
SAM
Science Arts & Métiers
Archive ouverte Arts & Métiers ParisTech
ARTS ET MÉTIERS ParisTech

Formability prediction of damageable elastic-viscoplastic media by a material stability analysis based on a linear perturbation method

G.Altmeyer[1], F.Abed-Meraim[1], T.Balan[1]

[1] *LPMM, FRE CNRS 3236, Arts et Métiers-ParisTech, France, {guillaume.altmeyer,farid.abed-meraim,tudor.balan}@metz.ensam.fr*

During deep drawing operations, diffuse and localized necking may lead to defective final products. Predicting accurately such phenomena has become a major industrial challenge. As the physics of necking and strain localization are still to be better understood for a large range of metallic materials, many material instability criteria based on different physical principles have been developed during the last decades. Although a large number of predictive criteria can be rapidly advanced by a bibliographic review of methods used to determine limits of formability, the most frequently used of them seem to derive from four main theoretical principles that have their own advantages and limits.

The earliest of these physical principles, known as the "Maximum Force", is based on the experimental observation according to which diffuse necking occurs when the load reaches its maximum during a uniaxiale tensile test [1]. Extensions to this one-directional necking criterion have been proposed to predict necking of metal sheets submitted to biaxial loadings [2], latter modified to predict strain localization phenomena [3-4] and enhanced to take into account more advanced materials and geometrical effects [5]. Although these criteria have been extensively used, their theoretical bases still have to be reinforced. To overcome these limitations, a zero-extension band criterion was developed, allowing a prediction of necking modes, but restricted to the left-hand side of the forming limit diagram (FLD) where such a band can exist [6]. In addition to this criterion, Marciniak-Kuczynski [7] criterion has been largely used in its first version to predict the right-hand side of FLDs, before being extended to the left one [8]. This criterion is based on the study of the evolution of a preexisting defect, introduced in the metal sheet under the form of a reduced thickness band. "M-K model" is applicable to a wide range of materials including elastic-viscoplastic materials. A major drawback of this formulation comes from the necessary use of extrinsic user defined parameters, such as the size of initial defect or the threshold value. This can however be overcome by the use of criteria based on bifurcation theory. Diffuse necking modes can be predicted by "General Bifurcation" criterion, based on the loss of positivity of the second order work [9-10], while Rudnicki and Rice's criterion, based on the loss of ellipticity of the acoustic tensor, was established to predict localized necking under shear bands [11]. However this last criterion is restricted to both rate independent materials and softening behavior for associative plasticity. This first restriction can now be bypassed by the use of criteria based on stability analysis by a linear perturbation method. Necking can then be seen as instability of the global or local mechanical equilibrium [12] ; such an approach has been used to predict formability of metal sheets during deep-drawing and hydroforming processes [13].

Recent methods based on the material stability analysis by linear perturbation methods have been developed and mostly applied to soil mechanics [14]. A small perturbation is introduced into the equilibrium equations of the system and the perturbed system can then be obtained. The linearization of this system leads to a problem whose conditions of absolute or relative stability are sought by evaluation of the growth rate of the introduced perturbation. Finally stability analysis by linear perturbation method leads to a condition depending on the tangent modulus in the local case and on the boundary conditions and the tangent modulus in the global one. It has been shown that Rice's criterion, requiring softening effects, is a limit case of stability criterion [15]. In soils mechanics applications, softening effects are classically introduced from non-associative plasticity laws [16]. In the case of phenomenological description of engineering materials these effects are usually introduced by a damage variable. Although general stability analysis methods taking into account damage effects exist [17], they remain seldom used [18].

The originality of the approach used here is then to present stability conditions of metallic materials modeled by general phenomenological laws and to apply them to the study of the formability of thin metallic sheets. The constitutive equations are based on an elastic-viscoplastic model coupled with a classical isotropic damage model. Various classical isotropic and kinematic hardening models are considered here to reproduce the hardening behavior and the induced anisotropy, while the initial anisotropy of metallic rolled sheets is introduced by Hill'48 anisotropic yield function. Softening is taken into account by coupling the constitutive model with a scalar damage variable. This phenomenological variable is defined within the framework of continuum damage mechanics and represents the degradation of the elastic properties during the forming operations. The stability of the constitutive damageable elastic-viscoplastic system is then sought by means of linear perturbation approach. Application of this method to a mild steel and a dual phase steel leads to the determination of their formability limit diagrams. On the one hand, FLDs obtained with Rice's criterion and with this stability criterion illustrate the theoretical relation between these criteria and on the other hand, a parametric study shows the influence of governing parameters of hardening, damage and strain rate sensitivity.

References

[1] A. Considère. *Ann. Ponts et Chaussées*, **9**, 574, 1885.
[2] H.W. Swift. *J. Mech. Phys. Sol.*, **1**, 1-18, 1952.
[3] P. Hora, L. Tong, J. Reissner. In: *Proc. of Numisheet'96*, eds. R. Wagonner et al., 252-256, 1996.
[4] K. Mattiasson, M. Sigvant, M. Larson. In: *Proc. of IDDRG'06*, eds. Santos and Barata da Rocha, Porto, 1-9, 2006.
[5] M. Brunet, F. Morestin. *J. Mech. Proc. Tech.*, **112**, 214-226, 2001.
[6] R. Hill. *J. Mech. Phys. Sol.*, **1**, 19-30, 1952.
[7] Z. Marciniak, K. Kuczynski. *Int. J. Mech. Sci.*, **9**, 609-620, 1967.
[8] J.W. Hutchinson, K.W. Neale. In: *Mechanics of sheet metal forming*, eds. Koistinen and Wang, New-York, 127-153, 1978.
[9] D.C. Drucker. *Q. Appl. Math.*, **16**, 35-42, 1956.
[10] R. Hill. *J. Mech. Phys. Sol.*, **6**, 236-249, 1958.
[11] J.W. Rudnicki, J.R. Rice. *J. Mech. Phys. Sol.*, **23**, 371-394, 1975.
[12] D. Dudzinski, A. Molinari. *Int. J. Sol. Str.*, **27**, 601-628, 1991.
[13] N. Boudeau, J. Gelin. *J. Mat. Proc. Tech.*, **32**, 521-530, 1992.
[14] A. Benallal. *Eur. J. Mech. A/Solids*, **19**, 45-60, 2000.
[15] G. Barbier, A. Benallal, V. Cano. *C. R. Acad. Sci. Paris*, **326**, Série IIb, 153-158, 1998.
[16] N.G. Diouta, I. Sharour. *Eur. Phys. J. Appl. Phys.*, **34**, 85-96, 2006.
[17] G. Roussellier. *C. R. Acad. Sci. Paris*, **320**, Série IIb, 265-270, 1995.
[18] J. Bikard, T. Désoyer. In : *Proc. of 16ème Congrès de Mécanique*, Nice, 2003.