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A NEW LOCKING-FREE FORMULATION FOR THE SHB8PS SOLID-SHELL ELEMENT: NON-LINEAR BENCHMARK PROBLEMS

FARID ABED-MERAIM[*] AND ALAIN COMBESCURE[†]

[*] Laboratoire de Physique et Mécanique des Matériaux (LPMM)
ENSAM CER de Metz, UMR CNRS 7554, 4 rue Augustin Fresnel, 57078 Metz, France
E-mail: farid.abed-meraim@metz.ensam.fr

[†] Laboratoire de Mécanique des Contacts et des Solides (LaMCoS)
INSA de Lyon, UMR CNRS 5514, 18-20 rue des Sciences, 69621 Villeurbanne, France

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Summary. *In this work, a new physically stabilized and locking-free formulation of the SHB8PS element is presented. This is a solid-shell element based on a purely 3D formulation. It has eight nodes as well as five integration points, all distributed along the "thickness" direction. Consequently, it can be used for the modeling of thin structures, while providing an accurate description of the various through-thickness phenomena. The reduced integration has been used in order to prevent some locking phenomena and to increase computational efficiency. The spurious zero-energy deformation modes due to the reduced integration are efficiently stabilized, whereas the strain components corresponding to locking modes are eliminated with a projection technique following the Enhanced Assumed Strain (EAS) method.*

1 INTRODUCTION

Over the last decade, considerable progress has been made in the development of three-dimensional finite elements capable of modeling thin structures (see references [1, 2, 3, 4, 5]). The coupling between solid and shell formulations is a good way to provide continuum finite element models that can be efficiently used for structural applications. These solid-shell elements have numerous advantages for the analysis of various complex structural forms that are common in many industrial applications. Their main advantage is to allow the meshing of complex structural forms without the classical problems of connecting zones meshed with different element types (continuum and structural elements for instance). Another important benefit of solid-shell elements is the avoidance of tedious and complex pure-shell element formulations. In this work, a new locking-free formulation for the SHB8PS solid-shell element is performed. More specifically, this work focuses on the elimination of the residual membrane and shear locking phenomena persisting in the previous formulations. By using orthogonal projection of the discrete gradient operator, these severe shear and membrane locking modes are removed. Several numerical experiments in linear and non linear benchmark problems show that this new formulation of the SHB8PS element is effective and allows fast convergence without locking phenomena.

2 FORMULATION OF THE SHB8PS ELEMENT

The element's coordinates x_i and displacements u_i $(i = 1,...,3)$ are interpolated using the isoparametric trilinear shape functions $N_I(\xi,\eta,\zeta)$ $(I = 1,...,8)$. By introducing the *Hallquist's* vectors ($\underline{b}_i$, i = 1,..., 3), defined in ref. [6] as:

$$\underline{b}_i^T = \underline{N}_{,i}(0,0,0) \qquad i = 1,2,3 \qquad \textit{Hallquist Form} \tag{1}$$

where $\underline{N}_{,i} = \partial \underline{N} / \partial x_i$, one can show that the discrete gradient operator, which relates the linear deformations to the nodal displacements (i.e., $\underline{\underline{\nabla}}_S(\underline{u}) = \underline{\underline{B}} \cdot \underline{d}$), is given by Eqn. (2). This $\underline{\underline{B}}$-matrix uses the following variables:

$$\begin{cases} \underline{\gamma}_\alpha = \frac{1}{8}\left[\underline{h}_\alpha - \sum_{j=1,3}(\underline{h}_\alpha^T \cdot \underline{x}_j)\underline{b}_j\right] \\ h_1 = \eta\zeta,\ h_2 = \zeta\xi,\ h_3 = \xi\eta,\ h_4 = \xi\eta\zeta \end{cases} ; \begin{cases} \underline{d}_i^T = (u_{i1}, u_{i2}, u_{i3,}, ..., u_{i8}), & \underline{x}_i^T = (x_{i1}, x_{i2}, x_{i3}, ..., x_{i8}) \\ \underline{h}_1^T = (1,1,-1,-1,-1,-1,1,1), & \underline{h}_2^T = (1,-1,-1,1,-1,1,1,-1) \\ \underline{h}_3^T = (1,-1,1,-1,1,-1,1,-1), & \underline{h}_4^T = (-1,1,-1,1,1,-1,1,-1) \end{cases}$$

$$\underline{\underline{B}} = \begin{bmatrix} \underline{b}_x^T + h_{\alpha,x}\underline{\gamma}_\alpha^T & \underline{0} & \underline{0} \\ \underline{0} & \underline{b}_y^T + h_{\alpha,y}\underline{\gamma}_\alpha^T & \underline{0} \\ \underline{0} & \underline{0} & \underline{b}_z^T + h_{\alpha,z}\underline{\gamma}_\alpha^T \\ \underline{b}_y^T + h_{\alpha,y}\underline{\gamma}_\alpha^T & \underline{b}_x^T + h_{\alpha,x}\underline{\gamma}_\alpha^T & \underline{0} \\ \underline{0} & \underline{b}_z^T + h_{\alpha,z}\underline{\gamma}_\alpha^T & \underline{b}_y^T + h_{\alpha,y}\underline{\gamma}_\alpha^T \\ \underline{b}_z^T + h_{\alpha,z}\underline{\gamma}_\alpha^T & \underline{0} & \underline{b}_x^T + h_{\alpha,x}\underline{\gamma}_\alpha^T \end{bmatrix} \left(\begin{array}{c}\text{The convention of implied summation} \\ \text{of repeated subscripts } \alpha \text{ is adopted}\end{array}\right) \tag{2}$$

Despite the geometry of the element (eight-node hexahedron), several modifications are introduced in order to provide it with shell features. Among them, a shell-like behavior is intended for the element, by modifying the three-dimensional constitutive law so that the plane-stress conditions are approached and by aligning all the integration points along a privileged direction, called the thickness. The stiffness matrix is then obtained by Gauss integration:

$$\underline{\underline{K}}_e = \int_{\Omega_e} \underline{\underline{B}}^T \cdot \underline{\underline{C}} \cdot \underline{\underline{B}}\, dv = \sum_{I=1}^{5} \omega(\zeta_I) J(\zeta_I) \underline{\underline{B}}^T(\zeta_I) \cdot \underline{\underline{C}} \cdot \underline{\underline{B}}(\zeta_I) \tag{3}$$

Because the $h_{\alpha,i}$ functions $(\alpha = 3,4;\ i = 1,2,3)$ vanish at the five Gauss points, of coordinates $\xi_I = \eta_I = 0,\ \zeta_I \neq 0,$ the $\underline{\underline{B}}$-matrix Eqn. (2) reduces at its $\underline{\underline{B}}_{12}$ part, with only the $h_{\alpha,i}$ terms $(\alpha = 1,2;\ i = 1,2,3)$. This leads to six hourglass modes generated by $\underline{h}_3$ and $\underline{h}_4$. These spurious modes are stabilized following the approach given in ref. [7]. Moreover, we apply an assumed strain method in order to eliminate locking. The $\underline{\underline{B}}$-matrix is thus projected onto $\overline{\underline{\underline{B}}}$ as:

$$\overline{\underline{\underline{B}}} = \underline{\underline{B}}_{12} + \overline{\underline{\underline{B}}}_{34} \tag{4}$$

Consequently, the stiffness matrix, Eqn. (3), can be rewritten as:

$$\underline{\underline{K}}_e = \underline{\underline{K}}_{12} + \underline{\underline{K}}_{STAB} \qquad (5)$$

The first term $\underline{\underline{K}}_{12}$ is obtained by Gauss integration, Eqn. (3). The second term $\underline{\underline{K}}_{STAB}$ represents the stabilization stiffness:

$$\underline{\underline{K}}_{STAB} = \int_{\Omega_e} \underline{\underline{B}}_{12}^T \cdot \underline{\underline{C}} \cdot \overline{\underline{\underline{B}}}_{34} dv + \int_{\Omega_e} \overline{\underline{\underline{B}}}_{34}^T \cdot \underline{\underline{C}} \cdot \underline{\underline{B}}_{12} dv + \int_{\Omega_e} \overline{\underline{\underline{B}}}_{34}^T \cdot \underline{\underline{C}} \cdot \overline{\underline{\underline{B}}}_{34} dv \qquad (6)$$

3 NUMERICAL RESULTS AND DISCUSSIONS

The performance of this new formulation has been tested through a variety of linear and non linear mechanical problems. In all of these tests, the new version showed better performance than the previous formulation. In particular, the improvement is significant in the pinched hemispherical shell test. This test has become very popular and is used by many authors. It is severe since the shear and membrane locking phenomena are very important and emphasized by the problem geometry (distorted, skewed elements). As reported by many authors, in this doubly-curved shell problem, the membrane locking is much more severe than shear locking. Fig. 1 shows the geometry, loading and boundary conditions for this problem.

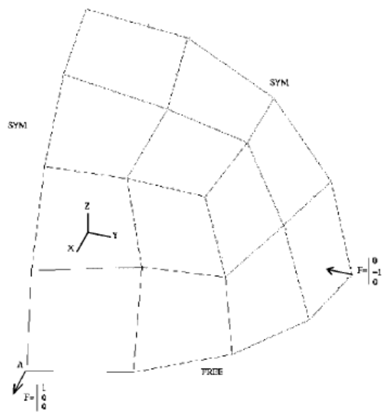

Figure 1: Schematic of hemispherical shell

The radius is R=10, the thickness t=0.04, the Young's modulus E=6.825×10^7, Poisson's ratio ν =0.3. Using the symmetry, only a quarter of the hemisphere is meshed using a single element through the thickness and with two unit loads along directions Ox and Oy. The analytical solution for the radial displacement at the load point is 0.0924. The convergence results are reported in Tab. 1 in terms of the normalized displacement at the load point. The new version of the SHB8PS element is compared to the former one and to the three elements HEX8, HEXDS and H8-ct-cp. The HEX8 element is a standard, 8-node, full integration solid element (8 Gauss points). The HEXDS element is an 8-node, four Gauss points solid element (see ref. [8]). The H8-ct-cp element was developed in ref. [9]. Tab. 1 shows that the new version of the SHB8PS element provides an excellent convergence and shows no locking.

This new version has also been tested on a variety of non linear, elastic and elastic-plastic problems. We demonstrate that the projection adopted in this formulation better eliminates the locking phenomena. As shown particularly in the pinched hemisphere test, Tab. 1, this element also demonstrates an excellent efficiency and convergence through numerous other tests.

Number of elements	SHB8PS previous version	HEX8	HEXDS	H8-ct-cp	SHB8PS new version
	Ux/Uref	Ux/Uref	Ux/Uref	Ux/Uref	Ux/Uref
12	0.0629	0.0005		0.05	0.8645
27	0.0474	0.0011			1.0155
48	0.1660	0.0023	0.408	0.35	1.0098
75	0.2252	0.0030	0.512	0.58	1.0096
192	0.6332	0.0076	0.701	0.95	1.0008
363	0.8592	0.0140	0.800		1.0006
768	0.9651	0.0287			1.0006
1462	0.9910	0.0520			1.0009

Table 1: Normalized displacement at the load point of the pinched hemispherical shell

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