COMPARISON OF FORMING LIMIT DIAGRAMS PREDICTED WITH DIFFERENT LOCALIZATION CRITERIA

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ABSTRACT

Automotive industries are more and more subject to restrictive environmental constraints. Weight reduction of structures seems to be an interesting way to satisfy these requirements. This can be achieved either by using new materials, such as high strength steels or by adopting appropriate dimensioning methods to predict the occurrence of strain localization. Forming Limit Diagram (FLD) is a concept widely used to characterize the formability of thin metal sheets.

Analytical determination of FLDs is usually based on the use of localization criteria. Some of the existing material instability criteria are for example based on empirical observations, on the maximum load principle [1-3], on the existence of an initial defect in the sheet [4], on a perturbation method [5] or on bifurcation analysis [6]. Although numerous criteria have been developed, they all have advantages but also drawbacks and limitations. Their confrontation on a wide range of materials is still insufficiently developed to compare their respective capability of accurately predicting FLDs for new materials. Adaptations of some criteria to advanced constitutive laws are also made necessary by the use of new high strength materials.

The aim of this paper is to give a general formulation of some localization criteria allowing the comparison of the predicted FLDs for a wide range of materials. An implementation of these criteria coupled with different phenomenological constitutive laws is presented and compared for different materials including an aluminium alloy, a brass and a dual phase steel.

Keywords: necking, strain localization, forming limit diagram.

1. CONSTITUTIVE MODEL

The adopted constitutive model is based on a phenomenological approach. It is applicable to elastic-plastic materials. As the main application is deep drawing, this model can take into account the anisotropy of the sheet but it is restricted to cold deformation. A hypo-elastic law, a yield criterion, a plastic flow rule and a set of internal variables are introduced in order to describe the behaviour of the material:

$$\mathbf{D} = \mathbf{D}^e + \mathbf{D}^p$$
$$\mathbf{D}^p = \dot{\lambda}\frac{\partial f}{\partial \boldsymbol{\sigma}} = \dot{\lambda}\mathbf{V} \qquad (1)$$

where $\mathbf{D}^e$, $\mathbf{D}^p$ respectively denote the elastic and plastic parts of the strain rate $\mathbf{D}$, $\dot{\lambda}$ and $\mathbf{V}$ are the plastic multiplier and the flow direction, normal to the yield surface defined by the potential f.

$$\dot{\boldsymbol{\sigma}} = \mathbf{C} : \mathbf{D}^e = \mathbf{C} : \left(\mathbf{D} - \dot{\lambda}\mathbf{V}\right)$$
$$f\left(\boldsymbol{\sigma}, \mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_n\right) \le 0 \text{ , } \dot{\lambda} \ge 0 \text{ , } \dot{\lambda} f = 0 \qquad (2)$$
$$\dot{\mathbf{y}}_i = \dot{\lambda}\mathbf{H}_{\mathbf{y}_i}$$

where $\boldsymbol{\sigma}$ represents the Cauchy stress and $\mathbf{y}_1, ..., \mathbf{y}_i, ..., \mathbf{y}_n$ internal variables. $\mathbf{C}$ is the fourth-order tensor of the elastic constants and $\mathbf{H}_{\mathbf{y}_i}$ a modulus describing the time evolution of the corresponding internal variable.

Using equations (1) and (2) together with the consistency condition, it is possible to write the relation between the stress and strain rates:

$$\dot{\boldsymbol{\sigma}} = \left(\mathbf{C} - \frac{(\mathbf{C}:\mathbf{V}) \otimes (\mathbf{V}:\mathbf{C})}{\mathbf{V}:\mathbf{C}:\mathbf{V} - \sum_i \frac{\partial f}{\partial \mathbf{y}_i} : \mathbf{H}_{\mathbf{y}_i}} \right) : \mathbf{D} = \mathbf{L} : \mathbf{D} \tag{3}$$

where $\mathbf{L}$ is the tangent modulus.

Modelling of a large class of materials is made possible by the use of this general formulation. In this way, it is for example possible to account for initial anisotropy, isotropic and kinematic hardening as well as damage phenomena. A more detailed description of these mechanisms will be given in the third section.

2. ANALYTICAL LOCALIZATION CRITERIA

As previously mentioned, the FLD is a widely used concept to characterize the formability of metallic sheets. These FLDs can either be determined by experimental tests (Nakazima and Marciniak tests are usually used), by numerical simulation or by the use of analytical criteria. As the aim of this paper is to give an overview of such analytical criteria and of their ability to predict FLDs, other techniques will not be considered.

Different analytical approaches have been proposed to predict both diffuse and localized necking, which occur during sheet metal forming processes. Some of the most important ones are considered in the sequel.

2.1 Maximum Force Criteria

These criteria are based on Considère's observation according to which the diffuse necking in a bar corresponds to the maximum of the applied load during the one dimensional tensile test [7]. This idea has been extended to the two dimensional case by Swift [1] and by Hill [2].

The hypothesis made by Swift to extend the model is that the diffuse necking corresponds to the simultaneous maximum of the two components of the load, which can be expressed by the following equations:

$$dF_1 = 0 \quad \text{and} \quad dF_2 = 0 \tag{4}$$

As the simultaneity of the maximum of loads is not usually observed, the most often used formulation of this criterion is:

$$dF_1 = 0 \tag{5}$$

This leads to the following expression, commonly known as the "Maximum Force Criterion" (MFC):

$$\frac{d\sigma_{11}}{d\varepsilon_{11}} = \sigma_{11} \tag{6}$$

In this criterion, no hypothesis is made on the form of the necking mode. Therefore it is a criterion predicting the diffuse necking. Hill's criterion [2] is based on the same principle, but other more restrictive conditions are introduced. In this formulation the necking is supposed to occur in a band with stationary extension (thus restraining the use of this criterion to the domain of negative minor strains). The resulting conditions on the possibility of occurrence of localization are the following:

$$\begin{aligned} &\tan^2(\theta) = -\beta \\ &dF_n = 0 \\ &\beta = \frac{\dot{\varepsilon}_2}{\dot{\varepsilon}_1} \end{aligned} \tag{7}$$

with θ the angle between the normal to the band and the direction of the major load, β the ratio of the minor and major strain rates and F_n the component of the force in the direction of the normal to the band. The criterion can be rewritten under the form:

$$\frac{d\sigma_n}{d\varepsilon_n} = \sigma_n \tag{8}$$

This criterion can be regarded as a special case of the previous one. This criterion is adapted to the prediction of the localized necking within a localized band in the stretching domain.

Another criterion was proposed by Hora et al. [3] to predict the localized necking in the left and right sides of the FLD. This criterion is a modification of the MFC to take into account the influence of strain or stress path changes during loading. With this additional effect, the criterion is called Modified Maximum Force Criterion (MMFC):

$$\frac{\partial \sigma_{11}}{\partial \varepsilon_{11}} + \frac{\partial \sigma_{11}}{\partial \beta}\frac{\partial \beta}{\partial \varepsilon_{11}} = \sigma_{11} \qquad (9)$$

The criterion is often used under the form:

$$\frac{\partial H}{\partial \bar{\varepsilon}} = \left(1 + \frac{f'(\alpha)g(\beta)}{\beta'(\alpha)\bar{\varepsilon}}\right)\frac{H}{g(\beta)} \qquad (10)$$

where H is the plastic hardening function, $\bar{\varepsilon}$ the equivalent plastic strain, α the ratio between σ_{22} and σ_{11} and $g(\beta)$ the ratio between $\bar{\varepsilon}$ and $\dot{\varepsilon}_{11}$.

As mentioned by Brunet et al. [8], the main drawback of this formulation is that it requires a local linearization to approximate the derivative of β:

$$\frac{\partial \beta}{\partial \varepsilon_{11}} = -\frac{\beta}{\varepsilon_{11}} \qquad (11)$$

The Extended Maximum Force Criterion (EMFC) is based on the equations of the MFC and allows the prediction of a localized necking mode in the left and right side of the FLD [9]. To take into account the evolution of the strain path after diffuse necking, the hypothesis of local verification of the maximum force principle is introduced. With this hypothesis, it is possible to recalculate the strain path for each loading increment. The calculus is stopped when the strain path is close to the plane strain state. It is however necessary to define arbitrarily the threshold. The authors propose:

$$\frac{\beta_0}{10} > \frac{\dot{\varepsilon}_2}{\dot{\varepsilon}_1} \qquad (12)$$

2.2 Marciniak-Kuczyński criterion

The Marciniak-Kuczyński model (M-K) is a criterion based on the semi empirical observation according to which the necking occurs at a defect of the structure [4]. In this criterion, a defect is introduced in the sheet metal. This defect can be a geometrical or a material defect, but it is often introduced in the structure as a band of reduced thickness. This defect area will be noted B and the homogeneous unaffected one A. The initial defect size f_0 can be arbitrarily defined as:

$$f_0 = \frac{t_0^B}{t_0^A} \qquad (13)$$

where t_0 is the initial thickness. The value of the initial defect is usually taken between 0.98 and 1. A major drawback of this approach is that the postulated initial imperfection has an influence on the FLD, while it does not have physical meaning. It can be viewed as an equivalent defect.

The basis of the M-K analysis is to compare the evolution of mechanical or geometrical properties of zones A and B. Plane stress and planar anisotropy are assumed. In this formulation the calculus of the mechanical states of zones A and B are performed separately. During the loading, components of stress and strain tensor are imposed to zone A, and the mechanical state is computed from equation (3). Equilibrium, compatibility equations and evolution of the defect f are then used to compute the mechanical state of zone B:

$$\begin{aligned} \sigma_{nn}^B t^B &= \sigma_{nn}^A t^A \\ \sigma_{nt}^B t^B &= \sigma_{nt}^A t^A \\ \dot{\varepsilon}_{tt}^B &= \dot{\varepsilon}_{tt}^A \\ f &= f_0 \exp\left(\varepsilon_{33}^B - \varepsilon_{33}^A\right) \end{aligned} \qquad (14)$$

where n and t denote the normal and the tangential direction to the band B. From equations (3) and (14) it is possible to determine the mechanical state of zone B.

Mechanical or geometrical properties of zones A and B are then compared. The choice of these mechanical properties has obviously an influence on the quality of the predicted FLD. A common choice is to use the ratio of major principal strain rates in zone B and A, but it is possible to choose the ratio of strain rates in the thickness direction or the ratio of equivalent strain rates:

$$\frac{\dot{\varepsilon}_{11}^{B}}{\dot{\varepsilon}_{11}^{A}} > S \qquad (15)$$

S is the threshold of the criterion (i.e. the critical value). Once inequality (15) is satisfied, the localization is predicted.
Hutchinson and Neale observed that the original formulation of the criterion leads to overestimated prediction of the FLD in the stretching domain [10]. To overcome this problem, the initial orientation of the band has to be taken into account. The orientation of the band has to be considered when the principal directions of the load and of the anisotropy are different, but this case will not be developed in this paper.
Three user defined parameters appear in this model and can be used as "fitting" parameters with little physical meaning. This is a major limitation of the method. The following criteria do not use such parameters.

2.3 Bifurcation analysis

Diffuse necking occurrence can be regarded as an evolution from a homogeneous deformation state to a heterogeneous one. Localized deformation occurrence can be seen as the transformation from a quasi homogeneous deformation mode to a localized one. Following this approach, a bifurcation analysis can be used to predict necking.
Drucker [11] and Hill [12] introduced a necessary condition for the loss of uniqueness of the solution of the boundary value problem for rate independent materials (without elastic unloading):

$$\int_{\Omega} \Delta\dot{\boldsymbol{\sigma}} : \Delta\dot{\boldsymbol{\varepsilon}} \, dV = 0 \qquad (16)$$

A local necessary condition can be written from the weak form of equation (16):

$$\dot{\boldsymbol{\varepsilon}} : \mathbf{H} : \dot{\boldsymbol{\varepsilon}} = 0 \quad \text{or} \quad \det\left(\mathbf{H}^{S}\right) = 0 \qquad (17)$$

where $\mathbf{H}^S$ is the symmetric part of the tangent modulus of the linear comparison solid, excluding possible elastic unloading. The second part of equation (17) is known as the general bifurcation condition and it is associated with the loss of positivity of the second order work.
The limit point bifurcation is known as a special case of the general bifurcation, associated with stationary stress state [13]:

$$\mathbf{H} : \dot{\boldsymbol{\varepsilon}} = 0 \quad \text{or} \quad \det\left(\mathbf{H}\right) = 0 \qquad (18)$$

Criteria (17) and (18) are generally associated with the prediction of possible occurrence of diffuse necking. Other criteria have been developed to predict localized necking.
The loss of strong ellipticity is another subset of general bifurcation [14]. In this criterion the possible discontinuous bifurcation modes must be kinematically compatible. With a kinematically compatible mode, this criterion can be written as follows:

$$\begin{aligned} &\mathbf{Q} = \mathbf{n}\mathbf{L}^{band}\mathbf{n} \\ &\det\left(\mathbf{Q}^{S}\right) = 0 \end{aligned} \qquad (19)$$

with $\mathbf{Q}$ the acoustic tensor, computed with the tangent modulus inside the band of localization $\mathbf{L}^{band}$ and the normal to the localization band $\mathbf{n}$.
The classical discontinuous bifurcation analysis is a subset of the previous criterion, with kinematically admissible modes and with identical tangent modulus inside and outside the band during the initiation of the localization:

$$\det\left(\mathbf{Q}\right) = 0 \qquad (20)$$

From the previous definitions, it is possible to notice that the limit point bifurcation and the loss of strong ellipticity will occur at the same time or after the general bifurcation and that the classical discontinuous bifurcation will occur at the same time or after the loss of strong ellipticity. In fact in the applications presented in the third part, with

associated plasticity and within small deformation formulation, on one hand limit point and general bifurcation and on the other hand loss of strong ellipticity and classical discontinuous bifurcation will occur at the same time.
More details can be found in [15].

2.4 Linear perturbation method

Localization phenomenon can be seen as instability of the local mechanical equilibrium [5]. The stability of equilibrium equations is evaluated by linearized perturbation theory.
The mechanical equilibrium equation may be written under the following form [16]:

$$\frac{\partial \mathbf{U}}{\partial t} = G\left(\mathbf{U}, \mu\right) \tag{21}$$

with **U** a mechanical state, *G* a non linear operator describing the equilibrium equations and μ the loading parameter.
The stability of this system is analysed on a finite period, during which the solution is supposed to exist, by looking at the evolution of the perturbed solution on this period:

$$\frac{\partial}{\partial t}\left(\mathbf{U}^0 + \delta\mathbf{U}\right) = G\left(\mathbf{U}^0, \mu\right) + \frac{\partial G}{\partial \mathbf{U}}\left(\mathbf{U}^0\right)\delta\mathbf{U}$$
$$\frac{\partial}{\partial t}\left(\delta\mathbf{U}\right) = \frac{\partial G}{\partial \mathbf{U}}\left(\mathbf{U}^0\right)\delta\mathbf{U} \tag{22}$$

Solutions of equation (22) may be written under the form:

$$\delta\mathbf{U} = \hat{\mathbf{U}}e^{\eta t} \tag{23}$$

and the system becomes:

$$\eta\hat{\mathbf{U}} = \frac{\partial G}{\partial \mathbf{U}}\left(\mathbf{U}^0\right)\hat{\mathbf{U}} \tag{24}$$

Instability analysis of equilibrium equations leads to the study of positive values of the real part of the eigenvalues η.
For the computation of FLDs a formulation with rigid-plastic orthotropic material assumption has been used (Lejeune et al. [17]).

3. APPLICATION OF LOCALIZATION CRITERIA

Various principles and formulations have been developed to predict formability of metal sheets. In this part, the above-mentioned localization criteria will be applied to different materials and their ability to predict forming limit curves will be discussed.

3.1 Materials properties

Hypo-elastic law with kinematic and isotropic hardening models is used to describe the behaviour of the aluminium alloy and steels used in the FLD simulation.
Two hardening laws are considered to describe the isotropic hardening, Swift equation (25) and Voce equation (26):

$$\dot{R} = nk\left(\varepsilon_0 + \bar{\varepsilon}_p\right)^{n-1}\dot{\lambda} = H_R\dot{\lambda} \tag{25}$$
$$\dot{R} = C_R\left(R_{sat} - R\right)\dot{\lambda} = H_R\dot{\lambda} \tag{26}$$

where *R* is the isotropic hardening variable and *n*, *k*, ε_0, C_R and R_{sat} are material parameters. Rate-sensitivity is introduced by coupling isotropic hardening and strain rate:

$$\dot{R} = nk\left(\varepsilon_0 + \bar{\varepsilon}_p\right)^{n-1}\left(\frac{\dot{\bar{\varepsilon}}}{\dot{\bar{\varepsilon}}_0}\right)^m \dot{\lambda} = H_R\dot{\lambda} \tag{27}$$

$\dot{\bar{\varepsilon}}_0$ and *m* are material parameters.
The yield surface is described by the von Mises isotropic criterion (28) or by the Hill'48 anisotropic criterion (29):

$$\bar{\sigma}_{\text{von Mises}} = \sqrt{\frac{3}{2}\left(\boldsymbol{\sigma}' - \mathbf{X}\right):\left(\boldsymbol{\sigma}' - \mathbf{X}\right)} \tag{28}$$

$$\bar{\sigma}_{\text{Hill'48}} = \sqrt{\left(\boldsymbol{\sigma}' - \mathbf{X}\right):\mathbf{M}:\left(\boldsymbol{\sigma}' - \mathbf{X}\right)} \tag{29}$$

where **M** is a fourth-order tensor containing the six anisotropy coefficients of Hill'48 criterion, which can be expressed with Lankford's coefficients r_0, r_{45} and r_{90}.
For the criteria based on bifurcation analysis, softening is necessary. In the current work, this effect is introduced by the cou-

pling of the previous equations with isotropic damage:

$$\dot{d} = \begin{cases} \dfrac{1}{(1-d)^{\beta}} \left(\dfrac{Y_e - Y_{ei}}{S} \right)^s \dot{\lambda} & \text{if } Y_e \geq Y_{ei} \\ 0 & \text{otherwise} \end{cases} \quad (30)$$

where d is the damage variable, Y_e and Y_{ei} are respectively the elastic energy and the corresponding threshold, β, S and s are material parameters.

3.2 Application of localization criteria to FLDs

In this section, FLDs obtained from numerical simulation of localization criteria are presented for different materials including an aluminium alloy, a brass and a dual phase steel.

The mechanical time-independent behaviour of the aluminium alloy is described with Hill'48 anisotropic yield criterion and the Swift isotropic hardening law.

Table 1. Material parameters of the aluminium alloy.

k(MPa)	n	ε_0	r_0	r_{45}	r_{90}
580	0.2	0.004	1	1.5	2

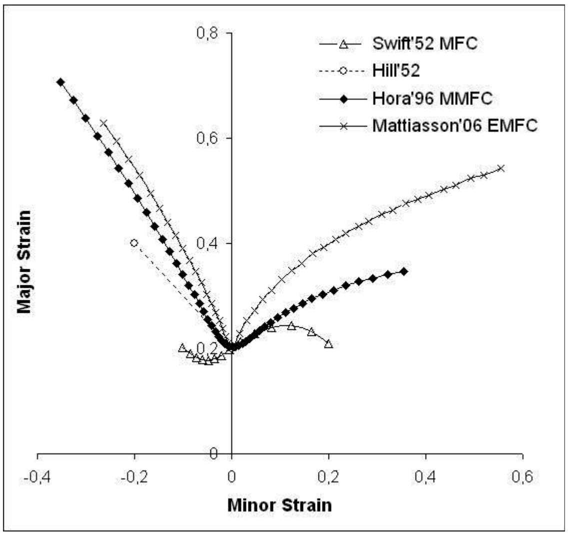

Figure 1. FLDs of the aluminium alloy.

In Figure 1, FLDs of the aluminium alloy modelled with isotropic hardening are obtained from criteria based on the maximum force principle.

Numerical simulation allows classifying the criteria to the order of necking prediction. Swift'52 MFC is intended to predict diffuse necking and then yields lower analytical FLD than the other criteria. It is then not suitable for the prediction of localized necking. Hill'52 criterion predicts localized necking in the left-hand side of the FLD. Hora'96 MMFC gives higher FLD than Swift'52 and Hill'52 criteria in both sides. This criterion is generally in good accordance with experimental data. Mattiasson'06 EMFC gives higher predictions than the MMFC and seems in this application to overestimate the formability of this alloy.

The second application is based on the study of a brass. On the diagram in Figure 2, FLDs obtained from M-K, MMFC and perturbation methods are presented.

Table 2. Parameters of the brass material [17].

k(Mpa)	N	ε_0	m
618.3	0.118	0.014	0
r_0	r_{45}	r_{90}	
1.8	1.3	2.0	

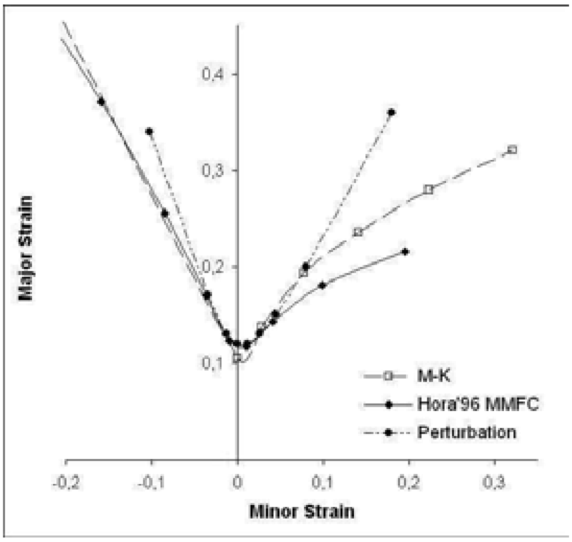

Figure 2. FLDs of the brass.

The M-K criterion tends to give lower FLD prediction than the other criteria near plane tension and higher near equibiaxial tension. This trend may however be controlled by user parameters (initial defect and critical value) to fit experimental data.

The perturbation method gives unrealistic predictions with the 2D criterion, but a 3D formulation of this criterion, not represented here, seems to give good prediction of FLD [17].
The third application is made on a dual phase steel. The behaviour is described by Hill'48 anisotropic yield criterion, Voce law for isotropic hardening and coupling with isotropic damage law.
The use of time-independent materials with softening behaviour is necessary for criteria based on bifurcation analysis. The softening behaviour can be obtained by the use of a non-associative flow rule or here by the use of damage. Numerical simulations confirm that classical discontinuous bifurcation and loss of strong ellipticity give the same result within the assumptions made in this formulation. Low bifurcation modes are predicted with general bifurcation and limit point bifurcation, these criteria do not seem to be adapted to the prediction of FLDs (Figure 3).

Table 3. Material parameters of the dual phase steel [18].

R_{sat}	C_r	r_0	r_{45}	r_{90}
551.4	9.3	1	1.5	2
B	S	s	Y_{ei}	
5	20	0.1	0	

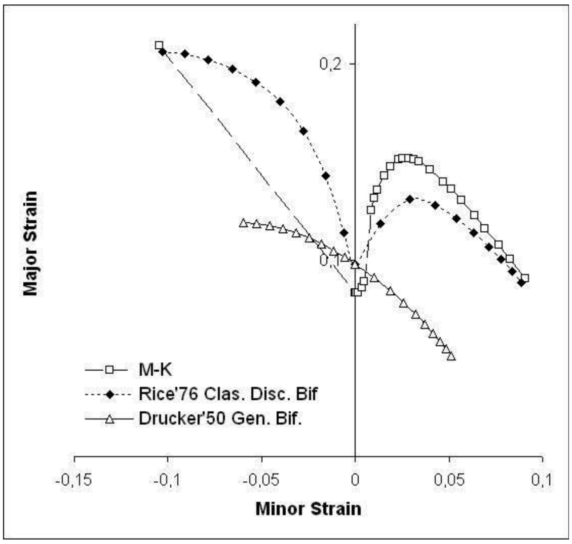

Figure 3. FLDs of the dual phase steel.

Close results are obtained with the use of M-K model and classical discontinuous bifurcation analysis. However M-K model gives lower FLD prediction near plane tension and then higher in the right-hand side of the FLD, which is in accordance with previous observations.

CONCLUSION

In this paper, theoretical comparisons of localization criteria are performed and allow classifying the criteria of the same group by order of localization prediction. Classification of criteria from different groups is made possible by additional numerical comparisons. Three materials with phenomenological material modelling are used for these numerical comparisons.
From this study, it can be seen that numerical FLDs strongly depend on the choice of both the localization criterion and the material model. Advanced material modelling, relevant localization criteria as well as better knowledge of localization mode seem necessary to accurately predict FLDs.
Further developments are concerned with the use of physically-based constitutive laws in order to improve the prediction of FLDs, the comparison of theoretical and numerical results with experimental data as well as the implementation of the most appropriate criteria into a FEM code for the prediction of localization in deep drawing processes.

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