

collection of definable concepts and the collection of objects are immaterial. The only cardinality restriction is that the size of the collection of definable concepts is no greater than the size of the language (at most countably many in the case of a countable language). This restriction follows not from the diagonalization argument but from what it is to be a definable concept. *Definable Basic Law V*, which forces \wedge to be injective only on the definable concepts, vividly demonstrates the fact that Russell's Paradox applies even if the collection of definable concepts is of the same size or even smaller than the collection of objects.⁸

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Dispositions and the principle of least action revisited

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1. Introduction

Some time ago, Joel Katzav (2004, 2005) and Brian Ellis (2005) debated the compatibility of dispositional essentialism with the principle of least action.

Surprisingly, very little has been said on the matter since,¹ even by the most naturalistically inclined metaphysicians. Here, we revisit the Katzav–Ellis arguments of 2004–05. We outline the two problems for the dispositionalist identified by Katzav in his 2004 (we call these the ‘contingent action-quantities’ and ‘explanatory’ objections), and claim they are not as problematic for the dispositional essentialist at it first seems – but not for the reasons espoused by Ellis.

2. *Dispositional essentialism*

Dispositional essentialism is the view that *at least some* properties have a dispositional essence. Dispositions are natural properties conferring ‘powers’ on the objects that instantiate them, which manifest when the relevant stimulus conditions are met. For example, salt has the disposition ‘solubility’: the power to dissolve when placed in water; thus, when one submerges a grain of salt in water, its disposition to dissolve manifests, forming a saline solution. Dispositional essentialists believe that all *explainable* change occurs in virtue of dispositional properties² – indeed, all physical changes just *are* the manifestation of dispositions.³ In short (at least insofar as Katzav, Ellis and we use the term in this debate), dispositions are fundamental natural properties that explain all physical change.

3. *Least action principles*

The laws of physics can be divided into two categories: first there are the instantaneous laws, most of which all students with a rudimentary understanding of physics are familiar with (these include the Newton’s law of gravitation, Maxwell’s laws of electromagnetism and the Schrödinger equation in quantum mechanics). If one knows the state of the system at a given time t , one can apply the instantaneous laws and predict the state of the system at $t + 1$. Then there are the least action principles. These are the fundamental meta-laws underlying an atemporal nomology; they state that for any given physical system, and any specified initial and final conditions, that

- 1 Alexander Bird briefly mentions least action principles as reason to doubt dispositional essentialism (2007: 214–5) in response to one of the objections presented in this article, but he devotes only half a page to the matter, and there are several other objections that Bird does not consider. These additional objections are also addressed in this article. We also reply to objections to dispositional essentialism that Ellis does not adequately deal with in his reply to Katzav, and reply to one possible objection not identified by Katzav in his original article.
- 2 Only explainable change, since as Handfield (2001) has argued, dispositional essentialism is compatible with ‘space-invaders’, which spontaneously pop into existence for no reason.
- 3 Proponents of dispositional essentialism include Shoemaker (1998), Bird (2007), Mumford (2004), Heil (2012) and Tugby (2013).

system will evolve through the sequence of states that minimize, or more accurately, extremize, a quantity called ‘action’.⁴

Generally, action principles are expressed in terms of a Lagrangian formulation. For each physical system one can uniquely specify a function called the ‘Lagrangian’. This function is determined by the nature of the physical system in question; that is, the properties and forces inherent in it. It applies, in one form or another, to all physical theories including general and special relativity, quantum mechanics, quantum field theory and even string theory.⁵ In the case of a Newtonian Mechanical system, for example, the Lagrangian L is simply given by the kinetic energy of the system T minus the potential energy of the system V . Once the Lagrangian is known, the action associated with a physical system passing through a sequence of states can be defined by integrating the Lagrangian over the sequence of states. The *actual* sequence of states, then, is that which extremizes action. Importantly, given the Lagrangian and the principle of least action, all the instantaneous laws applicable to the system in question can be derived.⁶

4. Katzav’s ‘contingent action-quantities’ objection

One can define the action of *any* possible sequence of states between two states of a physical system (we call these the initial and final states), and in general there are many – the principle of least action tells us which of the possible sequences the system follows. Katzav explicitly states that this principle *presupposes* that the ‘action of any given physical system could have taken various values, and thus that any such system could have been correctly described by different equations of motion’ (Katzav 2004: 210); the sequence of states the system instantiates, and hence the instantaneous laws of motion, must (according to Katzav) be contingent.

In considering the dispositionalist metaphysics outlined earlier, one can see that whereas some take a property’s identity to be fixed primitively by a ‘quiddity’, dispositional essentialists take properties to be identified purely by the causal role they play (if F is the property ‘solubility’, then in all

4 For more details on the historical, philosophical and formal foundations of action principles see Stöltzner (1994, 2003, 2009).

5 In the context of quantum theories, the connection to the Lagrangian and principle of least action is via the path integral formulation. This can be seen most clearly by considering the second postulate in the original derivation by Feynman (1948), wherein the quantum amplitude is assumed to be peaked upon the least action path.

6 More precisely, the PLA allows us to derive the Euler-Lagrange equations that in turn provide us with the equations of motion for any given Lagrangian. There are some rather technical subtleties that mean, strictly speaking, full mathematical equivalence with the instantaneous laws does not obtain. Rather, given these issues, it is only under the addition of certain formal sufficiency conditions that we can establish a rigorous proof of equivalence. See Brechtken-Manderscheid (1991: S6).

possible worlds, objects instantiating *F* will dissolve when placed in water). If the version of dispositional essentialism assumed by Katzav is correct, and dispositions are the *only* explanantia for physical change, then given the initial and final states of a physical system (i.e. the pattern of property instantiations at the specified times) there is only one possible quantity of action, and hence just one possible sequence of states that the system can follow. The laws of motion, for the dispositional essentialist, are thus necessary, and so according to Katzav's criterion dispositional essentialism is incompatible with the principle of least action.

4.1 Response 1. *Scientific essentialism*

Ellis responds to the contingent action-quantities objection by drawing a distinction between 'naïve' and 'sophisticated' forms of dispositional essentialism (Ellis calls this view 'Scientific Essentialism' – see (Ellis 2005: 90)); he concedes that Katzav's objection may hold some weight against the naïve conception Katzav assumes – that dispositions are the most fundamental natural properties, and these fundamental properties are the only real explanation for physical change) – but insists that the sophisticated dispositional essentialist's complex hierarchy of natural kinds adequately accommodates the principle of least action.

According to Ellis's 'sophisticated' scientific essentialism, there are several natural kind hierarchies: natural kinds of substances, natural kinds of processes/events and natural kinds of properties (for Ellis, these are tropes⁷) – to each of these natural kinds, essential properties are ascribed; that is, properties without which no substance/process/property could be a member of the natural kind in question. Laws of nature, says Ellis, describe the essential properties of natural kinds, and these laws have explanatory value (Ellis 2001).

Ellis suggests that the property 'Lagrangianism' is a 'truly universal property – one that is possessed by every object in the universe' (Ellis 2005: 91), and further, that given our empirical observations, it is reasonable to suppose this property is of the essence of all physical systems.⁸ The explanation for why physical systems are all Lagrangian, then, is simply that Lagrangianism is of the essence of physical systems; in other words, it is a law of nature that physical systems have extremized quantities of action.

7 See Ellis 2001.

8 The nature of the 'Lagrangian' property is never made particularly clear. Ellis writes that 'every continuing object must be Lagrangian, that is be disposed to evolve in accordance with the principle of least action' (2005: 91), so one must assume he takes the property to be dispositional in character. However, Ellis's view is that the PLA is a law, and further that it is not causal. Is Ellis supposing, then, that there are non-causal dispositions? Given the nature of, and motivation for the literature on dispositional properties, this strikes as odd, but not necessarily implausible.

This explanation for the principle of least action being true of the actual world, just like Katzav's, is nomic rather than causal – there are no stimulus conditions necessary for least action principles to kick in, for they are atemporal. But that is not to say the principle of least action is incompatible with dispositional essentialism – it can still be the case that substances instantiate fundamental dispositional properties, and that these dispositions manifest when their stimulus conditions are met. Ellis's view requires only that there is a particular 'global law' that ensures all physical systems do, in fact, adhere to the least action principles.

Unsurprisingly, this is wholly unsatisfactory for Katzav. The action-quantities are still not contingent. If anything, the situation is worse. Scientific essentialism is committed to the *a posteriori* necessity of action being extremized – it is of the essence of physical systems.

4.2 Response 2. Contingent sequence of states? whatever do you mean?

Ellis's argument rests on the strength of global kind explanations – that F being of the essence of the global kind is a good explanation for members of the global kind (including the actual world) having the property F. This seems somewhat *ad hoc*, for the question 'why is F of the essence of the global kind?' arises. It is true that explanations must end somewhere, of course, and further that for the Scientific Essentialist there is nowhere else to go, but nonetheless, we (and others (see Bird 2007: 213)) find Ellis's response somewhat unsatisfying. However, Katzav's argument is dubious nonetheless.

Alexander Bird questions in what sense the quantity of action being extremized must be contingent. If metaphysical contingency is genuinely required, then Katzav's objection holds, as on Ellis's view there are no two metaphysically possible systems with identical initial and final states, which differ in their quantities of action. But does the principle of least action *really* require metaphysical contingency? If logical contingency is insufficient, then the burden of proof is on Katzav to show this, and there is no obvious reason why it should not be. Given that the dispositional essentialist can happily endorse the logical contingency of the quantities of action of *any* physical system, Katzav's argument is diffused⁹ (Bird 2007: 214).

9 A world is logically possible only if a complete description of that world contains no contradictions. Metaphysical possibility, however, rests on the *nature* of particulars and their properties. To borrow Saul Kripke's (1980) example, water is H₂O. Stating that water is not H₂O involves no logical contradiction, but it is nonetheless metaphysically impossible for water *not* to be H₂O, since anything that is not H₂O is not water. Similarly, if dispositional essentialism is true, then (assuming no interfering factors) it is metaphysically necessary for dispositional properties to manifest when their stimulus conditions are met, but it is not logically necessary, since one can describe these situations without logical contradiction.

Katzav will no doubt respond that when we conceive of a physical system violating least action principles, it really *is* our kind of physical system that we are imagining (since if, as many argue, conceivability is a good guide to possibility,¹⁰ then the domains of metaphysical and logical possibility are the same). Ellis will counter, however, that Katzav is simply misled by the fact that the physical systems he is conceiving *look* similar to the physical systems we observe. Katzav is conceiving of a world with a different kind-membership to the actual world, since the two worlds have different essential properties.¹¹

5. *The first explanatory objection*

Katzav claims that the principle of least action affords important explanations incompatible with dispositional essentialism. He argues the fact that, in reality, action always takes the extremized possible value should lead us to believe that the principle of least action is no accident; further, that if it is no accident, then we can appeal to it in explanation.

As mentioned, given the formulation of a system's action integral (the Lagrangian) and the principle of least action, one can derive the instantaneous laws; the principle of least action, then, arguably explains why the objects comprising the system move in accordance with the laws of motion (Katzav, 2004: 211). These laws, he claims, in turn explain the dispositions of those objects. The dispositional essentialist conception assumed by Katzav, on the other hand, takes dispositional properties to be the ultimate explanantia. If dispositions are the ultimate explanantia, then clearly the principle of least action cannot be that which explains the dispositions, and so on the face of it, the explanation afforded by the principle of least action is incompatible with dispositionalism (211). We should therefore reject dispositional essentialism.

Of course, here the question 'which is the better explanation?' arises. Consider the case of Newtonian Mechanics not only can the instantaneous laws of motion be derived from the principle of least action, but given the instantaneous laws of motion, one can derive the principle of least action (212) – if Katzav can claim the principle of least action explains the dispositions, the dispositional essentialist can claim the dispositions of objects explain the principle of least action. Katzav acknowledges that this kind of response can be raised, but states that 'the fact that physicists typically use the principle of least action (PLAC)¹² to deduce the corresponding equations of motion and not vice versa illustrates, the explanation only proceeds from the

10 See Chalmers 2002 for a comprehensive discussion.

11 We thank an anonymous referee for raising this issue.

12 PLAC – the principle of least action as used in classical particle mechanics.

PLAC to the equations of motion' (213). We deem this unconvincing for the reasons expressed later.

5.1 *Response to the first explanatory problem*

That there are close relationships between our concepts of explanation, metaphysics and scientific practice is obvious, but clearly we cannot hope to untangle the intricacies in this short article. However, it is worth outlining a few, crude observations. Some philosophers (see Lewis 1986: 214) deem all explanations to be causal, that to provide an explanation for some event is to provide some information about its causal history; some admit of nomological explanations (see Hempel 1965; Salmon 1971); and some are even willing to grant that mere regularities, without any causal or nomological connotations, can have explanatory value (Smart 2013). The governing laws and dispositionalist metaphysics both invoke some form of 'necessary connection' in their explanations of physical change, and this comes not from current scientific practice, but from the mind-independent *nature* of the universe. For metaphysicians, the *real* explanantia for physical change should be taken to be mind-independent.

As Katzav rightly points out, deduction and explanation are not the same, but *that* scientists tend to move from the principle of least action to derive dispositions and not vice versa has no bearing on what is the real explanation of what sequences of states are taken by physical systems. This depends on the nature of the universe: does the principle of least action guide physical systems such that action is extremized, or do the fundamental dispositions of objects have a modal character that determines the extremized quantities of action inherent to all physical systems? The answer to this question may be hidden from us (hopefully not – as this is the focus of much endeavour in metaphysics), but Katzav's apparent motivation for choosing the principle of least action as the explanans is far from convincing.

Moreover, the fact that, *in the context of classical particle mechanics*, scientists often find it easier to work *from* action principles to equations of motion is not sufficient to warrant a general thesis about explanatory priority. Consider the case of the discovery of the field equations of general relativity (Renn 2007). Although the derivation of Hilbert *did* largely proceed along such a route, Einstein (who has historical precedence) found the equations that bear his name by following a different chain of reasoning. What is more, in the contemporary context, some of the most perspicuous *explanations* of why the field equations take the form they do (Lovelock 1971) use symmetry and uniqueness arguments, not action principles, as their basis.

6. *The second explanatory objection*

Katzav states that the principle of least action is no accident (2004: 212). There are lots of logically possible quantities of action for every physical

system, but only one physically possible one – that which extremizes action. But why is this? Why could the dispositions not have been such that it was metaphysically necessary for the quantities action to take other values? On the face of it, one is left with a cosmic coincidence, *unless* one takes the ‘principle of least action’ itself to be a governing law, *ensuring* that physical systems have extremized quantities of action.

6.1 Response to the second explanatory problem

The dispositional essentialist’s ontology cannot accommodate the ‘governing principle of least action’ response, but we do not take this to be problematic. Dispositional essentialism, *ex hypothesi*, is the thesis that dispositions are the most fundamental units – the most fundamental explanantia. One cannot ask, however, why we have the natural kinds (and hence the dispositions that are essential to them) that we do – these are questions to which the dispositional essentialist must deny there are answers. That action is extremized is indeed no coincidence; it flows from the dispositional properties inherent to the actual world.

7. The third explanatory objection¹³

Given that one can only explain the dispositions instantiated by any one object by appeal to the manifestation of properties at earlier times and that the laws of nature are determined by the dispositions of objects (as dispositions are fundamental), then any explanation for objects moving in accordance with the laws of motion must be a historical one. Least action principles, on the other hand, are by their nature atemporal, so this rather unappealing consequence is not encountered; one should therefore prefer least action principle explanations.

7.1 Response to the third explanatory problem

There are different forms of dispositional essentialism. Ellis has mentioned ‘naïve’ and ‘sophisticated’ dispositionalism, and suggested that Katzav’s article is (unbeknownst to him) directed purely at the naïve dispositionalist, that is, the dispositional essentialist who does not claim that laws are essential properties of natural kinds. Here, we focus on one of the ‘naïve’ dispositionalist ontologies – dispositional monism (Bird 2007; Mumford 2004).

There is, of course, a strong sense in which the dispositions an object possesses are determined causally. Causation involves the ‘passing around of powers’ (Mumford 2009) – for example, when billiard ball *a* hits billiard ball *b*, *a* loses momentum and the stationary ball *b* gains the power to move other billiard balls, as *b* now has momentum. However, that is not to say the

13 To our knowledge this has not been raised by Katzav, but it should be addressed as a potential objection nonetheless.

dispositional monist cannot provide an atemporal explanation for action being extremized.

Bird's form of dispositional monism asserts that all properties are universals, and that all natural properties have a dispositional essence. The view relies on an entire framework of universals, such that the nature of every property is fixed by its place in a network of natural properties, connected by second-order universals (relations between the natural properties). The causal role of a fundamental property, then, is determined by the stimulus and manifestation relations (the second-order properties) it bears to other natural properties. *Prima facie* this leads to an infinite regress, but due to the asymmetric nature of the network of relations, this is not the case.¹⁴ Given that the identity of any one property is wholly determined by its place in a network of universals, for that one property to exist at all, the structure of the network must be precisely as it is.

The network determining the nature of the natural properties does not exist in virtue of the history of the universe – its structure is very much atemporal.¹⁵ For the dispositional essentialist, this network of properties not only entails the Lagrangian but also guarantees that objects behave in accordance with the instantaneous laws; that is, the laws of motion.

8. Conclusion

Katzav attempts to undermine dispositional essentialism by arguing for its incompatibility with one of our most fundamental physical principles. Ellis responds by espousing a more sophisticated conception of the thesis, which he believes saves dispositional essentialism from Katzav's objections; as we have shown, this response succeeds only if one takes the principle of least action to assume only the *logical* contingency of physical systems' quantities of action. However, as we see no reason to think the principle should assume a more liberal metaphysical possibility, we dismiss Katzav's objections accordingly. We considered several further explanatory objections, but demonstrate that none of these are irresolvable for the dispositionalist. We do not claim, in this short article, to have demonstrated beyond doubt that dispositionalist ontologies are unthreatened by least action principles, but further argument is certainly required if one is to reject a 'powers' metaphysic on these grounds.¹⁶

14 See Bird 2007; Tugby 2013.

15 See Tugby 2013 for a good discussion of platonic dispositions.

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