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New Correlations for Thermal Resistances of Vertical Single U-Tube Ground Heat Exchanger

A new 2D numerical model of a single U-tube ground heat exchanger is proposed and a four-thermal-resistance model is adopted to evaluate the effective pipe-to-borehole, pipe-to-pipe, and borehole-to-borehole thermal resistances. The influence of temperature distributions on both borehole surface and outer diameter of two pipes to these thermal resistances has been thoroughly studied. The best-fit correlations of effective pipe-to-borehole, pipe-to-pipe, and borehole-to-borehole thermal resistances are proposed and compared with the available equations in the literature. It is found that the present correlations of thermal resistances for ground heat exchanger are more accurate than those of available formulas. Furthermore, based on these obtained thermal resistance correlations, an analytical model is proposed to evaluate the heat transfer performance of the ground heat changer. [DOI: 10.1115/1.4006516]

1 Introduction

Ground-source heat pump (GSHP) uses the earth as a heat source or heat sink to extract or reject the thermal energy. Since the annual temperature fluctuation of soil under the ground is relatively small, the GSHP system has been recognized as one of the most energy efficient systems for space heating and cooling in residential and commercial buildings. In a GSHP system, one of the most important components is the ground-coupled heat exchanger through which the thermal energy is exchanged between heat carrier fluid (i.e., water or water-antifreeze fluid) and soil. Since the ground heat exchanger is responsible for a major part of the initial cost of GSHP system and the efficiency of this system depends on the performance of ground heat exchanger, a careful design of ground heat exchanger is crucial for a successful application of GSHP system [1].

For a typical single U-tube ground heat exchanger (as shown in Fig. 1), a U-tube is vertically and symmetrically inserted in a borehole and the gap between pipes and borehole is filled by grout material. A heat carrier fluid is circulated in the U-tube and heat is exchanged between carrier fluid and soil through pipes and grout within the borehole.

Since the pipe-to-borehole thermal resistance (i.e., R_b), which is defined as a thermal resistance between the outer diameter of the pipes and the borehole surface for a unit length of ground heat exchanger, plays a dominant role to size the ground heat exchanger, some analytical and numerical models were proposed to estimate it based on the 2D heat conduction problem with different geometrical parameters and thermal properties of soil and grout. As shown in Fig. 1, the geometrical parameters of a single U-tube ground heat exchanger could be described by borehole diameter, D_b, outer diameter of the pipe, D_p, and shank spacing, S. The thermal conductivities of soil and grout are k_s and k_g, respectively. Therefore, four dimensionless variables of ground heat exchanger could be defined as follows:

$$\theta_1 = \frac{S}{D_b} \tag{1}$$

$$\theta_2 = \frac{D_b}{D_p} \tag{2}$$

$$\theta_3 = \frac{D_p}{2S} = \frac{1}{2\theta_1 \cdot \theta_2} \tag{3}$$

$$\sigma = \frac{k_g - k_s}{k_g + k_s} \tag{4}$$

Shonder and Beck [2] simplified the complicated geometrical parameters of ground heat exchanger and treated the U-tube as a single pipe with an equivalent diameter which has the same total area on the horizontal cross section as that of the U-tube. Therefore, the complex geometry of borehole is simplified as a coaxial pipe and the effective pipe-to-borehole thermal resistance was given as

$$R_b = \frac{1}{2\pi \cdot k_g} \ln\left(\frac{\theta_2}{\sqrt{n}}\right) \tag{5}$$

where n is the number of pipes on the horizontal cross section within the borehole and k_g is the thermal conductivity of grout material. Since this simple model neglects the thermal interference resistance (i.e., R_{12}) between the pipes, the pipe-to-borehole thermal resistance of Eq. (5) is not the function of shank spacing, i.e., S.

Sharqawy et al. [3] developed a 2D numerical model that assumes steady-state heat conduction within the borehole. The different geometries of U-tube ground heat exchanger and thermal

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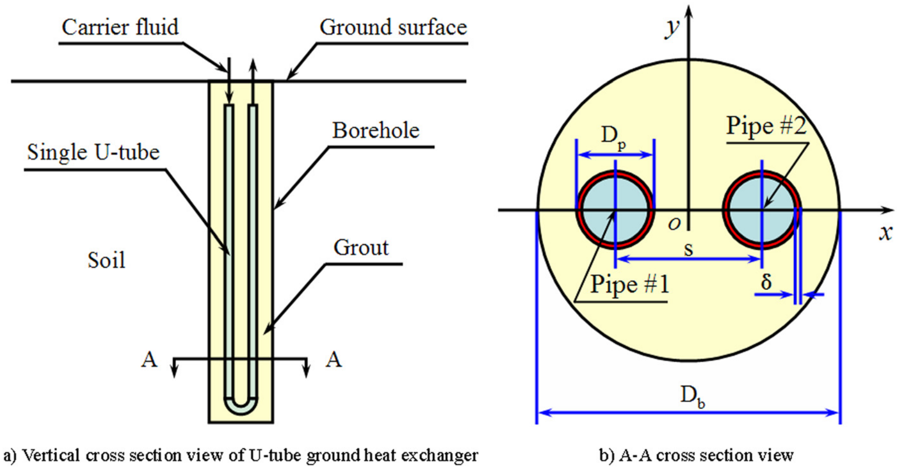


Fig. 1 Schematic diagram of a typical single U-tube ground heat exchanger

property of grout were taken into account in the simulations. After numerous simulations were carried out, a best-fit correlation was obtained for the effective pipe-to-borehole thermal resistance (i.e., R_b) as follows:

$$R_b = \frac{1}{2\pi \cdot k_g}[-1.49 \cdot \theta_1 + 0.656 \ln(\theta_2) + 0.436] \tag{6}$$

Although Sharqawy claimed that Eq. (6) has better accuracy to estimate the effective pipe-to-borehole thermal resistance than other available formulas in the literature, Lamarche et al. [4] pointed out that the boundary conditions (i.e., uniform temperature distributions on the outer diameter of the pipes and borehole surface, respectively) adopted in the 2D model by Sharqawy are *not* consistent with the real physical situation. An improved 2D numerical model was developed and solved by using COMSOL™ finite element software. In this improved 2D model, the soil that surrounds the borehole region is taken into account, and the distance between inner surface of borehole and outer boundary of computational domain is much greater than the diameter of borehole. Since the isothermal boundary conditions are still imposed at the outer diameter of the pipes and outer boundary of computational domain, there is no more constraint for the temperature distribution along the perimeter of borehole surface and the nonuniform temperature distribution on the borehole was observed. After comprehensive comparisons of pipe-to-borehole thermal resistance between the existing formulas and the numerical simulation data, a conclusion was drawn by Lamarche et al. that the equation proposed by Bennet et al. [5] gives the best estimation for the pipe-to-borehole thermal resistance and the root mean square error between simulation data and Bennet formula is less than 3.0×10^{-3}. The equation of pipe-to-borehole thermal resistance proposed by Bennet et al. is as follows:

$$R_b = \frac{1}{4\pi \cdot k_g}\left[\ln\left(\frac{\theta_2}{2\theta_1\left(1-\theta_1^4\right)^{\sigma}}\right) - \frac{\theta_3^2 \cdot \left(1 - \dfrac{4\sigma \cdot \theta_1^4}{1-\theta_1^4}\right)^2}{1 + \theta_3^2 \cdot \left(1 + \dfrac{16\sigma \cdot \theta_1^4}{\left(1-\theta_1^4\right)^2}\right)}\right] \tag{7}$$

where all the dimensionless parameters are defined in Eqs. (1)–(4), and k_g is the thermal conductivity of grout material.

Although the nonuniform temperature distribution on the perimeter of borehole surface was taken into account by the improved 2D model of Lamarche et al., the isothermal boundary conditions are still imposed at the outer diameter of the pipes. Unfortunately, in a real physical situation of ground heat exchanger, not only is the temperature distribution at borehole surface nonuniform but also that at the outer diameter of the pipes is nonuniform. All these angular variations of temperature at the borehole surface and the outer diameter of the pipes are due to the symmetrical arrangement of pipes; temperature differences of carrier fluid between pipes; and the thermal conductivities of pipe, grout, and soil. In order to consider the influence of nonuniform temperature distributions at both the outer diameter of the pipes and the borehole surface on the effective pipe-to-borehole thermal resistance, the thickness of pipes is taken into account in a new 2D numerical model, and the third kind of boundary condition (i.e., the temperature of carrier fluid and heat transfer coefficient are given) is imposed at the inner diameter of the pipes in this paper. After systematically choosing dimensionless geometrical variables and thermal properties of grout and soil, the new 2D numerical model is solved by FLUENT 6.3.26 software. Since the nonuniform temperature distribution on the perimeter of borehole surface could lead to negative thermal resistance between the two pipes (i.e., the thermal interference resistance, R_{12}) for the conventional three-thermal-resistance model of ground heat exchanger [6,7], a four-thermal-resistance model of ground heat exchanger in which the borehole surface temperature is divided into two parts along the symmetrical line is proposed in this paper. Based on this four-thermal-resistance model, all the thermal resistances, such as the effective pipe-to-borehole thermal resistance (i.e., R_b), pipe-to-pipe thermal interference resistance (i.e., R_{12}), and borehole-to-borehole thermal resistance (i.e., R_{b12}), are evaluated by using the numerical results. Eventually, new best-fit correlations for these thermal resistances (i.e., R_b, R_{12}, and R_{b12}) are proposed by using Nealder–Mead [8] method, and comprehensive comparisons between the present correlation (especially for the effective pipe-to-borehole thermal resistance) and the available formulas in the literature are presented in this paper. Furthermore, based on these obtained correlations of thermal resistance, an analytical model is proposed to evaluate the heat transfer performance of ground heat exchanger.

2 Physical and Mathematical Model

In order to take into account the nonuniform temperature distributions along the perimeter of both the borehole surface and the outer diameter of the pipes, a 2D numerical model that consists of soil, grout, and thickness of pipes was developed. The geometrical configuration and meshes of this new model are presented in Fig. 2.

The diameter of computational domain (D_{soil}), diameter of borehole (D_b), outer diameter of pipes (D_p), shank spacing (S),

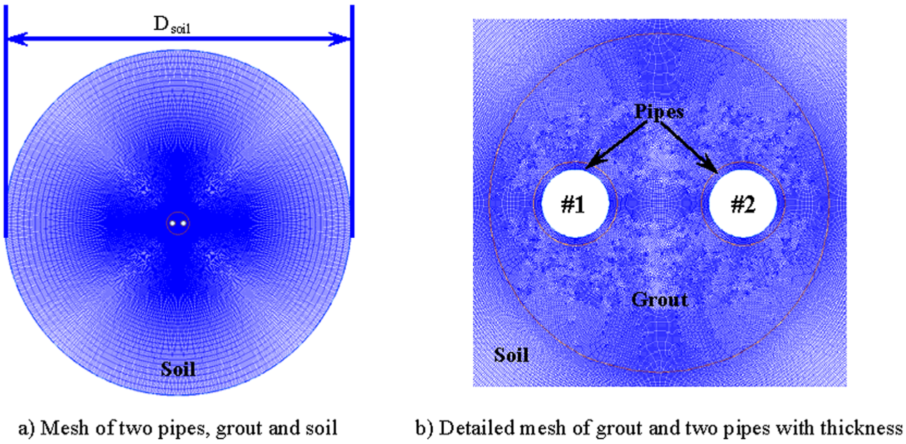


Fig. 2 Computational domain and meshes in a new 2D model

thickness of pipes (δ), and thermal conductivity of pipes (k_{pipe}) in the numerical model are given in Table 1. In a real U-tube, since the minimum bending diameter is 1.5 times of the outer diameter of the pipe, the range of shank spacing is between $1.5D_p$ and $D_b - D_p$.

In order to solve the above 2D heat conduction problem, some assumptions have to be made as follows:

(1) Steady-state 2D heat conduction is assumed for this numerical model.
(2) The materials (including soil, grout, and pipes) are homogenous, and all the thermal properties are independent of temperature.

Under the above assumptions, the governing equation of 2D steady-state heat conduction in a Cartesian x-y coordinate system could be written as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{8}$$

The boundary conditions for the above governing equation are as follows: Constant temperature is imposed at the outer boundary of computational domain and the third kind of boundary condition (i.e., the carrier fluid temperature and heat transfer coefficient are given) is imposed at the inner diameter of #1 and #2 pipe surfaces.

As shown in Fig. 3, a four-thermal-resistance model could be developed to determine the effective pipe-to-borehole thermal resistance, R_b, pipe-to-pipe thermal interference resistance, R_{12}, and borehole-to-borehole thermal resistance, R_{b12}. Due to the symmetrical arrangement of pipes within the borehole, the thermal resistance R_{b1} is equal to R_{b2} and both of them are equal to two times of the effective pipe-to-borehole thermal resistance R_b. Therefore, the above mentioned thermal resistances could be evaluated as follows:

$$R_b = \frac{R_{b1}}{2} = \frac{R_{b2}}{2} = \frac{(T_{p1} + T_{p2}) - (T_{b1} + T_{b2})}{2 \cdot (q_1 + q_2)} \tag{9}$$

$$R_{12} = \frac{2(T_{p1} - T_{p2})}{q_1 - q_2 - [(T_{p1} - T_{p2}) - (T_{b1} - T_{b2})]/R_{b1}} \tag{10}$$

Table 1 Range of parameters in present 2D numerical model

D_{soil} (m)	D_b (m)	D_p (m)	S (m)	δ (m)	k_{pipe} (W/m K)
4.0	0.13–0.2	0.025–0.065	$1.5D_p$ to $D_b - D_p$	0.003	0.44

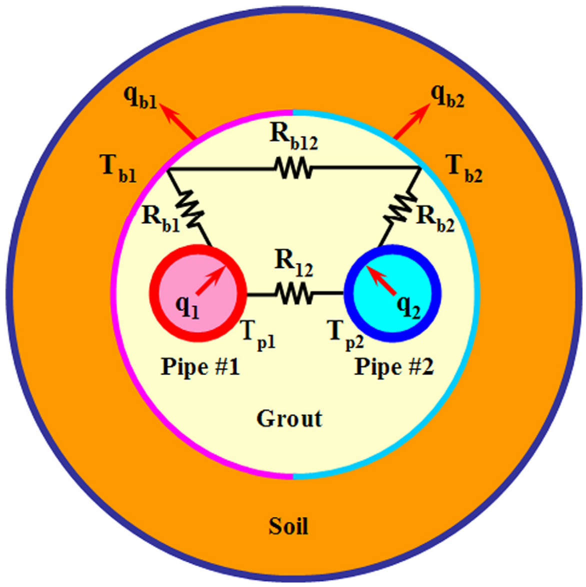


Fig. 3 Diagram of four-thermal-resistance model within borehole

$$R_{b12} = \frac{2(T_{p1} - T_{p2})}{q_{b1} - q_{b2} + [(T_{p1} - T_{p2}) - (T_{b1} - T_{b2})]/R_{b1}} \tag{11}$$

where T_{p1} and T_{p2} are the average temperatures at the outer diameter of pipe #1 and pipe #2, respectively; T_{b1} and T_{b2} are the average temperatures with half perimeter of borehole surface, as shown in Fig. 3. q_1 and q_2 are the rate of heat transfer from pipe #1 and pipe #2, respectively. q_{b1} and q_{b2} are the rate of heat transfer from half perimeter of borehole surface, respectively.

3 Validation of the Numerical Method

In order to validate the present numerical technique, the 2D numerical model of Lamarche was obtained by deleting the thickness of the pipes in the present 2D numerical model (as shown in Fig. 2), and the constant temperature boundary conditions are imposed at both the outer diameter of pipes and outer boundary of computational domain, respectively. The geometrical parameters and thermal properties of soil and grout in this validation numerical model are given in Table 2.

When all the parameters are set up, the numerical simulations of validation model are carried out with different thermal conductivities of grout, and the four-thermal-resistance model (as shown in Fig. 3) was adopted to evaluate the effective pipe-to-borehole thermal resistance, i.e., R_b. As shown in Fig. 4, the comparisons of dimensionless pipe-to-borehole thermal resistance between numerical simulation results in the validation model and Bennet formula, i.e., Eq. (7), are presented. From this figure, it is clearly shown that the numerical simulation results have very good agreements with the results of Bennet et al. The absolute maximum relative error between simulation results and Eq. (7) is less than 0.2% and this is consistent with the conclusions in the paper of Lamarche et al.

Therefore, it could reasonably be believed that the present numerical technique is reliable, and all the simulation results based on this technique are valid.

4 Results and Discussion

In order to demonstrate the nonuniform temperature distributions at the borehole surface and the outer diameter of the pipes, the 2D numerical model with $D_{soil} = 4.0$ m, $D_b = 0.153$ m, $D_p = 0.06$ m, $S = 0.09$ m, and $\delta = 0.003$ m was numerically solved by FLUENT 6.3.26 with 300.0 K constant temperature at the outer boundary of computational domain, 325.0 K fluid temperature for pipe #1, 320.0 K fluid temperature for pipe #2, 4000.0 W/m^2 K heat transfer coefficient for the inner diameter of #1 and #2 pipe surfaces, 0.44 W/m K thermal conductivity for pipes, 1.8 W/m K thermal conductivity for soil, and 2.0 W/m K thermal conductivity for grout. As shown in Fig. 5, the temperature distributions along the angular direction of the borehole surface and the outer diameter of pipe surfaces are presented. From this figure, it could be clearly shown that the temperature distributions at both the borehole surface and the outer diameter of pipe surfaces are indeed *not* uniform.

Since the numerical technique in this paper has been validated, the simulations of new 2D model for ground heat exchanger were systematically carried out for the different combinations of dimensionless parameters of θ_1, θ_2, and σ. And all the thermal resistances (i.e., R_b, R_{12}, and R_{b12}) were evaluated by using Eqs. (9)–(11), respectively. In this paper, 744 sets of numerical simulation have been conducted, and all the data of R_b, R_{12}, R_{b12}, θ_1, θ_2, and σ were collected. The Nealder–Mead method was adopted to obtain the relationship between the thermal resistances (i.e., R_b, R_{12}, and R_{b12}) and the primary variables (i.e., θ_1, θ_2, and σ). Eventually, the best-fit correlations for the effective pipe-to-borehole thermal resistance, pipe-to-pipe thermal interference resistance, and borehole-to-borehole thermal resistance are obtained as follows.

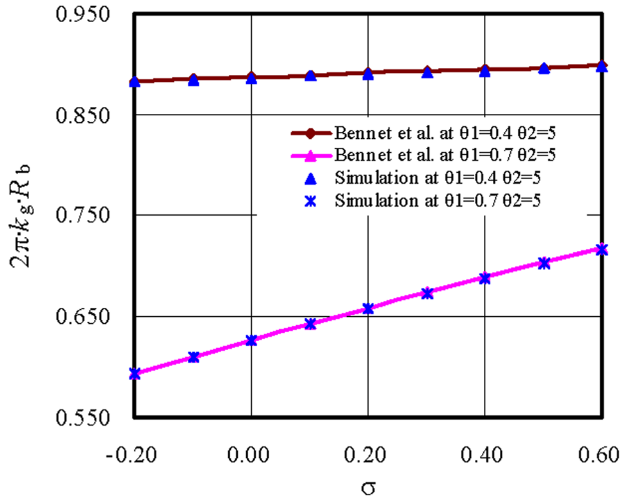


Fig. 4 Comparisons of dimensionless borehole thermal resistance between validation model and Bennet et al. Eq. (7)

$$R_b = \frac{1}{2\pi\lambda_g}\left[-0.50125\ln(\theta_1) + 0.51248\ln(\theta_2) + 0.51057\sigma\cdot\ln\left(\frac{1}{1-\theta_1^4}\right) - 0.36925\right] \tag{12}$$

$$R_{12} = \frac{1}{\lambda_g}(0.030455\cdot\theta_2 + 0.0020030\cdot\sigma^2 + 0.0065003\cdot\sigma - 0.026484)e^{(0.10664\sigma+4.7420)\theta_1} \tag{13}$$

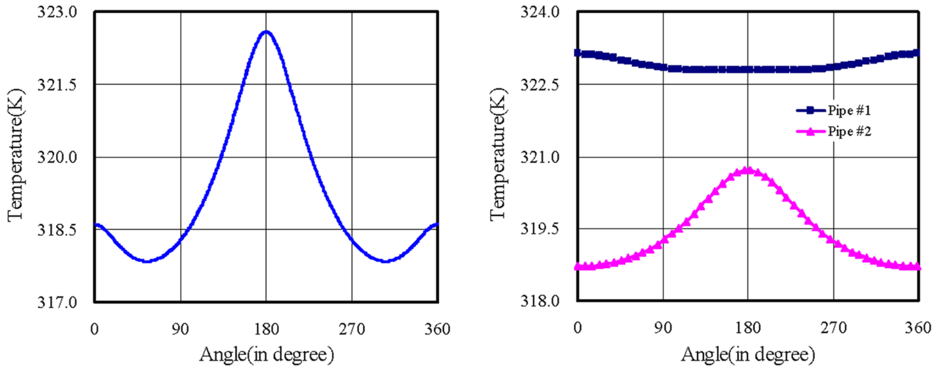
$$R_{b12} = \frac{1}{\lambda_g}\left(0.25920\cdot\sigma + 0.94137 - \frac{0.25161\cdot\sigma + 0.79423}{\ln(\theta_2)}\right)e^{\left(0.18494\cdot\sigma - 0.93402 + \frac{1.7547 - 0.16530\cdot\sigma}{\ln(\theta_2)}\right)\frac{1}{\theta_1}} \tag{14}$$

For all these 744 simulation data sets, the relative error range of correlations for pipe-to-borehole thermal resistance (i.e., R_b), pipe-to-pipe thermal interference resistance (i.e., R_{12}), and borehole-to-borehole thermal resistance (i.e., R_{b12}) is from -1.7% to 3.08%, from -6.68% to 7.29%, and from -14.5% to 11.4%, respectively. The root mean square errors for these three thermal resistance correlations are 3.06×10^{-4}, 1.66×10^{-2}, and 1.39×10^{-2}, respectively. Therefore, Eqs. (12)–(14) are relatively accurate enough to estimate the effective pipe-to-borehole thermal resistance, pipe-to-pipe thermal interference resistance, and borehole-to-borehole thermal resistance within the range of dimensionless parameters $-0.214 \le \theta_1 \le 0.85$, $2.5 \le \theta_2 \le 7.0$, and $-0.2 \le \sigma \le 0.6$.

Table 2 Range of parameters in the validation numerical model

D_{soil} (m)	D_p (m)	D_b (m)	S (m)	k_s (W/m K)	k_g (W/m K)
4.0	0.025	0.125	0.05/0.0875	1.8	1.2–7.2

In order to show the differences between the present correlation and the available formulas for the estimation of effective pipe-to-borehole thermal resistance, the comparisons between the present correlation, Bennet et al. equation, i.e., Eq. (7), and Sharqawy formula, i.e., Eq. (6), are presented with different combinations of dimensionless parameters θ_1, θ_2, and σ. As shown in Figs. 6 and 7, the dependence of $2\pi\cdot k_g\cdot R_b\cdot$ on σ is presented at the condition of $\theta_2 = 4$, $\theta_1 = 0.375$, or $\theta_1 = 0.7$. Since the correlation of Sharqawy is not the function of σ, the dimensionless thermal resistance



a) Temperature distribution along perimeter of borehole

b) Temperature distributions along outer perimeter of pipes

Fig. 5 Nonuniform temperature distributions along the perimeter of borehole and outer diameter of pipes

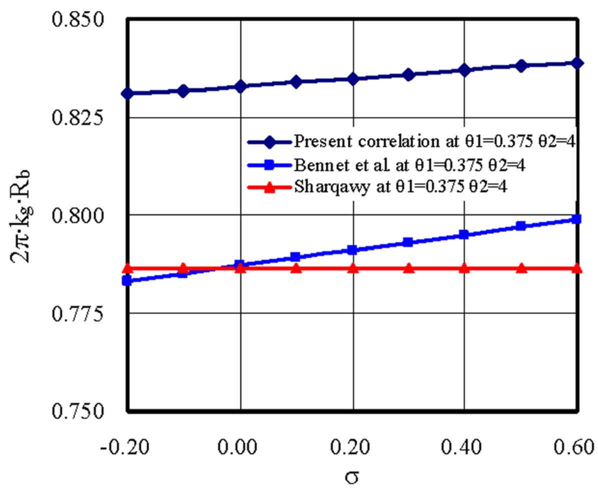


Fig. 6 Comparisons of dimensionless borehole thermal resistance at $\theta_1 = 0.375$ and $\theta_2 = 4$ when σ is between −0.2 and 0.6

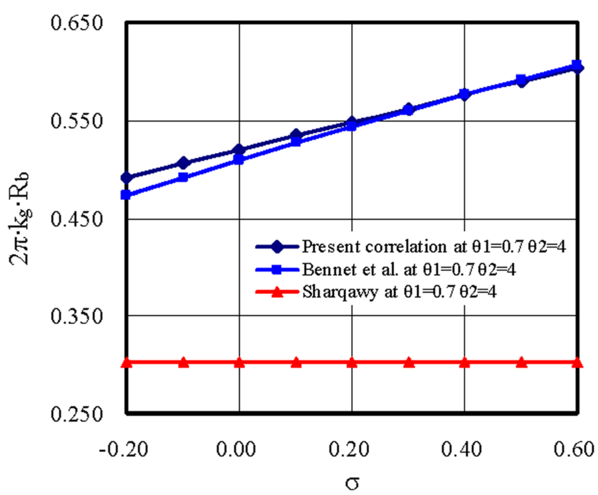


Fig. 7 Comparisons of dimensionless borehole thermal resistance at $\theta_1 = 0.7$ and $\theta_2 = 4$ when σ is between −0.2 and 0.6

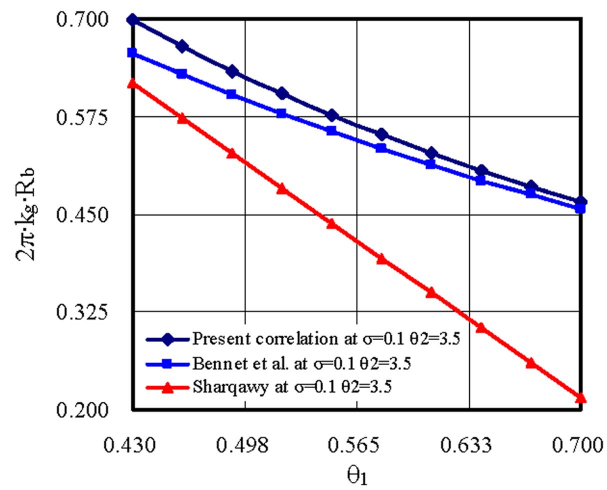


Fig. 8 Comparisons of dimensionless borehole thermal resistance at $\sigma = 0.1$ and $\theta_2 = 3.5$ when θ_1 is between 0.43 and 0.7

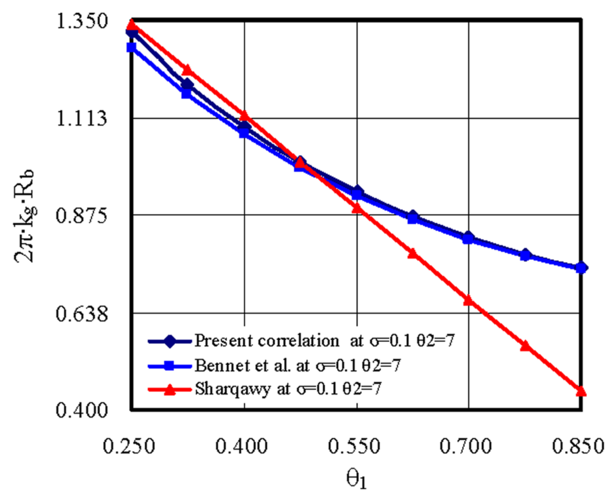


Fig. 9 Comparisons of dimensionless borehole thermal resistance at $\sigma = 0.1$ and $\theta_2 = 7.0$ when θ_1 is between 0.25 and 0.85

of Eq. (6) is independent of thermal conductivity of soil. Comparing Figs. 6 and 7, one could find that the differences of $2\pi \cdot k_g \cdot R_b$ between present correlation and Bennet et al. equation decrease with the increase in dimensionless parameters θ_1 and σ. This could be explained as follows. On the one hand, as θ_1 increases, the distance between two pipes is increased for a given borehole diameter, and the influence of temperature differences between the two carrier fluids in the pipes on the nonuniform temperature distributions at the outer diameter of pipe surfaces decreases. Therefore, the temperature distributions at the outer diameter of pipe surfaces become more uniform, and the differences between the third kind of boundary condition in the present model and the isothermal boundary condition in the model of Lamarche get decreased. On the other hand, as σ increases, the thermal conductivity of soil decreases for a given thermal conductivity of grout, and the total rate of heat transfer from borehole to soil decreases. Therefore, the temperature distributions at the outer diameter of the pipes become more uniform and the difference of boundary condition between the present model and that of Lamarche et al. decreases as well.

In Figs. 8 and 9, the dependence of $2\pi \cdot k_g \cdot R_b$ on θ_1 is presented at the condition of $\sigma = 0.1$, $\theta_2 = 3.5$, or $\theta_2 = 7.0$. From these two figures, it could be seen that with an increase in θ_1, the dimensionless effective pipe-to-borehole thermal resistance predicted by these three correlations decreases. However, the differences between the present correlation (or Bennet et al. equation) and Sharqawy formula increase with an increase in θ_1. This is due to the fact that the nonuniform temperature distributions along the perimeter of borehole surface become more prominent with the increase in θ_1. On the other hand, the change of differences between the present correlation and Bennet et al. equation in Figs. 8 and 9 decreases as θ_1 or θ_2 increases. This is due to the fact that the temperature distributions at the outer diameter of #1 and #2 pipe surfaces become more uniform as θ_1 or θ_2 increases. Therefore, the differences between isotherm boundary condition and the third kind of boundary condition, which is imposed at the inner diameter of #1 and #2 pipe surfaces, diminish for a given 2D model of ground heat exchanger.

Figures 10 and 11 show the variation of $2\pi \cdot k_g \cdot R_b$ on θ_2 for $\theta_1 = 0.5$, $\sigma = -0.2$, or $\sigma = 0.6$. It is seen that these three correlations show a similar trend for the dependence of $2\pi \cdot k_g \cdot R_b$ on θ_2, i.e., with the increase in θ_2 the dimensionless pipe-to-borehole thermal resistance gradually increases for a given σ. It is evident that the influence of σ on $2\pi \cdot k_g \cdot R_b$ is much smaller than that of θ_2. On the other hand, the value of $2\pi \cdot k_g \cdot R_b$ given by the present correlation is always greater than that predicted by Bennet et al. equation and Sharqawy formula. This is due to the fact that the temperature distributions at the outer diameter of #1 and #2 pipe surfaces in a real physical situation are *not* uniform; therefore, the isothermal boundary condition at the outer diameter of #1 and #2 pipe surfaces adopted in the model of both Bennet et al. and Sharqawy is not appropriate.

Although several expressions have been proposed for the effective pipe-to-borehole thermal resistance (i.e., R_b), few authors have suggested methods to evaluate the pipe-to-pipe thermal interference resistance (i.e., R_{12}) and borehole-to-borehole thermal resistance (i.e., R_{b12}). In order to demonstrate how to use these

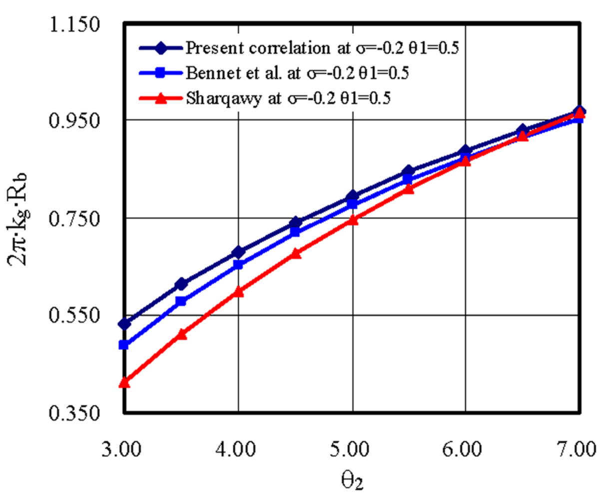


Fig. 10 Comparisons of dimensionless borehole thermal resistance at $\sigma = -0.2$ and $\theta_1 = 0.5$ when θ_2 is between 3.0 and 7.0

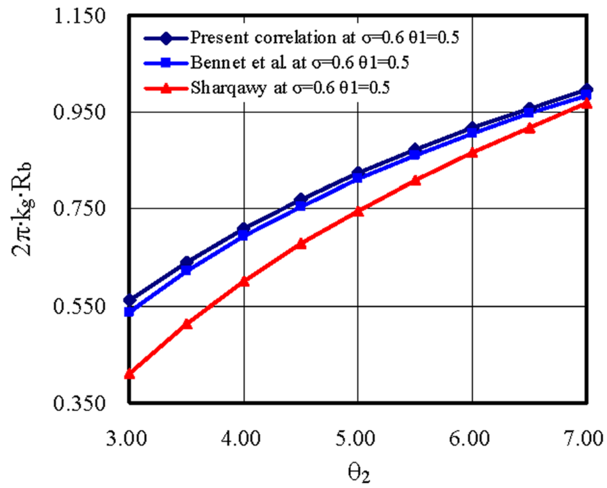


Fig. 11 Comparisons of dimensionless borehole thermal resistance at $\sigma = 0.6$ and $\theta_1 = 0.5$ when θ_2 is between 3.0 and 7.0

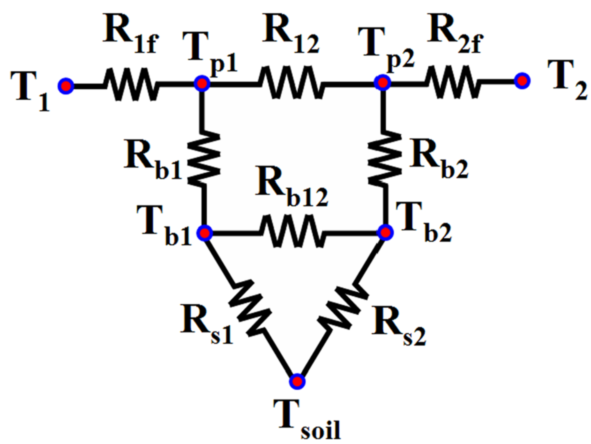


Fig. 12 Thermal resistance network for a ground heat exchanger

correlations (especially for R_{12} and R_{b12}) in a real engineering application, an analytical model is proposed for a ground heat exchanger based on the thermal resistance network, as shown in Fig. 12. In this figure, T_1 and T_2 stand for the temperature of heat carrier fluid for pipes #1 and #2, respectively; T_{p1} and T_{p2} are the average temperatures of outer diameter of pipes #1 and #2, respectively; T_{b1} and T_{b2} are the average temperatures of borehole with half length of perimeter as shown in Fig. 3; and T_{soil} stands for the undisturbed soil temperature. Since the effective pipe-to-borehole thermal resistance does not include the thermal resistance of pipe thickness (i.e., R_{pipe}) and convection heat transfer thermal resistance (i.e., $R_{\mathrm{convection}}$) on the inner diameter surface of pipes #1 and #2, the thermal resistances R_{1f} and R_{2f} are introduced to take into account these two thermal resistances for pipes #1 and #2, respectively. In Fig. 12, the thermal resistance between half perimeter of borehole surface and outer boundary computational domain (i.e., R_{s1} and R_{s2}) could be obtained by the line source model of ground heat exchanger [9]. Due to the symmetrical arrangement of the U-tube pipe, the values of R_{s1} and R_{s2} could be evaluated as follows:

$$R_{s1} = R_{s2} = 2R_s = \frac{1}{2\pi \cdot k_s} \ln\left(\frac{16\alpha_s t}{\gamma \cdot D_b^2}\right) \tag{15}$$

where k_s is the thermal conductivity of soil, α_s is the thermal diffusivity of soil, and γ is a constant and approximately equal to 1.78.

For a given single U-tube ground heat exchanger, all the geometrical parameters are given and all the thermal properties of pipe, grout, and soil are known. If this ground heat exchanger has a length of L, the mass flow rate of carrier fluid is m, and the temperature of carrier fluid at inlet of ground heat exchanger is T_{in}, the following equation could be obtained on the basis of thermal resistance network by using the conservation of energy for an elementary length of ground heat exchanger:

$$\begin{cases} mc_p \dfrac{dT_1}{dZ} = \dfrac{T_{p1} - T_1}{R_{1f}} \\ -mc_p \dfrac{dT_2}{dZ} = \dfrac{T_{p2} - T_2}{R_{2f}} \\ \dfrac{T_1 - T_{p1}}{R_{1f}} + \dfrac{T_{b1} - T_{p1}}{R_{b1}} + \dfrac{T_{p2} - T_{p1}}{R_{12}} = 0 \\ \dfrac{T_2 - T_{p2}}{R_{2f}} + \dfrac{T_{b2} - T_{p2}}{R_{b2}} + \dfrac{T_{p1} - T_{p2}}{R_{12}} = 0 \\ \dfrac{T_{p1} - T_{b1}}{R_{b1}} + \dfrac{T_{b2} - T_{b1}}{R_{b12}} + \dfrac{T_{soil} - T_{b1}}{R_{s1}} = 0 \\ \dfrac{T_{p2} - T_{b2}}{R_{b2}} + \dfrac{T_{b1} - T_{b2}}{R_{b12}} + \dfrac{T_{soil} - T_{b2}}{R_{s2}} = 0 \end{cases} \tag{16}$$

The variable Z represents depth, which increases downward. And the boundary conditions for the above equation are as follows:

$$\begin{cases} T_1 = T_{in} & @Z = 0 \\ T_2 = T_1 & @Z = L \end{cases} \tag{17}$$

In the above Eq. (16), the undisturbed soil temperature (i.e., T_{soil}) is an input parameter for the ground heat exchanger. Therefore, there are six unknowns (i.e., T_1, T_2, T_{p1}, T_{p2}, T_{b1}, and T_{b2}) and six equations with two boundary conditions. From the mathematical point of view, the above equation is closed and all the unknowns are solvable. Once Eq. (16) is solved, all the variables such as T_{b1}, T_{b2}, T_{p1}, and T_{p2} could be obtained, and the total heat transfer from carrier fluid to soil could be evaluated by the following equation:

$$Q = \int_0^L \frac{T_{p1} - T_1}{R_{1f}} dZ + \int_0^L \frac{T_{p2} - T_2}{R_{2f}} dZ \tag{18}$$

Therefore, the heat transfer performance could be obtained on the basis of this analytical model of ground heat exchanger.

5 Conclusions

A new 2D numerical model for a single U-tube ground heat exchanger has been developed. The best-fit correlations for the effective pipe-to-borehole, pipe-to-pipe, and borehole-to-borehole thermal resistances are obtained based on 744 numerical simulations on the basis of a four-thermal-resistance model for the ground heat exchanger. Comprehensive comparisons of effective pipe-to-borehole thermal resistance between the present correlation and the existing correlations in the literature are present. The following conclusions could be drawn:

(1) Temperature distributions at both the borehole surface and the outer diameter of the pipes are found to be nonuniform and are functions of the geometrical parameters θ_1 and θ_2 and the grout-soil thermal property parameter σ.
(2) Temperature distributions at both the borehole surface and the outer diameter of the pipes play an important role with regard to the thermal resistances of ground heat exchanger (i.e., R_b, R_{12}, and R_{b12}).
(3) Generalized correlations are proposed for pipe-to-borehole thermal resistance, pipe-to-pipe thermal interference resistance, and borehole-to-borehole thermal resistance to account for nonuniform outer diameter of pipe and borehole surface temperatures. The correlation for the pipe-to-borehole thermal resistance is shown to be more accurate than those in the literature.
(4) By using the obtained correlations of thermal resistance, an analytical model is proposed on the basis of thermal resistance network for a ground heat exchanger.

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Nomenclature

c = volumetric heat capacity (J/mK)
D = diameter (m)
k = thermal conductivity (W/mK)
m = mass flow rate (kg/s)
n = number of pipes
q = heat transfer rate per unit length (W/m)
Q = total amount of heat (W)
R = thermal resistance (mK/W)
S = shank spacing (m)
T = temperature (K)

Greek Letters

θ = dimensionless parameter of borehole geometry
σ = dimensionless parameter of thermal properties between grout and soil
δ = thickness of pipe (m)

Subscripts

b = borehole
p = pipe
g = grout
s = soil
1 = pipe #1
2 = pipe #2
$b1$ = pipe #1 to borehole
$b2$ = pipe #2 to borehole
$p1$ = pipe #1
$p2$ = pipe #2

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