Inductances and back-emf harmonics influence on the Torque/Speed characteristic of five-phase SPM machine

Franck Scuiller
Research Institute of Naval Academy
BCRM BREST - EN / GEP
CC 600 - 29240 BREST Cedex 9 - FRANCE
franck.scuiller@ecole-navale.fr

Eric Semail
Laboratory of Electrical Engineering
and Power Electronics of Lille (L2EP)
Arts et Métiers ParisTech, LILLE 59043 - FRANCE
eric.semail@ensam.eu

***Abstract*—In this paper, the study of torque/speed characteristic for five-phase Surface-mounted Permanent Magnet (SPM) machine is carried out. With considering several hypothesis (linear magnetic modelling, only first and third harmonic terms in the back-emf and current spectrums), an optimization problem that aims to maximize the torque for given maximum voltage and RMS current is formulated: the optimal torque sharing among the two virtual machines (the two dq-axis subspaces) that represent the real five-phase machine is thus calculated for any mechanical speed. For an inverter and a DC voltage sized with only considering the first harmonic of back-emf and current, the problem is solved with changing the ratio between the two virtual machine back-emfs and changing the ratio between the two virtual machine inductances. The results are examined by introducing particular speed points.**

NOMEMCLATURE

FW	Flux Weakening
MM	Main Machine (1st harmonic dq-subspace)
SM	Secondary Machine (3rd harmonic dq-subspace)
Ω_m	Mechanical speed (rad/s)
p	Pole pair number
R	Armature resistance
ϵ_1, ϵ_3	MM and SM no load back-emf ($\Omega_m = 1rad/s$)
L_1, L_3	MM and SM cyclic inductances
θ_1, θ_3	MM and SM back-emf to current angles
I_1, I_3	MM and SM currents
V_b, I_b	Base RMS voltage and current
Ω_b, T_b	Base speed and torque

I. INTRODUCTION

In electric vehicle application, high power density and fault tolerant capability are commonly required for the machine drive. Furthermore, a wide speed range capability is often wanted thus making the machine operating in the flux weakening region. All these constraints have to be satisfied with a low DC bus voltage. This context favours the use of multi-phase PM machines [1].

Numerous papers deal with the FW operation of three-phase PM machines fed by voltage source inverter. For instance, in [2], with considering classical dq-circuit model of three-phase PM machine (with or without saliency), the authors analytically determine the torque/speed characteristic for the whole speed range. In [3], the authors focus on the winding design influence on the FW ability. A similar analysis is undertaken in [4] for Surface-mounted PM machines (SPM).

On the contrary, few papers address the FW operation of multi-phase PM machine. This is mainly because five-phase drives are not used to the same extent as their three-phase counterparts. Another justification probably result from the fact that a multi-phase machine behaves as several dq-circuit machines (or dq subspaces), thus making difficult the analytical computation of the currents in FW mode. In addition, among the few papers analysing the FW ability of multi-phase PM machines, most of them focus on the control side for a given machine and for a particular speed point [5], [6], which restricts highly the applicability of the results to other multi-phase machines. It should be noted that, for three-phase PM machine, the per unit system used in [2] gives results naturally applicable for any machine, which is particularly useful for the designer.

Actually, in order to study the FW mode of multi-phase machine, a numerical approach is necessary: an optimization problem has to be formulated and solved to determine the currents sharing among the dq subspaces at a given speed for a given DC voltage [1], [7]. For given optimization problem and numerical method, the results will depend on the way to represent the machine and the inverter. The more accurate approach consists in modelling the machine with Finite Element Analysis and the inverter with time differential equations to account the commutations. Such an approach is hard to implement and computionally time-consuming. Another possibility consists in using the equivalent multi dq-circuit model for the multi-phase SPM machine [8] and an average model for the inverter. The steady-state torque/speed characteristic can thereby be estimated with considering a quite reduced number of parameters, which provides more applicable results for the designer. In practical terms, with the proposed approach, the designer can predict the change in the Torque/Speed characteristic when acting on the winding distribution and the magnet layer design.

II. Five-phase Machine Modelling

A. Hypothesis

If the magnetic saturations and the demagnetization issue are not considered, it can be shown that a star-connected five-phase SPM machine behaves as two two-phase virtual machines that are magnetically independent but electrically and mechanically coupled [9]. Furthermore, as the rotor saliency can be neglected with SPM machines, the space harmonics are distributed among the two virtual machines: the virtual machine sensitive to the fundamental is called Main Machine (MM) whereas the other sensitive to the third harmonic is called Secondary Machine (SM). Actually the virtual machine is a physical reading of the mathematical subspace usually represented with dq-axis circuit. As there is no saliency effect, no distinction has to be made between d-axis and q-axis inductance. Additional hypothesis will be taken:

- only 1st and 3rd space and time harmonics are considered thus meaning that each virtual machine owns a sinusoidal back-emf and is supplied with sinus current
- the machine has a low armature reaction which implies that the speed range under FW control is finite [2].

B. MM contribution to the phase voltage and the torque

If only first time and space harmonics are considered, the Main Machine contribution to one of the five real phase voltages is:

$$\begin{aligned} v_{MM}(t) = \quad & \Omega_m \epsilon_1 \sqrt{2} \sin(p\Omega_m t) \\ & + R I_1 \sqrt{2} \sin(p\Omega_m t + \theta_1) \\ & + p\Omega_m L_1 I_1 \sqrt{2} \sin\left(p\Omega_m t + \theta_1 + \frac{\pi}{2}\right) \end{aligned} \quad (1)$$

By isolating the electromagnetic power in relation (1), Main Machine contribution to the average torque can be estimated:

$$T_{MM} = 5\epsilon_1 I_1 \cos\theta_1 \quad (2)$$

C. SM contribution to the phase voltage and the torque

The MM contribution to the phase voltage is simply obtained by replacing subscribe 1 by subscribe 3 in relation (1) and changing the pole pair number:

$$\begin{aligned} v_{SM}(t) = \quad & \Omega_m \epsilon_3 \sqrt{2} \sin(3p\Omega_m t) \\ & + sgn(\epsilon_3) R I_3 \sqrt{2} \sin(3p\Omega_m t + \theta_3) \\ & + sgn(\epsilon_3) 3p\Omega_m L_3 I_3 \sqrt{2} \sin\left(3p\Omega_m t + \theta_3 + \frac{\pi}{2}\right) \end{aligned} \quad (3)$$

In the previous equation, sgn is the sign function. One must bear in mind that MM and SM are mechanically coupled. This property implies that MM d-axis and SM d-axis are always superimposed but not necessarily in the same direction: ϵ_3 and ϵ_1 can be in phase or in opposition. The chosen sinus description of the voltage allows to simply take into account the two possibilities by assuming that ϵ_1 is a positive number and ϵ_3 is a signed number: if ϵ_3 is positive, ϵ_3 and ϵ_1 are in phase; else, ϵ_3 and ϵ_1 are in opposition.

Secondary Machine contribution to the torque is similarly calculated by considering relation (2):

$$T_{SM} = 5\epsilon_3 I_3 \cos\theta_3 \quad (4)$$

D. Total phase voltage and electromagnetic torque

The phase voltage equation with first harmonic hypothesis for each virtual machine (with considering only the first and third time and space harmonic for the five-phase machine) is expressed from equations (1) and (3):

$$v(t) = v_{MM}(t) + v_{SM}(t) \quad (5)$$

The average electromagnetic torque is the sum of the torque of each virtual machine (given by (2) and (4)):

$$T = 5\epsilon_1 I_1 \cos\theta_1 + 5\epsilon_3 I_3 \cos\theta_3 \quad (6)$$

III. Torque/Speed Characteristic Estimation

A. Base point choice

As for a three-phase machine, it is considered that the base quantities of the five-phase machine are obtained with a perfect sinusoidal supply of the Main Machine because the Main Machine usually determines the real machine pole pair number. Therefore it is supposed that the five-phase machine can produce the base torque T_b as long as the mechanical speed is lower than the base mechanical speed Ω_b. Below the base speed, the machine is controlled according to Maximum Torque Per Ampere (MTPA) strategy. The corresponding base electrical speed is:

$$\omega_b = p\Omega_b \quad (7)$$

The base current is calculated from relation (2) in MTPA mode ($\theta_1 = 0$):

$$I_b = \frac{T_b}{5\epsilon_1} \quad (8)$$

As illustrated by figure 1, the base voltage is the voltage obtained at the base point (base current and MTPA mode):

$$V_b = \sqrt{(\epsilon_1\Omega_b + RI_b)^2 + (L_1 p\Omega_b I_b)^2} \quad (9)$$

Base armature resistance and inductance are classically defined:

$$\begin{cases} x_1 = \dfrac{L_1\omega_b}{V_b} \\ r = \dfrac{RI_b}{V_b} \end{cases} \quad (10)$$

Low armature reaction hypothesis means that x_1 is lower than 0.707 [2] (if armature resistance is neglected). The base voltage value is used to choose the DC bus voltage for the inverter.

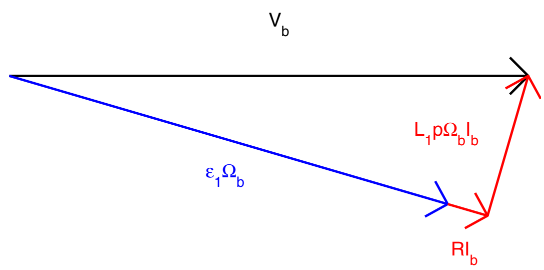

Fig. 1. Voltage vectorial diagram for the base point

B. Optimization problem

For a given mechanical speed Ω_m (i.e. for a given electrical speed ω), taking into account the maximum DC voltage (driven by base voltage V_b) and the maximum copper losses (driven by base current I_b), the goal consists in finding the MM and SM current distribution that maximizes the electromagnetic torque. To solve this problem is equivalent to find the optimal d-axis and q-axis references for each virtual machine. The optimization variable is defined as follows:

$$x = \begin{bmatrix} I_1 & \theta_1 & I_3 & \theta_3 \end{bmatrix}^T \qquad (11)$$

The optimization variable is lower and upper bounded according to the following relations:

$$X_{low} = \begin{bmatrix} 0 \\ -\pi \\ 0 \\ -\pi \end{bmatrix} \leq x \leq \begin{bmatrix} I_b \\ \pi \\ I_b \\ \pi \end{bmatrix} = X_{up} \qquad (12)$$

The objective is to maximize the electromagnetic torque. This goal is expressed in the following relation where the electromagnetic torque can be calculated considering equation (6):

$$x^* = argmin(-T(x)) \qquad (13)$$

The non linear constraint regarding the peak phase voltage is written in the following relation (where the voltage is given by relation (5)):

$$f_V(x) = \max\{v(p\Omega_m t, x), p\Omega_m t \in [0..2\pi]\} - V_{peak} \qquad (14)$$

The constraint relative to the maximum RMS current is quadratic and is defined by the following equation:

$$f_I(x) = x(1)^2 + x(3)^2 - I_b^2 \qquad (15)$$

The following expression summarizes the optimization problem under consideration:

$$\begin{aligned} x^* &= argmin(-T(x)) \\ \text{with} & \begin{cases} X_{low} \leq x \leq X_{up} \\ f_V(x) \leq 0 \\ f_I(x) \leq 0 \end{cases} \end{aligned} \qquad (16)$$

The choice of V_{peak} greatly influences the optimization results. In this study, the peak voltage is set with considering the base voltage V_b:

$$V_{peak} = \sqrt{2} V_b \qquad (17)$$

Relation (17) simply means that the DC bus voltage is sized for a sinusoidal supply of the Main Machine (if base voltage V_b is determined as explained in susbection III-A). The optimization problem is written such as the five-phase machine never operates with a modulation signal whose magnitude does not comply with the DC bus voltage (linear modulation operation).

C. Example

Figure 2 shows the optimized torque/speed characteristic when considering a five-phase machine with $\epsilon_3/\epsilon_1 = 0.3$ and $L_3/L_1 = 0.5$. Below the base speed, the results are analytically predictable: the virtual machine torque sharing and the torque increase depends on ϵ_3/ϵ_1 ratio [8]. Above the base speed, the numerical approach allows to determine the optimal torque sharing among the two virtual machines to maximize the torque with regard to the current and maximum voltage constraints: it is interesting to note that, when beginning the FW operation (between 1 and 1.2 p.u. speed), the SM contribution to the torque is increased. This considered example

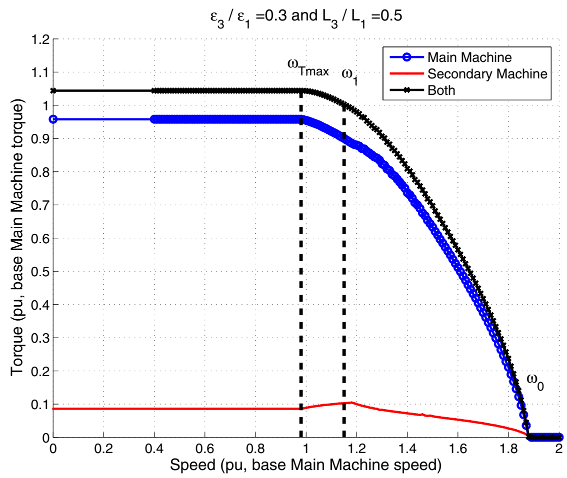

Fig. 2. Torque/Speed characteristics ($x_1 = 0.28$ and $r = 0.08$)

illustrates how the SM affects the Torque/Speed characteric in FW operation. However the observed results are not applicable to any five-phase machine. In the following part, a procedure to obtain general results is detailled.

IV. Influence of SM machine inductance and back-EMF amplitude on the Torque/Speed characteristic

A. Objectives

The optimization problem can be solved for a set of 5-phase SMPM machine with the same base values (same base voltage V_b, base current I_b, base mechanical speed Ω_b and base torque T_b), thus meaning that the Main Machine inductance L_1 and elementary back-emf ϵ_1 are invariant (according to the definition of base point given in subsection III-A). Therefore, the optimized torque/speed characteristic only depends on the Secondary Machine inductance L_3 and elementary back-emf ϵ_3. The influence of the SM parameters on the torque/speed characteristic can then be studied just by varying the ratio between L_3 and L_1 and the ratio between ϵ_3 and ϵ_1, respectively called inductance ratio and back-emf ratio [10], [11].

Such an approach means that it is possible to design a machine where the inductance ratio L_3/L_1 and the back-emf ratio ϵ_3/ϵ_1 are not correlated. The inductance ratio is mainly determined by the winding distribution and the slot shape whereas the no load back-emf mainly depends on the winding distribution and magnet layer properties (magnet shape, magnetization orientation): for a given winding, it is then theoretically possible to design a magnet layer to obtain the wanted back-emf ratio.

B. Torque/Speed characteristics particular points

To facilitate the results analysis when solving problem (16) for numerous values of inductance and back-emf ratios, three particular speed points are defined from the Torque/Speed characteristic:

- ω_{Tmax} is the highest speed where the maximum torque can be obtained

$$\omega_{Tmax} = \max\left\{\frac{\omega}{\omega_b}, T(\omega) = T_{max}\right\} \tag{18}$$

- ω_1 is the highest speed where the torque is higher than the base torque

$$\omega_1 = \max\left\{\frac{\omega}{\omega_b}, T(\omega) = T_b\right\} \tag{19}$$

- ω_0 is the lowest speed where the torque is zero (theoretical maximum reachable speed taking into account machine and inverter limitations)

$$\omega_0 = \min\left\{\frac{\omega}{\omega_b}, T(\omega) = 0\right\} \tag{20}$$

If the example of figure 2 is considered, ω_{Tmax}, ω_1 and ω_0 are respectively equal to 0.98, 1.15 and 1.89.

One must bear in mind that, when changing back-emf and inductance ratios, base torque T_b, base current I_b and base speed ω_b do not change: these three base quantities are defined for the common sinus control of the 5-phase machine which, according to the multimachine approach, means that only the Main Machine contributes to the torque. When the third harmonic term is not null, at low speed, the torque can be increased with controlling the Secondary Machine to provide torque (without increasing base RMS current I_b). The resulting maximum torque can be explicitly calculated:

$$T_{max} = T_b\sqrt{1 + \left(\frac{\epsilon_3}{\epsilon_1}\right)^2} \tag{21}$$

On the contrary, by considering optimization problem (16), it seems difficult to analytically estimate the speed ω_{Tmax} that corresponds to the highest speed where maximum torque T_{max} can be maintained. For the same reasons, a numerical approach seems necessary to calculate ω_1 and ω_0. If the back-emf is perfectly sinewave (no third harmonic term), ω_1 equals ω_{Tmax}.

C. Results

It is decided to solve optimization problem (16) when varying the back-emf ratio from -1 to 1 in three cases:

- a case where the inductance ratio is low ($L_3/L_1 = 0.5$); such a ratio can be obtained in case of integral-slot winding
- a case where the inductance ratio is one ($L_3/L_1 = 1$); such a ratio can be obtained in case of fractional-slot winding (and particularly concentrated fractional-slot winding)
- a case where the inductance ratio is high ($L_3/L_1 = 1.5$); such a ratio is possibly obtained for particular 5-phase machines [12], [13].

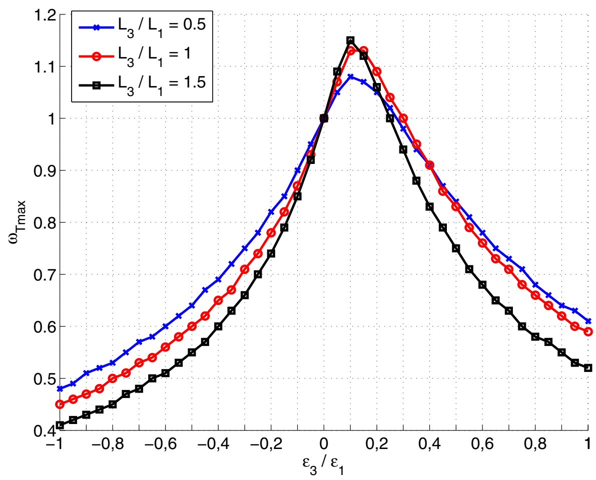

Fig. 3. Speed ω_{Tmax} with solving optimization problem (16) ($x_1 = 0.28$ and $r = 0.08$)

Figure 3 shows the obtained ω_{Tmax} curve according to the back-emf ratio for the three considered inductance ratios. This figure illustrates a quite predictable result: in MTPA mode (operated with the base current), for a given back-emf ratio, when the inductance ratio of the machine increases, the armature reaction is higher. Consequently the voltage inverter will saturate for a speed as low as the absolute value of the back-emf is large. However it should be noted that this property is not true for low positive value of the back-emf ratio (between 0 and 0.3): in this range, the third harmonic current (who generates an armature reaction as high as the inductance ratio is large) allows a better use of the DC bus voltage. In this case, it should be highlighted that this improvement comes with an increase of the torque which can be an interesting feature for the drive.

Figure 4 shows the obtained ω_1 curve according to the back-emf ratio for the three considered inductance ratios. Whatever the back-emf ratio is, ω_1 is as high as the inductance ratio is large. When reducing the back-emf ratio (for negative values), ω_1 is lower than the reference base speed ω_b and is decreasing. Furthermore, it can be supposed that, when inductance ratio increases, the maximum of ω_1 tends toward a finite value obtained for a particular back-emf ratio near $1/3$.

Figure 5 shows the obtained ω_0 curve according to the back-emf ratio for the three considered inductance ratios. One can observe that the ω_0 curve is smooth for the low inductance ratio but disturbed for the high inductance ratio. Actually the disturbance seems all the stronger as the inductance ratio is large. It can be inferred that this disturbance is not physical

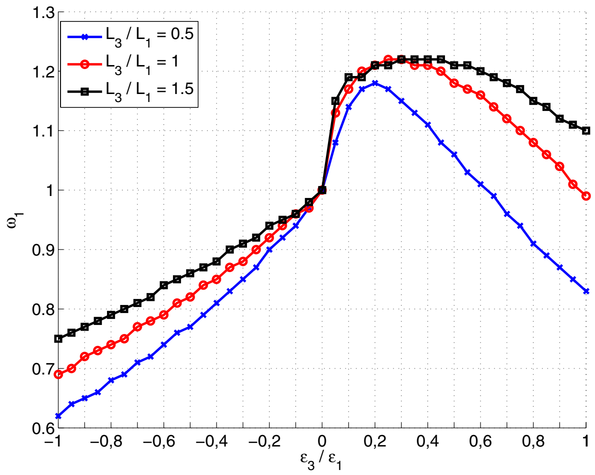

Fig. 4. Speed ω_1 with solving optimization problem (16) ($x_1 = 0.28$ and $r = 0.08$)

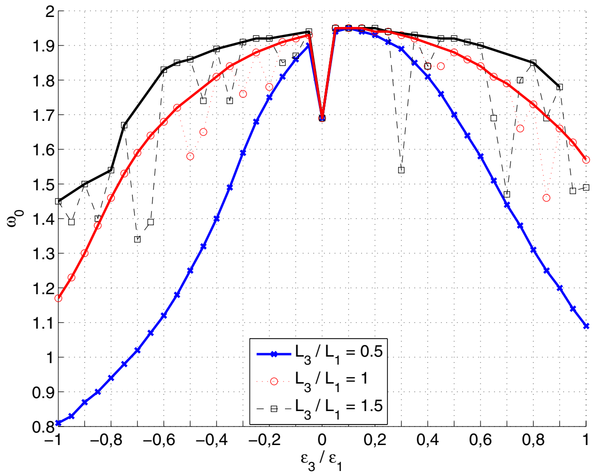

Fig. 5. Maximum reachable speed ω_0 with solving optimization problem (16) ($x_1 = 0.28$ and $r = 0.08$)

but numerical: it probably results from the optimization algorithm and parameters settings. Indeed, in the three cases, it seems possible to interpolate the ω_0 curve to obtain a smooth evolution: the resulting interpolations are drawn in figure 5. Therefore it clearly appears that ω_0 increases with the inductance ratio when the absolute value of the back-emf ratio is large. For instance, if the back-emf ratio is 0.8, ω_0 goes from 1.31 to 1.73 to 1.85 when the inductance ratio increases from 0.5 to 1 to 1.5.

The considered optimization problem (see relation (16)) constrains the RMS current (that drives the copper losses) but not the peak current whereas this value is a key parameter to size the inverter. By analyzing each optimal torque/speed characteristic (each computed (ϵ_3/ϵ_1, L_3/L_1) point), the maximum peak current change according to the back-emf ratio obtained for the three inductance ratios can be drawn. The results, reported in Figure 6, show that the peak current is never higher than 1.4 time the peak base current, which is acceptable.

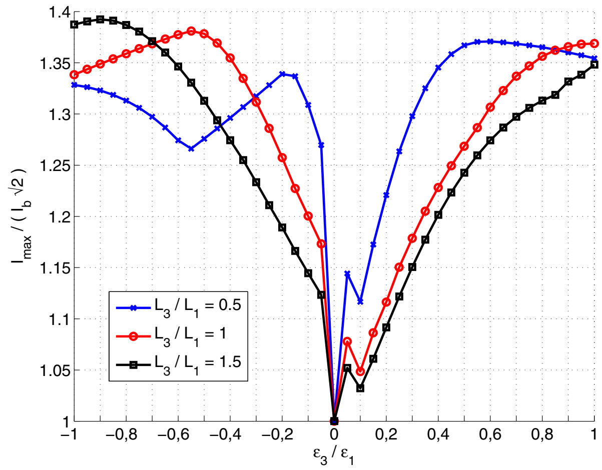

Fig. 6. Maximum peak current according to ϵ_3/ϵ_1

V. Conclusion

In this paper, the study of torque/speed characteristic for five-phase SPM machine is carried out. This analysis is realized with considering several hypotheses: linear magnetic modelling, star-connected machine, low armature reaction, back-emf and machine current only contains first and third harmonic terms. For an inverter and a DC voltage sized with only considering the first harmonic of back-emf and current (the virtual MM), an optimization problem that aims to maximize the torque for given maximum voltage and RMS current at a given mechanical speed is formulated. This problem is solved for several back-emf and inductance ratios. Some interesting conclusions can be drawn:

- the maximum speed where the maximum torque can be maintained is increased for positive back-emf ratio
- the maximum speed where the torque is higher than the base torque is higher than the base speed if the back-emf ratio is positive; and this maximum speed is all the higher as the inductance ratio L_3/L_1 is larger
- whatever the back-emf ratio is, a large inductance ratio allows to extend the speed range (the maximum reachable speed is increased).

The given results should be applicable to any five-phase SPM machines if the previously mentioned hypothesis can be done. However it should be highlighted that other results could be obtained if more back-emf harmonics are considered when calculating the torque/speed characteristic. For example, one can suppose that the fifth harmonic could help the machine to operate in FW mode. Furthermore the linear magnetic modelling is all the less acceptable as the speed is high thus making the here given results less useable. It is also necessary to underline that the way to size the inverter DC voltage in this paper is all the less justified as the back-emf ratio increases.

REFERENCES

[1] L. Lu, B. Aslan, L. Kobylanski, P. Sandulescu, F. Meinguet, X. Kestelyn, and E. Semail, “Computation of optimal current references for flux-weakening of multi-phase synchronous machines,” in *IECON 2012 - 38th Annual Conference on IEEE Industrial Electronics Society*, Oct 2012, pp. 3610–3615.

[2] R. Schiferl and T. Lipo, “Power capability of salient pole permanent magnet synchronous motors in variable speed drive applications,” in *Industry Applications Society Annual Meeting, 1988., Conference Record of the 1988 IEEE*, Oct 1988, pp. 23–31 vol.1.

[3] F. Magnussen, P. Thelin, and C. Sadarangani, “Performance evaluation of permanent magnet synchronous machines with concentrated and distributed windings including the effect of field-weakening,” in *Power Electronics, Machines and Drives, 2004. (PEMD 2004). Second International Conference on (Conf. Publ. No. 498)*, vol. 2, March 2004, pp. 679–685 Vol.2.

[4] A. M. El-Refaie, T. M. Jahns, and D. W. Novotny, “Analysis of surface permanent magnet machines with fractional-slot concentrated windings,” *IEEE Transactions on Energy conversion*, vol. 21, no. 1, pp. 34–43, March 2006.

[5] L. Parsa, N. Kim, and H. Toliyat, “Field weakening operation of high torque density five-phase permanent magnet motor drives,” in *Electric Machines and Drives, 2005 IEEE International Conference on*, May 2005, pp. 1507–1512.

[6] S. Xuelei, W. Xuhui, and C. Wei, “Research on field-weakening control of multiphase permanent magnet synchronous motor,” in *Electrical Machines and Systems (ICEMS), 2011 International Conference on*, Aug 2011, pp. 1–5.

[7] J. Gong, B. Aslan, F. Gillon, and E. Semail, “High-speed functionality optimization of five-phase PM machine using third harmonic current,” *COMPEL*, vol. 33, no. 3, pp. 879–893, April 2014.

[8] X. Kestelyn and E. Semail, “A vectorial approach for generation of optimal current references for multiphase permanent-magnet synchronous machines in real time,” *Industrial Electronics, IEEE Transactions on*, vol. 58, no. 11, pp. 5057 –5065, nov. 2011.

[9] E. Semail, A. Bouscayrol, and J.-P. Hautier, “Vectorial formalism for analysis and design of polyphase synchronous machines,” *Eur. Phys. J.*, vol. AP 22, pp. 207–220, 2003.

[10] F. Scuiller, E. Semail, J.-F. Charpentier, and P. Letellier, “Multi-criteria based design approach of multiphase permanent magnet low speed synchronous machines,” *IET Electric Power Applications*, vol. 3, no. 2, pp. 102–110, 2009.

[11] F. Scuiller, “Magnet shape optimization to reduce pulsating torque for a five-phase permanent-magnet low-speed machine,” *Magnetics, IEEE Transactions on*, vol. 50, no. 4, pp. 1–9, April 2014.

[12] B. Aslan and E. Semail, “New 5-phase concentrated winding machine with bi-harmonic rotor for automotive application,” in *International Congress on Electrical Machines, 2014*, September 2014.

[13] H. Zahr, E. Semail, and F. Scuiller, “Five-phase version of 12slots/8poles three-phase synchronous machine for marine-propulsion,” in *Vehicle Power and Propulsion Conference International, 2014 IEEE*, October 2014.