Medium earth orbit and inclined geosynchronous orbit satellite control strategies optimization based on the function approximation method

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Abstract: The article proposes an optimization method based on the function approximation in control strategies design of medium earth orbit (MEO) and inclined geosynchronous orbit (IGSO) satellites. As an extension of the functional approximation method (FAM), this method is suitable to solve a single-variable or a multivariable optimization question with equality or inequality constraints. This ensures that the optimal control strategies for MEO and IGSO satellites to manoeuvre along the ideal control arc can be easily determined, and finally make satellites enter the designed orbits as soon as possible after satellites being launched under restrictions of the limited propellant and number of revolutions around the earth. In the current article, the basic FAM model is first introduced, and then the method applications and the simulation results are discussed in detail. Compared with the conventionally adopted exhaust search in the process of the optimal strategy design for the MEO and IGSO satellites, this method has the advantages of simplicity, less dependence on the initial parameter range, and requires much less computational effort.

Keywords: medium earth orbit and inclined geosynchronous orbit satellites, control strategies, optimization, function approximation method

1 INTRODUCTION

Both medium earth orbit (MEO) and inclined geosynchronous orbit (IGSO) satellites have been widely used in many large-scale constellation systems in the field of navigation and positioning [**1–8**]. It is well known that, although these two types of satellites have absolutely different orbit altitudes, the eccentricity and inclination of these two satellites are nearly the same. In general, the same control strategy is applied to the orbital manoeuvre in the early flying phase of MEO and IGSO satellites. Conventionally, the multidimensional exhaust search (ES) is often adopted for this purpose [**9–15**]. In this method, every variable must be divided into many intervals, and the objective function must be computed at every discrete point of all system variables. Thus, considerable computational effort is required in order to obtain the optimal function value, even if the system contains only one single variable. One of the major advantages of this method is that it is very simple to implement. However, it depends too much on the range of every variable. This means that only the perfect selection to the range of all variables can guarantee that the optimal result of objective function can be achieved quickly. The most difficult task in this method is to identify the ideal variable range under the condition that it is not obvious for the range of every selected variable. Thus, most of the computing effort has been expended in solving a single-variable or a multivariable control problem with a set of complex dynamics model systems and special constraint conditions. The function approximation method is proposed to solve this kind of problem [**16**, **17**]. The biggest advantage of this method is that it can be

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implemented easily and effectively in the initial process of optimal manoeuvre of MEO and IGSO satellites. Compared with the multidimensional ES, this method is also simple to implement, and it does not depend on the variable range too much and requires much less CPU time in solving the same complex system problem.

The functional approximation method (FAM) mentioned above is not the same as the well-known sequential quadratic programming (SQP) method although the objective function for each of these two methods has non-linear characteristics. In general, SQP methods attempt to solve a non-linear program directly rather than convert it to a sequence of unconstrained minimization problems. To FAM here, the objective function is only approximated step by step with the aid of a simple quadratic function. Function approximation differs from both derivative-based and heuristic evolutionary optimization, and it is designed for optimization of computationally expensive functions like simulation models for which it is not feasible to run the model a large number of times. Function approximation is usually applied to uncertainty analysis and to groundwater transport optimization in water resource and environmental analysis both for design of remediation plans and for calibration.

In this article, the proposed system model about the function approximation is described at first, and then its use in the strategy optimization of MEO and IGSO satellites orbit manoeuvre. Finally, the simulation condition and the discussion of the results of the orbit control strategies of MEO and IGSO satellites are shown in detail.

2 BASIC CALCULATION MODEL

2.1 Function approximation method

In the function approximation method, the high-order objective function of control system is fitted with a polynomial; this means that the function value at some point of the variable range can be used to construct the low-order interpolation polynomial for finding the minimum of objective function. Then, the minimum of the polynomial can be viewed as the approximate value of the minimum of the objective function. Generally, the simple second-order parabola interpolation is adopted as an example to introduce the basic model of the function approximation method.

Here, suppose that $f(x)$ is a convex function and $[x_1, x_2]$ is the range of x_k, and $x_0 \in [x_1, x_2]$. All these satisfy the following conditions

$$x_1 < x_0 < x_2 \tag{1}$$

$$f(x_1) > f(x_0) < f(x_1) \tag{2}$$

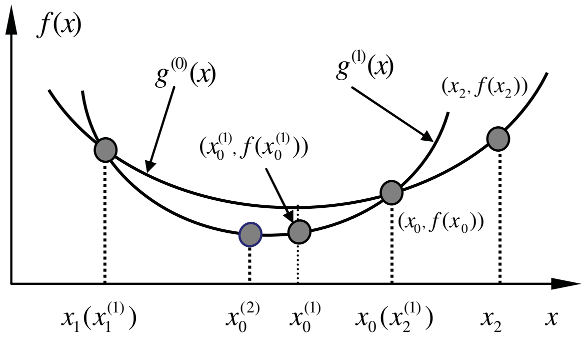


Fig. 1 Schematic diagram for FAM

As per the schematic diagram is shown in Fig. 1, the main procedure for function approximation is followed below.

First, construct the second-order interpolation polynomial $g^{(0)}(x)$ by using three initial given points $(x_i, f(x_i))$ $(i = 0, 1, 2)$. The minimum point $x_{\min}^{(0)}$ is obtained according to

$$\frac{\mathrm{d}g^{(0)}(x)}{\mathrm{d}x} = 0 \tag{3}$$

Second, select three points $x_0^{(1)}, x_1^{(1)}$, and $x_2^{(1)}$ from the known points x_0, x_1, x_2 (i.e. $x_0^{(0)}, x_1^{(0)}, x_2^{(0)}$), and $x_{\min}^{(0)}$, and make these three values satisfy the condition inequalities (1) and (2) and the following inequality

$$x_2^{(1)} - x_1^{(1)} < x_2^{(0)} - x_1^{(0)} \tag{4}$$

Repeatedly, the range $(x_1^{(m)}, x_2^{(m)})$ and $x_{\min}^{(m)}$, which satisfy the condition inequalities (1) and (2), can be obtained, and they satisfy the following conditions too

$$(x_1^{(m)}, x_2^{(m)}) \subset (x_1^{(m-1)}, x_2^{(m-1)}) \cdots \subset (x_1^{(0)}, x_2^{(0)}) \tag{5}$$

$$x_{\min}^{(m)} \in (x_1^{(m)}, x_2^{(m)}) \tag{6}$$

According to conditions (5) and (6), obviously when $m \to \infty, x_1^{(m)} \to x_2^{(m)}$, the final minimum point $x_{\min}$ can be obtained by $x_{\min} = 1/2(x_1^{(m)} + x_2^{(m)})$.

Generally, the objective function is complicated and it is difficult to express them by a perfect explicit formulation. However, it can be calculated according to the orbital dynamics equation as mentioned in the next section.

2.2 Orbital dynamics equation

When the satellite performs an orbit manoeuvre, the thrust force that acts on the satellite can be generally regarded as the perturbation force. Relatively, the acceleration created by the thrust force can be considered as the perturbation acceleration. Here, the variation equations of Gauss, which provide a convenient set of equations relating the effect of a control acceleration vector u to the oscillating orbital element time derivatives [**18**], are applied to describe the orbital elements variation of satellite

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2a^2}{h}\left(e\sin f\ u_\mathrm{r} + \frac{p}{r}u_\theta\right) \tag{7}$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{1}{h}[p\sin f\ u_\mathrm{r} + ((p+r)\cos f + re)u_\theta] \tag{8}$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = \frac{r\cos\theta}{h}\ u_\mathrm{h} \tag{9}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{r\sin\theta}{h\sin i}u_\mathrm{h} \tag{10}$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{1}{he}[-p\cos f\ u_\mathrm{r} + (p+r)\sin f\ u_\theta] - \frac{r\sin\theta\cos i}{h\sin i}u_\mathrm{h} \tag{11}$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} = n + \frac{\eta}{he}[(p\cos f - 2re)u_\mathrm{r} - (p+r)\sin f\ u_\theta] \tag{12}$$

The majority nomenclature for these and subsequent equations is explained in the notation section of this article. Here, $\boldsymbol{u} = (u_\mathrm{r}, u_\theta, u_\mathrm{h})$ is the control acceleration vector, written in the Local-Vertical-Local-Horizontal frame, u_r is the radial component of the perturbation acceleration, which points in the radial direction away from the earth, u_h is the normal component of the perturbation acceleration, which is aligned with the orbit angular momentum vector, and u_θ is the lateral component of the perturbation acceleration, which is orthogonal to both u_r and u_h. r is the scalar orbit radius, p is the semi-latus rectum; here, $p = a(1-e^2)$, $\theta = \omega + f$, $h = \sqrt{pr}$, $\eta = \sqrt{1-e^2}$, and $n = \sqrt{\mu/a^3}$. The mass variation equation of the satellite is

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -\frac{F}{I_\mathrm{sp}g} \tag{13}$$

In addition, the thrust vector can be considered to be perpendicular to the radius vector of the satellite during the orbit manoeuvre around apogee or perigee. The perturbation force (or perturbation acceleration) can be divided into three orthogonal components, which are defined as the radial component, the lateral component, and the normal one. Thus, the constraint relation between the motor thrust vector $\boldsymbol{F}$ and the satellite's orbit radius vector $\boldsymbol{r}$ (from the geocentre to the satellite, $r = |\boldsymbol{r}|$) can be expressed by the following equation

$$F\cdot r = 0 \tag{14}$$

where

$$\boldsymbol{F} = \{F\cos\delta\cos\alpha, F\cos\delta\sin\alpha, F\sin\delta\}$$

$$\boldsymbol{r} = \{x, y, z\}$$

Here, $F = |\boldsymbol{F}|$, α is the attitude longitude of the thrust vector $\boldsymbol{F}$ and δ is the attitude latitude of the thrust vector F in the geocentric equator inertia coordinate system. According to equation (14), the perpendicular relation between the thrust vector $\boldsymbol{F}$ and the radius vector $\boldsymbol{r}$ can be expressed as

$$x\cos\delta\cos\alpha + y\cos\delta\sin\alpha + z\sin\delta = 0 \tag{15}$$

Generally, for MEO and IGSO satellites, the orbit inclination does not need to be adjusted, and so the main task of orbit manoeuvre is in the orbit plane, and in the process of simulating to orbit manoeuvre to MEO and IGSO satellites, the lateral component F_h (it is vertical to the orbit plane) of the motor thrust vector $\boldsymbol{F}$ meets the following condition

$$F_\mathrm{h} = 0 \tag{16}$$

Obviously, it is difficult to express the objective function for optimal orbit manoeuvre with an explicit analytical formulation. However, every variable can be calculated by using numerical integration, and the results of the numerical integrations are used to calculate the optimal function value, which will be described in the next section.

3 CONTROL STRATEGY OPTIMIZATION FOR MEO AND IGSO SATELLITES

Generally, the orbit error or deflection surely exists between a satellite's practical orbit after the satellite is launched and its theoretical design orbit. Even though the orbit error may be quite small, in order to meet the function design requirement of the satellite and to obtain the longer application time, the orbit control implemented to the satellite is necessary and important. Designing a flexible, dependable, and efficient orbit control strategy is one of the key factors ensuring the satellite's flight on the precise orbit. The main task in designing a control strategy for a satellite's orbit manoeuvre is to obtain all controlled parameters before every orbit manoeuvre is performed. These parameters are used to determine when the orbit manoeuvre must be performed, how much orbit parameters adjustment must be achieved, and so on. Of course, for different satellites, the orbit control objective is often different.

For MEO and IGSO satellites, the longitude drift rate of the satellite is usually used as the reference benchmark in designing a control strategy to determine the objective orbit parameters after every orbit transfer. Its definition is given as follows

$$\dot{\alpha} = \left(1 - \frac{a}{a_\mathrm{obj}}\right)^{3/2} \tag{17}$$

where a_obj is a constant and represents the final objective semi-major axis of MEO or IGSO satellites.

Obviously, the parameter $\dot{\alpha}$ can be determined by the semi-major axis a, and so the semi-major axis a is used as the benchmark to determine the controlled quantity in place of the objective longitude drift rate of every orbit transfer. The amount of the parameter $\dot{\alpha}$ reflects the angle at which the current position of satellite deviates from the final objective position in one revolution. If $\dot{\alpha} > 0$, it means the satellite drifts more quickly than its final objective drift rate. If $\dot{\alpha} < 0$, it means the satellite drifts more slowly than its final objective drift rate, and only when $\dot{\alpha} = 0$, it means that the satellite drifts at the final objective drift rate.

3.1 Analysis of initial orbit characteristics of MEO and IGSO satellites

Normally, for MEO and IGSO satellites, irrespective of whether there are errors between the practical initial orbit and the designed initial orbit, orbit control is always necessary because of the requirements of orbit transfer (usually, from an initial larger elliptic orbit to an approximate circle one).

1. The initial orbit altitude of the satellite entirely exceeds the altitude range of the theoretical design orbit, as shown in Fig. 2(a). The quasi-optimized strategy is to continuously to decelerate to the satellite until both the perigee and the apogee altitudes meet the requirement. The most efficient chance is when the satellite is around the perigee where the thrust is in the local horizontal plane and the orientation of the thrust is anti-collinear to the velocity. After about several decelerations, manoeuvres to the satellite are implemented in the same perigee. For simplicity, two times of deceleration are shown here: the initial perigee will become the apogee and the new perigee altitude will then meet the design requirement. The last step (which may need several similar steps in practice) of the application is to implement the last deceleration to the satellite in the new perigee. In the end, the orbit shape of the satellite will become approximately circular, and all orbit elements can meet the design requirements for the objective orbit of the satellite.
2. The initial orbit altitude of the satellite partly exceeds the altitude range of the theoretical design orbit. In this case, the quasi-optimized strategy increases the perigee altitudes and decreases the apogee altitudes until the circular MEO/IGSO orbit is reached. First, the continuous acceleration to the satellite (in the apogee, i.e. increasing the perigee altitude continually) can be implemented repeatedly when the satellite is around the apogee where the thrust is in the local horizontal plane and it points toward the same direction of the satellite flight direction. Second, when the perigee altitude meets the orbit design requirements, the necessary manoeuvre that decrease the initial apogee altitude must be implemented in the new perigee, where the orientation of the thrust must be anti-collinear to the velocity. Finally, the satellite's orbit will also take on an approximately circular contour (usually, this is also the objective orbit).
3. The initial orbit altitude of the satellite is entirely smaller than the altitude of the theoretical design orbit. The schematic diagram of this case is shown in Fig. 2(b). In this case, the altitude of the perigee and the apogee is increased continuously until the MEO/IGSO circular orbit is reached. Each time of acceleration manoeuvre to the satellite is implemented in the apogee because the initial perigee may eventually become an apogee after several rounds of acceleration; however, it is also possible that all orbit acceleration controls are implemented in almost the same position, and usually it

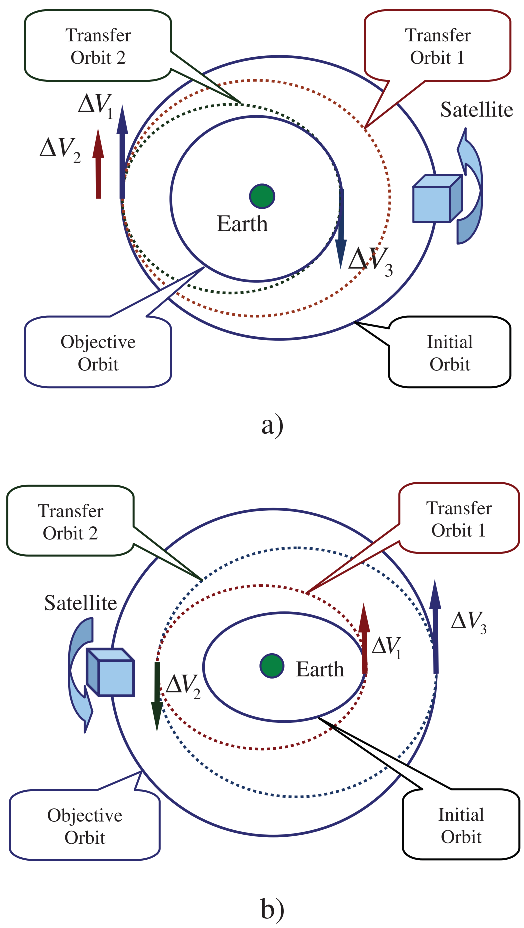


Fig. 2 Schematic diagram of the initial orbit characteristics of an MoEO/IGSO satellite: (a) Initial orbit altitude entirely exceeds the design orbit altitude; and (b) Initial orbit altitude is entirely smaller than the design orbit altitude

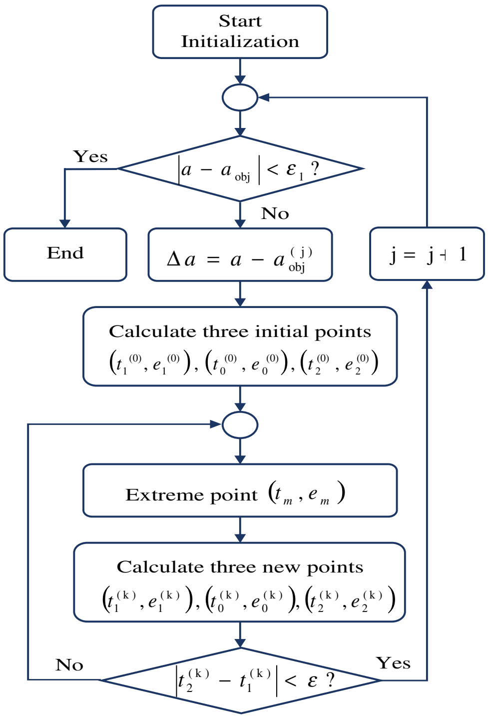


Fig. 3 Logic diagram for FAM application in orbit manoeuvre

is around the apogee. In this kind of orbital control, the orientation of the motor thrust is almost always in the local horizontal and is collinear to the velocity around the apogee, and thus the eccentricity becomes smaller and smaller until the objective circular orbit of the MEO/IGSO satellite is reached.

Especially, when the initial orbit altitude at the apogee is equal to the objective orbit altitude, almost every manoeuvre will be implemented near this initial apogee. According to the analyses mentioned above, obviously, irrespective of the kind of initial orbit used for the MEO/IGSO satellite, the eccentricity parameters must be satisfied after the whole orbit manoeuvres task is completed under the orbit altitude constraint. The function approximation method (FAM) can be applied perfectly in the strategies design of the MEO/IGSO satellite.

3.2 Simulation procedure for parameters calculation in orbit manoeuvre

For the general objective orbit design about MEO and IGSO satellites, the semi-major axis a is often used as the constraint condition to solve the optimization problem for the orbit manoeuvre in the orbital plane, and the eccentricity e is selected as the objective variable or function so that its value becomes minimum when the orbit semi-major axis a is equal to the objective semi-major axis a_{obj} finally. The main procedure of one orbit manoeuvre, for FAM to be applied in the control strategies design of MEO and IGSO satellites, is introduced as follows.

1. Initialization: obtain the initial orbital elements and all the constants required for the orbit manoeuvre calculation, and then transfer these orbital parameters to the selected computational coordinate system that is suitable for the integrals of equations (7) to (13).
2. According to the constraint of the time interval between two orbit manoeuvres and the longest time limit Δt_{max} for the motor to continuously work in each orbit manoeuvre, two different initial times t_1 and t_2 can be treated as the boundary points of the time range for searching the optimal start time of each orbit manoeuvre. Generally, the time interval Δt_{12} between t_1 and t_2 must be longer than the Δt_{max}, and so the Δt_{max} may be larger than one-half the orbit time period, and the optimal start time for the manoeuvre motor to work can be obtained between the initial time t_1 and t_2 by the following calculation.
3. Calculate three different objective elements of an orbit, which correspond to three different start time parameters t_1, t_2, and t_0 (where $t_0 = (t_1 + t_2)/(2)$) for an orbit manoeuvre, respectively. This can be accomplished using the numerical integration in the dynamics equations for satellite orbit manoeuvre under a pre-determined constraint condition, and three different eccentricity parameters e_1, e_2, and e_0 can be obtained, respectively. For the MEO/IGSO satellite, the constraint condition is the objective semi-major axis of the satellite. Here, $a_{obj}^{j}(j = 1, 2, \ldots J_{max})$ indicates the objective semi-major axis of the jth orbit manoeuvre.
4. In terms of the calculated three points (t_1, e_1), (t_0, e_0), and (t_2, e_2) in the plane $t - e$, the unique conic shape can be determined, and all the coefficients of this parabola can also be obtained by calculation.
5. Calculate the extreme point of this parabola and obtain the time variable t_m at the minimum point. According to the dynamics equations (7) to (13), which are used to describe the satellite orbit manoeuvre, the eccentricity e_m after orbit manoeuvre can be obtained by numerical integration of these dynamics equations. Here, the integration start time is the time t_m; the calculation process of e_m by numerical integration is similar to that described in step 3. Thus, these three new time parameters points t_1^*, t_0^*, and t_2^box*, all together with the corresponding orbit eccentricity parameters e_1^*, e_0^*, and e_2^*, can be selected from points (t_1, e_1), (t_2, e_2), (t_2, e_2), and (t_m, e_m) with the aid of the function approximation method described above. Furthermore, the three new points must meet the following

conditions

$$t_1^* < t_0^* < t_2^* \tag{18}$$

$$e_1^* > e_0^* < e_2^* \tag{19}$$

$$|t_2^* - t_1^*| < |t_2 - t_1| \tag{20}$$

6. Repeat the above steps 4 and 5 until the following condition is satisfied

$$|t_2^* - t_1^*| < \varepsilon \tag{21}$$

And then, the ideal start time t_{ost}, for the orbit manoeuvre, can be obtained by the following expression

$$t_{\mathrm{ost}} = \frac{(t_1^* + t_2^*)}{2} \tag{22}$$

This time will be used to calculate all other related orbit parameters. The eccentricity e_{m} after orbit manoeuvre can be obtained when the absolute error between the current semi-major axis a and the objective semi-major axis $a_{\mathrm{obj}}^{(j)}$ of the jth orbit manoeuvre is satisfied. The convergence criterion is set and must be satisfied. Otherwise the time variable t_{m} at the next minimum point has to be calculated again, and the theoretical orbit eccentricity after orbit manoeuvre can be obtained by solving the dynamics equations set (7) to (13) under the imposed initial condition iteratively. This iterative process ends when the convergence criterion is satisfied.

Irrespective of the kind of satellite (MEO or IGSO), considering that the numerical integration process for the dynamics simulation in every orbit manoeuvre is almost similar, the calculation process of every orbit manoeuvre for the MEO or IGSO satellite will not be repeatedly mentioned here. However, after $J_{\max}$ times of orbit manoeuvres, the final objective orbit can be obtained successfully, and the main procedure is shown in Fig. 3.

4 SIMULATION RESULT DISCUSSIONS

In this article, the application of FAM in the optimal strategy design for an MEO/IGSO satellite orbit manoeuvre is carried out with the aid of a numerical simulation. For the simulation calculation of the optimality design of orbit control strategies of MEO/IGSO satellites to be implemented successfully, it is necessary to make some assumptions about the initial parameters of an MEO/IGSO satellite. The main orbit parameters, which will be used in the numerical simulations of the orbit manoeuvres of MEO and IGSO satellites, are shown in Table 1, and some main parameters of a satellite thrust system are also listed in Table 1. For simplification, the motor thrust and specific are also proposed to be kept constant during the orbit manoeuvre of the MEO/IGSO satellite. However, the mass variation of both MEO and IGSO satellites must be considered in the simulation process of orbit manoeuvre. In this article, a more typical kind of initial orbit is used to simulate the process of control strategies optimization, and a discussion of some results follows.

1. MEO satellite manoeuvre simulation using FAM: according to all the assumptions, constraints, and computational model described above, the optimal control strategies for orbit manoeuvres of MEO satellite can be obtained conveniently. An MEO satellite's main orbital variation in three orbit manoeuvres is shown in Table 2. From Table 2, it is obvious that the eccentricity of MEO satellite decreases continually and becomes nearly zero at the end of the final manoeuvre and the final object orbit is obtained successfully.
2. IGSO satellite manoeuvre simulation using FAM: as a similar simulation to MEO satellite, the control strategies for IGSO satellites to manoeuvre

Table 1 Assumed initial and objective orbit elements and thrust system parameters of MEO and IGSO satellites

	MEO		IGSO	
F (N)	500.0		600.0	
I_{SP} (s)	3000.0		3000.0	
Ms (kg)	2500.0		3000.0	
	Initial orbit	Objective orbit	Initial orbit	Objective orbit
a (m)	17 215 000	27 960 000	28 000 000	42 165 000
e	0.62	$\to$ 0.00	0.51	$\to$ 0.00
i (rad)	1.0472	1.0472	1.0472	1.0472
λ_{G} (rad)	0.6370	–	2.1049	–
ω (rad)	2.9775	–	2.9932	–
M (rad)	0.3578	–	2.8135	–

'–' means null in Tables 1–3.

Table 2 Main parameters and their variations at the beginning and the end of the whole manoeuvre to a MEO satellite

	Δa (m)	e	λ_{G} (rad)	M (rad)
t_{beg}	17 215 000	0.620	3.0222	2.7142
Δ_1	2 890 700	−0.231	−0.2073	0.7187
Δ_2	6 274 100	−0.328	−0.1876	0.4821
Δ_3	1 580 100	−0.061	−0.0270	0.0578
t_{end}	27 960 000	0.000	3.0365	3.1660
	Ms (kg)	Δt (s)	Δv (m/s)	$\dot{\alpha}$ (rad/rev)
t_{beg}	2500.000	–	–	4.6445
Δ_1	479.794	2852.72	631.052	−0.4316
Δ_2	431.305	2565.51	711.204	−1.0396
Δ_3	60.314	369.34	114.603	−0.2829
t_{end}	1528.587	–	–	2.8904

Table 3 Main parameters and their variations at the beginning and the end of the whole manoeuvre to an IGSO satellite

	Δa (m)	e	λ_G (rad)	M (rad)
t_{beg}	28 000 000	0.506	1.4837	3.0011
Δ_1	4 600 000	−0.212	−0.1447	0.2419
Δ_2	8 900 000	−0.278	−0.1372	0.1735
Δ_3	665 000	−0.016	−0.0073	0.0079
t_{end}	42 165 000	0.000	1.4504	3.1453
	Ms (kg)	Δt (s)	Δv (m/s)	$\dot{\alpha}$ (rad/rev)
t_{beg}	3000.000	–	–	2.8831
Δ_1	395.895	1984.34	423.300	−0.8713
Δ_2	375.388	1881.90	465.592	−1.8636
Δ_3	18.366	102.58	24.749	−0.1482
t_{end}	2210.351	–	–	0.0000

can be obtained by numerical calculation and the orbital variation of the IGSO satellite in three orbit manoeuvres is shown in Table 3. Obviously, after three orbit manoeuvres are completed, the eccentricity of the IGSO satellite decreases to nearly zero and the longitude drift rate decreases to 0.0 rad/rev. All these indicate that the FAM can be used to simulate the IGSO satellite manoeuvre quickly and accurately.

3. Simulation result comparison between FAM and ES. Considering that the final end time for MEO and IGSO satellites to finish all manoeuvres is probably different when the orbit manoeuvre is simulated in two different algorithms, it is generally meaningless to compare two orbit parameters that are in the same kind but in different epoch times. Thus, every orbit parameter in Table 4 has to be adjusted to the same time by orbit extrapolation. Here, according to the simulation results from two algorithms, the later final end time in two kinds of results is selected as the benchmark time for orbit extrapolation.

As shown in Table 4, 11 parameters are selected to reflect the difference of simulation results between the function approximate method (FAM) and ES; for every parameter, the error is equal to that the value by FAM minus the value by ES. It is obvious that the angle error for satellite attitude longitude and latitude, footprint geographic longitude, and latitude based on FAM and ES are slightly larger than that of inclination and variation in terms of anomaly. However, this kind of angle error is mainly brought about by the difference in the manoeuvre start and end time between FAM and ES, and their maximum error is only slightly higher than 8.7266×10^{-4} rad, and this kind of difference for MEO and IGSO satellites manoeuvre simulation results is insignificant. The semi-major axis error in the simulation results of two methods is less than 200 m. The maximum error of the longitude drift rate is about 3.4906×10^{-5} rad/rev. For other parameters, such as propellant consumption, duration for manoeuvre motor to run and velocity increment, the differences between FAM and ES are very small. The main reason for the error between the two methods is that it is not possible for the variable interval between discrete points to be divided into infinitely small when ES is applied. From these errors analyses and the successful application of ES, it is clear that the function approximate method (FAM) can be applied for designing control strategies for MEO and IGSO satellites.

Table 4 Main parameters difference between FAM and ES at the end of the whole manoeuvre to MEO and IGSO satellites

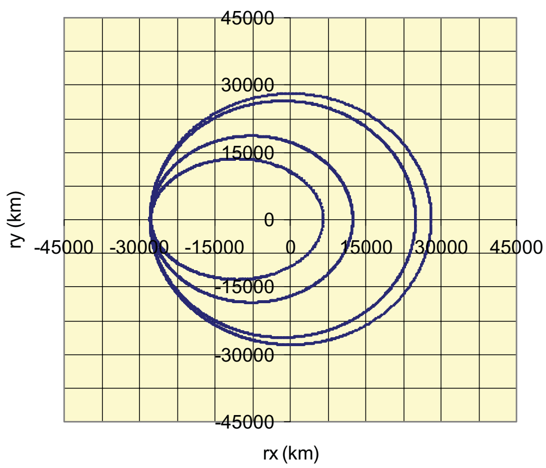
Output parameters in the end of whole manoeuvre	Error on MEO (FAM–ES)	Error on IGSO (FAM–ES)
α (rad) Attitude longitude	−0.000 75 4.341 04–4.341 79	−0.000 66 2.861 82–2.862 48
δ (rad) Attitude latitude	−0.000 44 1.031 16–1.031 60	−0.000 37 1.164 94–1.165 31
λ (rad) Geographic longitude of satellite footprint	−0.000 98 2.968 18–2.969 16	−0.000 75 1.330 59–1.331 34
φ (rad) Geographic latitude of satellite footprint	−0.000 54 −0.118 02 − (−0.117 48)	−0.000 44 −0.069 83 − (−0.069 39)
Δa (m) Semi-major axis variation	190 10 745 000–10 744 810	130 14 165 000–14 164 870
Δe ($\Delta e = e_{end} - e_{beg}$) Eccentricity variation	−0.0008 −0.620 − (−0.6186)	−0.0007 −0.5056 − (−0.5049)
$\Delta\omega$ (rad) Variation in argument of perigee	3.4906×10^{-5} 0.001 28–0.003 40	5.2360×10^{-5} 0.005 81–0.005 76
$\dot{\alpha}$ (rad/rev) Longitude drift rate	-3.4906×10^{-5} 2.890 39–2.890 42	-3.4906×10^{-5} 0.0–3.4906×10^{-5}
Δm (kg) Propellant consumption	−0.109 970.922–971.031	−0.120 790.649–790.751
ΔT (s) Total manoeuvre time	−0.53 5786.77–5787.30	−0.60 3968.82–3969.42
Δv (m/s) Velocity increment	−0.180 1455.90–1456.08	−0.140 913.641–913.781

Table 5 Comparison of CPU consumption between FAM and ES

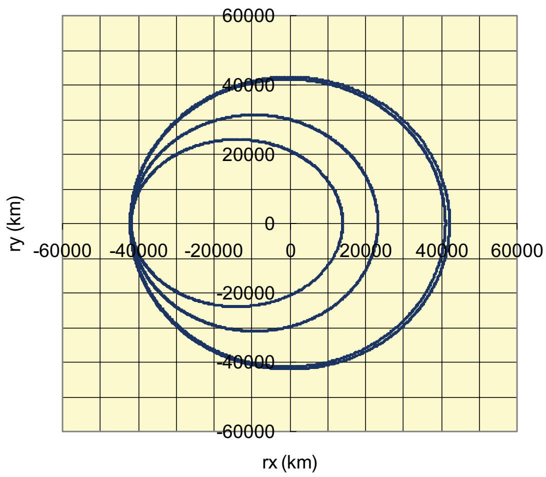
Time step length in orbit manoeuvre simulation (*s*)		CPU consumption (*s*) FAM	ES
MEO	10	68	795
	6	106	1571
IGSO	10	82	1693
	6	129	2689

In this article, the base time step length for an orbit manoeuvre simulation using ES is about 10 s, and the same time span (about half one orbit period) is selected for the time variable. It is worth pointing out that the ES requires at least 10 times more CPU than the function approximate method does for all the manoeuvres of an MEO satellite, and needs at least 20 times more CPU time than the function approximate method does for all the maneuvers of an IGSO satellite, as shown in Table 5. Moreover, the precision for the final result of a simulation manoeuvre by ES is not very accurate. Especially, the CPU consumption by using ES will increase dramatically when a higher precision is required for the simulation of the MEO and IGSO satellites. In contrast, the CPU consumption increases only slightly when the FAM is used when a higher precision simulation is needed in orbit manoeuvre simulation. Hence, the FAM has more advantages than ES, in particular in CPU consumption, corresponding to the same simulation precision.

In Fig. 4(a) it is shown that the geocentric radius projection of an MEO satellite in the orbit plane varies with time during the whole orbit manoeuvre; there are four projection circles in this figure. The smallest

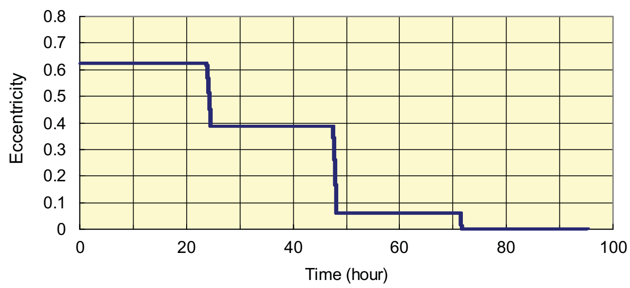


(a)

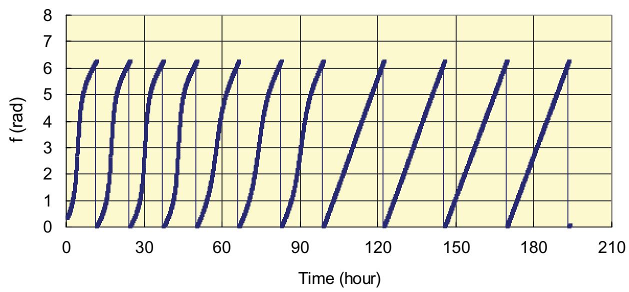


(b)

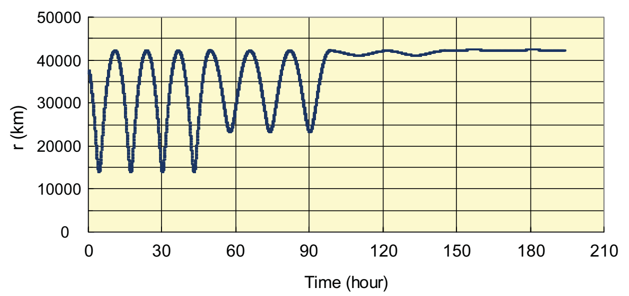
Fig. 4 Simulation results of control strategies for MEO and IGSO satellites to manoeuvre: (a) orbit control strategy for MEO satellite; and (b) orbit control strategy for IGSO satellite



(a)



(b)



(c)

Fig. 5 Some parameters variation with time during all manoeuvres of MEO and IGSO satellites: (a) eccentricity of MEO satellite varies during all maneuvers; (b) true anomaly of IGSO satellite varies with the time during all orbit maneuvers; and (c) geocentric distance of IGSO satellite varies with the time during all orbit maneuvers circle represents the initial case before all manoeuvres, obviously in that time the orbit eccentricity is the largest, and this can also be found directly in Fig. 5(a), which shows that the eccentricity varies gradually from $e = 0.62$ to approximately $e = 0.0$ during all orbit manoeuvres. The other three circles from small to large, respectively, display an orbit shape variation, i.e. from a larger eccentricity ellipse orbit to an approximately circular orbit (see from Fig. 4(a)). All these are almost similar to an IGSO satellite (see Fig. 4(b)). However, the semi-major axis variation in the last orbit control is slightly smaller, and so the last two circles almost overlap, and this can also be seen in Fig. 5(c), which shows that the radius varies with time during all manoeuvres. From this figure, it can easily be found that the variation of geocentric distance r becomes smaller and smaller with time after several rounds of apogee control to satellite, and in the end the IGSO satellite enters the designed objective orbit ($a_{\mathrm{obj}} = 42165$ km).

Figure 5(b) shows the variation of true anomaly with time. From this figure, it is obvious that the curve shape of the true anomaly variation with time gradually becomes one approximate straight line, and this means that the angle velocities of IGSO satellites will become regular too. According to the simulation results of MEO and IGSO satellites, the FAM is well selected to ensure that every control to be implemented near the apogee and the consumption of the propellant also becomes the smallest. For MEO and IGSO satellites, the application of FAM is simple and efficient. It can ensure that MEO and IGSO satellites enter their theoretical orbit on the most optimal control strategies.

5 CONCLUSION

Design of the optimal control strategies for MEO and IGSO satellites plays an important role in finishing all orbit manoeuvre tasks of these satellites in the early flight phase; it is the key factor to ensure that these satellites can be successfully put into the normal orbits. In this article, one type of FAM is introduced and applied in the control strategies simulation to MEO and IGSO satellites. Compared with the conventionally adopted multidimensional ES, this function approximation method described in this article has several main advantages.

1. It is simple to apply, and there is no need to divide the system variable into a series of discrete points.
2. Compared with the ES, this function approximation method has the same excellent precision in the control strategy optimization design.
3. In terms of the computational efficiency, this method needs much less CPU time and it depends much less on the capacity of the computer. With the aid of this algorithm, the optimal control strategy for orbit manoeuvre of MEO and IGSO satellites can be achieved quickly and efficiently.
4. This method can be easily extended to other satellites series such as GEO and LEO satellites.

In the next phase, the control strategy optimization issue for a high-dimensionality (6 DOF) satellite to manoeuvre with complex constraints will be studied, which involves solving a non-linear program using a linearization of the cost function, dynamics, and constraints of the initial feasible solution; it includes non-linear attitude dynamics, difficult non-convex constraints, and so on.

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APPENDIX

Notation

a	semi-major axis (m)
a_{obj}	final objective semi-major axis (m)
$a_{\text{obj}}^{(j)}$	objective semi-major axis for the jth manoeuvre (m)
e	eccentricity
e_{beg}	eccentricity at the beginning of the whole orbit manoeuvre
e_{end}	eccentricity at the end of the whole orbit manoeuvre
F	thrust value of the orbit manoeuvre motor (N)
$\boldsymbol{F}$	thrust vector of the orbit manoeuvre motor (N)
f	true anomaly (rad)
g	gravitational acceleration (m/s^2)
i	inclination (rad)
I_{SP}	specific impulse (s)
$J_{\max}$	the maximum times of whole orbit manoeuvres
k	superscript
m	current satellite mass (kg) or special subscript (for example: t_m, e_m)
M	mean anomaly (rad)
Ms	initial mass of satellite (kg)
n	mean angular velocity (rad/s)
p	semi-latus rectum (m)
r	geocentric distance (m)
$\boldsymbol{r}$	radius vector of satellite from the geocentre to the satellite (m)
rx	x component of geocentric distance of satellite in orbital plane (km)
ry	y component of geocentric distance of satellite in orbital plane (km)
t_{beg}	beginning time of whole manoeuvres (s)
t_{end}	end time of whole manoeuvre (s)
x	x component of the radius vector of satellite in geocentric equator inertial coordinate system (m)
y	y component of the radius vector of satellite in geocentric equator inertial coordinate system (m)
z	z component of the radius vector of satellite in geocentric equator inertial coordinate system (m)
α	attitude longitude (i.e. geographic longitude corresponding to the orientation of motor thrust) (rad)
$\dot{\alpha}$	longitude drift rate (rad/rev)
δ	attitude latitude (i.e. geographic latitude corresponding to the orientation of motor thrust) (rad)
Δa	controlled variable of orbit semi-major axis (m)
Δm	variation of satellite mass (kg)
Δ_i	parameter variation during the ith orbit manoeuvre ($i = 1, 2, 3, \ldots, J_{\max}$)
Δt	time length (s)
ΔT	total manoeuvre time length (s)
Δv	velocity increment (m/s)
$\Delta\omega$	variation in argument of perigee (rad)
ε	convergence criterion for optimal start time of orbit manoeuvre (s)
ε_1	convergence criterion for final objective semi-major axis (m)
λ	footprint geographic longitude (rad)
λ_{G}	geographic longitude of the ascending node (rad)
φ	footprint geographic latitude (rad)
ω	argument of perigee (rad)
Ω	right ascension of the ascending node (rad)