Calibration of Triaxial Accelerometer and Triaxial Magnetometer for Tilt Compensated Electronic Compass

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Abstract. This research paper describes the method for the calibration of accelerometer and magnetometer for tilt compensated electronic compass. The electronic compass is implemented using triaxial MEMS accelerometer and triaxial MEMS magnetometer. The heading of the compass is generally influenced by scale factors, offsets and misalignment errors of these sensors. The proposed calibration method determines twelve calibration parameters in six stationary positions for accelerometer and twelve calibration parameters in 3D rotations for magnetometer.

Keywords: accelerometer, calibration, electronic compass, magnetometer, micro-electro-mechanical-system.

1   Introduction

The electronic compasses are a crucial navigation tool in many areas even in present time of the global positioning system (GPS). They are used as a component for dead reckoning for determining the location. However, the electronic compasses are vulnerable to a variety of external influences such as hard iron and soft iron interferences and accelerations errors which can affect the calculated heading.

The electronic compasses are based on the measurement of the Earth's geomagnetic field. The Earth can be considered as a magnetic dipole with poles near the North Pole and the South Pole. The magnetic vectors points toward the North Pole and near the equator, the magnetic vectors are parallel to the surface (Fig. 1.). [1]

This inclination, angle between the magnetic vector and the horizontal plane, is highly dependent on the geographical latitude (Fig. 2); same as the intensity. The intensity of the Earth's magnetic field is approximately between 25 µT and 65 µT (0.25 – 0.65 gauss). [2]
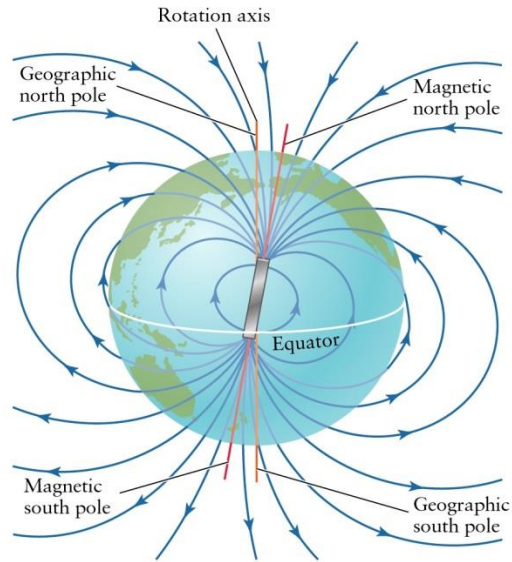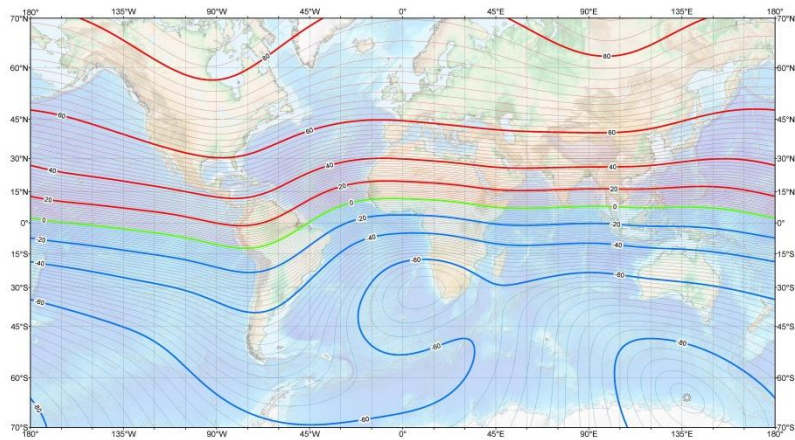
Fig. 1. Earth's magnetic field [3]

Fig. 2. Magnetic inclination map [4]

The common approach measures two orthogonal components of the magnetic vector from magnetometer to estimate the heading. However, if the sensor is tilted, the measured values of these components change and the calculated heading is not accurate. To avoid this error, a triaxial accelerometer must be coupled to compensate the tilt. [1]

In this research paper, we proposed the method for the calibration of accelerometer and magnetometer for the implementation of tilt compensated electronic compass.

The electronic compass is implemented using triaxial MEMS accelerometer and triaxial MEMS magnetometer.

The reminder of this paper is organized as follows. The accelerometer calibration and magnetometer calibration is described in section 2 and section 3. Section 4 explains the procedure of tilt compensation algorithm.

2   Accelerometer

2.1   Accelerometer Error Model

The accelerometers suffer from a variety of error sources which are slightly different depending on the type of accelerometer.

Ordinarily, the accelerometer measurement (R_X) can be represented in form of the acceleration along its sensitive axis ($\tilde{A}_X$) and the accelerations along the pendulum and hinge axes, $\tilde{A}_Y$ and $\tilde{A}_Z$ respectively, by equation [5]:

$$R_X = (1 + S_X) \cdot \tilde{A}_X + M_Y \cdot \tilde{A}_Y + M_Z \cdot \tilde{A}_Z + B_f + B_v \cdot \tilde{A}_X \cdot \tilde{A}_Y + n_X \tag{1}$$

where

S_X is the scale factor,

M_Y, M_Z are the cross-axis coupling factors,

B_f is the measurement bias,

B_v is the vibro-pendulous error coefficient,

n_X is the random noise.

For MEMS accelerometers, it is expected that the cross-axis coupling factors and vibro-pendulous error are negligible [6]. Then equation (1) can be simplified to

$$R_X = \tilde{A}_X + S_X \cdot \tilde{A}_X + B_f + n_X \tag{2}$$

2.2   Accelerometer Calibration

The relation between the raw accelerometer readings and the correct accelerometer outputs is described as [2]:

$$\begin{bmatrix} A_X \\ A_Y \\ A_Z \end{bmatrix} = [Am]_{3x3} \cdot \begin{bmatrix} \dfrac{1}{S_X} & 0 & 0 \\ 0 & \dfrac{1}{S_Y} & 0 \\ 0 & 0 & \dfrac{1}{S_Z} \end{bmatrix} \cdot \begin{bmatrix} R_X - O_X \\ R_Y - O_Y \\ R_Z - O_Z \end{bmatrix} \tag{3}$$

where

A_X, A_Y, A_Z are known correct accelerometer outputs,

Am is a 3x3 misalignment matrix between accelerometer axes and device axes,

S_X, S_Y, S_Z are the scale factors,

R_X, R_Y, R_Z are the raw accelerometer readings,

O_X, O_Y, O_Z are the offsets.

Equation (3) can be simplified to

$$\begin{bmatrix} A_X \\ A_Y \\ A_Z \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} \cdot \begin{bmatrix} R_X \\ R_Y \\ R_Z \end{bmatrix} + \begin{bmatrix} P_{10} \\ P_{20} \\ P_{30} \end{bmatrix} \tag{4}$$

The goal of this calibration is to determine twelve unknown calibration parameters P_i. These parameters are typically obtained by the least square method. [2]

The basic calibration is performed at two stationary positions in each sensitive axis which are showed in Table 1. In order to get more precise output values, we can add more stationary positions.

Table 1. Sign definition of raw accelerometer readings

Stationary position	A_X	A_Y	A_Z
1	0	0	+ g
2	0	0	- g
3	0	+ g	0
4	0	- g	0
5	+ g	0	0
6	- g	0	0

Equation (4) can be rewritten as

$$\begin{bmatrix} A_X & A_Y & A_Z \end{bmatrix} = \begin{bmatrix} R_X & R_Y & R_Z & 1 \end{bmatrix} \cdot \begin{bmatrix} P_{11} & P_{21} & P_{31} \\ P_{12} & P_{22} & P_{32} \\ P_{13} & P_{23} & P_{33} \\ P_{10} & P_{20} & P_{30} \end{bmatrix} \tag{5}$$

or also to

$$Y = w \cdot X \tag{6}$$

where

Y is known correct gravity vector,

w is the raw accelerometer reading at each positon,

X is the matrix containing twelve calibration parameters.

3   Magnetometer

3.1   Magnetometer error model

The magnetometer data are influenced by wide band measurement noise, stochastic biases, installation errors and magnetic interferences in the vicinity of the sensors.

The magnetic interference can be divided up into two groups. The first group, known as hard iron interference, consists of fixed or slightly time-varying field generated by ferromagnetic materials. [7]

In the second group, soft iron interference, the magnetic field is generated within the device itself.

$$R = C \cdot \left[M_M \cdot S \cdot SI \cdot (M + B + n) \right] \tag{7}$$

where

C is the matrix of misalignment between magnetometer axes and device axes,

M_M is the matrix of misalignment errors,

S is the matrix of scale factors,

SI is the matrix of soft iron biases

M is vector of magnetic field along sensitive axis,

B is vector of hard iron biases,

n is the wideband noise.

3.2   Magnetometer calibration

The relation between the correct magnetometer data and the raw measurements of magnetometer is expressed as [1], [2]

$$\begin{bmatrix} M_X \\ M_Y \\ M_Z \end{bmatrix} = [Mm]_{3x3} \cdot \begin{bmatrix} \dfrac{1}{S_X} & 0 & 0 \\ 0 & \dfrac{1}{S_Y} & 0 \\ 0 & 0 & \dfrac{1}{S_Z} \end{bmatrix} \cdot [SI]_{3x3} \cdot \begin{bmatrix} R_X - O_X \\ R_Y - O_Y \\ R_Z - O_Z \end{bmatrix} \tag{8}$$

where

M_X, M_Y, M_Z are the correct magnetometer outputs,

Mm is a 3x3 misalignment matrix between the magnetometer axes and the device axes,

S_X, S_Y, S_Z are the scale factors,

R_X, R_Y, R_Z are the raw magnetometer readings,
SI is a 3x3 matrix of offsets caused by soft-iron interference,
O_X, O_Y, O_Z are the offsets caused by hard-iron interference.

Equation (8) can be simplified to [2]

$$\begin{bmatrix} M_X \\ M_Y \\ M_Z \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} \cdot \begin{bmatrix} R_X - P_{10} \\ R_Y - P_{20} \\ R_Z - P_{30} \end{bmatrix} \qquad (9)$$

The goal of magnetometer calibration is to determine twelve unknown calibration parameters P_i to known correct magnetometer outputs which can be obtained at random positions. [2]

4 Electronic compass

The tilt compensated electronic compass uses triaxial accelerometer and triaxial magnetometer. The accelerometer measures components of gravity and magnetometer measures parts of geomagnetic field. The accelerometer readings provide roll and pitch angles which are used for magnetometer correction. That allows accurate calculation of compass heading. The tilt compensated electronic compass is not operating under freefall, low-g and high-g accelerations. [2], [8]

4.1 Tilt Compensation Algorithm

The electronic compass coordinate system is represented on Fig. 3, where X_B, Y_B, Z_B are the device body axes and X, Y, Z are the accelerometer and magnetometer sensing axes. [2], [9]
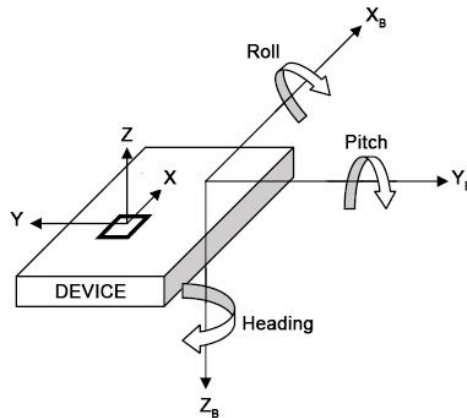

Fig. 3. Electronic compass coordinate system

Rotations can be expressed as quaternions, Euler angles, roll-pitch-heading and rotations matrix. We used the rotations matrix. This method defines the roll γ as the angle between the Y_B axis and the horizontal plane. The pitch ρ is defined as the angle between X_B axis and horizontal plane and the heading ψ is the angle with respect to the magnetic north pole. [9]

To obtain the rotation matrixes, we need to measure the rotations around each sensitive axis. These rotations can be described by the roll matrix R_γ (equation (10)), the pitch matrix R_ρ (equation (11)) and the heading matrix R_ψ (equation (12)). [2], [7], [8]

$$R_\gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{bmatrix} \tag{10}$$

$$R_\rho = \begin{bmatrix} \cos\rho & 0 & -\sin\rho \\ 0 & 1 & 0 \\ \sin\rho & 0 & \cos\rho \end{bmatrix} \tag{11}$$

$$R_\psi = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

The relationship between the arbitrary position X'_B, Y'_B, Z'_B and the device body axes is [2], [7], [8]

$$\begin{bmatrix} X'_B \\ Y'_B \\ Z'_B \end{bmatrix} = R_\gamma \cdot R_\rho \cdot R_\psi \cdot \begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} \tag{13}$$

In the horizontal plane $X_B = Y_B = 0, Z_B = +1g$ and X'_B, Y'_B, Z'_B are raw accelerometer readings. Then we can rewrite equation (13) to [2], [7], [8]

$$\begin{bmatrix} A_X \\ A_Y \\ A_Z \end{bmatrix} = R_\gamma \cdot R_\rho \cdot R_\psi \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{14}$$

Therefore, the roll and the pitch angle can be computed as

$$Pitch = \rho = \arcsin(-A_X) \tag{15}$$

$$Roll = \gamma = \arcsin\left(\frac{A_Y}{\cos\rho}\right) \tag{16}$$

The equations for tilt compensated x and y axis are expressed on equation (17), where M_X, M_Y, M_Z are the components of the magnetometer measurements. [2], [7], [8]

$$X_H = M_X \cdot \cos\rho + M_Z \sin\rho$$
$$Y_H = M_X \cdot \sin\gamma \cdot \cos\rho + M_Y \cdot \cos\gamma - M_Z \sin\gamma \cdot \cos\rho \tag{17}$$

The heading angle is calculated as [1], [2]

$$Heading = \psi = \arctan\frac{Y_H}{X_H} \tag{18}$$

5   Conclusion

This research paper has presented calibration methods for accelerometer and magnetometer. These sensors are implemented in tilt compensated electronic compass. The roll and pitch angles are calculated from accelerometer readings that provides tilt compensation for the compass and magnetometer measures the components of the geomagnetic field.

The proposed calibration methods for accelerometer and magnetometer follow simple steps. After the calibration, the tilt compensated electronic compass is capable of providing precise accuracy which is below 2°.

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