplitudes U, V. The condition of a nontrivial solution is det(A)=0, where

$$A=\begin{pmatrix} -C\zeta k^2 + \zeta k^2 - C\bar{\zeta}k^2 - i\eta k + i\bar{\eta}k & C\psi k^4 - \psi k^4 - C\bar{\psi}k^4 + C\lambda k^2 - \lambda k^2 - C\bar{\lambda}k^2 - \omega \\ -QC\bar{\zeta}k^2 + iQ\bar{\eta}k - \omega & -C\bar{\psi}k^4 - QC\bar{\lambda}k^2 - CQ\omega \end{pmatrix}$$

and,

$C_B^0 = C$, $\qquad P_A M_A(0) = \zeta$, $\qquad S_A M_A(0) = \psi$,

$T_A M_A(0) = \eta$, $\qquad T_A M_A'(0) = \lambda$, $\qquad P_B M_B(0) = \bar{\zeta}$,

$S_B M_B(0) = \bar{\psi}$, $\qquad T_B M_B(0) = \bar{\eta}$, $\qquad T_B M_B'(0) = \bar{\lambda}$

The condition det(A)=0 is equivalent to the quadratic equation for the growth rate function $\omega(k)$:

$$
\begin{aligned}
0 =&\omega^2 + C_0^2k^6QS_AM_{\mathrm{A0}}P_BM_{\mathrm{B0}} + C_0k^6QS_AM_{\mathrm{A0}}P_BM_{\mathrm{B0}} - C_0^2k^6P_AM_{\mathrm{A0}}S_BM_{\mathrm{B0}} + \\
&C_0k^6P_AM_{\mathrm{A0}}S_BM_{\mathrm{B0}} + iC_0k^5QS_AM_{\mathrm{A0}}T_BM_{\mathrm{B0}} - iC_0k^5T_AM_{\mathrm{A0}}S_BM_{\mathrm{B0}} - \\
&C_0^2k^4QT_A\mathrm{MP}_{\mathrm{A0}}P_BM_{\mathrm{B0}} + C_0k^4QT_A\mathrm{MP}_{\mathrm{A0}}P_BM_{\mathrm{B0}} - C_0^2k^4QP_AM_{\mathrm{A0}}T_B\mathrm{MP}_{\mathrm{B0}} + \\
&C_0k^4QP_AM_{\mathrm{A0}}T_B\mathrm{MP}_{\mathrm{B0}} + iC_0k^3QT_A\mathrm{MP}_{\mathrm{A0}}T_BM_{\mathrm{B0}} - iC_0k^3QT_AM_{\mathrm{A0}}T_B\mathrm{MP}_{\mathrm{B0}} - \\
&ik^5QS_AM_{\mathrm{A0}}T_BM_{\mathrm{B0}} - ik^3QT_A\mathrm{MP}_{\mathrm{A0}}T_BM_{\mathrm{B0}} - C_0k^4\omega S_AM_{\mathrm{A0}} - C_0^2k^2Q\omega P_AM_{\mathrm{A0}} + \\
&C_0k^2Q\omega P_AM_{\mathrm{A0}} - C_0k^2\omega T_A\mathrm{MP}_{\mathrm{A0}} - iC_0kQ\omega T_AM_{\mathrm{A0}} + k^4\omega S_AM_{\mathrm{A0}} + k^2\omega T_A\mathrm{MP}_{\mathrm{A0}} + \\
&C_0^2k^6QP_BS_BM_{\mathrm{B0}}^2 - C_0^2k^6P_BS_BM_{\mathrm{B0}}^2 - iC_0k^5QS_BT_BM_{\mathrm{B0}}^2 + iC_0k^5S_BT_BM_{\mathrm{B0}}^2 + \\
&C_0k^4\omega S_BM_{\mathrm{B0}} - C_0^2k^2Q\omega P_BM_{\mathrm{B0}} + C_0k^2Q\omega P_BM_{\mathrm{B0}} + C_0k^2\omega T_B\mathrm{MP}_{\mathrm{B0}} + iC_0kQ\omega T_BM_{\mathrm{B0}} - \\
&ikQ\omega T_BM_{\mathrm{B0}}.
\end{aligned}
$$

The dispersion relation is the complex function, $\omega(k) = \omega_r(k) + i\ \omega_i(k)$. The analysis of $\omega_i(k)$ for the characteristic values of the parameters shows that $|\omega_i(k)| \ll 1$ for all k. Moreover, $\omega_i(k)$ has no effect on surface stability (from (2.1.2), $\xi, \hat{C_B} \sim e^{\omega(k)t}$ and $e^{\omega(k)t} = e^{\omega_r(k)t + i\omega_i(k)t} = e^{\omega_r(k)t}\left[\mathrm{Cos}\left(\omega_i(k)t\right) + i\mathrm{Sin}\left(\omega_i(k)t\right)\right]$). The latter form shows that only the value and sign of $\omega_r(k)$ matter for amplification or de-amplification of a perturbation. Thus in what follows, we will call $\omega_r(k)$ the dispersion relation. We show the dispersion relation in Appendix 3, as it is cumbersome. In the plots we will use ω for $\omega_r(k)$.

2.2. Analysis of The Dispersion Relation

After assigning numerical values to the parameters of the dispersion relation and solving the equation $\omega_r(k) = 0$ we can find the positive root, k_c, called the cut-off wavenumber. An example of the dispersion curve is Figure 2.2.1, where the cut-off wavenumber is 158.114 for the typical parameter values from Table 2.1.1.

Physical Parameters	Typical Values	Range	Physical Meaning
h_0	$1.0 \times 10^{-5} cm$	Fixed	Initial Height of the Film (same for $\forall x$)
Ω	$2.0 \times 10^{-23} cm^3$	Fixed	Volume of an Adatom
ν	$1.0 \times 10^{15} cm^{-2}$	Fixed	Surface Density of Adatoms
δ	$2.0 \times 10^{-8} cm^{-1}$	Fixed	Thickness of a Surface Layer
D_A	$1.5 \times 10^{-6} \frac{cm^2}{s}$	Fixed	Diffusivity of A species
D_B	$1.5 \times 10^{-6} \frac{cm^2}{s}$	Fixed	Diffusivity of B species
α	1.0	Fixed	Sets electric field orientation
q	$5.0 \times 10^{-11} C$	Fixed	Effective Charge of Adatoms
V	1 V	$5 \times 10^{-3} \leq V \leq 10$	Applied Voltage
L	$1.0 \times 10^{-4} cm$	Fixed	Lateral Extent of the Film
kT	$1.12 \times 10^{-13} erg$	Fixed	Boltzmann Factor
γ_A^f	$2.0 \times 10^3 \frac{erg}{cm^2}$	Fixed	Surface Energy of A Species
γ_B^f	$2.0 \times 10^3 \frac{erg}{cm^2}$	Fixed	Surface Energy of B Species
S_A	$\frac{1}{14000}$	Fixed	Energy parameter
S_B	$\frac{1}{14000}$	Fixed	Energy parameter
T_A	$\frac{25}{28}$	$\frac{1}{224} \leq T_A \leq \frac{125}{14}$	Electric field parameter
T_B	$\frac{25}{28}$	$\frac{1}{224} \leq T_B \leq \frac{125}{14}$	Electric field parameter
$M_A(0)$	1	Fixed	Diffusional Mobility of A Species on planar surface
$M_B(0)$	1	Fixed	Diffusional Mobility of B Species on planar surface
$M_A'(0)$	-3	$-10 \leq M_A'(0) \leq -1$	Derivative of Atomic Mobility for A Species on planar surface
$M_B'(0)$	-3	$-10 \leq M_B'(0) \leq -1$	Derivative of Atomic Mobility for B Species on planar surface
$C_A(0) = C_B(0)$	0.5	Fixed	Initial Fraction of Species on planar surface

TABLE 2.1.1. The physical parameter values used in our study.

The surface is stable with respect to perturbations of any wavenumber if $\omega_r(k) < 0 \ \forall k$, otherwise the surface is long-wave unstable, Figures 2.2.2 (b) and (a) shows both cases respectively. The degree of instability is measured by the width of

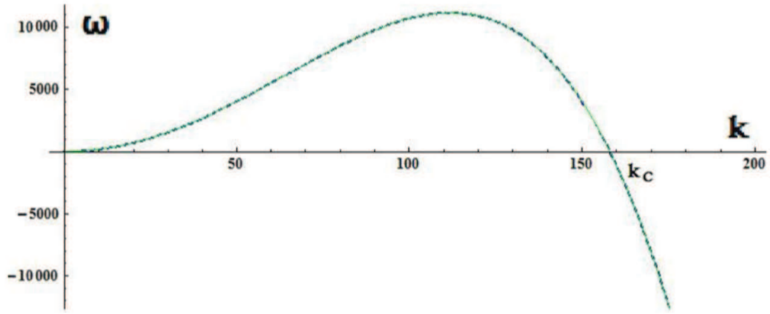


FIGURE 2.2.1. Sketch of the growth rate $\omega(k)$ corresponding to long-wave instability of the film surface

the domain under the dispersion curve $\omega_r(k)$. Therefore, the instability is greater whenever k_c is larger.

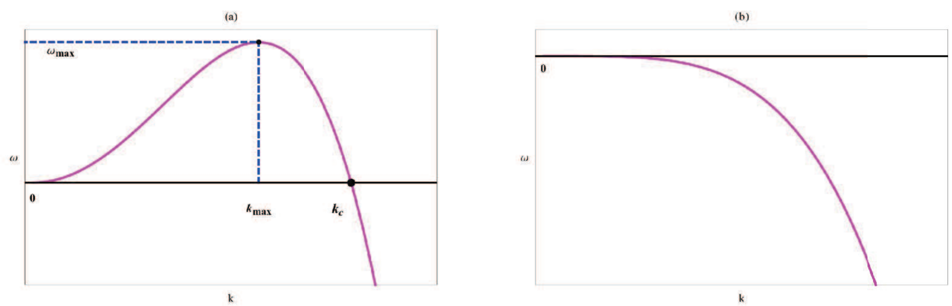
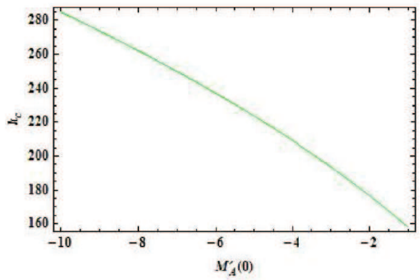


FIGURE 2.2.2. Two cases of the typical growth rate $\omega(k)$. (a) Long-wave instability; (b) $\omega(k) < 0$, $\forall k$: stability.

By solving the equation $d\omega_r/dk = 0$ for k numerically, we will get $k_{max} = 111.803$, which matches the expected value $k_{max} = \frac{k_c}{\sqrt{2}}$ for $k_c = 158.114$.

Figure 2.2.3 shows the variation of the cut-off wavenumber when $M_A'(0)$ and $M_B'(0)$ varies between -10 and -1. As we can see k_c decreases when the absolute value of the derivative of the atomic mobility for both species increases.



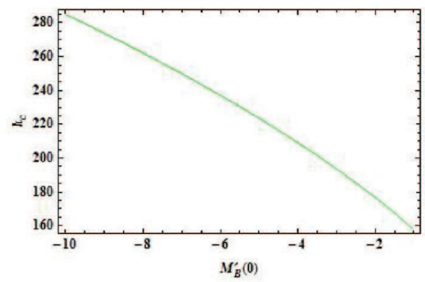


FIGURE 2.2.3. Variation of k_c (the instability cut-off wavenumber) **Left:** with change in $M'_A(0)$; **Right:** with change in $M'_B(0)$.

Since any change in applied voltage will affect parameters T_A and T_B, the cut-off wavenumber is a function of T_A and T_B. The contour plot of the function $k_c(T_A, T_B)$ is shown in Figure 2.2.4. To find the k_c function, we plugged the typical values of physical parameters in the dispersion relation where ω as a function of k is shown in Appendix 3. The contour plot in Figure 2.2.4 shows the overall pattern of variation of the cut-off wavenumber with respect to T_A and T_B. However, as we can see k_c is not displayed for some values since the function $k_c(T_A, T_B)$ seems to possess a non-zero imaginary part for those values.

The variation of k_c when either T_A or T_B is fixed is shown in Figure 2.2.5. It can be seen that with the increase of T_B, k_c increases, thus the surface becomes more unstable. With the increase of T_A up to 3, the cut-off wavenumber rapidly decreases, which makes the surface more stable.

In another attempt to derive values of k_c for values of T_A and T_B where there is a "white spot" in Figure 2.2.4, we substituted T_A=T_B=4, as well as other parameters, into the dispersion relation $\omega(k)$, and plotted this function (Figure 2.2.6). In this approach, it can be seen that k_c is well-defined real value. Thus additional study, perhaps using increased numerical precision, should be undertaken in the future to resolve the ambiguity in determination of missing k_c values.

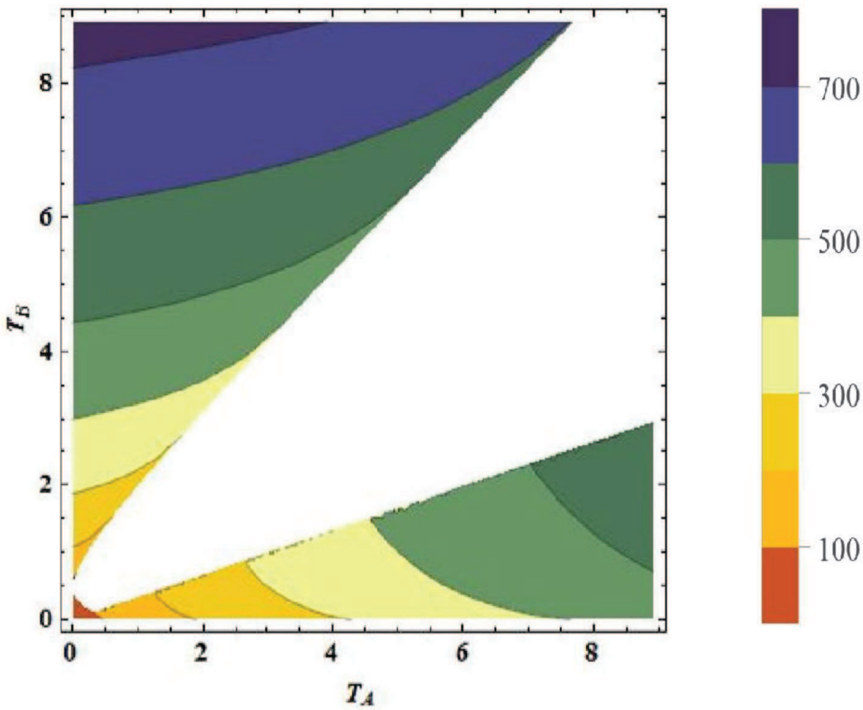


FIGURE 2.2.4. Contour plot of $k_c(T_A,T_B)$

.

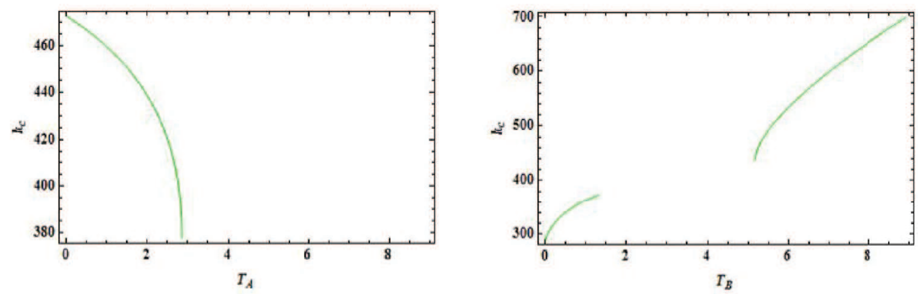


FIGURE 2.2.5. Variation of k_c (the instability cut-off wavenumber) **Left:** with change in T_A ($T_B = 4$); **Right:** with change in T_B ($T_A = 4$).

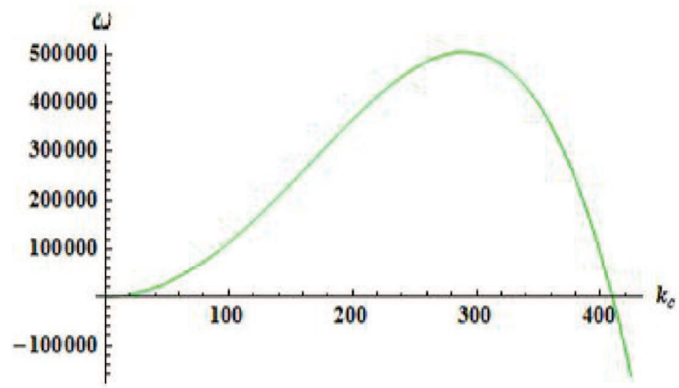


FIGURE 2.2.6. Longwave instability (k_c) for $T_A = 4$ and $T_B = 4$

CHAPTER 3

CONCLUSION AND FUTURE WORK

Chapter one started with physical definition and applied examples of electromigration, then we stated the problem of surface electromigration in the two-component metal films, where electric field is horizontal.

In chapter two, after linearizing two PDEs from first chapter, we used the perturbation method for linear stability analysis of the coupled PDE model. In the method we used a small perturbation of the initial film thickness and the initial concentration of B-type atomic species. Then we derived the dispersion relation by solving the growth rate function. The dispersion curve has a characteristic long-wave shape. Finally, we studied the variations of the cut-off wave number with physical parameters. We found that the surface stability decreases with the increase of the absolute value of the derivative of the atomic mobility for both species. Also, the surface stability decreases with the increase of applied voltage.

Investigating impacts of electromigration in the presence of both horizontal and vertical electric fields may be worth future work. Studies of the same problem by considering mechanical stresses in the film would be also interesting.

APPENDIX 1: BASIC *MATHEMATICA* CODE FOR LINEARIZATION OF THE FIRST PDE (EQ. 1.1.14)

$$\mu_A = \left(1 + (h'[x])^2\right)^{\frac{-1}{2}} \left(-h''[x] \left(1 + h'[x]^2\right)^{-1}\right)$$

$$\mu_{Ax} = D[\mu_A, x]$$

$$\mu_B = \left(1 + (h'[x])^2\right)^{\frac{-1}{2}} \left(-h''[x] \left(1 + h'[x]^2\right)^{-1}\right)$$

$$\mu_{Bx} = D[\mu_B, x]$$

$$M_A[h'[x]\_] := M_{A0} + MP_{A0} * h'[x]$$

$$M_B[h'[x]\_] := M_{B0} + MP_{B0} * h'[x]$$

$$\text{rhsEQ14} = D\Big[\Big(\left(1 + (h'[x])^2\right)^{\frac{-1}{2}} * (M_A[h'[x]\_] * (1 - C[x]) * (S_A * \mu_{Ax} - P_A * D[C[x], x]) + M_B[h'[x]\_] * C[x] * (S_B * \mu_{Bx} + P_B * D[C[x], x]) + T_A * M_A[h'[x]\_]\,(1 - C[x]) + T_B * M_B[h'[x]\_] * C[x])\Big), x\Big]$$

$$\text{rhsEQ14} = \text{Simplify}[\text{rhsEQ14}]$$

$$\text{rhsEQ14} = \text{rhsEQ14} \;/.\; \{h'[x] \to \xi'[x],\ h''[x] \to \xi''[x],\ h'''[x] \to \xi'''[x],\ h''''[x] \to \xi''''[x],\ C'[x] \to \hat{C}'[x],\ C''[x] \to \hat{C}''[x]\}$$

$$\text{rhsEQ14} = \text{Simplify}[\text{ExpandAll}[\text{rhsEQ14}]]$$

$$\text{d1} = \text{Series}\left[\frac{1}{\left(1 + \xi'[x]^2\right)^4}, \{\xi'[x], 0, 3\}\right] \text{ // Normal;}$$

$$\text{d2} = \text{Series}\left[\left(1 + \xi'[x]^2\right)^{\frac{1}{2}}, \{\xi'[x], 0, 3\}\right] \text{ // Normal;}$$

$$\text{rhsEQ14} = \text{Simplify}\left[\text{rhsEQ14} \;/.\; \left\{\frac{1}{\left(1 + \xi'[x]^2\right)^4} \to \text{d1},\ \left(1 + \xi'[x]^2\right)^{\frac{1}{2}} \to \text{d2}\right\}\right]$$

$$\text{rhsEQ14} = \text{Simplify}\Big[\text{rhsEQ14} \;/.\; \{\xi[x] \to \epsilon * \xi[x],\ \xi'[x] \to \epsilon * \xi'[x],\ \xi''[x] \to \epsilon * \xi''[x],\ \xi^{(3)}[x] \to \epsilon * \xi^{(3)}[x],\ \xi^{(4)}[x] \to \epsilon * \xi^{(4)}[x],\ \hat{C}[x] \to \epsilon * \hat{C}[x],\ \hat{C}'[x] \to \epsilon * \hat{C}'[x],\ \hat{C}''[x] \to \epsilon * \hat{C}''[x]\}\Big]$$

$$\text{rhsEQ14} = \text{rhsEQ14} \;/.\; \text{Table}\left[\epsilon^i \to 0, \{i, 2, 20\}\right]$$

$$\text{rhsEQ14} = \text{rhsEQ14} \;/.\; \epsilon \to 1$$

APPENDIX 2: BASIC *MATHEMATICA* CODE FOR LINEARIZATION OF THE FIRST PDE (EQ. 1.1.15)

```
μ_A = (1 + (h'[x])^2)^(-1/2) (-h''[x] (1 + h'[x]^2)^-1)

μ_Ax = D[μ_A, x]

μ_B = (1 + (h'[x])^2)^(-1/2) (-h''[x] (1 + h'[x]^2)^-1)

μ_Bx = D[μ_B, x]

M_A[h'[x] _] := M_A0 + MP_A0 * h'[x]

M_B[h'[x] _] := M_B0 + MP_B0 * h'[x]

W1 = D[M_B[h'[x] _] * (1 + (h'[x])^2)^(-1/2) * C[x] * ((S_B * μ_Bx + P_B * Q * D[C[x], x]) + T_B * Q), x]

rhsEQ15 = Simplify[- (1 + (h'[x])^2)^(-1/2) * (Q * C[x] * h_t - W1)]

rhsEQ15 = rhsEQ15 /. {h'[x] → ξ'[x], h''[x] → ξ''[x],
    h'''[x] → ξ'''[x], h''''[x] → ξ''''[x], C'[x] → Ĉ'[x], C''[x] → Ĉ''[x] }

rhsEQ15 = Simplify[ExpandAll[rhsEQ15]]

d1 = Series[1/(1 + ξ'[x]^2)^(9/2), {ξ'[x], 0, 3}] // Normal;

d2 = Series[(1 + ξ'[x]^2)^(1/2), {ξ'[x], 0, 3}] // Normal;

rhsEQ15 = Simplify[rhsEQ15 /. {1/(1 + ξ'[x]^2)^(9/2) → d1, (1 + ξ'[x]^2)^(1/2) → d2}]

rhsEQ15 = Simplify[
  rhsEQ15 /. {ξ[x] → ϵ * ξ[x], ξ'[x] → ϵ * ξ'[x], ξ''[x] → ϵ * ξ''[x], ξ^(3)[x] → ϵ * ξ^(3)[x],
    ξ^(4)[x] → ϵ * ξ^(4)[x], Ĉ[x] → ϵ * Ĉ[x], Ĉ'[x] → ϵ * Ĉ'[x], Ĉ''[x] → ϵ * Ĉ''[x]}]

rhsEQ15 = rhsEQ15 /. Table[ϵ^i → 0, {i, 2, 20}]

rhsEQ15 = ExpandAll[rhsEQ15]

rhsEQ15 = rhsEQ15 /. ϵ → 1
```

APPENDIX 3: BASIC *MATHEMATICA* CODE FOR FINDING GROWTH RATE FUNCTION USING PERTURBATION METHOD

$$\mathrm{rhsEQ14} = \mathrm{rhsEQ14}\ /.\ \left\{\hat{C}'[x] \to (I*k)*V*e^{\omega*t}\,e^{I*k*x},\ \hat{C}''[x] \to -(k)^2*V*e^{\omega*t}\,e^{I*k*x},\right.$$

$$\left.\xi''[x] \to -(k)^2*U*e^{\omega*t}\,e^{I*k*x},\ \xi''''[x] \to k^4*U*e^{\omega*t}\,e^{I*k*x}\right\}$$

$$\mathrm{rhsEQ14} = \mathrm{Collect}\left[\mathrm{rhsEQ14},\ e^{i\,k\,x+t\,\omega}\right]$$

$$\mathrm{lhsEQ14} = U*\omega*e^{\omega*t}*e^{I*k*x}$$

$$\mathrm{rhsEQ15} = \mathrm{rhsEQ15}\ /.$$

$$\left\{\xi_t \to \omega*U*e^{\omega*t}\,e^{I*k*x},\ \hat{C}'[x] \to (I*k)*V*e^{\omega*t}\,e^{I*k*x},\ \hat{C}''[x] \to -(k)^2*V*e^{\omega*t}\,e^{I*k*x}\right.,$$

$$\left.\xi''[x] \to -(k)^2*U*e^{\omega*t}\,e^{I*k*x},\ \xi''''[x] \to k^4*U*e^{\omega*t}\,e^{I*k*x}\right\}$$

$$\mathrm{rhsEQ15} = \mathrm{Collect}\left[\mathrm{rhsEQ15},\ e^{i\,k\,x+t\,\omega}\right]$$

$$\mathrm{lhsEQ15} = V*\omega*e^{\omega*t}*e^{I*k*x}$$

```
EQ1 = rhsEQ14 - lhsEQ14

EQ1 = Collect[EQ1, {U, V}]

EQ2 = rhsEQ15 - lhsEQ15

EQ2 = Collect[EQ2, {U, V}]
```

$$d = \mathrm{Det}\begin{bmatrix} k^2\,\varsigma - k^2\,C\,\varsigma - k^2\,C\,\overline{\varsigma} - i\,k\,\eta + i\,k\,\overline{\eta} & -\omega - k^4\,\psi + k^4\,C\,\psi - k^4\,C\,\overline{\psi} - k^2\,\lambda + k^2\,C\,\lambda - k^2\,C\,\overline{\lambda} \\ -\omega - k^2\,Q\,C\,\overline{\varsigma} + i\,k\,Q\,\overline{\eta} & -Q\,\omega\,C - k^4\,C\,\overline{\psi} - k^2\,Q\,C\,\overline{\lambda} \end{bmatrix}$$

$$C = C_B\ (0)$$

$$\varsigma = M_A\ (0)\ P_A$$

$$\psi = M_A\ (0)\ S_A$$

$$\eta = M_A\ (0)\ T_A$$

$$\overline{\varsigma} = M_B\ (0)\ P_B$$

$$\overline{\psi} = M_B\ (0)\ S_B$$

$$\overline{\eta} = M_B\ (0)\ T_B$$

$$\lambda = M_A'\ (0)\ T_A$$

$$\overline{\lambda} = M_B'\ (0)\ T_B$$

$$d = \mathrm{ExpandAll}[d\ /.\ \omega \to (\omega\mathrm{Re} + I*\omega\mathrm{Im})]$$

$$d = \mathrm{ComplexExpand}[d]$$

$$\begin{aligned} \mathrm{dRe} = \mathrm{Collect}\Big[&\omega\mathrm{Im}^2 - \omega\mathrm{Re}^2 - k^2\,Q\,\omega\mathrm{Re}\,C_0\,M_{A0}\,P_A + k^2\,Q\,\omega\mathrm{Re}\,C_0^2\,M_{A0}\,P_A - k^2\,Q\,\omega\mathrm{Re}\,C_0\,M_{B0}\,P_B + \\ & k^2\,Q\,\omega\mathrm{Re}\,C_0^2\,M_{B0}\,P_B - k^4\,\omega\mathrm{Re}\,M_{A0}\,S_A + k^4\,\omega\mathrm{Re}\,C_0\,M_{A0}\,S_A - k^6\,Q\,C_0\,M_{A0}\,M_{B0}\,P_B\,S_A + \\ & k^6\,Q\,C_0^2\,M_{A0}\,M_{B0}\,P_B\,S_A - k^4\,\omega\mathrm{Re}\,C_0\,M_{B0}\,S_B - k^6\,C_0\,M_{A0}\,M_{B0}\,P_A\,S_B + k^6\,C_0^2\,M_{A0}\,M_{B0}\,P_A\,S_B + \\ & k^6\,C_0^2\,M_{B0}^2\,P_B\,S_B - k^6\,Q\,C_0^2\,M_{B0}^2\,P_B\,S_B - k\,Q\,\omega\mathrm{Im}\,C_0\,M_{A0}\,T_A - k^2\,\omega\mathrm{Re}\,MP_{A0}\,T_A + k^2\,\omega\mathrm{Re}\,C_0\,MP_{A0}\,T_A - \\ & k^4\,Q\,C_0\,M_{B0}\,MP_{A0}\,P_B\,T_A + k^4\,Q\,C_0^2\,M_{B0}\,MP_{A0}\,P_B\,T_A - k\,Q\,\omega\mathrm{Im}\,M_{B0}\,T_B + k\,Q\,\omega\mathrm{Im}\,C_0\,M_{B0}\,T_B - \\ & k^2\,\omega\mathrm{Re}\,C_0\,MP_{B0}\,T_B - k^4\,Q\,C_0\,M_{A0}\,MP_{B0}\,P_A\,T_B + k^4\,Q\,C_0^2\,M_{A0}\,MP_{B0}\,P_A\,T_B,\ \omega\mathrm{Im}\Big] \end{aligned}$$

$$
\begin{aligned}
\mathtt{dIm} = \mathtt{Collect}\Big[&\big(-2\,\omega\mathrm{Im}\,\omega\mathrm{Re} - k^2\,Q\,\omega\mathrm{Im}\,C_0\,M_{A0}\,P_A + k^2\,Q\,\omega\mathrm{Im}\,C_0^2\,M_{A0}\,P_A - k^2\,Q\,\omega\mathrm{Im}\,C_0\,M_{B0}\,P_B + k^2\,Q\,\omega\mathrm{Im}\,C_0^2\,M_{B0}\,P_B - \\
&k^4\,\omega\mathrm{Im}\,M_{A0}\,S_A + k^4\,\omega\mathrm{Im}\,C_0\,M_{A0}\,S_A - k^4\,\omega\mathrm{Im}\,C_0\,M_{B0}\,S_B + k\,Q\,\omega\mathrm{Re}\,C_0\,M_{A0}\,T_A - k^2\,\omega\mathrm{Im}\,MP_{A0}\,T_A + \\
&k^2\,\omega\mathrm{Im}\,C_0\,MP_{A0}\,T_A + k^5\,C_0\,M_{A0}\,M_{B0}\,S_B\,T_A + k\,Q\,\omega\mathrm{Re}\,M_{B0}\,T_B - k\,Q\,\omega\mathrm{Re}\,C_0\,M_{B0}\,T_B - \\
&k^2\,\omega\mathrm{Im}\,C_0\,MP_{B0}\,T_B + k^5\,Q\,M_{A0}\,M_{B0}\,S_A\,T_B - k^5\,Q\,C_0\,M_{A0}\,M_{B0}\,S_A\,T_B - k^5\,C_0\,M_{B0}^2\,S_B\,T_B + \\
&k^5\,Q\,C_0\,M_{B0}^2\,S_B\,T_B + k^3\,Q\,M_{B0}\,MP_{A0}\,T_A\,T_B - k^3\,Q\,C_0\,M_{B0}\,MP_{A0}\,T_A\,T_B + k^3\,Q\,C_0\,M_{A0}\,MP_{B0}\,T_A\,T_B\big),\ \omega\mathrm{Im}\Big]
\end{aligned}
$$

```
solIm = Simplify[Solve[dIm == 0, ωIm]]
```

$$
\begin{aligned}
\omega\mathrm{Imm} = \\
-\Big(&k\,\big(-Q\,M_{B0}\,(\omega\mathrm{Re} + k^4\,M_{A0}\,S_A + k^2\,MP_{A0}\,T_A)\,T_B + C_0\,\big(M_{B0}\,\big(-k^4\,(-1+Q)\,M_{B0}\,S_B + Q\,(\omega\mathrm{Re} + k^2\,MP_{A0} \\
&T_A)\big)\,T_B - M_{A0}\,\big(-k^4\,Q\,M_{B0}\,S_A\,T_B + T_A\,(Q\,\omega\mathrm{Re} + k^4\,M_{B0}\,S_B + k^2\,Q\,MP_{B0}\,T_B)\big)\big)\big)\Big)\,\Big/ \\
&\big(2\,\omega\mathrm{Re} - k^2\,Q\,C_0^2\,(M_{A0}\,P_A + M_{B0}\,P_B) + k^4\,M_{A0}\,S_A + k^2\,MP_{A0}\,T_A + k^2\,C_0 \\
&\quad(M_{A0}\,(Q\,P_A - k^2\,S_A) + M_{B0}\,(Q\,P_B + k^2\,S_B) - MP_{A0}\,T_A + MP_{B0}\,T_B)\big)
\end{aligned}
$$

```
dRe = Simplify[dRe /. ωIm -> ωImm]

dRe = ExpandAll[dRe]

dRe = Simplify[dRe]

dRe = Collect[dRe, ωRe^4]

solRe = Solve[dRe == 0, ωRe]
```

$$
\begin{aligned}
\omega = \frac{1}{16}\Big(&-8\,k^2\,Q\,C_0\,M_{A0}\,P_A + 8\,k^2\,Q\,C_0^2\,M_{A0}\,P_A - 8\,k^2\,Q\,C_0\,M_{B0}\,P_B + 8\,k^2\,Q\,C_0^2\,M_{B0}\,P_B - 8\,k^4\,M_{A0}\,S_A + \\
&8\,k^4\,C_0\,M_{A0}\,S_A - 8\,k^4\,C_0\,M_{B0}\,S_B - 8\,k^2\,MP_{A0}\,T_A + 8\,k^2\,C_0\,MP_{A0}\,T_A - 8\,k^2\,C_0\,MP_{B0}\,T_B + \\
&\sqrt{}\Big(\big(\big(-8\,k^2\,Q\,C_0\,M_{A0}\,P_A + 8\,k^2\,Q\,C_0^2\,M_{A0}\,P_A - 8\,k^2\,Q\,C_0\,M_{B0}\,P_B + 8\,k^2\,Q\,C_0^2\,M_{B0}\,P_B - 8\,k^4\,M_{A0}\,S_A + \\
&\quad 8\,k^4\,C_0\,M_{A0}\,S_A - 8\,k^4\,C_0\,M_{B0}\,S_B - 8\,k^2\,MP_{A0}\,T_A + 8\,k^2\,C_0\,MP_{A0}\,T_A - 8\,k^2\,C_0\,MP_{B0}\,T_B\big)^2 + \\
&32\,\big(-k^4\,Q^2\,C_0^2\,M_{A0}^2\,P_A^2 + 2\,k^4\,Q^2\,C_0^3\,M_{A0}^2\,P_A^2 - k^4\,Q^2\,C_0^4\,M_{A0}^2\,P_A^2 - 2\,k^4\,Q^2\,C_0^2\,M_{A0}\,M_{B0}\,P_A\,P_B + \\
&\quad 4\,k^4\,Q^2\,C_0^3\,M_{A0}\,M_{B0}\,P_A\,P_B - 2\,k^4\,Q^2\,C_0^4\,M_{A0}\,M_{B0}\,P_A\,P_B - k^4\,Q^2\,C_0^2\,M_{B0}^2\,P_B^2 + \\
&\quad 2\,k^4\,Q^2\,C_0^3\,M_{B0}^2\,P_B^2 - k^4\,Q^2\,C_0^4\,M_{B0}^2\,P_B^2 - 2\,k^6\,Q\,C_0\,M_{A0}^2\,P_A\,S_A + 4\,k^6\,Q\,C_0^2\,M_{A0}^2\,P_A\,S_A - \\
&\quad 2\,k^6\,Q\,C_0^3\,M_{A0}^2\,P_A\,S_A - 6\,k^6\,Q\,C_0\,M_{A0}\,M_{B0}\,P_B\,S_A + 8\,k^6\,Q\,C_0^2\,M_{A0}\,M_{B0}\,P_B\,S_A - \\
&\quad 2\,k^6\,Q\,C_0^3\,M_{A0}\,M_{B0}\,P_B\,S_A - k^8\,M_{A0}^2\,S_A^2 + 2\,k^8\,C_0\,M_{A0}^2\,S_A^2 - k^8\,C_0^2\,M_{A0}^2\,S_A^2 - \\
&\quad 4\,k^6\,C_0\,M_{A0}\,M_{B0}\,P_A\,S_B + 4\,k^6\,C_0^2\,M_{A0}\,M_{B0}\,P_A\,S_B - 2\,k^6\,Q\,C_0^2\,M_{A0}\,M_{B0}\,P_A\,S_B + \\
&\quad 2\,k^6\,Q\,C_0^3\,M_{A0}\,M_{B0}\,P_A\,S_B + 4\,k^6\,C_0^2\,M_{B0}^2\,P_B\,S_B - 6\,k^6\,Q\,C_0^2\,M_{B0}^2\,P_B\,S_B + 2\,k^6\,Q\,C_0^3\,M_{B0}^2\,P_B\,S_B - \\
&\quad 2\,k^8\,C_0\,M_{A0}\,M_{B0}\,S_A\,S_B + 2\,k^8\,C_0^2\,M_{A0}\,M_{B0}\,S_A\,S_B - k^8\,C_0^2\,M_{B0}^2\,S_B^2 - 2\,k^4\,Q\,C_0\,M_{A0}\,MP_{A0}\,P_A\,T_A + \\
&\quad 4\,k^4\,Q\,C_0^2\,M_{A0}\,MP_{A0}\,P_A\,T_A - 2\,k^4\,Q\,C_0^3\,M_{A0}\,MP_{A0}\,P_A\,T_A - 6\,k^4\,Q\,C_0\,M_{B0}\,MP_{A0}\,P_B\,T_A + \\
&\quad 8\,k^4\,Q\,C_0^2\,M_{B0}\,MP_{A0}\,P_B\,T_A - 2\,k^4\,Q\,C_0^3\,M_{B0}\,MP_{A0}\,P_B\,T_A - 2\,k^6\,M_{A0}\,MP_{A0}\,S_A\,T_A + \\
&\quad 4\,k^6\,C_0\,M_{A0}\,MP_{A0}\,S_A\,T_A - 2\,k^6\,C_0^2\,M_{A0}\,MP_{A0}\,S_A\,T_A - 2\,k^6\,C_0\,M_{B0}\,MP_{A0}\,S_B\,T_A + \\
&\quad 2\,k^6\,C_0^2\,M_{B0}\,MP_{A0}\,S_B\,T_A - k^2\,Q^2\,C_0^2\,M_{A0}^2\,T_A^2 - k^4\,MP_{A0}^2\,T_A^2 + 2\,k^4\,C_0\,MP_{A0}^2\,T_A^2 - k^4\,C_0^2\,MP_{A0}^2\,T_A^2 - \\
&\quad 4\,k^4\,Q\,C_0\,M_{A0}\,MP_{B0}\,P_A\,T_B + 2\,k^4\,Q\,C_0^2\,M_{A0}\,MP_{B0}\,P_A\,T_B + 2\,k^4\,Q\,C_0^3\,M_{A0}\,MP_{B0}\,P_A\,T_B - \\
&\quad 2\,k^4\,Q\,C_0^2\,M_{B0}\,MP_{B0}\,P_B\,T_B + 2\,k^4\,Q\,C_0^3\,M_{B0}\,MP_{B0}\,P_B\,T_B - 2\,k^6\,C_0\,M_{A0}\,MP_{B0}\,S_A\,T_B + \\
&\quad 2\,k^6\,C_0^2\,M_{A0}\,MP_{B0}\,S_A\,T_B - 2\,k^6\,C_0^2\,M_{B0}\,MP_{B0}\,S_B\,T_B - 2\,k^2\,Q^2\,C_0\,M_{A0}\,M_{B0}\,T_A\,T_B + \\
&\quad 2\,k^2\,Q^2\,C_0^2\,M_{A0}\,M_{B0}\,T_A\,T_B - 2\,k^4\,C_0\,MP_{A0}\,MP_{B0}\,T_A\,T_B + 2\,k^4\,C_0^2\,MP_{A0}\,MP_{B0}\,T_A\,T_B - \\
&\quad k^2\,Q^2\,M_{B0}^2\,T_B^2 + 2\,k^2\,Q^2\,C_0\,M_{B0}^2\,T_B^2 - k^2\,Q^2\,C_0^2\,M_{B0}^2\,T_B^2 - k^4\,C_0^2\,MP_{B0}^2\,T_B^2 + \\
&\quad \sqrt{}\Big(\big(\big(k^4\,Q^2\,C_0^2\,M_{A0}^2\,P_A^2 - 2\,k^4\,Q^2\,C_0^3\,M_{A0}^2\,P_A^2 + k^4\,Q^2\,C_0^4\,M_{A0}^2\,P_A^2 + 2\,k^4\,Q^2\,C_0^2\,M_{A0}\,M_{B0}\,P_A\,P_B - \\
&\qquad 4\,k^4\,Q^2\,C_0^3\,M_{A0}\,M_{B0}\,P_A\,P_B + 2\,k^4\,Q^2\,C_0^4\,M_{A0}\,M_{B0}\,P_A\,P_B + k^4\,Q^2\,C_0^2\,M_{B0}^2\,P_B^2 - \\
&\qquad 2\,k^4\,Q^2\,C_0^3\,M_{B0}^2\,P_B^2 + k^4\,Q^2\,C_0^4\,M_{B0}^2\,P_B^2 + 2\,k^6\,Q\,C_0\,M_{A0}^2\,P_A\,S_A - 4\,k^6\,Q\,C_0^2\,M_{A0}^2\,P_A\,S_A + \\
&\qquad 2\,k^6\,Q\,C_0^3\,M_{A0}^2\,P_A\,S_A + 6\,k^6\,Q\,C_0\,M_{A0}\,M_{B0}\,P_B\,S_A - 8\,k^6\,Q\,C_0^2\,M_{A0}\,M_{B0}\,P_B\,S_A +
\end{aligned}
$$

$$2 k^6 Q C_0^3 M_{A0} M_{B0} P_B S_A + k^8 M_{A0}^2 S_A^2 - 2 k^8 C_0 M_{A0}^2 S_A^2 + k^8 C_0^2 M_{A0}^2 S_A^2 +$$
$$4 k^6 C_0 M_{A0} M_{B0} P_A S_B - 4 k^6 C_0^2 M_{A0} M_{B0} P_A S_B + 2 k^6 Q C_0^2 M_{A0} M_{B0} P_A S_B -$$
$$2 k^6 Q C_0^3 M_{A0} M_{B0} P_A S_B - 4 k^6 C_0^2 M_{B0}^2 P_B S_B + 6 k^6 Q C_0^2 M_{B0}^2 P_B S_B -$$
$$2 k^6 Q C_0^3 M_{B0}^2 P_B S_B + 2 k^8 C_0 M_{A0} M_{B0} S_A S_B - 2 k^8 C_0^2 M_{A0} M_{B0} S_A S_B +$$
$$k^8 C_0^2 M_{B0}^2 S_B^2 + 2 k^4 Q C_0 M_{A0} MP_{A0} P_A T_A - 4 k^4 Q C_0^2 M_{A0} MP_{A0} P_A T_A +$$
$$2 k^4 Q C_0^3 M_{A0} MP_{A0} P_A T_A + 6 k^4 Q C_0 M_{B0} MP_{A0} P_B T_A - 8 k^4 Q C_0^2 M_{B0} MP_{A0} P_B T_A +$$
$$2 k^4 Q C_0^3 M_{B0} MP_{A0} P_B T_A + 2 k^6 M_{A0} MP_{A0} S_A T_A - 4 k^6 C_0 M_{A0} MP_{A0} S_A T_A +$$
$$2 k^6 C_0^2 M_{A0} MP_{A0} S_A T_A + 2 k^6 C_0 M_{B0} MP_{A0} S_B T_A - 2 k^6 C_0^2 M_{B0} MP_{A0} S_B T_A +$$
$$k^2 Q^2 C_0^2 M_{A0}^2 T_A^2 + k^4 MP_{A0}^2 T_A^2 - 2 k^4 C_0 MP_{A0}^2 T_A^2 + k^4 C_0^2 MP_{A0}^2 T_A^2 +$$
$$4 k^4 Q C_0 M_{A0} MP_{B0} P_A T_B - 2 k^4 Q C_0^2 M_{A0} MP_{B0} P_A T_B - 2 k^4 Q C_0^3 M_{A0} MP_{B0} P_A T_B +$$
$$2 k^4 Q C_0^2 M_{B0} MP_{B0} P_B T_B - 2 k^4 Q C_0^3 M_{B0} MP_{B0} P_B T_B + 2 k^6 C_0 M_{A0} MP_{B0} S_A T_B -$$
$$2 k^6 C_0^2 M_{A0} MP_{B0} S_A T_B + 2 k^6 C_0^2 M_{B0} MP_{B0} S_B T_B + 2 k^2 Q^2 C_0 M_{A0} M_{B0} T_A T_B -$$
$$2 k^2 Q^2 C_0^2 M_{A0} M_{B0} T_A T_B + 2 k^4 C_0 MP_{A0} MP_{B0} T_A T_B - 2 k^4 C_0^2 MP_{A0} MP_{B0} T_A T_B +$$
$$k^2 Q^2 M_{B0}^2 T_B^2 - 2 k^2 Q^2 C_0 M_{B0}^2 T_B^2 + k^2 Q^2 C_0^2 M_{B0}^2 T_B^2 + k^4 C_0^2 MP_{B0}^2 T_B^2 \big)^2 -$$
$$16 \big(k^{10} Q^3 C_0^3 M_{A0}^3 M_{B0} P_A^2 P_B S_A - 3 k^{10} Q^3 C_0^4 M_{A0}^3 M_{B0} P_A^2 P_B S_A + 3 k^{10} Q^3$$
$$C_0^5 M_{A0}^3 M_{B0} P_A^2 P_B S_A - k^{10} Q^3 C_0^6 M_{A0}^3 M_{B0} P_A^2 P_B S_A + 2 k^{10} Q^3 C_0^3 M_{A0}^2 M_{B0}^2 P_A P_B^2 S_A -$$
$$6 k^{10} Q^3 C_0^4 M_{A0}^2 M_{B0}^2 P_A P_B^2 S_A + 6 k^{10} Q^3 C_0^5 M_{A0}^2 M_{B0}^2 P_A P_B^2 S_A - 2 k^{10} Q^3 C_0^6 M_{A0}^2 M_{B0}^2$$
$$P_A P_B^2 S_A + k^{10} Q^3 C_0^3 M_{A0} M_{B0}^3 P_B^3 S_A - 3 k^{10} Q^3 C_0^4 M_{A0} M_{B0}^3 P_B^3 S_A + 3 k^{10} Q^3 C_0^5 M_{A0}$$
$$M_{B0}^3 P_B^3 S_A - k^{10} Q^3 C_0^6 M_{A0} M_{B0}^3 P_B^3 S_A + 2 k^{12} Q^2 C_0^2 M_{A0}^3 M_{B0} P_A P_B S_A^2 - 6 k^{12} Q^2 C_0^3$$
$$M_{A0}^3 M_{B0} P_A P_B S_A^2 + 6 k^{12} Q^2 C_0^4 M_{A0}^3 M_{B0} P_A P_B S_A^2 - 2 k^{12} Q^2 C_0^5 M_{A0}^3 M_{B0} P_A P_B S_A^2 +$$
$$2 k^{12} Q^2 C_0^2 M_{A0}^2 M_{B0}^2 P_B^2 S_A^2 - 6 k^{12} Q^2 C_0^3 M_{A0}^2 M_{B0}^2 P_B^2 S_A^2 + 6 k^{12} Q^2 C_0^4 M_{A0}^2 M_{B0}^2 P_B^2 S_A^2 -$$
$$2 k^{12} Q^2 C_0^5 M_{A0}^2 M_{B0}^2 P_B^2 S_A^2 + k^{14} Q C_0 M_{A0}^3 M_{B0} P_B S_A^3 - 3 k^{14} Q C_0^2 M_{A0}^3 M_{B0} P_B S_A^3 +$$
$$3 k^{14} Q C_0^3 M_{A0}^3 M_{B0} P_B S_A^3 - k^{14} Q C_0^4 M_{A0}^3 M_{B0} P_B S_A^3 + k^{10} Q^2 C_0^3 M_{A0}^3 M_{B0} P_A^3 S_B -$$
$$3 k^{10} Q^2 C_0^4 M_{A0}^3 M_{B0} P_A^3 S_B + 3 k^{10} Q^2 C_0^5 M_{A0}^3 M_{B0} P_A^3 S_B - k^{10} Q^2 C_0^6 M_{A0}^3 M_{B0} P_A^3 S_B +$$
$$2 k^{10} Q^2 C_0^3 M_{A0}^2 M_{B0}^2 P_A^2 P_B S_B - 7 k^{10} Q^2 C_0^4 M_{A0}^2 M_{B0}^2 P_A^2 P_B S_B + k^{10} Q^3 C_0^4 M_{A0}^2 M_{B0}^2 P_A^2$$
$$P_B S_B + 8 k^{10} Q^2 C_0^5 M_{A0}^2 M_{B0}^2 P_A^2 P_B S_B - 2 k^{10} Q^3 C_0^5 M_{A0}^2 M_{B0}^2 P_A^2 P_B S_B - 3 k^{10} Q^2$$
$$C_0^6 M_{A0}^2 M_{B0}^2 P_A^2 P_B S_B + k^{10} Q^3 C_0^6 M_{A0}^2 M_{B0}^2 P_A^2 P_B S_B + k^{10} Q^2 C_0^3 M_{A0} M_{B0}^3 P_A P_B^2 S_B -$$
$$5 k^{10} Q^2 C_0^4 M_{A0} M_{B0}^3 P_A P_B^2 S_B + 2 k^{10} Q^3 C_0^4 M_{A0} M_{B0}^3 P_A P_B^2 S_B + 7 k^{10} Q^2 C_0^5$$
$$M_{A0} M_{B0}^3 P_A P_B^2 S_B - 4 k^{10} Q^3 C_0^5 M_{A0} M_{B0}^3 P_A P_B^2 S_B - 3 k^{10} Q^2 C_0^6 M_{A0} M_{B0}^3 P_A P_B^2 S_B +$$
$$2 k^{10} Q^3 C_0^6 M_{A0} M_{B0}^3 P_A P_B^2 S_B - k^{10} Q^2 C_0^4 M_{B0}^4 P_B^3 S_B + k^{10} Q^3 C_0^4 M_{B0}^4 P_B^3 S_B +$$
$$2 k^{10} Q^2 C_0^5 M_{B0}^4 P_B^3 S_B - 2 k^{10} Q^3 C_0^5 M_{B0}^4 P_B^3 S_B - k^{10} Q^2 C_0^6 M_{B0}^4 P_B^3 S_B +$$
$$k^{10} Q^3 C_0^6 M_{B0}^4 P_B^3 S_B + 2 k^{12} Q C_0^2 M_{A0}^3 M_{B0} P_A^2 S_A S_B - 6 k^{12} Q C_0^3 M_{A0}^3 M_{B0} P_A^2 S_A S_B +$$
$$6 k^{12} Q C_0^4 M_{A0}^3 M_{B0} P_A^2 S_A S_B - 2 k^{12} Q C_0^5 M_{A0}^3 M_{B0} P_A^2 S_A S_B + 2 k^{12} Q C_0^2 M_{A0}^2 M_{B0}^2 P_A$$
$$P_B S_A S_B - 8 k^{12} Q C_0^3 M_{A0}^2 M_{B0}^2 P_A P_B S_A S_B + 4 k^{12} Q^2 C_0^3 M_{A0}^2 M_{B0}^2 P_A P_B S_A S_B +$$
$$10 k^{12} Q C_0^4 M_{A0}^2 M_{B0}^2 P_A P_B S_A S_B - 8 k^{12} Q^2 C_0^4 M_{A0}^2 M_{B0}^2 P_A P_B S_A S_B - 4 k^{12} Q C_0^5 M_{A0}^2$$
$$M_{B0}^2 P_A P_B S_A S_B + 4 k^{12} Q^2 C_0^5 M_{A0}^2 M_{B0}^2 P_A P_B S_A S_B - 2 k^{12} Q C_0^3 M_{A0} M_{B0}^3 P_B^2 S_A S_B +$$
$$4 k^{12} Q^2 C_0^3 M_{A0} M_{B0}^3 P_B^2 S_A S_B + 4 k^{12} Q C_0^4 M_{A0} M_{B0}^3 P_B^2 S_A S_B - 8 k^{12} Q^2 C_0^4$$
$$M_{A0} M_{B0}^3 P_B^2 S_A S_B - 2 k^{12} Q C_0^5 M_{A0} M_{B0}^3 P_B^2 S_A S_B + 4 k^{12} Q^2 C_0^5 M_{A0} M_{B0}^3 P_B^2 S_A S_B +$$
$$k^{14} C_0 M_{A0}^3 M_{B0} P_A S_A^2 S_B - 3 k^{14} C_0^2 M_{A0}^3 M_{B0} P_A S_A^2 S_B + 3 k^{14} C_0^3 M_{A0}^3 M_{B0} P_A S_A^2 S_B -$$
$$k^{14} C_0^4 M_{A0}^3 M_{B0} P_A S_A^2 S_B - k^{14} C_0^2 M_{A0}^2 M_{B0}^2 P_B S_A^2 S_B + 3 k^{14} Q C_0^2 M_{A0}^2 M_{B0}^2 P_B S_A^2 S_B +$$
$$2 k^{14} C_0^3 M_{A0}^2 M_{B0}^2 P_B S_A^2 S_B - 6 k^{14} Q C_0^3 M_{A0}^2 M_{B0}^2 P_B S_A^2 S_B - k^{14} C_0^4 M_{A0}^2 M_{B0}^2 P_B S_A^2 S_B +$$
$$3 k^{14} Q C_0^4 M_{A0}^2 M_{B0}^2 P_B S_A^2 S_B + 2 k^{12} Q C_0^3 M_{A0}^2 M_{B0}^2 P_A^2 S_B^2 - 4 k^{12} Q C_0^4 M_{A0}^2 M_{B0}^2 P_A^2 S_B^2 +$$
$$2 k^{12} Q C_0^5 M_{A0}^2 M_{B0}^2 P_A^2 S_B^2 + 2 k^{12} Q C_0^3 M_{A0} M_{B0}^3 P_A P_B S_B^2 - 6 k^{12} Q C_0^4 M_{A0} M_{B0}^3$$
$$P_A P_B S_B^2 + 2 k^{12} Q^2 C_0^4 M_{A0} M_{B0}^3 P_A P_B S_B^2 + 4 k^{12} Q C_0^5 M_{A0} M_{B0}^3 P_A P_B S_B^2 -$$
$$2 k^{12} Q^2 C_0^5 M_{A0} M_{B0}^3 P_A P_B S_B^2 - 2 k^{12} Q C_0^4 M_{B0}^4 P_B^2 S_B^2 + 2 k^{12} Q^2 C_0^4 M_{B0}^4 P_B^2 S_B^2 +$$
$$2 k^{12} Q C_0^5 M_{B0}^4 P_B^2 S_B^2 - 2 k^{12} Q^2 C_0^5 M_{B0}^4 P_B^2 S_B^2 + 2 k^{14} C_0^2 M_{A0}^2 M_{B0}^2 P_A S_A S_B^2 -$$
$$4 k^{14} C_0^3 M_{A0}^2 M_{B0}^2 P_A S_A S_B^2 + 2 k^{14} C_0^4 M_{A0}^2 M_{B0}^2 P_A S_A S_B^2 - 2 k^{14} C_0^3 M_{A0} M_{B0}^3 P_B S_A S_B^2 +$$
$$3 k^{14} Q C_0^3 M_{A0} M_{B0}^3 P_B S_A S_B^2 + 2 k^{14} C_0^4 M_{A0} M_{B0}^3 P_B S_A S_B^2 -$$

$$3 k^{14} Q C_0^4 M_{A0} M_{B0}^3 P_B S_A S_B^2 + k^{14} C_0^3 M_{A0} M_{B0}^3 P_A S_B^3 - k^{14} C_0^4 M_{A0} M_{B0}^3 P_A S_B^3 -$$
$$k^{14} C_0^4 M_{B0}^4 P_B S_B^3 + k^{14} Q C_0^4 M_{B0}^4 P_B S_B^3 + k^8 Q^3 C_0^3 M_{A0}^2 M_{B0} MP_{A0} P_A^2 P_B T_A -$$
$$3 k^8 Q^3 C_0^4 M_{A0}^2 M_{B0} MP_{A0} P_A^2 P_B T_A + 3 k^8 Q^3 C_0^5 M_{A0}^2 M_{B0} MP_{A0} P_A^2 P_B T_A -$$
$$k^8 Q^3 C_0^6 M_{A0}^2 M_{B0} MP_{A0} P_A^2 P_B T_A + 2 k^8 Q^3 C_0^3 M_{A0} M_{B0}^2 MP_{A0} P_A P_B^2 T_A -$$
$$6 k^8 Q^3 C_0^4 M_{A0} M_{B0}^2 MP_{A0} P_A P_B^2 T_A + 6 k^8 Q^3 C_0^5 M_{A0} M_{B0}^2 MP_{A0} P_A P_B^2 T_A -$$
$$2 k^8 Q^3 C_0^6 M_{A0} M_{B0}^2 MP_{A0} P_A P_B^2 T_A + k^8 Q^3 C_0^3 M_{B0}^3 MP_{A0} P_B^3 T_A -$$
$$3 k^8 Q^3 C_0^4 M_{B0}^3 MP_{A0} P_B^3 T_A + 3 k^8 Q^3 C_0^5 M_{B0}^3 MP_{A0} P_B^3 T_A - k^8 Q^3 C_0^6 M_{B0}^3 MP_{A0} P_B^3 T_A +$$
$$4 k^{10} Q^2 C_0^2 M_{A0}^2 M_{B0} MP_{A0} P_A P_B S_A T_A - 12 k^{10} Q^2 C_0^3 M_{A0}^2 M_{B0} MP_{A0} P_A P_B S_A T_A +$$
$$12 k^{10} Q^2 C_0^4 M_{A0}^2 M_{B0} MP_{A0} P_A P_B S_A T_A - 4 k^{10} Q^2 C_0^5 M_{A0}^2 M_{B0} MP_{A0} P_A P_B S_A T_A +$$
$$4 k^{10} Q^2 C_0^2 M_{A0} M_{B0}^2 MP_{A0} P_B^2 S_A T_A - 12 k^{10} Q^2 C_0^3 M_{A0} M_{B0}^2 MP_{A0} P_B^2 S_A T_A +$$
$$12 k^{10} Q^2 C_0^4 M_{A0} M_{B0}^2 MP_{A0} P_B^2 S_A T_A - 4 k^{10} Q^2 C_0^5 M_{A0} M_{B0}^2 MP_{A0} P_B^2 S_A T_A +$$
$$3 k^{12} Q C_0 M_{A0}^2 M_{B0} MP_{A0} P_B S_A^2 T_A - 9 k^{12} Q C_0^2 M_{A0}^2 M_{B0} MP_{A0} P_B S_A^2 T_A +$$
$$9 k^{12} Q C_0^3 M_{A0}^2 M_{B0} MP_{A0} P_B S_A^2 T_A - 3 k^{12} Q C_0^4 M_{A0}^2 M_{B0} MP_{A0} P_B S_A^2 T_A +$$
$$2 k^{10} Q C_0^2 M_{A0}^2 M_{B0} MP_{A0} P_A^2 S_B T_A - 6 k^{10} Q C_0^3 M_{A0}^2 M_{B0} MP_{A0} P_A^2 S_B T_A +$$
$$6 k^{10} Q C_0^4 M_{A0}^2 M_{B0} MP_{A0} P_A^2 S_B T_A - 2 k^{10} Q C_0^5 M_{A0}^2 M_{B0} MP_{A0} P_A^2 S_B T_A +$$
$$2 k^{10} Q C_0^2 M_{A0} M_{B0}^2 MP_{A0} P_A P_B S_B T_A - 8 k^{10} Q C_0^3 M_{A0} M_{B0}^2 MP_{A0} P_A P_B S_B T_A +$$
$$4 k^{10} Q^2 C_0^3 M_{A0} M_{B0}^2 MP_{A0} P_A P_B S_B T_A + 10 k^{10} Q C_0^4 M_{A0} M_{B0}^2 MP_{A0} P_A P_B S_B T_A -$$
$$8 k^{10} Q^2 C_0^4 M_{A0} M_{B0}^2 MP_{A0} P_A P_B S_B T_A - 4 k^{10} Q C_0^5 M_{A0} M_{B0}^2 MP_{A0} P_A P_B S_B T_A +$$
$$4 k^{10} Q^2 C_0^5 M_{A0} M_{B0}^2 MP_{A0} P_A P_B S_B T_A - 2 k^{10} Q C_0^3 M_{B0}^3 MP_{A0} P_B^2 S_B T_A +$$
$$4 k^{10} Q^2 C_0^3 M_{B0}^3 MP_{A0} P_B^2 S_B T_A + 4 k^{10} Q C_0^4 M_{B0}^3 MP_{A0} P_B^2 S_B T_A - 8 k^{10} Q^2 C_0^4 M_{B0}^3$$
$$MP_{A0} P_B^2 S_B T_A - 2 k^{10} Q C_0^5 M_{B0}^3 MP_{A0} P_B^2 S_B T_A + 4 k^{10} Q^2 C_0^5 M_{B0}^3 MP_{A0} P_B^2 S_B T_A +$$
$$2 k^{12} C_0 M_{A0}^2 M_{B0} MP_{A0} P_A S_A S_B T_A - 6 k^{12} C_0^2 M_{A0}^2 M_{B0} MP_{A0} P_A S_A S_B T_A +$$
$$6 k^{12} C_0^3 M_{A0}^2 M_{B0} MP_{A0} P_A S_A S_B T_A - 2 k^{12} C_0^4 M_{A0}^2 M_{B0} MP_{A0} P_A S_A S_B T_A -$$
$$2 k^{12} C_0^2 M_{A0} M_{B0}^2 MP_{A0} P_B S_A S_B T_A + 6 k^{12} Q C_0^2 M_{A0} M_{B0}^2 MP_{A0} P_B S_A S_B T_A +$$
$$4 k^{12} C_0^3 M_{A0} M_{B0}^2 MP_{A0} P_B S_A S_B T_A - 12 k^{12} Q C_0^3 M_{A0} M_{B0}^2 MP_{A0} P_B S_A S_B T_A -$$
$$2 k^{12} C_0^4 M_{A0} M_{B0}^2 MP_{A0} P_B S_A S_B T_A + 6 k^{12} Q C_0^4 M_{A0} M_{B0}^2 MP_{A0} P_B S_A S_B T_A +$$
$$2 k^{12} C_0^2 M_{A0} M_{B0}^2 MP_{A0} P_A S_B^2 T_A - 4 k^{12} C_0^3 M_{A0} M_{B0}^2 MP_{A0} P_A S_B^2 T_A +$$
$$2 k^{12} C_0^4 M_{A0} M_{B0}^2 MP_{A0} P_A S_B^2 T_A - 2 k^{12} C_0^3 M_{B0}^3 MP_{A0} P_B S_B^2 T_A + 3 k^{12} Q C_0^3 M_{B0}^3$$
$$MP_{A0} P_B S_B^2 T_A + 2 k^{12} C_0^4 M_{B0}^3 MP_{A0} P_B S_B^2 T_A - 3 k^{12} Q C_0^4 M_{B0}^3 MP_{A0} P_B S_B^2 T_A +$$
$$2 k^8 Q^2 C_0^2 M_{A0} M_{B0} MP_{A0}^2 P_A P_B T_A^2 - 6 k^8 Q^2 C_0^3 M_{A0} M_{B0} MP_{A0}^2 P_A P_B T_A^2 +$$
$$6 k^8 Q^2 C_0^4 M_{A0} M_{B0} MP_{A0}^2 P_A P_B T_A^2 - 2 k^8 Q^2 C_0^5 M_{A0} M_{B0} MP_{A0}^2 P_A P_B T_A^2 +$$
$$2 k^8 Q^2 C_0^2 M_{B0}^2 MP_{A0}^2 P_B^2 T_A^2 - 6 k^8 Q^2 C_0^3 M_{B0}^2 MP_{A0}^2 P_B^2 T_A^2 + 6 k^8 Q^2 C_0^4 M_{B0}^2 MP_{A0}^2$$
$$P_B^2 T_A^2 - 2 k^8 Q^2 C_0^5 M_{B0}^2 MP_{A0}^2 P_B^2 T_A^2 + 3 k^{10} Q C_0 M_{A0} M_{B0} MP_{A0}^2 P_B S_A T_A^2 -$$
$$9 k^{10} Q C_0^2 M_{A0} M_{B0} MP_{A0}^2 P_B S_A T_A^2 + 9 k^{10} Q C_0^3 M_{A0} M_{B0} MP_{A0}^2 P_B S_A T_A^2 -$$
$$3 k^{10} Q C_0^4 M_{A0} M_{B0} MP_{A0}^2 P_B S_A T_A^2 + k^8 Q^2 C_0^3 M_{A0}^3 M_{B0} P_A S_B T_A^2 - k^8 Q^2 C_0^4 M_{A0}^3 M_{B0}$$
$$P_A S_B T_A^2 + k^{10} C_0 M_{A0} M_{B0} MP_{A0}^2 P_A S_B T_A^2 - 3 k^{10} C_0^2 M_{A0} M_{B0} MP_{A0}^2 P_A S_B T_A^2 +$$
$$3 k^{10} C_0^3 M_{A0} M_{B0} MP_{A0}^2 P_A S_B T_A^2 - k^{10} C_0^4 M_{A0} M_{B0} MP_{A0}^2 P_A S_B T_A^2 +$$
$$k^8 Q^2 C_0^3 M_{A0}^2 M_{B0}^2 P_B S_B T_A^2 - k^8 Q^2 C_0^4 M_{A0}^2 M_{B0}^2 P_B S_B T_A^2 - k^{10} C_0^2 M_{B0}^2 MP_{A0}^2 P_B S_B T_A^2 +$$
$$3 k^{10} Q C_0^2 M_{B0}^2 MP_{A0}^2 P_B S_B T_A^2 + 2 k^{10} C_0^3 M_{B0}^2 MP_{A0}^2 P_B S_B T_A^2 - 6 k^{10} Q C_0^3 M_{B0}^2$$
$$MP_{A0}^2 P_B S_B T_A^2 - k^{10} C_0^4 M_{B0}^2 MP_{A0}^2 P_B S_B T_A^2 + 3 k^{10} Q C_0^4 M_{B0}^2 MP_{A0}^2 P_B S_B T_A^2 +$$
$$k^{10} Q C_0^2 M_{A0}^3 M_{B0} S_A S_B T_A^2 - k^{10} Q C_0^3 M_{A0}^3 M_{B0} S_A S_B T_A^2 - k^{10} C_0^2 M_{A0}^2 M_{B0}^2 S_B^2 T_A^2 +$$
$$k^{10} Q C_0^3 M_{A0}^2 M_{B0}^2 S_B^2 T_A^2 + k^8 Q C_0 M_{B0} MP_{A0}^3 P_B T_A^3 - 3 k^8 Q C_0^2 M_{B0} MP_{A0}^3 P_B T_A^3 +$$
$$3 k^8 Q C_0^3 M_{B0} MP_{A0}^3 P_B T_A^3 - k^8 Q C_0^4 M_{B0} MP_{A0}^3 P_B T_A^3 + k^8 Q C_0^2 M_{A0}^2 M_{B0} MP_{A0} S_B T_A^3 -$$
$$k^8 Q C_0^3 M_{A0}^2 M_{B0} MP_{A0} S_B T_A^3 + k^8 Q^3 C_0^3 M_{A0}^3 MP_{B0} P_A^3 T_B - 3 k^8 Q^3 C_0^4 M_{A0}^3 MP_{B0} P_A^3 T_B +$$
$$3 k^8 Q^3 C_0^5 M_{A0}^3 MP_{B0} P_A^3 T_B - k^8 Q^3 C_0^6 M_{A0}^3 MP_{B0} P_A^3 T_B + 2 k^8 Q^3 C_0^3 M_{A0}^2 M_{B0} MP_{B0} P_A^2$$
$$P_B T_B - 6 k^8 Q^3 C_0^4 M_{A0}^2 M_{B0} MP_{B0} P_A^2 P_B T_B + 6 k^8 Q^3 C_0^5 M_{A0}^2 M_{B0} MP_{B0} P_A^2 P_B T_B -$$
$$2 k^8 Q^3 C_0^6 M_{A0}^2 M_{B0} MP_{B0} P_A^2 P_B T_B + k^8 Q^3 C_0^3 M_{A0} M_{B0}^2 MP_{B0} P_A P_B^2 T_B -$$
$$3 k^8 Q^3 C_0^4 M_{A0} M_{B0}^2 MP_{B0} P_A P_B^2 T_B + 3 k^8 Q^3 C_0^5 M_{A0} M_{B0}^2 MP_{B0} P_A P_B^2 T_B -$$
$$k^8 Q^3 C_0^6 M_{A0} M_{B0}^2 MP_{B0} P_A P_B^2 T_B + 2 k^{10} Q^2 C_0^2 M_{A0}^3 MP_{B0} P_A^2 S_A T_B -$$

$6 k^{10} Q^2 C_0^3 M_{A0}^3 MP_{B0} P_A^2 S_A T_B + 6 k^{10} Q^2 C_0^4 M_{A0}^3 MP_{B0} P_A^2 S_A T_B -$

$2 k^{10} Q^2 C_0^5 M_{A0}^3 MP_{B0} P_A^2 S_A T_B + 2 k^{10} Q^2 C_0^2 M_{A0}^2 M_{B0} MP_{B0} P_A P_B S_A T_B -$

$4 k^{10} Q^2 C_0^3 M_{A0}^2 M_{B0} MP_{B0} P_A P_B S_A T_B + 2 k^{10} Q^2 C_0^4 M_{A0}^2 M_{B0} MP_{B0} P_A P_B S_A T_B +$

$2 k^{10} Q^2 C_0^3 M_{A0} M_{B0}^2 MP_{B0} P_B^2 S_A T_B - 4 k^{10} Q^2 C_0^4 M_{A0} M_{B0}^2 MP_{B0} P_B^2 S_A T_B +$

$2 k^{10} Q^2 C_0^5 M_{A0} M_{B0}^2 MP_{B0} P_B^2 S_A T_B + k^{12} Q C_0 M_{A0}^3 MP_{B0} P_A S_A^2 T_B -$

$3 k^{12} Q C_0^2 M_{A0}^3 MP_{B0} P_A S_A^2 T_B + 3 k^{12} Q C_0^3 M_{A0}^3 MP_{B0} P_A S_A^2 T_B -$

$k^{12} Q C_0^4 M_{A0}^3 MP_{B0} P_A S_A^2 T_B + 2 k^{12} Q C_0^2 M_{A0}^2 M_{B0} MP_{B0} P_B S_A^2 T_B -$

$4 k^{12} Q C_0^3 M_{A0}^2 M_{B0} MP_{B0} P_B S_A^2 T_B + 2 k^{12} Q C_0^4 M_{A0}^2 M_{B0} MP_{B0} P_B S_A^2 T_B +$

$2 k^{10} Q C_0^3 M_{A0}^2 M_{B0} MP_{B0} P_A^2 S_B T_B + 2 k^{10} Q^2 C_0^3 M_{A0}^2 M_{B0} MP_{B0} P_A^2 S_B T_B -$

$4 k^{10} Q C_0^4 M_{A0}^2 M_{B0} MP_{B0} P_A^2 S_B T_B - 4 k^{10} Q^2 C_0^4 M_{A0}^2 M_{B0} MP_{B0} P_A^2 S_B T_B +$

$2 k^{10} Q C_0^5 M_{A0}^2 M_{B0} MP_{B0} P_A^2 S_B T_B + 2 k^{10} Q^2 C_0^5 M_{A0}^2 M_{B0} MP_{B0} P_A^2 S_B T_B +$

$2 k^{10} Q C_0^3 M_{A0} M_{B0}^2 MP_{B0} P_A P_B S_B T_B + 2 k^{10} Q^2 C_0^3 M_{A0} M_{B0}^2 MP_{B0} P_A P_B S_B T_B -$

$6 k^{10} Q C_0^4 M_{A0} M_{B0}^2 MP_{B0} P_A P_B S_B T_B - 2 k^{10} Q^2 C_0^4 M_{A0} M_{B0}^2 MP_{B0} P_A P_B S_B T_B +$

$4 k^{10} Q C_0^5 M_{A0} M_{B0}^2 MP_{B0} P_A P_B S_B T_B - 2 k^{10} Q C_0^4 M_{B0}^3 MP_{B0} P_B^2 S_B T_B +$

$2 k^{10} Q^2 C_0^4 M_{B0}^3 MP_{B0} P_B^2 S_B T_B + 2 k^{10} Q C_0^5 M_{B0}^3 MP_{B0} P_B^2 S_B T_B -$

$2 k^{10} Q^2 C_0^5 M_{B0}^3 MP_{B0} P_B^2 S_B T_B + 2 k^{12} C_0^2 M_{A0}^2 M_{B0} MP_{B0} P_A S_A S_B T_B +$

$2 k^{12} Q C_0^2 M_{A0}^2 M_{B0} MP_{B0} P_A S_A S_B T_B - 4 k^{12} C_0^3 M_{A0}^2 M_{B0} MP_{B0} P_A S_A S_B T_B -$

$4 k^{12} Q C_0^3 M_{A0}^2 M_{B0} MP_{B0} P_A S_A S_B T_B + 2 k^{12} C_0^4 M_{A0}^2 M_{B0} MP_{B0} P_A S_A S_B T_B +$

$2 k^{12} Q C_0^4 M_{A0}^2 M_{B0} MP_{B0} P_A S_A S_B T_B - 2 k^{12} C_0^3 M_{A0} M_{B0}^2 MP_{B0} P_B S_A S_B T_B +$

$4 k^{12} Q C_0^3 M_{A0} M_{B0}^2 MP_{B0} P_B S_A S_B T_B + 2 k^{12} C_0^4 M_{A0} M_{B0}^2 MP_{B0} P_B S_A S_B T_B -$

$4 k^{12} Q C_0^4 M_{A0} M_{B0}^2 MP_{B0} P_B S_A S_B T_B + 2 k^{12} C_0^3 M_{A0} M_{B0}^2 MP_{B0} P_A S_B^2 T_B +$

$k^{12} Q C_0^3 M_{A0} M_{B0}^2 MP_{B0} P_A S_B^2 T_B - 2 k^{12} C_0^4 M_{A0} M_{B0}^2 MP_{B0} P_A S_B^2 T_B -$

$k^{12} Q C_0^4 M_{A0} M_{B0}^2 MP_{B0} P_A S_B^2 T_B - 2 k^{12} C_0^4 M_{B0}^3 MP_{B0} P_B S_B^2 T_B +$

$2 k^{12} Q C_0^4 M_{B0}^3 MP_{B0} P_B S_B^2 T_B + 2 k^8 Q^2 C_0^2 M_{A0}^2 MP_{A0} MP_{B0} P_A^2 T_A T_B -$

$6 k^8 Q^2 C_0^3 M_{A0}^2 MP_{A0} MP_{B0} P_A^2 T_A T_B + 6 k^8 Q^2 C_0^4 M_{A0}^2 MP_{A0} MP_{B0} P_A^2 T_A T_B -$

$2 k^8 Q^2 C_0^5 M_{A0}^2 MP_{A0} MP_{B0} P_A^2 T_A T_B + 2 k^8 Q^2 C_0^2 M_{A0} M_{B0} MP_{A0} MP_{B0} P_A P_B T_A T_B -$

$4 k^8 Q^2 C_0^3 M_{A0} M_{B0} MP_{A0} MP_{B0} P_A P_B T_A T_B + 2 k^8 Q^2 C_0^4 M_{A0} M_{B0} MP_{A0} MP_{B0} P_A P_B$

$\quad T_A T_B + 2 k^8 Q^2 C_0^3 M_{B0}^2 MP_{A0} MP_{B0} P_B^2 T_A T_B - 4 k^8 Q^2 C_0^4 M_{B0}^2 MP_{A0} MP_{B0} P_B^2 T_A T_B +$

$2 k^8 Q^2 C_0^5 M_{B0}^2 MP_{A0} MP_{B0} P_B^2 T_A T_B + k^8 Q^3 C_0^2 M_{A0}^3 M_{B0} P_A S_A T_A T_B -$

$2 k^8 Q^3 C_0^3 M_{A0}^3 M_{B0} P_A S_A T_A T_B + k^8 Q^3 C_0^4 M_{A0}^3 M_{B0} P_A S_A T_A T_B +$

$2 k^{10} Q C_0 M_{A0}^2 MP_{A0} MP_{B0} P_A S_A T_A T_B - 6 k^{10} Q C_0^2 M_{A0}^2 MP_{A0} MP_{B0} P_A S_A T_A T_B +$

$6 k^{10} Q C_0^3 M_{A0}^2 MP_{A0} MP_{B0} P_A S_A T_A T_B - 2 k^{10} Q C_0^4 M_{A0}^2 MP_{A0} MP_{B0} P_A S_A T_A T_B +$

$k^8 Q^3 C_0^2 M_{A0}^2 M_{B0}^2 P_B S_A T_A T_B - 2 k^8 Q^3 C_0^3 M_{A0}^2 M_{B0}^2 P_B S_A T_A T_B +$

$k^8 Q^3 C_0^4 M_{A0}^2 M_{B0}^2 P_B S_A T_A T_B + 4 k^{10} Q C_0^2 M_{A0} M_{B0} MP_{A0} MP_{B0} P_B S_A T_A T_B -$

$8 k^{10} Q C_0^3 M_{A0} M_{B0} MP_{A0} MP_{B0} P_B S_A T_A T_B + 4 k^{10} Q C_0^4 M_{A0} M_{B0} MP_{A0} MP_{B0}$

$\quad P_B S_A T_A T_B + k^{10} Q^2 C_0 M_{A0}^3 M_{B0} S_A^2 T_A T_B - 2 k^{10} Q^2 C_0^2 M_{A0}^3 M_{B0} S_A^2 T_A T_B +$

$k^{10} Q^2 C_0^3 M_{A0}^3 M_{B0} S_A^2 T_A T_B + k^8 Q^2 C_0^2 M_{A0}^2 M_{B0}^2 P_A S_B T_A T_B - 3 k^8 Q^2 C_0^3 M_{A0}^2$

$\quad M_{B0}^2 P_A S_B T_A T_B + k^8 Q^3 C_0^3 M_{A0}^2 M_{B0}^2 P_A S_B T_A T_B + 2 k^8 Q^2 C_0^4 M_{A0}^2 M_{B0}^2 P_A S_B T_A T_B -$

$k^8 Q^3 C_0^4 M_{A0}^2 M_{B0}^2 P_A S_B T_A T_B + 2 k^{10} C_0^2 M_{A0} M_{B0} MP_{A0} MP_{B0} P_A S_B T_A T_B +$

$2 k^{10} Q C_0^2 M_{A0} M_{B0} MP_{A0} MP_{B0} P_A S_B T_A T_B - 4 k^{10} C_0^3 M_{A0} M_{B0} MP_{A0} MP_{B0}$

$\quad P_A S_B T_A T_B - 4 k^{10} Q C_0^3 M_{A0} M_{B0} MP_{A0} MP_{B0} P_A S_B T_A T_B + 2 k^{10} C_0^4 M_{A0}$

$\quad M_{B0} MP_{A0} MP_{B0} P_A S_B T_A T_B + 2 k^{10} Q C_0^4 M_{A0} M_{B0} MP_{A0} MP_{B0} P_A S_B T_A T_B +$

$k^8 Q^2 C_0^2 M_{A0} M_{B0}^3 P_B S_B T_A T_B - 3 k^8 Q^2 C_0^3 M_{A0} M_{B0}^3 P_B S_B T_A T_B +$

$k^8 Q^3 C_0^3 M_{A0} M_{B0}^3 P_B S_B T_A T_B + 2 k^8 Q^2 C_0^4 M_{A0} M_{B0}^3 P_B S_B T_A T_B -$

$k^8 Q^3 C_0^4 M_{A0} M_{B0}^3 P_B S_B T_A T_B - 2 k^{10} C_0^3 M_{B0}^2 MP_{A0} MP_{B0} P_B S_B T_A T_B +$

$4 k^{10} Q C_0^3 M_{B0}^2 MP_{A0} MP_{B0} P_B S_B T_A T_B + 2 k^{10} C_0^4 M_{B0}^2 MP_{A0} MP_{B0} P_B S_B T_A T_B -$

$4 k^{10} Q C_0^4 M_{B0}^2 MP_{A0} MP_{B0} P_B S_B T_A T_B - k^{10} Q C_0 M_{A0}^2 M_{B0}^2 S_A S_B T_A T_B -$

$k^{10} Q C_0^2 M_{A0}^2 M_{B0}^2 S_A S_B T_A T_B + 2 k^{10} Q^2 C_0^2 M_{A0}^2 M_{B0}^2 S_A S_B T_A T_B +$

$2 k^{10} Q C_0^3 M_{A0}^2 M_{B0}^2 S_A S_B T_A T_B - 2 k^{10} Q^2 C_0^3 M_{A0}^2 M_{B0}^2 S_A S_B T_A T_B +$

$$2 k^{10} C_0^2 M_{A0} M_{B0}^3 S_B^2 T_A T_B - k^{10} Q C_0^2 M_{A0} M_{B0}^3 S_B^2 T_A T_B - 2 k^{10} Q C_0^3 M_{A0} M_{B0}^3$$
$$S_B^2 T_A T_B + k^{10} Q^2 C_0^3 M_{A0} M_{B0}^3 S_B^2 T_A T_B + k^6 Q^3 C_0^2 M_{A0}^2 M_{B0} MP_{A0} P_A T_A^2 T_B -$$
$$2 k^6 Q^3 C_0^3 M_{A0}^2 M_{B0} MP_{A0} P_A T_A^2 T_B + k^6 Q^3 C_0^4 M_{A0}^2 M_{B0} MP_{A0} P_A T_A^2 T_B +$$
$$k^6 Q^3 C_0^3 M_{A0}^3 MP_{B0} P_A T_A^2 T_B - k^6 Q^3 C_0^4 M_{A0}^3 MP_{B0} P_A T_A^2 T_B +$$
$$k^8 Q C_0 M_{A0} MP_{A0}^2 MP_{B0} P_A T_A^2 T_B - 3 k^8 Q C_0^2 M_{A0} MP_{A0}^2 MP_{B0} P_A T_A^2 T_B +$$
$$3 k^8 Q C_0^3 M_{A0} MP_{A0}^2 MP_{B0} P_A T_A^2 T_B - k^8 Q C_0^4 M_{A0} MP_{A0}^2 MP_{B0} P_A T_A^2 T_B +$$
$$k^6 Q^3 C_0^2 M_{A0} M_{B0}^2 MP_{A0} P_B T_A^2 T_B - 2 k^6 Q^3 C_0^3 M_{A0} M_{B0}^2 MP_{A0} P_B T_A^2 T_B +$$
$$k^6 Q^3 C_0^4 M_{A0} M_{B0}^2 MP_{A0} P_B T_A^2 T_B + k^6 Q^3 C_0^3 M_{A0}^2 M_{B0} MP_{B0} P_B T_A^2 T_B -$$
$$k^6 Q^3 C_0^4 M_{A0}^2 M_{B0} MP_{B0} P_B T_A^2 T_B + 2 k^8 Q C_0^2 M_{B0} MP_{A0}^2 MP_{B0} P_B T_A^2 T_B -$$
$$4 k^8 Q C_0^3 M_{B0} MP_{A0}^2 MP_{B0} P_B T_A^2 T_B + 2 k^8 Q C_0^4 M_{B0} MP_{A0}^2 MP_{B0} P_B T_A^2 T_B +$$
$$2 k^8 Q^2 C_0 M_{A0}^2 M_{B0} MP_{A0} S_A T_A^2 T_B - 4 k^8 Q^2 C_0^2 M_{A0}^2 M_{B0} MP_{A0} S_A T_A^2 T_B +$$
$$2 k^8 Q^2 C_0^3 M_{A0}^2 M_{B0} MP_{A0} S_A T_A^2 T_B + k^8 Q^2 C_0^2 M_{A0}^3 MP_{B0} S_A T_A^2 T_B -$$
$$k^8 Q^2 C_0^3 M_{A0}^3 MP_{B0} S_A T_A^2 T_B - k^8 Q C_0 M_{A0} M_{B0}^2 MP_{A0} S_B T_A^2 T_B -$$
$$k^8 Q C_0^2 M_{A0} M_{B0}^2 MP_{A0} S_B T_A^2 T_B + 2 k^8 Q^2 C_0^2 M_{A0} M_{B0}^2 MP_{A0} S_B T_A^2 T_B +$$
$$2 k^8 Q C_0^3 M_{A0} M_{B0}^2 MP_{A0} S_B T_A^2 T_B - 2 k^8 Q^2 C_0^3 M_{A0} M_{B0}^2 MP_{A0} S_B T_A^2 T_B -$$
$$2 k^8 Q C_0^2 M_{A0}^2 M_{B0} MP_{B0} S_B T_A^2 T_B + k^8 Q C_0^3 M_{A0}^2 M_{B0} MP_{B0} S_B T_A^2 T_B +$$
$$k^8 Q^2 C_0^3 M_{A0}^2 M_{B0} MP_{B0} S_B T_A^2 T_B + k^6 Q^2 C_0 M_{A0} M_{B0} MP_{A0}^2 T_A^3 T_B -$$
$$2 k^6 Q^2 C_0^2 M_{A0} M_{B0} MP_{A0}^2 T_A^3 T_B + k^6 Q^2 C_0^3 M_{A0} M_{B0} MP_{A0}^2 T_A^3 T_B +$$
$$k^6 Q^2 C_0^2 M_{A0}^2 MP_{A0} MP_{B0} T_A^3 T_B - k^6 Q^2 C_0^3 M_{A0}^2 MP_{A0} MP_{B0} T_A^3 T_B + 2 k^8 Q^2 C_0^3$$
$$M_{A0}^2 MP_{B0}^2 P_A^2 T_B^2 - 4 k^8 Q^2 C_0^4 M_{A0}^2 MP_{B0}^2 P_A^2 T_B^2 + 2 k^8 Q^2 C_0^5 M_{A0}^2 MP_{B0}^2 P_A^2 T_B^2 +$$
$$2 k^8 Q^2 C_0^3 M_{A0} M_{B0} MP_{B0}^2 P_A P_B T_B^2 - 4 k^8 Q^2 C_0^4 M_{A0} M_{B0} MP_{B0}^2 P_A P_B T_B^2 +$$
$$2 k^8 Q^2 C_0^5 M_{A0} M_{B0} MP_{B0}^2 P_A P_B T_B^2 + k^8 Q^3 C_0 M_{A0}^2 M_{B0}^2 P_A S_A T_B^2 -$$
$$3 k^8 Q^3 C_0^2 M_{A0}^2 M_{B0}^2 P_A S_A T_B^2 + 3 k^8 Q^3 C_0^3 M_{A0}^2 M_{B0}^2 P_A S_A T_B^2 - k^8 Q^3 C_0^4 M_{A0}^2 M_{B0}^2$$
$$P_A S_A T_B^2 + 2 k^{10} Q C_0^2 M_{A0}^2 MP_{B0}^2 P_A S_A T_B^2 - 4 k^{10} Q C_0^3 M_{A0}^2 MP_{B0}^2 P_A S_A T_B^2 +$$
$$2 k^{10} Q C_0^4 M_{A0}^2 MP_{B0}^2 P_A S_A T_B^2 + k^8 Q^3 C_0 M_{A0} M_{B0}^3 P_B S_A T_B^2 - 3 k^8 Q^3 C_0^2$$
$$M_{A0} M_{B0}^3 P_B S_A T_B^2 + 3 k^8 Q^3 C_0^3 M_{A0} M_{B0}^3 P_B S_A T_B^2 - k^8 Q^3 C_0^4 M_{A0} M_{B0}^3 P_B S_A T_B^2 +$$
$$k^{10} Q C_0^3 M_{A0} M_{B0} MP_{B0}^2 P_B S_A T_B^2 - k^{10} Q C_0^4 M_{A0} M_{B0} MP_{B0}^2 P_B S_A T_B^2 -$$
$$k^{10} Q^2 C_0 M_{A0}^2 M_{B0}^2 S_A^2 T_B^2 + 2 k^{10} Q^2 C_0^2 M_{A0}^2 M_{B0}^2 S_A^2 T_B^2 - k^{10} Q^2 C_0^3 M_{A0}^2 M_{B0}^2 S_A^2 T_B^2 -$$
$$k^8 Q^2 C_0^2 M_{A0} M_{B0}^3 P_A S_B T_B^2 + k^8 Q^3 C_0^2 M_{A0} M_{B0}^3 P_A S_B T_B^2 +$$
$$2 k^8 Q^2 C_0^3 M_{A0} M_{B0}^3 P_A S_B T_B^2 - 2 k^8 Q^3 C_0^3 M_{A0} M_{B0}^3 P_A S_B T_B^2 -$$
$$k^8 Q^2 C_0^4 M_{A0} M_{B0}^3 P_A S_B T_B^2 + k^8 Q^3 C_0^4 M_{A0} M_{B0}^3 P_A S_B T_B^2 +$$
$$k^{10} C_0^3 M_{A0} M_{B0} MP_{B0}^2 P_A S_B T_B^2 + 2 k^{10} Q C_0^3 M_{A0} M_{B0} MP_{B0}^2 P_A S_B T_B^2 -$$
$$k^{10} C_0^4 M_{A0} M_{B0} MP_{B0}^2 P_A S_B T_B^2 - 2 k^{10} Q C_0^4 M_{A0} M_{B0} MP_{B0}^2 P_A S_B T_B^2 -$$
$$k^8 Q^2 C_0^2 M_{B0}^4 P_B S_B T_B^2 + k^8 Q^3 C_0^2 M_{B0}^4 P_B S_B T_B^2 + 2 k^8 Q^2 C_0^3 M_{B0}^4 P_B S_B T_B^2 -$$
$$2 k^8 Q^3 C_0^3 M_{B0}^4 P_B S_B T_B^2 - k^8 Q^2 C_0^4 M_{B0}^4 P_B S_B T_B^2 + k^8 Q^3 C_0^4 M_{B0}^4 P_B S_B T_B^2 -$$
$$k^{10} C_0^4 M_{B0}^2 MP_{B0}^2 P_B S_B T_B^2 + k^{10} Q C_0^4 M_{B0}^2 MP_{B0}^2 P_B S_B T_B^2 + k^{10} Q C_0 M_{A0} M_{B0}^3 S_A S_B T_B^2 -$$
$$2 k^{10} Q^2 C_0^2 M_{A0} M_{B0}^3 S_A S_B T_B^2 - k^{10} Q C_0^3 M_{A0} M_{B0}^3 S_A S_B T_B^2 +$$
$$2 k^{10} Q^2 C_0^3 M_{A0} M_{B0}^3 S_A S_B T_B^2 - k^{10} C_0^2 M_{B0}^4 S_B^2 T_B^2 + k^{10} Q C_0^2 M_{B0}^4 S_B^2 T_B^2 +$$
$$k^{10} Q C_0^3 M_{B0}^4 S_B^2 T_B^2 - k^{10} Q^2 C_0^3 M_{B0}^4 S_B^2 T_B^2 + k^6 Q^3 C_0 M_{A0} M_{B0}^2 MP_{A0} P_A T_A T_B^2 -$$
$$3 k^6 Q^3 C_0^2 M_{A0} M_{B0}^2 MP_{A0} P_A T_A T_B^2 + 3 k^6 Q^3 C_0^3 M_{A0} M_{B0}^2 MP_{A0} P_A T_A T_B^2 -$$
$$k^6 Q^3 C_0^4 M_{A0} M_{B0}^2 MP_{A0} P_A T_A T_B^2 + k^6 Q^3 C_0^2 M_{A0}^2 M_{B0} MP_{B0} P_A T_A T_B^2 -$$
$$2 k^6 Q^3 C_0^3 M_{A0}^2 M_{B0} MP_{B0} P_A T_A T_B^2 + k^6 Q^3 C_0^4 M_{A0}^2 M_{B0} MP_{B0} P_A T_A T_B^2 +$$
$$2 k^8 Q C_0^2 M_{A0} MP_{A0} MP_{B0}^2 P_A T_A T_B^2 - 4 k^8 Q C_0^3 M_{A0} MP_{A0} MP_{B0}^2 P_A T_A T_B^2 +$$
$$2 k^8 Q C_0^4 M_{A0} MP_{A0} MP_{B0}^2 P_A T_A T_B^2 + k^6 Q^3 C_0 M_{B0}^3 MP_{A0} P_B T_A T_B^2 -$$
$$3 k^6 Q^3 C_0^2 M_{B0}^3 MP_{A0} P_B T_A T_B^2 + 3 k^6 Q^3 C_0^3 M_{B0}^3 MP_{A0} P_B T_A T_B^2 -$$
$$k^6 Q^3 C_0^4 M_{B0}^3 MP_{A0} P_B T_A T_B^2 + k^6 Q^3 C_0^2 M_{A0} M_{B0}^2 MP_{B0} P_B T_A T_B^2 -$$
$$2 k^6 Q^3 C_0^3 M_{A0} M_{B0}^2 MP_{B0} P_B T_A T_B^2 + k^6 Q^3 C_0^4 M_{A0} M_{B0}^2 MP_{B0} P_B T_A T_B^2 +$$
$$k^8 Q C_0^3 M_{B0} MP_{A0} MP_{B0}^2 P_B T_A T_B^2 - k^8 Q C_0^4 M_{B0} MP_{A0} MP_{B0}^2 P_B T_A T_B^2 -$$
$$2 k^8 Q^2 C_0 M_{A0} M_{B0}^2 MP_{A0} S_A T_A T_B^2 + 4 k^8 Q^2 C_0^2 M_{A0} M_{B0}^2 MP_{A0} S_A T_A T_B^2 -$$

$$
\begin{aligned}
&2 k^{8} Q^{2} C_0^{3} M_{A0} M_{B0}^{2} MP_{A0} S_A T_A T_B^{2} - k^{8} Q^{2} C_0 M_{A0}^{2} M_{B0} MP_{B0} S_A T_A T_B^{2} + \\
&k^{8} Q^{2} C_0^{2} M_{A0}^{2} M_{B0} MP_{B0} S_A T_A T_B^{2} + k^{8} Q C_0 M_{B0}^{3} MP_{A0} S_B T_A T_B^{2} - \\
&2 k^{8} Q^{2} C_0^{2} M_{B0}^{3} MP_{A0} S_B T_A T_B^{2} - k^{8} Q C_0^{3} M_{B0}^{3} MP_{A0} S_B T_A T_B^{2} + \\
&2 k^{8} Q^{2} C_0^{3} M_{B0}^{3} MP_{A0} S_B T_A T_B^{2} + 3 k^{8} Q C_0^{2} M_{A0} M_{B0}^{2} MP_{B0} S_B T_A T_B^{2} - \\
&k^{8} Q^{2} C_0^{2} M_{A0} M_{B0}^{2} MP_{B0} S_B T_A T_B^{2} - 2 k^{8} Q C_0^{3} M_{A0} M_{B0}^{2} MP_{B0} S_B T_A T_B^{2} - \\
&k^{6} Q^{2} C_0 M_{B0}^{2} MP_{A0}^{2} T_A^{2} T_B^{2} + 2 k^{6} Q^{2} C_0^{2} M_{B0}^{2} MP_{A0}^{2} T_A^{2} T_B^{2} - k^{6} Q^{2} C_0^{3} M_{B0}^{2} MP_{A0}^{2} T_A^{2} T_B^{2} - \\
&k^{6} Q^{2} C_0 M_{A0} M_{B0} MP_{A0} MP_{B0} T_A^{2} T_B^{2} + k^{6} Q^{2} C_0^{2} M_{A0} M_{B0} MP_{A0} MP_{B0} T_A^{2} T_B^{2} - \\
&k^{6} Q^{2} C_0^{2} M_{A0}^{2} MP_{B0}^{2} T_A^{2} T_B^{2} + k^{6} Q^{2} C_0^{3} M_{A0}^{2} MP_{B0}^{2} T_A^{2} T_B^{2} + k^{8} Q C_0^{3} M_{A0} MP_{B0}^{3} P_A T_B^{3} - \\
&k^{8} Q C_0^{4} M_{A0} MP_{B0}^{3} P_A T_B^{3} + k^{8} Q^{2} C_0 M_{A0} M_{B0}^{2} MP_{B0} S_A T_B^{3} - \\
&2 k^{8} Q^{2} C_0^{2} M_{A0} M_{B0}^{2} MP_{B0} S_A T_B^{3} + k^{8} Q^{2} C_0^{3} M_{A0} M_{B0}^{2} MP_{B0} S_A T_B^{3} - \\
&k^{8} Q C_0^{2} M_{B0}^{3} MP_{B0} S_B T_B^{3} + k^{8} Q^{2} C_0^{2} M_{B0}^{3} MP_{B0} S_B T_B^{3} + k^{8} Q C_0^{3} M_{B0}^{3} MP_{B0} S_B T_B^{3} - \\
&k^{8} Q^{2} C_0^{3} M_{B0}^{3} MP_{B0} S_B T_B^{3} + k^{6} Q^{2} C_0 M_{B0}^{2} MP_{A0} MP_{B0} T_A T_B^{3} - \\
&2 k^{6} Q^{2} C_0^{2} M_{B0}^{2} MP_{A0} MP_{B0} T_A T_B^{3} + k^{6} Q^{2} C_0^{3} M_{B0}^{2} MP_{A0} MP_{B0} T_A T_B^{3} + \\
&k^{6} Q^{2} C_0^{2} M_{A0} M_{B0} MP_{B0}^{2} T_A T_B^{3} - k^{6} Q^{2} C_0^{3} M_{A0} M_{B0} MP_{B0}^{2} T_A T_B^{3} \big) \Big) \Big) \Big) \Big)
\end{aligned}
$$

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